Games with noisy signals about emotions

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Introduction: Motivation

- Starting point: well-being depends on **experienced emotions**.
- Key idea: **noisy signals** (e.g., facial cues) may betray emotions.
  - Everyday experience: blushing = embarrassment, gaze contact = interest, smiling = happiness, etc.
  - Literature:
    - Emotional leakage is associated with **lies and deception** (Porter et al. 2012; Matsumoto and Hwang 2018);
    - Nonverbal communication expresses **(dis)liking** (Givens 1978);
    - Gesture informs an audience of a speaker's **(unspoken) thoughts** (Goldin-Meadow 1999);
    - Facial cues allow to recognize others' **trustworthiness** or **predisposition to anger** (Van Leeuwen et al. 2018; Stirrat and Perrett, 2010)
    - **Emotional contagion** occurs when people recognize and mimic others' emotions (Hatfield et al. 2014; see Vasquez and Weretka 2020 for an economic analysis).
- Question: can emotional signals **shape behavior** when individuals **reason strategically**?

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- We formalize a **general framework** to model emotional feedback.
- We show how to derive behavioral predictions:
  - Definition of **rationality features**: both *cognitive* (coherence of beliefs, belief updating consistent with evidence and according to the rules of conditional probabilities), and *behavioral* (rational planning, consistent implementation of plans).
  - These features hold *only at some states of the world*: each requirement is an explicit assumption, and players can entertain the possibility of cognitive or behavioral failures of their opponents.
  - We prove that **rationality** is a well-defined **event**.
  - We define a version of the **strong rationalizability** solution procedure: we show it captures the implications of rationality and common strong belief in rationality.

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Introduction: Related literature (non-exhaustive)

Psychological games and belief-dependent preferences

- Articles: Geanakoplos, Pearce, and Stacchetti (1989); Battigalli and Dufwenberg (2009); Battigalli, Corrao and Dufwenberg (2019).
- Survey: Battigalli and Dufwenberg (2022).

Epistemic game theory

- Epistemic analysis without type structures: Battigalli, Corrao and Sanna (2020).
- Consistency between behavior and intentions: Battigalli and De Vito (2021).
- Strong rationalizability (and ancestors): Pearce (1984); Battigalli (1997); Battigalli and Siniscalchi (2002); Battigalli and Prestipino (2013).

What's next?

1. Definition of a **general framework**
2. Description of **inferences**
3. Formalization of a notion of **rationality**
4. Definition of a **solution concept** to derive predictions
5. Concluding remarks

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Framework: Warm-up

To give an idea of the phenomena we model, here is a heuristic example (called buy me an ice-cream).

Situation:

- Child is at home alone: can choose *Homework* or *Video-games*.
- Mom gets back home. Child: "Mom, can you please buy me an ice-cream?" Mom: "Did you do your homework?".
  Child can choose to answer *Yes* or *No* (note: *Yes* = "I did my homework"). *But* Child may *blush* if he lies.
- Mom decides whether to *Buy* the ice-cream or *Not*.

More general and relevant problem: *disclosure of information*.
Is lying worth it if lies can be spotted?

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Mechanisms at play:

1. Emotions determine **utilities** and **emotional feedback**: these mechanisms are embedded in an interactive setting.
2. Emotions are triggered by the **game unfolding** and **endogenous beliefs**.
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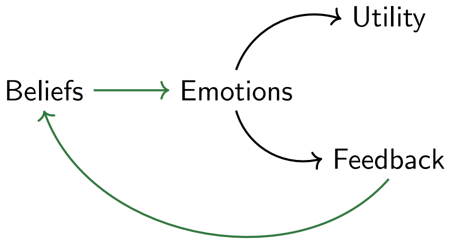

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- Tie with **beliefs**:
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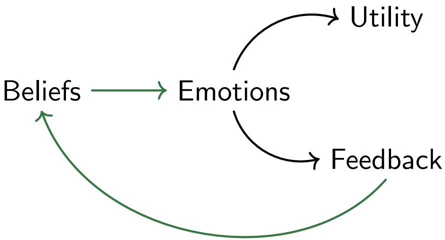

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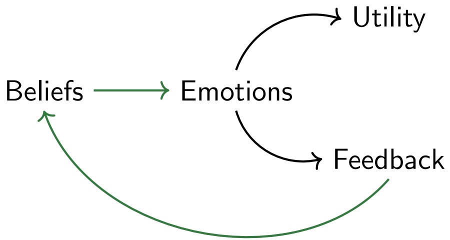

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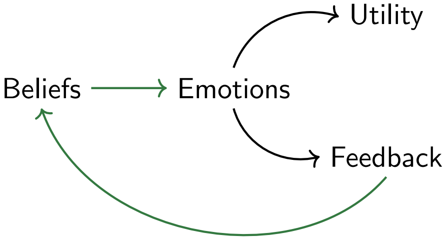

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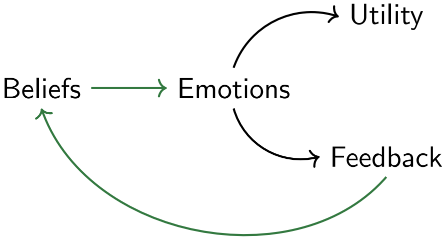

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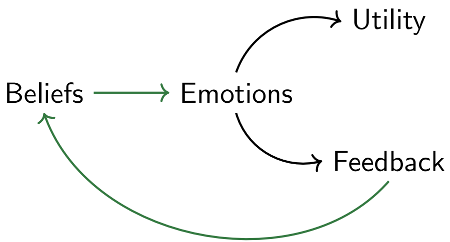

