Multistage Games: Representation, Strategic Form

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Game Theory: Analysis of Strategic Thinking

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Abstract

In this lecture I present the *mathematical description of multistage games with observable actions*. A heuristic example about a multistage variation of the Battle of the Sexes (BoS) introduces the topic. Another, more complex multistage variation of the BoS further illustrates the abstract concepts and their mathematical representation. *The formal analysis requires familiarity with mathematical notation about sets, functions, sequences, and sets of such objects* (see the note on *Mathematical Language and Game Theory*). Strategies and the strategic-form representations are derived from the primitives. The application to the strategic form of solution concepts introduced for static games is discussed.

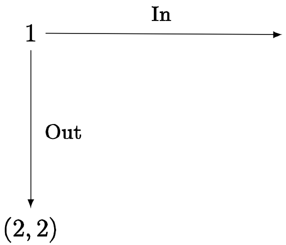
[These slides mostly summarize and in part complement Chapter 9 of GT-AST. See also the part on weak dominance of Chapter 3.]

Introduction

- After the analysis of simultaneous-move (i.e., static) games, we *allow for sequential moves*. Yet, moves may be simultaneous in some stages.
- We focus on a simple class of games encompassing static games as a special case: **multistage games with observable actions**. The *play proceeds through stages*.
    - In each stage, a subset of players is active; if the subset is not a singleton, then there are simultaneous moves in that stage. *Actions chosen in earlier stages are publicly observed.*
    - If there is only one stage, the game is static.
    - The set of *active players and their feasible actions may depend on past moves*, as in many board games (e.g., chess) and economic games (e.g., a firm first determines its capacity, then chooses output within the capacity constraint).
- First we have to introduce a *mathematical language* to represent such games. Then we will be able to analyze them with solution concepts and present algorithms to compute the (sets of) solutions.

Battle of the Sexes with an Outside Option (BoSOO)

- Pl. 1 (Ann) can choose an outside option (Out) or go into a BoS "subgame" with pl. 2 (In):



$1\backslash 2$	B_2	S_2
B_1	$3,1$	$0,0$
S_1	$0,0$	$1,3$

- The play goes through at most 2 stages. Duration is endogenous.
- The set of active players and their feasible actions after the first stage depend on earlier choices.
- Payoffs depend on the sequence of actions or action profiles until termination.

Heuristic analysis of BoSOO: description

- **Rules of interaction:** Summarized by the set Z of possible **plays/terminal histories** (terminating sequences of action pairs). At first, pl. 2 (Bob) can only *wait* (pseudo-action w); 5 possible plays: $((\mathrm{Out}, \mathrm{w}))$, $((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{B}_2))$, $((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{S}_2))$, ..., $\mathcal{A}_i((\mathrm{In}, \mathrm{w})) = \{\mathrm{B}_i, \mathrm{S}_i\}$ is the feasible set of pl. i given $(\mathrm{In}, \mathrm{w})$.
- **Outcome function** ($g : Z \to Y$): Y contains 4 outcomes: *Default Outcome (DO), Together at B (TB), Together at S (TS), Not Together (NT)*. With this, $g((\mathrm{Out}, \mathrm{w})) = DO$, $g((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{B}_2)) = TB$, $g((\mathrm{In}, \mathrm{w}), (\mathrm{S}_1, \mathrm{S}_2)) = TS$, $g((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{S}_2)) = g((\mathrm{In}, \mathrm{w}), (\mathrm{S}_1, \mathrm{B}_2)) = NT$.
- **Utility functions/preferences** ($v_i : Y \to \mathbb{R}$): $v_i(DO) = 2$ and $v_i(NT) = 0$ $(i = 1, 2)$, $v_1(TB) = v_2(TS) = 3$, $v_1(TS) = v_2(TB) = 1$.
- **Payoff functions** ($u_i = v_i \circ g : Z \to \mathbb{R}$): e.g., $u_1((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{B}_2)) = v_1(g((\mathrm{In}, \mathrm{w}), (\mathrm{B}_1, \mathrm{B}_2))) = 3$.

