Path agreements and off-path uncertainty

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Motivation

Pre-play, non-binding agreements among players often specify the path to follow, but not the off-path behavior.

Compliance with the agreement may rely more on the aversion to the uncertainty that a deviation entails, rather than on certainty of less advantageous re-coordination. We explore this possibility.

A deviation from the agreed-upon path may be rationalized as an attempt to achieve a higher payoff, a form of forward-induction reasoning.

We elucidate the following question:

as risk or ambiguity aversion increase, can strategically sophisticated players credibly agree on a larger set of paths?

Literature - comparative risk and ambiguity aversion

Battigalli, Cerreia-Vioglio, Maccheroni and Marinacci (2015):

the set of self-confirming equilibria (SCE) of essentially simultaneous games expands as ambiguity aversion increases.

Battigalli, Catonini, Lanzani and Marinacci (2019) extend the analysis to sequential games.
We have a different (and partial) source of coordination (agreement vs learning), therefore we use a different solution concept. (Yet, if the path-agreement is credible, it must also be an SCE path.)

Weinstein (2016): *the set of rationalizable actions in simultaneous games expands as risk aversion increases.*

Battigalli, Cerreia-Vioglio, Maccheroni and Marinacci (2016):

the set of justifiable (and rationalizable) actions expands as risk or ambiguity aversion *increase.*
For our analysis, we need to extend these results to sequential games under forward-induction reasoning.

Literature - strategic reasoning in dynamic games

Pearce (1984) and Battigalli (1997) capture forward-induction reasoning based on interactive beliefs in rationality with **strong rationalizability** (a.k.a. extensive-form rationalizability).

Battigalli (2003) and Battigalli and Siniscalchi (2003) introduce first-order belief restrictions with the notion of **strong directed rationalizability** (a.k.a. strong Δ-rationalizability).
(See also the references therein on the *epistemic foundations* of this solution concept.)

Catonini (2021) analyses forward induction under non-binding agreements.

Catonini (2020) shows that, for a path-agreement, directed rationalizability is well-suited for the analysis.

We adopt strong directed rationalizability to characterize forward-induction reasoning assuming transparency of the initial belief in the agreed-upon path.

Literature - equilibrium refinements

Kohlberg and Mertens (1986) put forward "strategic stability" to capture instances of forward-induction reasoning based on the idea that the deviator is trying to achieve a higher payoff than her equilibrium payoff.
Cho and Kreps (1987) and Banks and Sobel (1987) apply this idea to signaling games.

Govindan and Wilson (2009) provide a notion of forward-induction equilibrium that is simpler than strategic stability and retains some crucial properties.
All these works focus on FI-refined Nash/sequential equilibrium, whereas *we do not "neutralize" off-path uncertainty by assuming a commonly understood continuation equilibrium*.

Example - 1: low (zero) risk aversion

In this presentation, we only argue informally that a credible path-agreement remains credible for higher risk/ambiguity aversion, despite non-monotonic changes in rationalizable reactions to deviations.

Suppose that *Ann* and **Bob** agree on a path at the end of which *Ann*'s utility=payoff is 9.

Ann has a unilateral deviation from the path that leads to a subgame with monetary payoffs=utilities as follows:

$$\underset{\pi_a = 9}{z} \overset{\text{agreement}}{\longleftarrow} Ann \xrightarrow{\text{dev}}$$

$a \backslash \mathbf{b}$	C	D	E	F
G	*4*, **2**	*4*, **5**	*4*, **16**	*4*, **0**
H	*4*, **2**	*4*, **5**	*4*, **0**	*4*, **16**
J	*0*, **2**	*16*, **1**	*0*, **16**	*0*, **0**
K	*16*, **2**	*0*, **1**	*0*, **0**	*0*, **16**

Ann can profit from the deviation only if Bob plays C or D, but those are dominated (by a mixed action).

The "rationalizable" reactions to the deviation are E and F, and the z-agreement is credible.