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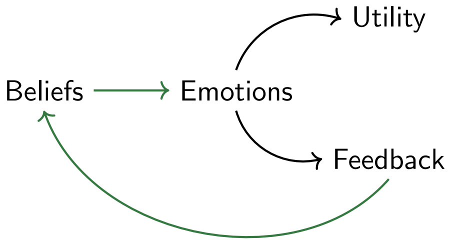

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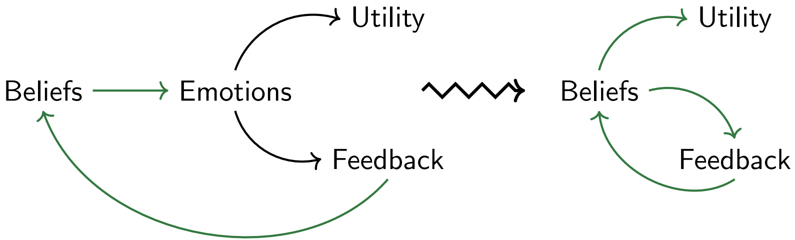

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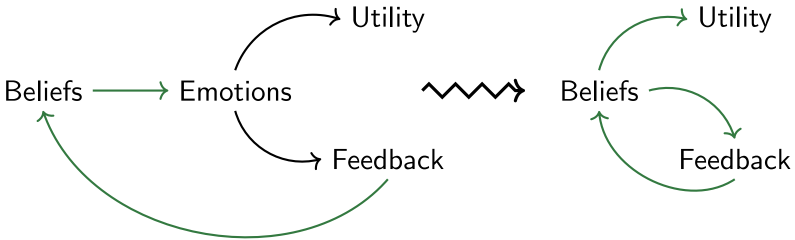

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Framework: Basic ingredients

Notation: for a generic indexed profile of sets $(X_i)_{i \in I}$, $X := \times_{i \in I} X_i$. For a generic set X, X^n ($n \in \mathbb{N}$) is the set of sequences of elements of X of length n.

Standard ingredients (all finite):

- Set of **players** I.
- Set of **actions** of i: A_i.
- Set of **personal traits** of i: Θ_i. Player i knows θ_i (informal assumption).
- Set of **outcomes** for i: Y_i.
- Set of **messages** i may observe: M_i.

Non-standard ingredients (compact metrizable):

- Set of **emotions** of i: E_i.
- Set of **streams of emotions** of i: $E_i^{\leq T+1} := \bigcup_{t=1}^{T+1} E_i^t$ ($T \in \mathbb{N}$).

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Framework: Emotions, utilities, feedback

We start with:

- Continuous **feedback function**: $\tilde{f} : A \times \Theta \times E^{\leq T+1} \to \Delta(M)$.
  Interpretation: messages are *stochastic* because messages about emotions are *noisy*.
- Profile of continuous **psychological utility functions**: $(\tilde{v}_i : Y \times \Theta \times E^{\leq T+1} \to \mathbb{R})_{i \in I}$.

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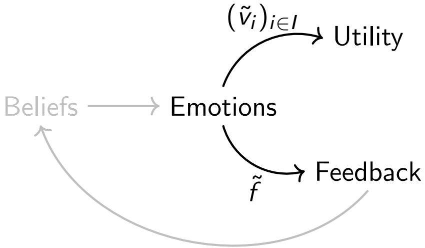

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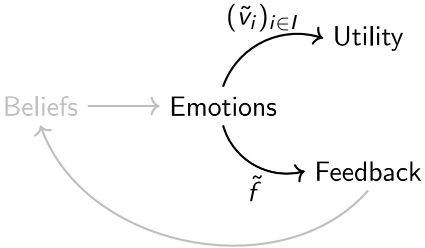

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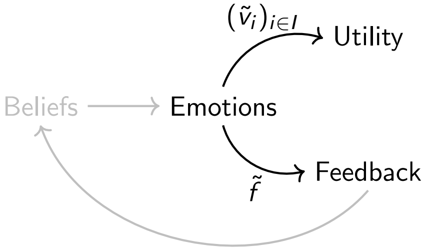

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Next, focus on:

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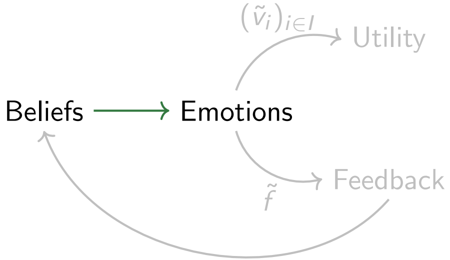

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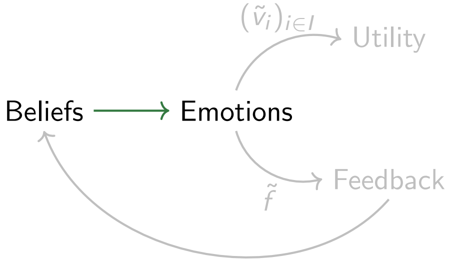

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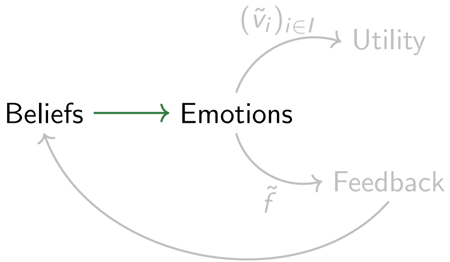

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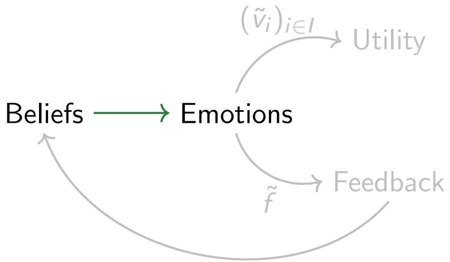