Heuristic analysis of BoSOO: strategies and behavior

- Players plan their **strategies** in advance: a strategy specifies a feasible action for each partial history (decision node), including the empty history (root). *Assuming that strategies are executed*, we obtain the **strategic** (or **normal**) **form** (an *auxiliary*, *fictitious* game):

$1\backslash 2$	$\mathrm{w.B}_2$	$\mathrm{w.S}_2$
$\mathrm{Out.B}_1$	2, 2	2, 2
$\mathrm{Out.S}_1$	2, 2	2, 2
$\mathrm{In.B}_1$	3, 1	0, 0
$\mathrm{In.S}_1$	0, 0	1, 3

- If Ann (pl. 1) conjectures $\mathrm{w.B}_2$, she chooses action In, then B_1. If Ann conjectures $\mathrm{w.S}_2$, she chooses action Out.
- If Bob conjectures $\mathrm{In.B}_1$ (resp. $\mathrm{In.S}_1$) and observes In, he chooses B_2 (resp. S_2). *What if he conjectures* $\mathrm{Out.B}_1$, but *then observes* In? Answering such questions is the *key new aspect of the theory* of games with sequential moves.

Multistage game trees

Primitives 1/2

We consider **multistage game trees with observable actions**:

$$\left\langle I, (A_i, \mathcal{A}_i(\cdot))_{i\in I} \right\rangle .$$

Without loss of generality, we assume that all players *simultaneously take an action at each stage* (the action of inactive players is "wait"):

- $i \in I$ **players**;
- $a_i \in A_i$ potentially feasible **actions** of i;
- $A := \times_{i\in I} A_i$, $A^t = \underbrace{A \times ... \times A}_{t \text{ times}}$ set of all sequences of action profiles of length t;
- by convention $A^0 := \{\varnothing\}$ where $\varnothing$ denotes the **empty sequence**.

Multistage game trees

Primitives 2/2

- $h \in A^{<\mathbb{N}_0} := \bigcup_{t \geq 0} A^t$ **finite sequences** of action profiles.
- $z \in A^{\mathbb{N}} := \{(a^t)_{t=1}^{\infty} : \forall t \in \mathbb{N}, a^t \in A\}$ **infinite sequences**.
- $A^{\leq \mathbb{N}_0} := A^{<\mathbb{N}_0} \cup A^{\mathbb{N}}$.
- $\mathcal{A}_i(\cdot) : A^{<\mathbb{N}_0} \rightrightarrows A_i$ **constraint correspondence** of i.
- *Assumption:* $\forall h \in A^{<\mathbb{N}_0}$, either $(\forall i, \mathcal{A}_i(h) \neq \emptyset)$ or $(\forall i, \mathcal{A}_i(h) = \emptyset)$ ($\mathcal{A}_i(h) = \emptyset$ for any i means "**game over**").
- For $h' \in A^{<\mathbb{N}_0}$, $h'' \in A^{\leq \mathbb{N}_0}$, $h = (h', h'') \in A^{\leq \mathbb{N}_0}$ **concatenation,** h' **prefix** of h.
- **Partial order:** $h' \preceq h \Leftrightarrow (\exists h'' \in A^{\leq \mathbb{N}_0}, h = (h', h''))$, $h' \prec h \Leftrightarrow (h' \preceq h \wedge h' \neq h)$.

Multistage game trees

Derived elements

Given $\langle I, (A_i, \mathcal{A}_i(\cdot))_{i\in I}\rangle$, *define*

- $h \in \bar{H} \subseteq A^{\leq \mathbb{N}_0}$ **histories**=sequences of action profiles allowed by the rules. Let $\mathcal{A}(h) := \times_{i\in I}\mathcal{A}_i(h)$, *then,*

