Reciprocity and Dynamic Consistency

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Abstract

Reciprocity theory assumes that people wish to be kind towards those they perceive to be kind, and unkind towards those they perceive to be unkind. Rabin (1993) argues that the *kindness is based on intentions*: the **kindness** of i towards j is measured by the difference between how much i expects to make j earn and an "equitable payoff" of j. Hence, *kindness depends on* (1^{st}-order) *beliefs*, making this a PGT model. Here we present a *theory of reciprocity in sequential games* that satisfies dynamic consistency, differenlty from Dufwenberg & Kirchsteiger (2004).

Introduction

- PGT shows how:
  - guilt avoidance can make agents keep materially costly promises;
  - frustration and anger can make agents carry out materially costly threats.
- Both effects are also promoted by **reciprocity**, the action tendency of being kind (resp. unkind) towards those whom we perceive as kind (resp. unkind) with us.
- The idea that people wish to be (un)kind towards those they perceive to be (un)kind is age-old. Early academic discussions can be found in anthropology, sociology, social psychology, biology, and economics (see references Akerlof 1982, and in the survey by B&D 2022).
- Rabin (1993) argues that the *kindness is based on intentions*: the **kindness** of i towards j is measured by the difference between how much i expects to make j earn and an "equitable payoff" of j. Hence, *kindness depends on* (1^{st}-order) *beliefs.*

Reciprocity and dynamic consistency

- According to the *general* theory of Dufwenberg & Kirchsteiger (D&K, 2004), *reciprocity is a reactive action tendency*. This is modeled with players having *different psychological utility functions at different nodes of the game*, which may yield *dynamic inconsistency* of preferences (cf. D&K 2004, Battigalli, Corrao & Dufwenberg 2019).
- Such *dynamic inconsistency* may be psychologically plausible, but it *is not a necessary feature of the intuitive notion of reciprocity*.
- We present below a *dynamically consistent model of reciprocity* for general game forms (in these slides, we restrict attention for simplicity to game forms with observed actions, like D&K 2004).

A dynamically consistent model of reciprocity, 1

- **Kindness of** i (*at the beginning of the game*): we take as given that the **equitable payoff** of j from i's perspective is determined by some belief-dependent function $\pi_j^e(\alpha_{i,-i})$ (where $\alpha_{i,-i}$ is the 1^{st}-order belief of i about $-i$, e.g., as in D&K 2004, or 2019). The (initial) **kindness of** i **towards** j is

$$K_{ij}(\alpha_i) = \mathrm{E}_{\alpha_i}(\pi_j) - \pi_j^e(\alpha_{i,-i})$$

where $\alpha_i = (\alpha_{ii}, \alpha_{i,-i})$ is the 1^{st}-order belief of i, including his plan α_{ii}.

- **Note:** In Leader-Follower game forms the D&K-utilty function of the follower is $u_2^{DK}(a_1, a_2, \alpha_1) =$

$$\begin{aligned} & \pi_2(a_1, a_2) + \theta_2 K_{12}(a_1, \alpha_{12})(\pi_1(a_1, a_2) - \pi_1^e(a_1)) \\ = \quad & \pi_2(a_1, a_2) + \theta_2 K_{12}(a_1, \alpha_{12})\pi_1(a_1, a_2) - \underbrace{\theta_2 K_{12}(a_1, \alpha_{12})\pi_1^e(a_1)}_{\text{independent of } a_2} \end{aligned}$$

A dynamically consistent model of reciprocity, 2

- Generalizing the previous observation, the only way to be kind (unkind) toward co-players is to increase (decrease), in expectation, their material payoff.
- With this, we (i) give up the product of (perceived) kindnesses of D&K, and (ii) we consider the (inital and updated/revised) *perception of* the co-players' *initial kindenss*: we propose to model reciprocity concerns with

$$u_i\left(z,\alpha_{-i}\right)=\pi_i\left(z\right)+\sum_{j\neq i}\theta_{ij}K_{ji}\left(\alpha_j\right)\pi_j\left(z\right),$$

a kind of *"state-dependent" utility function*, which yields—under SEU—*dynamically consistent conditional preferences*.