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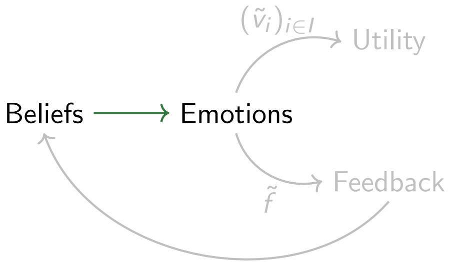

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- Assumption: players **need not observe** others' moves; they only get imperfect information about how the game is being played.
- Innovation: we model game-specific information as a **stream**; after each stage, players receive messages about co-players' moves.
- Set of **previous play messages** (PPM) of player i: $M_{i,p}$ (finite). PPMs generated based on the game unfolding (through a function $P : \bigcup_{t=0}^{T} A^t \to M_p$).
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- Assumption: players **need not observe** others' moves; they only get imperfect information about how the game is being played.
- Innovation: we model game-specific information as a **stream**; after each stage, players receive messages about co-players' moves.
- Set of **previous play messages** (PPM) of player i: $M_{i,p}$ (finite). PPMs generated based on the game unfolding (through a function $P : \bigcup_{t=0}^{T} A^t \to M_p$).
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- Retrieve the set $\bar{H}$ of **feasible histories** (sequences of profiles of actions, PPMs, and emotional messages). (a^t, m_p^t, m^t) is feasible if:
  1. action profiles are feasible given the last PPM profile;
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- $\bar{H}$ is partitioned into the set of **terminal histories** Z and the set of **non-terminal histories** H.
- Note: a given player has information only about the *actions she took* and the *messages she received*. If the history is $(a_j^t, m_{j,p}^t, m_j^t)_{j\in I}$, i "knows"/observes $(a_i^t, m_{i,p}^t, m_i^t)$.
- Set of **personal histories** of player i, $\bar{H}_i$: (feasible) sequences of actions played and messages received by i. Non-terminal personal histories of i: H_i.
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Framework: Application to “buy me an ice-cream”

Feedback: Child blushes (b) only if he says *Yes* after *Video-games*, with probability equal to the probability with which he believes Mom would not get him the ice-cream if he blushes.
Intuition: if he is confident that he can get away with his lie even if spotted by Mom, more likely to keep a poker face.

Histories: the timeline is as follows.

Stage 1 Child chooses $a_{C,1} \in \{Homework, Video\text{-}games\}$.
Mom does not observe it.

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Mom observes $a_{C,2}$ and m.

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So, a terminal history is $z = (a_{C,1}, a_{C,2}, m, a_M)$, but Mom observes only $z_M = (a_{C,2}, m, a_M)$.

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The unique play after which Child *may* blush is highlighted in green.

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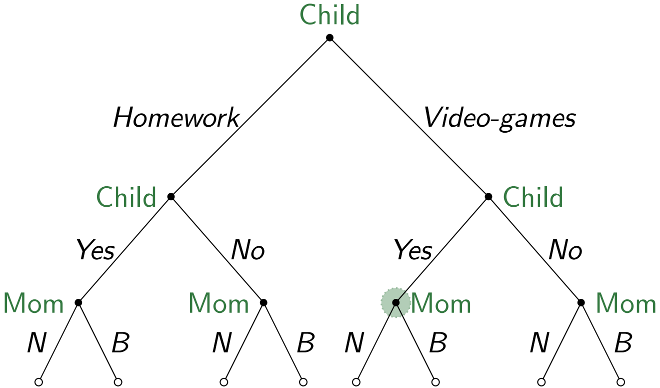

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Framework: Describing behavior

- Set of **personal external states** of i: $S_i := \times_{h_i \in H_i} A_i(h_i)$, with $A_i(h_i)$ set of actions available at h_i.
- Interpretation: $s_i \in S_i$ is a *complete objective description of how player i would behave* at each possible game-specific contingency (i.e., personal history she may observe).
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- Note: we allow players to form beliefs on *their own behavior*. Why? Players cannot know how they will behave in the future (because they are not committed), and they may also be uncertain about it. Beliefs about how they would act form a **plan**.
- Hierarchies of beliefs defined inductively. $\mathcal{T}_i^n$ is the set of hierarchical systems of beliefs of i of order $n \in \mathbb{N}$. τ_i^n is a map $h_i \mapsto \left(\tau_{i,k}(\cdot|h_i)\right)_{k=1}^n$, with $\tau_{i,k}(\cdot|h_i) \in \Delta(S \times \Theta_{-i} \times \mathcal{T}_{-i}^{k-1})$.
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Framework: States of the world

- Set of **states of the world**: $\Omega^\infty := \times_{i\in I}\left(S_i \times \Theta_i \times \mathcal{T}_i^\infty\right)$.
- Interpretation: a profile $(s_i, \theta_i, \tau_i^\infty)_{i\in I}$ describes *every relevant aspect of strategic interaction*.
- **Events** are Borel measurable subsets of Ω^∞.
- Interpretation of measurability: measurable sets are those players can *conceive*, *assess*, and *form beliefs about*. Hence, *players can reason only about events*.

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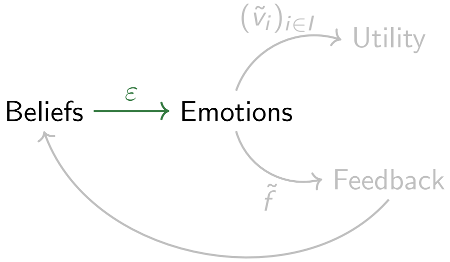
Framework: Emotions and game unfolding

Continuous **emotion-generating function** $\varepsilon : \bar{H} \times \mathcal{T}^\infty \to \Delta(E^{\leq T+1})$.