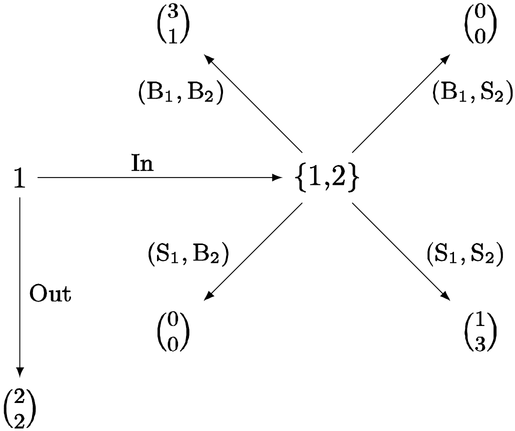
1. $\varnothing \in \bar{H}$ (**empty sequence** $\varnothing$ represents the *initial situation*),
2. $\forall (h,a) \in A^{<\mathbb{N}_0} \times A$, $(h,a) \in \bar{H} \Leftrightarrow (h \in \bar{H} \wedge a \in \mathcal{A}(h))$ (thus, *each prefix of a history must be a history*),
3. $\forall (a^t)_{t=1}^{\infty} \in A^{\mathbb{N}}$,
$(a^t)_{t=1}^{\infty} \in \bar{H} \Leftrightarrow \left(\left(a^1 \in \mathcal{A}(\varnothing)\right) \wedge \left(\forall t \in \mathbb{N}, a^{t+1} \in \mathcal{A}(a^1,...,a^t)\right)\right)$
(infinite feasible sequences are histories).

- $h \in H := \{h \in \bar{H} \backslash A^{\mathbb{N}} : \mathcal{A}(h) \neq \emptyset\}$, **non-terminal (partial)** histories (note: partial histories are *finite*).
- $z \in Z := \bar{H} \backslash H$, **terminal** histories, or **paths** (note: all infinite histories are "terminal").
- By 1-3, $(\bar{H}, \preceq)$ is a **tree** with root $\varnothing$: h is the *immediate predecessor* of $(h,a) \in H \times \mathcal{A}(h)$.

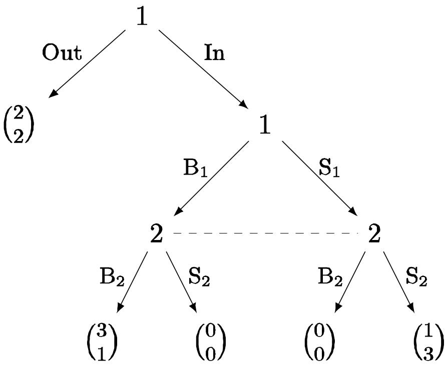
Multistage game trees

Faithful, but non-traditional tree-pictures

- Tree-pictures featuring *truly simultaneous* moves are not traditional in Game Theory, but they are perfectly legitimate. Here is the *BoSOO* picture:



- The traditional tree-pictures in GT *misrepresent simultaneous moves as if they were sequential* (e.g., Ann before Bob in the BoS), but add *"information sets"* to *make the misrepresentation innocuous* (e.g., Bob cannot observe Ann's choice in the BoS). Here is BoSOO:



Multistage games with complete information

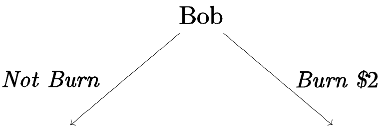
A **multistage game** (with *complete information* and *observable actions*) is a structure

$$\Gamma = \left\langle I, (A_i, \mathcal{A}_i(\cdot), u_i)_{i \in I} \right\rangle$$

where

- $\left\langle I, (A_i, \mathcal{A}_i(\cdot))_{i \in I} \right\rangle$ **multistage game tree**,
- for each i, $u_i : Z \to \mathbb{R}$ **payoff function**,
- u_i derivable from $g : Z \to Y$ and $v_i : Y \to \mathbb{R}$. (Game tree $\left\langle I, (A_i, \mathcal{A}_i(\cdot))_{i \in I} \right\rangle$ and outcome function g yield a **game form**; cf. static games).

Example: BoS with Burning option (dissipative action)



Ann\Bob	L	R
U	$4, 1$	$0, 0$
D	$0, 0$	$1, 4$

Ann\Bob	l	r
u	$4, -1$	$0, -2$
d	$0, -2$	$1, 2$

Figure 1: BoS with dissipative action.