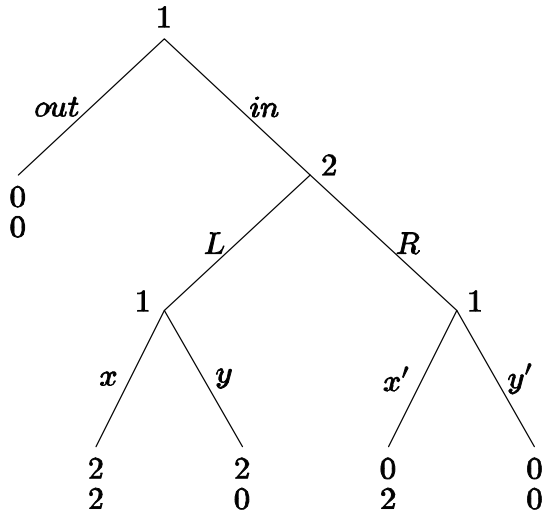
One-Deviation Principle

- By standard dynamic programming arguments (*One-Deviation Principle*), $\alpha_{i,i}$ (i's plan/strategy) *maximizes* $\mathrm{E}_{\alpha_{i,i},\beta_{i,-i}}(u_i|h)$ *starting from every* $h \in H$ *IFF* $\alpha_{i,i}$ *is an intra-personal equilibrium* (one-step optimal) given $\beta_{i,-i}$, that is, IFF, for every $h \in H$ and $a_i^* \in A_i(h)$,

$$\alpha_{i,i}(a_i^*|h) > 0 \Rightarrow a_i^* \in \arg\max_{a_i \in A_i(h)} \bar{u}_{i,h}\left(a_i, \alpha_{i,i}, \beta_{i,-i}\right),$$

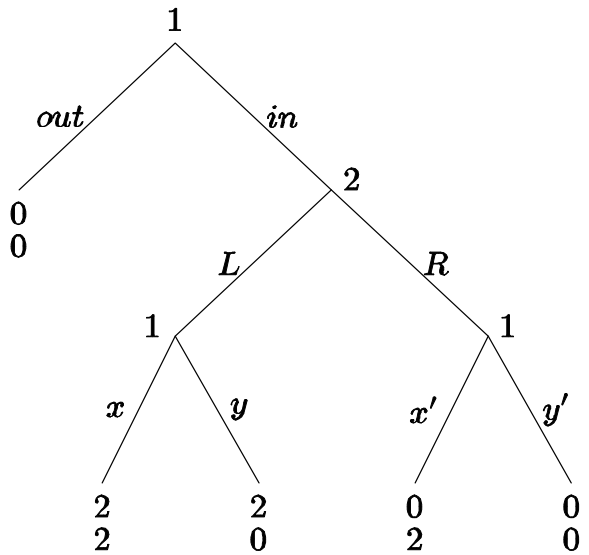
  where $\bar{u}_{i,h}\left(a_i, \alpha_{i,i}, \beta_{i,-i}\right) = \mathrm{E}_{\alpha_{i,i},\beta_{i,-i}}(u_i|h, a_i)$.
- **Intuition**: Obtain planned actions and corresponding values by "**folding back.**"
  1. Start from histories of height 1 (i.e., "pre-terminal") and maximize expected psychological utility given the updated/revised perception of others' kindness.
  2. Taking the resulting values into account go back to histories of height 2 ("pre-pre-terminal"), and so on.
  3. Resulting strategy (strategies, if there are ties) is both one-step optimal and sequentially optimal given $\beta_{i,-i}$. (Without ties, the