- Interpretation: *what happened*, and players' *ways of thinking* determine the emotions they experience (e.g., surprise, frustration).
- Assumption: only *realized* beliefs matter (not counterfactual ones).
- Assumption: only beliefs of order *up to* $K \in \mathbb{N}$ matter. So, we can focus on $S \times \Theta \times \mathcal{T}^K$ (**utility-relevant states**, drop K for simplicity).

More details

Framework: Emotions and game unfolding



Continuous **emotion-generating function** $\varepsilon : \bar{H} \times \mathcal{T}^\infty \to \Delta(E^{\leq T+1})$.

- Interpretation: *what happened*, and players' *ways of thinking* determine the emotions they experience (e.g., surprise, frustration).
- Assumption: only *realized* beliefs matter (not counterfactual ones).
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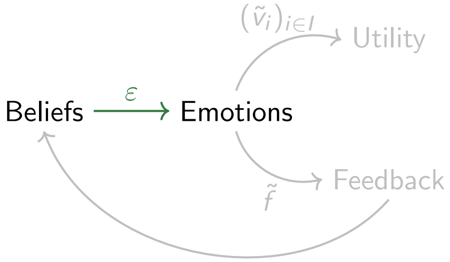

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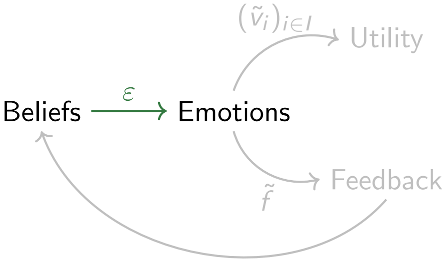

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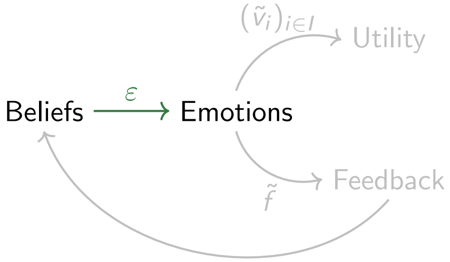

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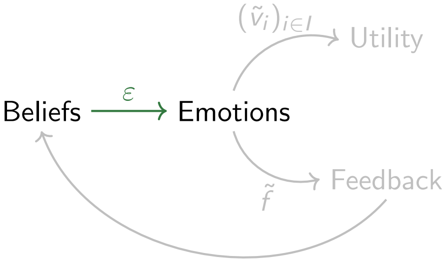

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Framework: Reduced form representation

We can derive a "reduced form" representation of feedback and utilities:

- (Continuous) **game-dependent feedback functions** $f := (f_h : S \times \Theta \times \mathcal{T} \to \Delta(M))_{h\in H}$. Interpretation: $f_h(s,\theta,\tau)[m]$ is the probability of m conditional on (s,θ,τ) and given h.
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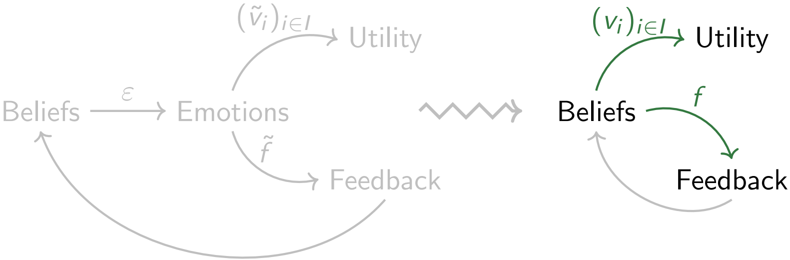

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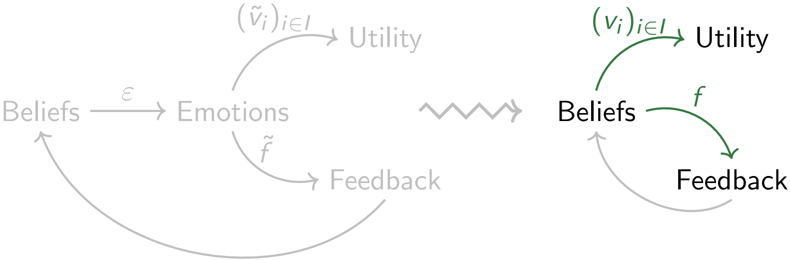

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Framework: Application to “buy me an ice-cream”

Child’s utility: $\mathbf{1}(I, z)$ and $\mathbf{1}(V, z)$ indicator functions for getting the ice-cream and playing video-games during terminal history z. $L = \{Homework.No, Video\text{-}games.Yes\}$ set of “lies”.

$$v_C(z, \theta, \tau) = \underbrace{\mathbf{1}(I, z)}_{\text{likes ice-cream}} + \overbrace{\theta_C}^{\text{video-games appreciation}} \cdot \underbrace{\mathbf{1}(V, z)}_{\text{likes video-games}} - \overbrace{\tau_M(L|z_M)}^{\text{image concern}}.$$

Mom’s utility: $\mathbf{1}(H, z)$ indicator function for doing *Homework* during z.