- $\mathcal{A}(\varnothing) = \{w\} \times \{N, B\}$, $\mathcal{A}((w, N)) = \{U, D\} \times \{L, R\}$, $\mathcal{A}((w, B)) = \{u, d\} \times \{l, r\}$ ($w =$"wait", $\mathcal{A}(h) := \mathcal{A}_a(h) \times \mathcal{A}_b(h)$);
- $H = \{\varnothing, ((w, N)), ((w, B))\}$;
- $Z = (\{((w, N))\} \times \mathcal{A}((w, N))) \cup (\{((w, B))\} \times \mathcal{A}((w, B)))$, $\bar{H} = H \cup Z$;
- 8 chains of histories, e.g.: $\varnothing \prec ((w, B)) \prec ((w, B), (u, l)) \in Z$.

Strategies, path function

- $s_i \in S_i := \times_{h \in H} \mathcal{A}_i(h)$ **strategies** (pure): s_i (a function) associates each $h \in H$ with a feasible action $a_i = s_i(h) \in \mathcal{A}_i(h)$.
- $s \in S := \times_{i \in I} S_i$ **strategy profiles**.
- **Path function** ζ (Greek zeta): it specifies the *path of play induced by* each $s \in S$ (if executed)

$$\begin{array}{rccc} \zeta : & S & \to & Z \\ & s & \mapsto & \left(s(\varnothing), s(s(\varnothing)), s(s(\varnothing), s(s(\varnothing))), ...\right) \end{array}$$

- $\forall h \in H$, $S(h) := \{s \in S : h \prec \zeta(s)\}$ strategy profiles inducing h.
- **Note:** $S(h) = \times_{i \in I} S_i(h)$, where $S_i(h) := \mathrm{proj}_{S_i} S(h)$, is the set of i's strategies allowing (not preventing) h. Try to prove it!

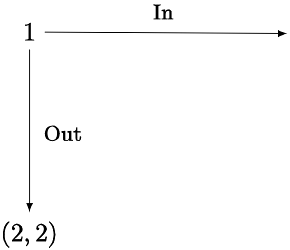
Behavioral and realization equivalence between strategies

- Let $H_i(s_i)$ denote the set of *non-terminal histories allowed* by s_i. **Note** the following characterizations:
  - $H_i(s_i) = \{h \in H : s_i \in S_i(h)\}$,
  - $H_i(s_i) = \{h \in H : \exists s_{-i} \in S_{-i}, h \prec \zeta(s_i, s_{-i})\}$.
- **Behavioral equivalence**: s_i and s_i' induce the *same behavior* of i independently of the behavior of $-i$, written $s_i \approx_i s_i'$, IFF $(H_i(s_i) = H_i(s_i')$ and $\forall h \in H_i(s_i), s_i(h) = s_i'(h))$ [**note**: also, $s_i \approx_i s_i'$ IFF $\forall h \in H_i(s_i) \cap H_i(s_i')$, $s_i(h) = s_i'(h)$].
- **Realization equivalence**: for every $s_{-i} \in S_{-i}$, the paths induced by s_i and s_i' coincide : $\forall s_{-i} \in S_{-i}, \zeta(s_i, s_{-i}) = \zeta(s_i', s_{-i})$.

Lemma

(Equivalence) Behavioral and realization equivalence coincide.

Strategy equivalence in the BoSOO



$1\backslash 2$	B_2	S_2
B_1	$3, 1$	$0, 0$
S_1	$0, 0$	$1, 3$

- In the BoSOO, $\mathrm{Out.B}_1 \approx_1 \mathrm{Out.S}_1$: $H_1(\mathrm{Out.B}_1) = \{\varnothing\} = H_1(\mathrm{Out.S}_1)$ and both choose Out at $\varnothing$. For each s_2, $\zeta(\mathrm{Out.B}_1, s_2) = ((\mathrm{Out}, \mathrm{w})) = \zeta(\mathrm{Out.S}_1, s_2)$.
- There is no other equivalence between strategies in the BoSOO.