Example - 2: intermediate risk aversion

Transform all payoffs with the square root: $u_i = \sqrt{\pi_i}$. Ann's on-path utility-payoff is now 3.

$$\begin{array}{c} z \\ u_a = 3 \end{array} \overset{\text{agreement}}{\dashleftarrow} Ann \xrightarrow{\text{dev}}$$

$a \backslash \mathbf{b}$	C	D	E	F
G	$2, \sqrt{\mathbf{2}}$	$2, \sqrt{\mathbf{5}}$	$2, \mathbf{4}$	$2, \mathbf{0}$
H	$2, \sqrt{\mathbf{2}}$	$2, \sqrt{\mathbf{5}}$	$2, \mathbf{0}$	$2, \mathbf{4}$
J	$0, \sqrt{\mathbf{2}}$	$4, \mathbf{1}$	$0, \mathbf{4}$	$0, \mathbf{0}$
K	$4, \sqrt{\mathbf{2}}$	$0, \mathbf{1}$	$0, \mathbf{0}$	$0, \mathbf{4}$

Now only C is dominated for Bob. Therefore, J survives Ann's second step of reasoning (prediction of a justifiable reaction). But Bob's best reply to J is E.

The "best-rationalizable" reaction to the deviation is E, the agreement is still credible.

Example - 3: high risk aversion

Transform the payoffs with the square root once more. Ann's payoff on path is now $\sqrt{3}$

$$\underset{u_a = \sqrt{3}}{z} \xleftarrow{\text{agreement}} Ann \xrightarrow{\text{dev}}$$

$a \backslash \mathbf{b}$	C	D	E	F
G	$\sqrt{2}$, $\mathbf{2\frac{1}{4}}$	$\sqrt{2}$, $\mathbf{5\frac{1}{4}}$	$\sqrt{2}$, $\mathbf{2}$	$\sqrt{2}$, $\mathbf{0}$
H	$\sqrt{2}$, $\mathbf{2\frac{1}{4}}$	$\sqrt{2}$, $\mathbf{5\frac{1}{4}}$	$\sqrt{2}$, $\mathbf{0}$	$\sqrt{2}$, $\mathbf{2}$
J	0, $\mathbf{2\frac{1}{4}}$	2, $\mathbf{1}$	0, $\mathbf{2}$	0, $\mathbf{0}$
K	2, $\mathbf{2\frac{1}{4}}$	0, $\mathbf{1}$	0, $\mathbf{0}$	0, $\mathbf{2}$

No action is dominated for Bob. Both J and K survive Ann's second step of reasoning. Only D is not a best reply to any belief over J and K. But then, Ann (3rd step) eliminates J. Bob's best reply to K is F.

The z-agreement is still credible, but the "best-rationalizable" reaction to the deviation is F — **disjoint from those obtained with lower risk aversion.**

Framework

A multistage game with finitely many actions at each stage, observable actions and *finite or infinite horizon.*

Monetary payoffs are common knowledge.

Risk and ambiguity attitudes are either common knowledge,
or have commonly known upper bounds.

We represent them with a vNM utility function and a "2nd-order utility" as in the smooth ambiguity model (Klibanoff, Marinacci and Mukherjee 2015).

There are no chance moves, and players cannot delegate their choices to randomization devices $\Longrightarrow$ no objective randomness $\Longrightarrow$ risk & ambiguity aversion cannot be disentangled, ambiguity-averse players are dynamically consistent.

Solution concept

We fix an agreed-upon path z and adopt Strong Directed Rationalizability with the following first-order belief restrictions:

every player initially believes that the co-players will not deviate from the agreed-upon path z.

Strong-z-rationaliz. captures *common strong belief* in rationality and the above.

Strong-z-rationalizability yields:

- the empty set, if believing in the path is at odds with strategic reasoning;
- the behavioral consequences of the z-agreement, otherwise.

We say that a z-agreement is **credible** if Strong-z-rationalizability is non-empty.

Main result

Theorem

The set of credible paths expands as risk or ambiguity aversion increase.

[The proof is tricky because *strong belief* (believing whenever possible that an event is true) *is non-monotone*, and yet we want to prove a monotonicity result.]

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