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Inferences: Warm-up

- Players should use of the signals they observe to make inferences about others' behavior or ways of thinking.
- In buy me an ice-cream, Child may *blush* only when he lies saying *Yes*.
- Mom wants to reward Child only if he has done his homework. Should she use the emotional signal to decide what to do?
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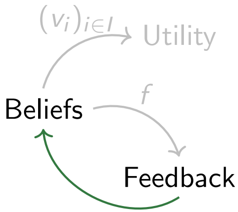

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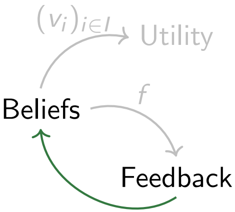

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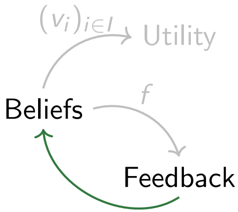

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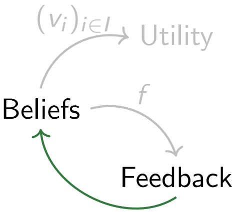

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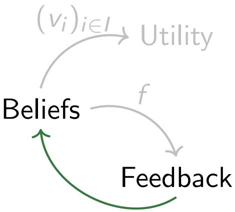

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Inferences: Reasoning about the game unfolding

- Fixing a utility-relevant state (s, θ, τ), several histories may occur: emotional messages are stochastic, and behavior may depend on their realizations. $\mathcal{H}_i(s, \theta, \tau) \subseteq \bar{H}_i$ is the set of personal histories of i that may realize given (s, θ, τ).
- We informally assume players know their beliefs and traits (i.e., i "knows" θ_i and τ_i): so, player i will make inferences about elements of $S \times \Theta_{-i} \times \mathcal{T}_{-i}$.
- When player i observes h_i (recall, she knows θ_i and τ_i), she **infers** that the set of elements of $S \times \Theta_{-i} \times \mathcal{T}_{-i}$ consistent with evidence is: $\mathcal{I}_{(\theta_i, \tau_i)}(h_i) := \left\{(s, \theta_{-i}, \tau_{-i}) : h_i \in \mathcal{H}_i(s, (\theta_i, \theta_{-i}), (\tau_i, \tau_{-i}))\right\}$.
- Under regularity conditions, inferences are "well-behaved" (i.e., for each i, h_i, τ_i, θ_i, $\mathcal{I}_{(\theta_i, \tau_i)}(h_i)$ is measurable).

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Rationality: Warm-up

Studying rationality we want to answer questions like the following ones (in the context of buy me an ice-cream):

- When Mom sees Child's blushing she should infer that he is lying. But assume he says *Yes* without blushing: is he telling the truth or did he manage to keep a poker face? Is there a reasonable way for Mom to update her beliefs?
- If Child did his *Homework*, he can answer *Yes* without blushing afterwards, because he would be telling the truth. Then, does it make sense for him to plan do say *No* after *Homework*?

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Rationality: Cognitive side

Rationality is defined as the conjunction of several cognitive and behavioral features.

Cognitive rationality

- Coherence (C): beliefs of different orders are coherent (recall that we work with infinite hierarchies of beliefs, τ_i^∞).
- Believe-what-you-observe (BO): beliefs over utility-relevant states (s, θ, τ) assign probability 1 to states consistent with evidence.
- Correct belief updating (CBU): at each stage, players receive two pieces of information; a_i about their *own behavior*, and $(m_{i,p}, m_i)$ about *others*' behavior and ways of thinking. CBU holds if these pieces of information are used to update beliefs about self and others "correctly" (following the rules of conditional probabilities).

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Plans and behavior

- Rational planning (RP): player *plans* to choose "optimal actions" at each contingency. Here, a plan is a player's belief on how she would behave: RP means that a player expects herself to act in a way that satisfies an "intra-personal equilibrium" condition.
- Consistency (Con): player implements her plans (i.e., planned and actual behavior coincide).

The set of states where i is **rational** is $R_i = C_i \cap BO_i \cap CBU_i \cap RP_i \cap Con_i \subseteq \Omega^\infty$.
Under regularity conditions, it R_i is Borel (hence, an event that can be assessed and expressed by players).

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Under regularity conditions, it R_i is Borel (hence, an event that can be assessed and expressed by players).

More details

Rationality: Behavioral side

Plans and behavior

- Rational planning (RP): player *plans* to choose "optimal actions" at each contingency. Here, a plan is a player's belief on how she would behave: RP means that a player expects herself to act in a way that satisfies an "intra-personal equilibrium" condition.
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What's next?

1. Definition of a **general framework**
2. Description of **inferences**
3. Formalization of a notion of **rationality**
4. Definition of a **solution concept** to derive predictions
5. Concluding remarks

Strong rationalizability: Warm-up

With our solution concept, we aim to capture **strategic thinking**. For example (based on buy me an ice-cream):

- If Mom is rational, she believes what she observes: if she sees Child blush, she will conclude he lied. If Child thinks that Mom is sophisticated enough, he understands that blushing means (1) no ice-cream, and (2) being labeled as a liar.
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Strong rationalizability: Preliminaries

- We are going to propose a procedure of iterated deletion of utility-relevant states (s_i, θ_i, τ_i) for each player $i \in I$.
- In the following, ρ_i is a system of beliefs of order $K+1$ of i: so, ρ_i is a map $h_i \mapsto \rho_i(\cdot|h_i) \in \Delta(S \times \Theta_{-i} \times \mathcal{T}_{-i})$.
- Key concept: **strong belief**. Strongly believing an event = assigning probability 1 to it as long as it is consistent with evidence.
- $(\theta_i, \tau_i, \rho_i)$ strongly believes $F \subseteq S \times \Theta_{-i} \times \mathcal{T}_{-i}$ if $\rho_i(F|h_i) = 1$ for each h_i such that $F \cap \mathcal{I}_{(\theta_i, \tau_i)}(h_i) \neq \emptyset$.

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- Can define events in Ω^∞ like:
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For each $i \in I$ and $n \in \mathbb{N}_0$: $\mathbf{P}_i(n) = \text{proj}_{S_i\times\Theta_i\times T_i} \mathbf{R}_i(n)$.