Interpretation: strategies and memory (I)

- The term "strategy" in the natural languange refers to a **plan** in the mind of the player.
    - (Originally, the plan of the "strategos," the general leading the army in ancient Greece).
- The interpretation of the mathematical object (function) $s_i \in \times_{h \in H} \mathcal{A}_i(h)$ as a plan in the mind of player i requires that either (1) i is always able to remember the history h that just occured, or (2) the rules of the game are such that she is reminded every time of everything that happened.
- Our mathematical representation does not distinguish between these two situations. But the latter is very rare in applications. Hence, our default interpretation is that *players have perfect memory* (a cognitive trait).

Interpretation: strategies, memory, and behavior (II)

- Note, also the traditional mathematical representation of games with *imperfectly* observable actions is unable to distinguish between objective features of the information structure and subjective traits concerning cognition and memory.
  - Fortunately, a better mathematical formalism is now available, although not yet widespread, and it formally clarifies that to make sense of the standard we should assume that players have perfect memory (in-depth study: Battigalli & Generoso, IGIER wp. 678).
- Functions $s_i \in \times_{h \in H} \mathcal{A}_i(h)$ may also be interpreted as *descriptions of behavior*: if h occurred, i would take action $s_i(h) \in \mathcal{A}_i(h)$ (this interpretation does not rely on perfect memory). Note, from the perspective of $-i$, the differences between behaviors described by two equivalent strategies s_i' and s_i'' are immaterial.
  - In most of game theory there is a *conflation of the two interpretations of "strategy as plan" and "strategy as behavior,"* with no clear conceptual reflection about the distinction (in-depth study: Battigalli & De Vito, 2021).

Reduced strategies and strategic forms

- Let $\mathbf{S}_i^r = S_i | \approx_i$ denote the collection of equivalence classes of $\approx_i$, this is the set of **reduced strategies**. (Thus, a reduced strategy of i is a *subset* of S_i: $\mathbf{s}_i^r \subseteq S_i$ for each $\mathbf{s}_i^r \in \mathbf{S}_i^r$.)
- $\left\langle I, (S_i, U_i)_{i\in I} \right\rangle$, with $U_i = u_i \circ \zeta : S \to \mathbb{R}$ is the **normal,** or **strategic form** of Γ.
- $\left\langle I, (\mathbf{S}_i^r, U_i^r)_{i\in I} \right\rangle$, with

$$\forall \mathbf{s}^r \in \times_{j\in I} \mathbf{S}_j^r, \forall s \in \mathbf{s}^r,\ U_i^r\left(\mathbf{s}^r\right) = U_i\left(s\right),$$

  is the **reduced strategic form** of Γ (by the Equivalence Lemma above, $U_i\left(s_{-j}, \cdot\right)$ is constant on each cell $\mathbf{s}_j^r \in \mathbf{S}_j^r$).
- *What is the reduced strategic form of the BoSOO?*

Reduced strategic form of BoS with Burning option

Bob

Not Burn | *Burn \$2*

Ann\Bob	L	R
U	$4,1$	$0,0$
D	$0,0$	$1,4$

Ann\Bob	l	r
u	$4,-1$	$0,-2$
d	$0,-2$	$1,2$

$a\backslash b$	$N.L$	$N.R$	$B.\ell$	$B.r$
$U.u$	4,1	0,0	4,-1	0,-2
$U.d$	4,1	0,0	0,-2	1,2
$D.u$	0,0	1,4	4,-1	0,-2
$D.d$	0,0	1,4	0,-2	1,2

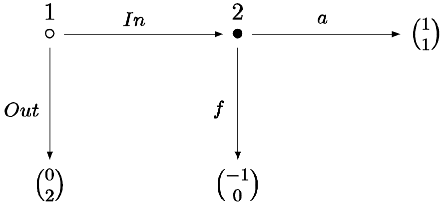
- $S_a = \{U.u, U.d, D.u, D.d\} \cong \mathbf{S}^r_a$,
- $S_b = \{N.L.\ell, N.R.\ell, N.L.r, N.R.r, B.L.\ell, B.R.\ell, B.L.r, B.R.r\}$,
- $\mathbf{S}^r_b = \{N.L, N.R, B.\ell, B.r\}$.

