Reciprocity: Theory
Lecture 17, *Experimental Econ. & Psychology*

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11 November 2020

Abstract
Reciprocity theory assumes that people wish to be kind towards those they perceive to be kind, and unkind towards those they perceive to be unkind. Rabin (1993) argues that the *kindness is based on intentions*: the **kindness** of i towards j is measured by the difference between how much i expects to make j earn and an "equitable payoff" of j. Hence, *kindness depends on* (1^{st}-order) *beliefs*, making this a PGT model. Here we present the *theory of sequential reciprocity* of Dufwenberg & Kirchsteiger (2004) for leader-follower game forms. We also hint at the theory of negative reciprocity and its application to the hold-up problem (Dufwenberg, Smith & Van Essen. 2013).

Introduction

- We studied how:
    - guilt avoidance can make agents keep materially costly promises;
    - frustration and anger can make agents carry out materially costly threats.
- Both effects are also promoted by **reciprocity**, the action tendency of being kind (resp. unkind) towards those whom we perceive as kind (resp. unkind) with us.
- The idea that people wish to be (un)kind towards those they perceive to be (un)kind is age-old. Early academic discussions can be found in anthropology, sociology, social psychology, biology, and economics (see references in the forthcoming survey by BD and in Sobel 2005). Akerlof (1982) analyzed "gift-exchange" in labor markets, that should imply a monotone wage-effort relationship.
- Rabin (1993) argues that the *kindness is based on intentions*: the **kindness** of i towards j is measured by the difference between how much i expects to make j earn and an "equitable payoff" of j. Hence, *kindness depends on* (1^{st}-order) *beliefs*.

Modeling kindness in leader-follower (LF) game forms

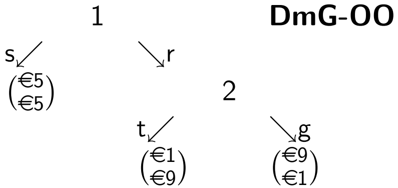
- In an **LF** game form, first pl. 1 (L) chooses $a_1 \in A_1$, next pl. 2 (F) chooses $a_2 \in A_2(a_1)$. (Let $A_2(a_1) = \{\text{wait}\}$ if a_1 is a terminating action.)
- The **kindness** of 1 towards 2 when choosing a_1 depends on its intended effects given the 1^{st}-order belief α_{12}:
  - let A_1^* [resp. $A_2^*(a_1)$] denote the set of 1's [resp. 2's] actions that cannot lead to Pareto-dominated outcomes [in the examples below, $A_1^* = A_1$, $\forall a_1$, $A_2^*(a_1) = A_2(a_1)$]; recall that $\mathbb{E}_{\alpha_{12}}(\pi_2|a_1) = \sum_{a_2 \in A_2(a_1)} \pi_2(a_1, a_2)\,\alpha_{12}(a_2|a_1)$;
  - then, for any given $\bar{a}_1 \in A_1$, the **leader's kindness** is

$$\kappa_{12}(\bar{a}_1, \alpha_{12}) = \mathbb{E}_{\alpha_{12}}(\pi_2|\bar{a}_1) - \frac{1}{2}\left(\max_{a_1 \in A_1^*} \mathbb{E}_{\alpha_{12}}(\pi_2|a_1) + \min_{a_1 \in A_1^*} \mathbb{E}_{\alpha_{12}}(\pi_2|a_1)\right)$$

    where $\frac{1}{2}(\ldots)$ is the "**equitable payoff**" (see discussion in Dufwenberg & Kirchsteiger 2019).
- **Follower's kindness** of $\bar{a}_2$ given a_1: $\kappa_{21}(a_1, \bar{a}_2) = \pi_1(a_1, \bar{a}_2) - \frac{1}{2}\left(\max_{a_2 \in A_2^*(a_1)} \pi_1(a_1, a_2) + \min_{a_2 \in A_2^*(a_1)} \pi_1(a_1, a_2)\right)$

Kindness in the Dictator mini-Game with Outside Option

- Consider the following Dictator mini-Game with an Outside Option (DmG-OO):