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Strong rationalizability: Application to “buy me an ice-cream”

Assume Child’s appreciation for video-games belongs to $\Theta_C = \{\theta', \theta''\}$ with $0 < \theta' < 1 < \theta''$.

Solution procedure:

1. **Child** Does not make sense to (plan to) say *No* after *Homework*.
   Mom if Child blushes, he must have played *Video-games* $\rightarrow$ she does *Not* buy the ice-cream if (Yes, b).
2. **Child** Consider step 1: if he blushes, Mom will conclude he is a liar $\rightarrow$ this undermines confidence $\rightarrow$ optimal action after *Video-games* is to say *No* $\rightarrow$ it is always optimal to tell the truth.
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   Mom if Child blushes, he must have played *Video-games* $\rightarrow$ she does *Not* buy the ice-cream if (Yes, b).
2. **Child** Consider step **1**: if he blushes, Mom will conclude he is a liar $\rightarrow$ this undermines confidence $\rightarrow$ optimal action after *Video-games* is to say *No* $\rightarrow$ it is always optimal to tell the truth.
   Mom Consider step **1**: if Child says *No*, he must have played *Video-games* $\rightarrow$ Mom does *Not* buy the ice-cream if $(No, \neg b)$.

Strong rationalizability: Application to “buy me an ice-cream”

Assume Child’s appreciation for video-games belongs to $\Theta_C = \{\theta', \theta''\}$ with $0 < \theta' < 1 < \theta''$.

Solution procedure:

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Strong rationalizability: Application to "buy me an ice-cream"

3. **Mom** Consider step 2: Child always tells the truth $\rightarrow$ Mom *Buys* him the ice-cream if $(Yes, \neg b)$.
4. **Child** Consider step 3: Child has to choose between *Homework* plus ice-cream and *Video-games* without ice-cream $\rightarrow$ if θ' he prefers *Homework* plus ice-cream, if θ'' he prefers *Video-games* without ice-cream.

Takeaway: signals about emotions and image concerns yield "full disclosure" (Child honestly reports his action, and Mom believes him).

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What's next?

1. Definition of a **general framework**
2. Description of **inferences**
3. Formalization of a notion of **rationality**
4. Definition of a **solution concept** to derive predictions
5. Concluding remarks

Concluding remarks: Future research

Applications

- Emotional leakage and disclosure in face-to-face interactions? Think of negotiations, political speeches, court hearings.
- More likely to accept unfair offers if the proposer smiles?
- More aggressive (conciliatory) when negotiating with happy (angry) counterparts?

Bounded rationality and strategic thinking

- We introduced a rich and expressive language to analyze failures of rationality on both the cognitive and the behavioral side. Here, focus on rationality and common strong belief in rationality.
- Interesting to allow agents to reason strategically about failures of rationality, and to capture behavioral implications of different epistemic assumptions.

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Thank you!

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Game tree: Derivation

- **Action feasibility correspondence** of i: $\mathcal{A}_i : M_{i,p} \rightrightarrows A_i$. Define $\mathcal{A}((m_{i,p})_{i\in I}) := \bigtimes_{i\in I} \mathcal{A}_i(m_{i,p})$.
- First PPM profile is $P(a^0) = P(\varnothing_A) =: m_{p,0}$ (neglected in notation).
- Histories are sequences of profiles of actions and messages: $(a^t, m_p^t, m^t) = (a_k, m_{p,k}, m_k)_{k=1}^t$ is **feasible** if for each $k \in \{1, \ldots, t\}$:
  1. $a_k \in \mathcal{A}(m_{p,k-1})$;
  2. $m_{p,k} = P(a^k)$;
  3. there exists (θ, e^{k+1}) such that $m_k \in \operatorname{supp} \tilde{f}(a_k, \theta, e^{k+1})$.

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Game tree: Application to "buy me an ice-cream"

Stage 1 Child chooses $a_{C,1} \in \{Homework, Video\text{-}games\}$.
Mom's PPM is uninformative: $(a_{C,1}) \mapsto \bar{m}_{M,p}$.
No emotional messages.

Stage 2 Child chooses $a_{C,2} \in \{Yes, No\}$, and $m \in \{b, \neg b\}$ realizes.
Mom's PPM reveals stage-2 action: $(a_{C,1}, a_{C,2}) \mapsto a_{C,2}$.
Mom's emotional message is m.

Stage 3 Mom chooses $a_M \in \{Buy, Not\}$.
Child's PPM reveals Mom's action: $(a_{C,1}, a_{C,2}, a_M) \mapsto a_M$.
No emotional messages.

A terminal history as the form $(a_{C,1}, \bar{m}_{M,p}, a_{C,2}, m, a_M)$.

Mom's only length-1 personal history is $(\bar{m}_{M,p})$ (henceforth neglected in notation).
Length-2 personal histories: (Yes, b), $(Yes, \neg b)$, $(No, \neg b)$.
Terminal personal histories: $\{(Yes, b), (Yes, \neg b), (No, \neg b)\} \times \{Buy, Not\}$.