"Static" solutions on the strategic form: a good idea?

- Let $\mathcal{N}(\Gamma) = \left\langle I, (S_i, U_i)_{i \in I} \right\rangle$ denote the **normal**, or **strategic form** of multistage game Γ; similarly, $\mathcal{N}^r(\Gamma) = \left\langle I, (\mathbf{S}_i^r, U_i^r)_{i \in I} \right\rangle$ denotes the **reduced normal (strategic) form** of Γ. These are mathematical representations of auxiliary, fictitious games, not of the game truly played, which is represented by Γ.
- It is *natural to try to apply known solution concepts defined for static games to these auxiliary games*. Is this a *good idea?* Does it depend on which "static" solution concept we use?
- **Remark**: If $s_i' \approx_i s_i''$, then $U_j(s_i', s_{-i}) = U_j(s_i'', s_{-i})$ for every $j \in I$ and $s_{-i} \in S_{-i}$, that is, *behavioral equivalence implies payoff equivalence* (*for all* players' payoffs). Therefore, *when we apply solution concepts defined for static games, we may as well work with the reduced strategic form* which is simpler.
  - [Try to understand by yourselves the exact meaning of this for Nash equilibrium, rationalizability, and the iterated deletion of weakly dominated strategies.]

A simple example: the Entry Game

- The picture represents a stylized situation where a potential entrant (pl. 1) has to decide whether to enter (*In*) or not (*Out*) in a market with an incumbent (pl. 2). The latter may fight entry (f), or acquiesce (a).



- Under complete information, the intuitive solution is obvious: *given In*, fighting is dominated; hence, it violates rationality: Anticipating a rational response (i.e., *a*cquiescence), pl. 1 goes *In*.
- Yet, $(Out, \mathbf{f})$ (where $\mathbf{f}$ denotes strategy "f if *In*") is a Nash equilibrium of the strategic form!

Restricted admissibility (weak dominance)

- An interesting way to "solve" multistage games is the iterated deletion of weakly dominated (inadmissible) strategies. For example, **f** is weakly dominated for pl. 2 in the Entry Game, after deleting **f**, the best reply of pl. 1 is *In*.
- Let $\langle I, (S_i, U_i)_{i\in I}\rangle$ be the strategic form of a multistage game Γ. *Preliminary* definition:

Definition

Fix a (nonempty) Cartesian subset of strategy profiles $C \subseteq S$. A strategy $\bar{s}_i \in C_i$ is **weakly dominated** (*inadmissible*) **within** C if there exists a mixed strategy $\sigma_i \in \Delta(C_i)$ such that

$$\begin{aligned} \forall s_{-i} &\in C_{-i},\ U_i(\bar{s}_i, s_{-i}) \leq U_i(\sigma_i, s_{-i}), \\ \exists \bar{s}_{-i} &\in C_{-i},\ U_i(\bar{s}_i, \bar{s}_{-i}) < U_i(\sigma_i, \bar{s}_{-i}), \end{aligned}$$

and it is **admissible** (not weakly dominated) **within** C otherwise.

Iterated admissibility

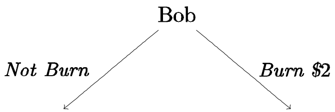
- The set of profiles of not weakly dominated (i.e., admissible) strategies given Cartesian subset $C \subseteq S$ is denoted $\mathrm{NWD}(C)$.
- **Note:** NWD is *not monotone*. But, by inspection of the definition, $\mathrm{NWD}(C) \subseteq C$ (NWD is a restriction operator). Thus, the sequence of subsets $\left(\mathrm{NWD}^k(S)\right)_{k=0}^{\infty}$ is weakly decreasing (where, $\mathrm{NWD}^k = \mathrm{NWD} \circ \mathrm{NWD}^{k-1}$, with $\mathrm{NWD}^0 = \mathrm{Id}_{\mathcal{C}}$).

Definition

The set of profiles of **iteratively admissible** (iteratively not weakly dominated) strategies is $\mathrm{NWD}^{\infty}(S) = \bigcap_k \mathrm{NWD}^k(S)$.