- To **g**ive (**t**ake) if 1 **r**eached is kind (unkind): $\kappa_{21}(\mathrm{r},\mathrm{g}) = 9 - \frac{1}{2}(1+9) = 4 = -\kappa_{21}(\mathrm{r},\mathrm{t})$.
- *Is* **r***eaching kind or unkind*? Pl. 1 is **kind** towards pl. 2 when **r**eaching, if he does so with the *intention* of making pl. 2 get, in expectation, more than the "equitable payoff": Let $p = \alpha_{12}(\mathrm{t}|\mathrm{r})$; since $\kappa_{12}(\mathrm{r}, p) = 9p + (1-p) - \frac{1}{2}(5 + 9p + (1-p)) = 4p - 2$, **r**eaching is kind (unkind) if $p > \frac{1}{2}$ ($p < \frac{1}{2}$).

Modeling reciprocity (leader-follower game forms)

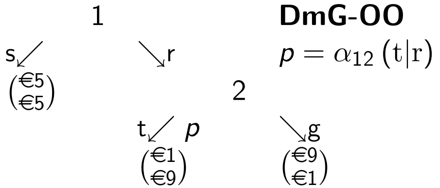
- **Reciprocity** is the action tendency of meting (un)kindness with (un)kindness.
- Such action tendency is captured by the following psychological utility functions [recall, only the kindness of pl. 1 (leader) is belief-dependent]:

$$u_i\left(a_1, a_2, \alpha_{12}\right) = \pi_i\left(a_1, a_2\right) + \theta_i \kappa_{12}\left(a_1, \alpha_{12}\right) \kappa_{21}\left(a_1, a_2\right).$$

- The follower must consult his (conditional) 2^{nd}-order belief $\beta_{21}\left(\cdot|a_1\right)$ to maximize the expected utility of his response to a_1:

$$\max_{a_2 \in A_2(a_1)} \left[\pi_2\left(a_1, a_2\right) + \theta_2 \mathbb{E}_{\beta_{21}}\left(\kappa_{12}\left(a_1, \alpha_{12}\right) | a_1\right) \kappa_{21}\left(a_1, a_2\right)\right]$$

Reciprocity in the Dictator mini-Game with Outside Option



- $\kappa_{21}(\mathrm{r},\mathrm{g}) = 4 = -\kappa_{21}(\mathrm{r},\mathrm{t})$.
- Let $q = \mathbb{E}_{\beta_{21}}(\widetilde{p}|\mathrm{r})$. Since $\kappa_{12}(\mathrm{r},p) = 4p - 2$, then $\mathbb{E}_{\beta_{21}}(\kappa_{12}(\mathrm{r},\widetilde{p})\,|\mathrm{r}) = 4q - 2$, and
  - $\bar{u}_{2,\mathrm{r}}(\mathrm{g};\beta_{21}) = 1 + \theta_2(4q-2)4$,
  - $\bar{u}_{2,\mathrm{r}}(\mathrm{t};\beta_{21}) = 9 + \theta_2(4q-2)(-4)$;
  - $\bar{u}_{2,\mathrm{r}}(\mathrm{g};\beta_{21}) > \bar{u}_{2,\mathrm{r}}(\mathrm{t};\beta_{21})$ IFF $\theta(4q-2) > 1$ ONLY IF $q > \frac{1}{2}$ (for high θ_2, pl. 2 wants to surprise pl. 1, **g**iving if 1 expects him to **t**ake).
  - This implies there is no "pure equilibrium" where pl. 1 correctly anticipates how 2 would respond and 2 understands this.