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Hierarchical systems of beliefs: Construction

- **Basic space of uncertainty**: $\Omega^0_{-i} := S \times \Theta_{-i}$.
- Set of **first-order systems of beliefs** of i: $\mathcal{T}_{i,1} := \left[\Delta(\Omega^0_{-i})\right]^{\bar{H}_i}$. Generic element $\tau_{i,1}$ is a map $h_i \mapsto \tau_{i,1}(\cdot|h_i)$.
- Define $\Omega^1_{-i} := \Omega^0_{-i} \times \left(\times_{j\neq i} \mathcal{T}_{j,1}\right)$. Set of **second-order systems of beliefs** of i: $\mathcal{T}_{i,2} := \left[\Delta(\Omega^1_{-i})\right]^{\bar{H}_i}$. Generic element $\tau_{i,2}$ is a map $h_i \mapsto \tau_{i,2}(\cdot|h_i)$ and $\tau_{i,2}(\cdot|h_i)$ is a belief over Ω^0_{-i} *and* others' first-order system of beliefs.
- Proceed by induction to retrieve $\mathcal{T}_{i,n} := \left[\Delta(\Omega^{n-1}_{-i})\right]^{\bar{H}_i}$ $(n \in \mathbb{N})$.
- Set of ***n*-th-order hierarchical systems of beliefs** of i: $\mathcal{T}^n_i := \times_{k=1}^n \mathcal{T}_{i,k}$. Generic element τ^n_i is a map $h_i \mapsto (\tau_{i,k}(\cdot|h_i))_{k=1}^n$.
- Set of **epistemic types** of i: $\mathcal{T}^\infty_i = \times_{k\in\mathbb{N}} \mathcal{T}_{i,k}$.

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Emotion-generating function: Assumptions

Assumptions about ε:

- Counterfactual beliefs do not matter. Define realized-beliefs map: for each $h = (h_i)_{i\in I}$, β_h is the map $(\tau_i^\infty)_{i\in I} \mapsto ((\tau_i^\infty(\cdot|h_i'))_{h_i' \preceq h_i})_{i\in I}$. Then, for each $h \in H$, ε_h can be written as $\bar{\varepsilon}_h \circ \beta_h$ for some $\bar{\varepsilon}_h$.
- Only beliefs of order up to K matter. For each h, τ^∞, and $\bar{\tau}^\infty$, $\tau^K = \bar{\tau}^K \Longrightarrow \varepsilon(h, \tau^\infty) = \varepsilon(h, \bar{\tau}^\infty)$.

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Game-dependent feedback: Derivation

Let $\mathbf{E} = E^{\leq T+1}$ and note that $\varepsilon(h, \tau) \in \Delta(\mathbf{E})$.

Recall that inputs of $\tilde{f}$ are actions, traits, streams of emotions.

Then, the game-dependent representation of feedback is (in green the game-specific ingredients):

$$f_h(s, \theta, \tau)[m] := \int_{\mathbf{E}} \tilde{f}(s(h), \theta, \mathbf{e})[m] \cdot \mathrm{d}\varepsilon(h, \tau).$$

If ε is deterministic, then simply:

$$f_h(s, \theta, \tau)[m] = 1 \text{ iff } m = \tilde{f}(s(h), \theta, \varepsilon(h, \tau)).$$

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Psychological utility functions: Hints

Some examples of psychological motivations. For simplicity, consider $I = \{i, j\}$ and emotions profiles in $E = E_i \times E_j$.

1. **Image concerns**: a player dislikes being thought of as "bad". Assume $\Theta_i = \Theta_i^S \times \{Nice_i, Rude_i\}$.
   Game-independent utility is:

$$\tilde{v}_i(y, \theta, e) = \underbrace{y_i}_{\text{own outcome}} - \overbrace{\theta_i^S}^{\text{sensitivity to } j\text{'s opinion}} \cdot \underbrace{e_i}_{\text{image concern}} .$$

   Game-dependent utility is:

$$v_i(z, \theta, \tau) = \underbrace{\pi_i(z)}_{\text{own outcome after } z} - \overbrace{\theta_i^S}^{\text{sensitivity to } j\text{'s opinion}} \cdot \underbrace{\tau_j(Rude_i | z)}_{j\text{'s opinion after } z} .$$

   Note: here, opinions about unobserved traits of others, but there are also opinions about unobserved actions (as in the running example).

Psychological utility functions: Hints

❷ **Guilt aversion**: a player dislikes failing others' expectations. *Game-independent* utility is:

$$\tilde{v}_i(y,\theta,e) = \underbrace{y_i}_{\text{outcome}} - \overbrace{\theta_i^S}^{\text{sensitivity to } j\text{'s disappointment}} \cdot \underbrace{e_j}_{j\text{'s disapp.}} .$$

Game-dependent utility is:

$$v_i(z,\theta,\tau) = \underbrace{\pi_i(z)}_{\text{outcome after } z} - \overbrace{\theta_i^S}^{\text{sensitivity to } j\text{'s disappointment}} \cdot \underbrace{\left[\mathbb{E}_{\tau_j}(\pi_j|\varnothing) - \pi_j(z)\right]^+}_{j\text{'s disapp. after } z} .$$

Back

Inferences: Properties of feedback

- We need conditions that ensure that players' inferential reasoning is well-defined.
- Assumption 1 (Own-belief independence, OBI): *feedback is about others*; at each history, the probabilities of realization of messages about i's co-players do not depend on i's beliefs.
- Assumption 2 (Regularity, Reg): *upon observing some message, players can always "discern" the set of states that did not prevent such message*; the set of utility-relevant states that allow for some message at some history is a measurable rectangle.

Lemma

Assume OBI and Reg hold. Then, $\mathcal{I}_{(\theta_i,\tau_i^K)}(h_i)$ is Borel for each i, h_i, θ_i and τ_i^K.

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Rationality: Coherence, believe-what-you-observe

Rationality is defined as the conjunction of several cognitive and behavioral features.

1. **Coherence**: beliefs of different orders are be coherent.
   - Epistemic type $\tau_i^\infty = (\tau_{i,n})_{n\in\mathbb{N}}$, with $\tau_{i,n}$ system of n-th-order beliefs.
   - $\tau_{i,n}$ is a map $h_i \mapsto \tau_{i,n}(\cdot|h_i) \in \Delta(S \times \Theta_{-i} \times \mathcal{T}_{-i}^{n-1})$. Call $S \times \Theta_{-i} \times \mathcal{T}_{-i}^{n-1} = \Omega_{-i}^{n-1}$.
   - τ_i^∞ is **coherent** if, for each h_i and n, $\tau_{i,n}(\cdot|h_i) = \mathrm{marg}_{\Omega_{-i}^{n-1}}\tau_{i,n+1}(\cdot|h_i)$.
   - $C_i \subseteq \Omega^\infty$ is the set of states where i's epistemic type is coherent.