- **Note:** By the Equivalence Lemma, we can just look at the *reduced* strategic form $\left\langle I, (\mathbf{S}_i^r, U_i^r)_{i \in I} \right\rangle$ of Γ.

Iterated admissibility in BoS w/ Burning option: Steps 1-2



Ann\Bob	L	R
U	$4,1$	$0,0$
D	$0,0$	$1,4$

Ann\Bob	l	r
u	$4,-1$	$0,-2$
d	$0,-2$	$1,2$

$a\backslash b$	$N.L$	$N.R$	$B.\ell$	$B.r$
$U.u$	4,1	0,0	4,-1	0,-2
$U.d$	4,1	0,0	0,-2	1,2
$D.u$	0,0	1,4	4,-1	0,-2
$D.d$	0,0	1,4	0,-2	1,2

- $\mathrm{NWD}^1(\mathbf{S}^r) = \mathbf{S}^r_a \times \{N.L, N.R, B.r\}$ ($B.\ell$ is dominated).
- $\mathrm{NWD}^2(\mathbf{S}^r) = \{U.d, D.d\} \times \{N.L, N.R, B.r\}$.

Iterated admissibility in BoS w/ Burning option: Steps 3-5

- $\mathrm{NWD}^2(\mathbf{S}^r) = \{U.d, D.d\} \times \{N.L, N.R, B.r\}$ (Bob would *Burn* only to "signal" r, delete $B.\ell$, Ann interprets *Burn* as a "signal" of r: delete $U.u$, $D.u$).
- $\mathrm{NWD}^3(\mathbf{S}^r) = \{U.d, D.d\} \times \{N.R, B.r\}$ (Bob expects more from $B.r$ than $N.L$).

$a \backslash b$	$N.L$	$N.R$	$B.r$
$U.d$	4,1	0,0	1,2
$D.d$	0,0	1,4	1,2

$\overset{3}{\Rightarrow}$

$a \backslash b$	$N.R$	$B.r$
$U.d$	0,0	1,2
$D.d$	1,4	1,2

$\overset{4\text{-}5}{\Rightarrow}$

$a \backslash b$	$N.R$
$D.d$	1,4

- $\mathrm{NWD}^4(\mathbf{S}^r) = \{D.d\} \times \{N.R, B.r\}$ (Ann expects R after N: hence, D after N).
- $\mathrm{NWD}^5(\mathbf{S}^r) = \{D.d\} \times \{N.R\}$ (no sacrifice is needed: Bob can "have the cake and eat it").

Solution concepts and strategic form: take home message

- Starting with the "founding fathers" von Neumann & Morgenstern, *some game theorists argued that the strategic (normal) form of a game is a sufficient representation*, that is, one can apply solution concepts to it as if she were analyzing a game with simultaneous moves (in such game, each player secretly and irreversibly commits ex ante to her strategy, which is then mechanically executed).
- Yet, even in simple games (e.g., the Entry Game), Nash equilibrium and rationalizability on the strategic form allow for counterintuitive behavior.
- Iterated admissibility, instead, yields meaningful results that seem to capture sophisticated strategic reasoning, whereby players "rationalize" earlier moves by co-players even if they are unexpected. We will see that, *except for a "negligible" class of games, iterated admissibility coincides with a strong version of "rationalizability" defined for sequential games* (Ch. 11, L 15).

References

- Battigalli, P., E. Catonini, and N. De Vito (2023): *Game Theory: Analysis of Strategic Thinking*. Typescript, Bocconi University.
- Battigalli, P. (2023): *Mathematical Language and Game Theory*. Typescript, Bocconi University.
- Battigalli, P., and N. De Vito (2021): "Beliefs, plans and perceived intentions in dynemic games," *Journal of Economic Theory*, 195, 105283.
- Battigalli, P., and N. Generoso. "Information flows and memory in games." IGIER w.p. 678. Bocconi University.
- Von Neumann, J., and O. Morgenstern (1953): *Theory of Games and Economic Behavior* (Third Ed.). Princeton: Princeton University Press.