Reciprocity vs Anger in the Ultimatum mini-Game

UmG 1

f ↙ ↓g $p = \alpha_{12}(\mathrm{r}|\mathrm{g})$

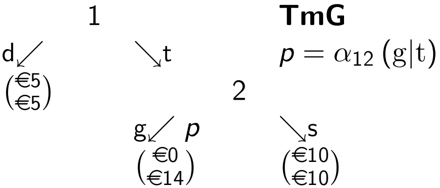
$\binom{€5}{€5}$ 2

r ↙ p ↘a

$\binom{€0}{€0}$ $\binom{€9}{€1}$

- $\kappa_{12}(\mathrm{g}, p) = (1-p) - \frac{1}{2}[5 + (1-p)] = -2 - \frac{1}{2}p$.
- Let $q = \mathbb{E}_{\beta_{21}}(\widetilde{p}|\mathrm{g})$, then
  - $\bar{u}_{2,\mathrm{g}}(\mathrm{r}, \beta_{21}) = \theta_2\left(-2 - \frac{1}{2}q\right)\left(0 - \frac{9}{2}\right) = \theta_2\left(9 + \frac{9}{4}q\right)$,
  - $\bar{u}_{2,\mathrm{g}}(\mathrm{a}, \beta_{21}) = 1 + \theta_2\left(-2 - \frac{1}{2}q\right)\left(9 - \frac{9}{2}\right) = 1 - \theta_2\left(9 + \frac{9}{4}q\right)$
  - $\bar{u}_{2,\mathrm{g}}(\mathrm{r}, \beta_{21}) > \bar{u}_{2,\mathrm{g}}(\mathrm{a}, \beta_{21})$ IFF $2\theta_2\left(9 + \frac{9}{4}q\right) > 1$ IFF $\theta_2\left(18 + \frac{9}{2}q\right) > 1$ IF $\theta_2 > \frac{1}{18}$.
- If $p = 1$, pl. 1 deems 2 unkind given **g**. For θ_1 large (how large?), 1 makes the **g**reedy offer to harm 2, who reciprocates **r**ejecting even if he expected. This *"miserable equilibrium"* is impossible according to the FA model: 2 does not feel angry if he expected **g**.

Reciprocity in the Trust mini-Game



- Can reciprocity support cooperative behavior? Yes, because **t**rust is a kind action, independently of 1st-order belief $p = \alpha_{12}(\mathrm{g}|\mathrm{t})$, and to **s**hare is a kind reply:
  - $\kappa_{12}(\mathrm{t}, p) = 14p + 10(1-p) - \frac{1}{2}[14p + 10(1-p) + 5] = 2p + \frac{5}{2}$;
  - $\kappa_{21}(\mathrm{t}, \mathrm{s}) = 10 - \frac{1}{2}(0 + 10) = 5 = -\kappa_{21}(\mathrm{t}, \mathrm{g})$.
- **Note**: pl. 2 has the *lowest incentive to* **s***hare* if *he believes that* $p = 0$, i.e., that *pl. 1 trusted him to* **s***hare*. Let $q = \mathbb{E}_{\beta_{21}}(\widetilde{p}|\mathrm{t})$,
  - $\bar{u}_{2,\mathrm{t}}(\mathrm{s}, \beta_{21}) = 10 + \theta_2\left(2q + \frac{5}{2}\right)5$,
  - $\bar{u}_{2,\mathrm{t}}(\mathrm{g}, \beta_{21}) = 14 + \theta_2\left(2q + \frac{5}{2}\right)(-5)$;
  - compute the threshold $\hat{\theta}_2$ such that pl. 2 certainly **s**hares if $\theta_2 > \hat{\theta}_2$.

Negative reciprocity

- According to *negative reciprocity theory*, players meet unkindness with unkindness, but (positive) kindness does not matter. Let $[x]^- = \min\{0, x\}$, then

$$\begin{aligned} u_1(a_1, a_2, \alpha_{12}) &= \pi_1(a_1, a_2) + \theta_1 \kappa_{12}(a_1, \alpha_{12}) [\kappa_{21}(a_1, a_2)]^-, \\ u_2(a_1, a_2, \alpha_{12}) &= \pi_2(a_1, a_2) + \theta_2 [\kappa_{12}(a_1, \alpha_{12})]^- \kappa_{21}(a_1, a_2). \end{aligned}$$

- Dufwenberg, Smith & Van Essen (2013) derive interesting predictions about *hold-up problems* by extending negative reciprocity theory to 3-stage game forms where:
    - pl. 1 can **in**vest in a relationship (non-binding contract), or stay **o***ut*,
    - pl. 2 can **d***eliver* (comply with the contract) or *renego*tiate, holding 1 up,
    - pl. 1 can *accept* (**y**es) or *reject* (**n**o).