2. **Believe-what-you-observe**: beliefs over utility-relevant states assign probability 1 to the set of states consistent with evidence.
   - τ_i^∞ satisfies **believe-what-you-observe** (BO) if, for each h_i, $\tau_{i,K+1}\big(\mathcal{I}_{\tau_i}(h_i)|h_i\big) = 1$.
   - $BO_i \subseteq \Omega^\infty$ is the set of states where i's epistemic type satisfies BO.

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Rationality: Belief updating

3. **Correct belief updating**: a player updates her beliefs about herself and her opponents according to the rules of conditional probabilities.
   - At each stage, i observes two pieces of information: first, a_i is chosen; then, $(m_{i,p}, m_i)$ realizes.
   - a_i is used to update beliefs on *own external states* (S_i); $(m_{i,p}, m_i)$ is used to update beliefs about *others* $(S_{-i} \times \Theta_{-i} \times \mathcal{T}_{-i})$.
   - Call $S_i(h_i, a_i)$ the set of s_i that allow for h_i and that are such that $s_i(h_i) = a_i$. **Chain rule**: for each h_i, $a_i \in A_i(h_i)$, $s_i \in S_i(h_i, a_i)$,

$$\underbrace{\tau_{i,K+1}(s_i|h_i')}_{\text{prob. of } s_i \text{ after } a_i} \cdot \underbrace{\tau_{i,K+1}(S_i(h_i, a_i)|h_i)}_{\text{prob. of } a_i} = \underbrace{\tau_{i,K+1}(s_i|h_i)}_{\text{prob. of } s_i \text{ before } a_i},$$

   where h_i' is any immediate successor of h_i where a_i is played.

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❸ **Correct belief updating** (continued)

- Now assume a_i has been played at h_i. Call $\mu(\cdot|h_i)$ the marginal of $\tau_{i,K+1}(\cdot|h_i)$ on $S_{-i} \times \Theta_{-i} \times \mathcal{T}_{-i} =: X$.
  Bayes rule: for each h_i, a_i, $(m_{i,p}, m_i)$, and Borel $F \subseteq X$,

$$\overbrace{\tau_{i,K+1}(F|h_i')}^{\text{prob. of } F \text{ after } (m_{i,p},m_i)} \cdot \underbrace{\int_X g_{h_i,a_i}(m_{i,p}, m_i|x)\mu(\mathrm{d}x|h_i)}_{\text{prob. of } (m_{i,p},m_i)} = \underbrace{\int_F g_{h_i,a_i}(m_{i,p}, m_i|x)\mu(\mathrm{d}x|h_i)}_{\text{prob. of } F \text{ before } (m_{i,p},m_i)},$$

  where $h_i' = (h_i, (a_i, m_{i,p}, m_i))$, and $g_{h_i,a_i}\big((m_{i,p}, m_i)|x\big)$ is the probability of $(m_{i,p}, m_i)$ given x after a_i was played at h_i (retrieved from p and f).
- Note: two "parallel" belief updating procedures. This way, if player i is surprised by her behavior, she still updates her beliefs about others correctly.
- τ_i^∞ satisfies **correct belief updating** if the chain rule and the Bayes rule hold.
- $CBU_i \subseteq \Omega^\infty$ is the set of states where i's epistemic type satisfies correct belief updating.

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Rationality: Rational planning

4. **Rational planning**: a player plans to choose only optimal actions, at each possible contingency.
   - Derive a *plan* of epistemic type τ_i^∞, $\sigma(\tau_i^\infty) \in \times_{h_i \in H_i} \Delta(A_i(h_i))$, based on i's beliefs about herself.
   - Derive the *decision utility* of i, $u_{i,h_i} : A_i \times \Theta_i \times \mathcal{T}_i^{K+1} \to \mathbb{R}$. Interpretation: $u_{i,h_i}(a_i, \theta_i, \tau_i^{K+1})$ is the expected utility i from choosing a_i at h_i when her trait is θ_i. This is derived from the fact that beliefs τ_i^{K+1} determine a continuation plan after (h_i, a_i).
   - Let $A_i^*(h_i, \theta_i, \tau_i^{K+1}) = \arg\max_{a_i \in A_i(h_i)} u_{i,h_i}(\cdot, \theta_I, \tau_i^{K+1})$ be the set of optimal actions at h_i when player i's trait and belief system are θ_i and τ_i^{K+1}.
   - Player i **plans rationally** at $(s_i, \theta_i, \tau_i^\infty)$ if, for each h_i, $\text{supp}\sigma(\tau_i^\infty)(\cdot|h_i) \subseteq A_i^*(h_i, \theta_i, \tau_i^{K+1})$.
   - $RP_i \subseteq \Omega^\infty$ is the set of states where i plans rationally.

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Rationality: Consistency

5. **Consistency**: planned and actual behavior coincide.
   - Player i is **consistent** at $(s_i, \theta_i, \tau_i^\infty)$ if, for each h_i, $\sigma(\tau_i^\infty)(s_i(h_i)|h_i) > 0$.
   - $Con_i \subseteq \Omega^\infty$ is the set of states where i is consistent.

The set of states where i is **rational** is $R_i = C_i \cap BO_i \cap CBU_i \cap RP_i \cap Con_i$.

Theorem

Assume OBI and Reg hold. Then, R_i is Borel for each i.

Back