Negative reciprocity: Hold-up mini-Game

$$\begin{array}{ccccccc}
1 & \xrightarrow{\text{in}} & 2 & \xrightarrow{\text{reneg}} & 1 & \xrightarrow{\text{yes}} & \binom{1}{9} \\
\downarrow \text{o} & & \downarrow \text{d} & & \downarrow \text{n} & & \\
\binom{2}{2} & & \binom{5}{5} & & \binom{0}{\omega} & &
\end{array}$$

- ω is the value for pl. 2 after a rejection and depends on *residual rights of control*:
  - if pl. 1 provided a service, he cannot take it back, ω can be as high as 10;
  - if pl. 1 produced a good (having no value for him), he can keep it, $\omega = 0$.
- According to the *residual rights of control*, negative reciprocity can make rejection an effective threat (if $\omega < 5$) and promote cooperation $(\mathrm{in}, \mathrm{d})$, or not (if $\omega > 5$).

Reciprocity and dynamic consistency (optional)

- According to the general theory of Dufwenberg & Kirchsteiger (DK), reciprocity is a reactive action tendency. This is modeled with players having different psychological utility functions at different nodes of the game, which may yield dynamic inconsistency of preferences.
- Such dynamic inconsistency may be psychologically plausible, but it is not a necessary feature of the intuitive notion of reciprocity.
- I present below a dynamically consistent model of reciprocity for general game forms (like DK, I restrict attention for simplicity to game forms with observable actions).

A dynamically consistent model of reciprocity (optional)

- **Kindness of** i (at the beginning of the game): I take as given that the equitable payoff of j from i's perspective is determined by some belief-dependent function $\pi_j^e(\alpha_{i,-i})$ (e.g., as in DK). The kindness of i towards j is

$$\kappa_{ij}(\alpha_i) = \mathbb{E}_{\alpha_i}(\pi_j) - \pi_j^e(\alpha_{i,-i})$$

- **Note:** In LF game forms $u_2(a_1, a_2, \alpha_1) =$

$$\begin{aligned} & \pi_2(a_1, a_2) + \theta_2 \kappa_{12}(a_1, \alpha_{12})(\pi_1(a_1, a_2) - \pi_1^e(a_1)) \\ = \; & \pi_2(a_1, a_2) + \theta_2 \kappa_{12}(a_1, \alpha_{12}) \pi_1(a_1, a_2) - \underbrace{\theta_2 \kappa_{12}(a_1, \alpha_{12}) \pi_1^e(a_1)}_{\text{independent of } a_2}. \end{aligned}$$

Thus,

$$\begin{aligned} & \arg \max_{a_2 \in A_2(a_1)} \bar{u}_{2,a_1}(a_2, \beta_{21}) \\ = \; & \arg \max_{a_2 \in A_2(a_1)} \pi_2(a_1, a_2) + \theta_2 \mathbb{E}_{\beta_{21}}(\kappa_{12}(a_1, \alpha_{12}) | a_1) \pi_1(a_1, a_2). \end{aligned}$$

A dynamically consistent model of reciprocity (optional)

- Given the previous observation, I propose to model reciprocity concerns with

$$u_i(z, \alpha_{-i}) = \pi_i(z) + \sum_{j \neq i} \theta_{ij} \kappa_{ji}(\alpha_j) \pi_j(z),$$

  a kind of "state-dependent" utility function, which yields dynamically consistent conditional preferences.

- By standard dynamic programming arguments, $\alpha_{i,i}$ (i's strategy) maximizes $\mathbb{E}_{\alpha_{i,i}, \beta_{i,-i}}(u_i|h)$ starting from every $h \in H$ IFF $\alpha_{i,i}$ is an intrapersonal equilibrium given $\beta_{i,-i}$, that is, for every $h \in H$ and $a_i^* \in A_i(h)$,

$$\alpha_{i,i}(a_i^*|h) > 0 \Rightarrow a_i^* \in \arg\max_{a_i \in A_i(h)} \bar{u}_{i,h}(a_i, \alpha_{i,i}, \beta_{i,-i}),$$

  where $\bar{u}_{i,h}(a_i, \alpha_{i,i}, \beta_{i,-i}) = \mathbb{E}_{\alpha_{i,i}, \beta_{i,-i}}(u_i|h, a_i)$.

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Additional references on reciprocity (all optional)

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