Example: rational planning in D&K



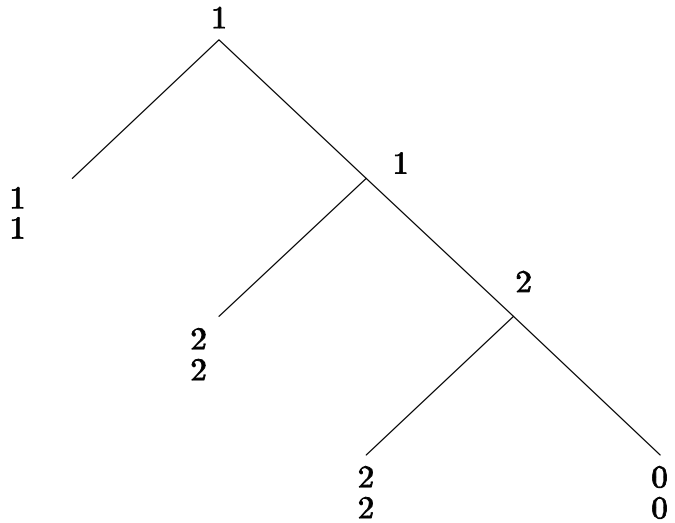
- 2^{nd}-order beliefs of pl. 1, β_{12}:
  - he is *certain* that *pl.2 plans* to choose L with prob. $\alpha_{22}(L|in) = p \in \left(\frac{1}{2}, 1\right)$ given *in*. Also, $\alpha_{12}(L|in) = p$.
  - Then $\mathrm{E}^{DK}_{\beta_{12}}(K_{21})$, $\mathrm{E}^{DK}_{\beta_{12}}(K_{21}|(in, L)) > 0$, $\mathrm{E}^{DK}_{\beta_{12}}(K_{21}|(in, R)) < 0$.
  - *Observed actions perceived as (if) intentional and planned with prob. 1.* At the root pl. 1 wish to committ to strategy $in.x.x'$.
  - But he cannot committ and rationally plans $in.x.y'$, as *he predicts he will want to be unkind given* (in, R).

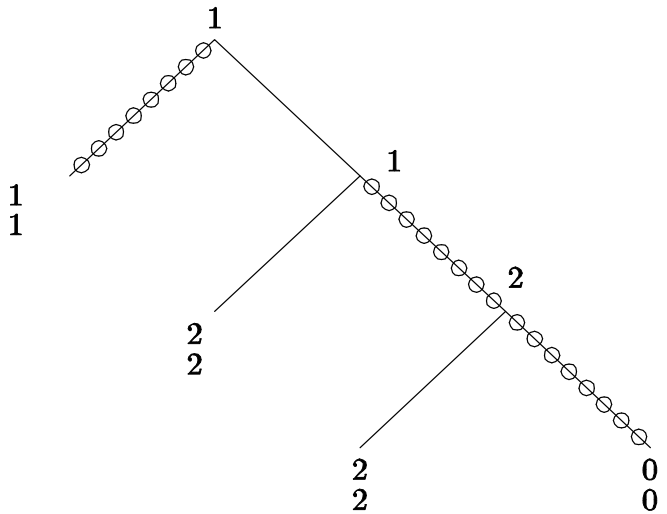
Example: rational planning in our model



- (2^{nd}-order) beliefs of pl. 1 about pl. 2, β_{12}:
  - he is *certain* that *pl.2 plans* to choose L with prob. $\alpha_{22}(L|in) = p \in \left(\frac{1}{2}, 1\right)$.
  - For simplicity, also assume $\alpha_{12}(L|in) = p$.
  - Then, *trivial updating* about $K_{21}(\alpha_2)$: $\mathrm{E}_{\beta_{12}}(K_{21}) = \mathrm{E}_{\beta_{12}}(K_{21}|(in, L)) = \mathrm{E}_{\beta_{12}}(K_{21}|(in, R)) > 0$.
  - For us (B&M), seq. & folding-back BR is $in.x.x'$.

Other example, equilibria





- On the right: one DK sequential reciprocity equilibrium, with "miserable eq." in subgame.
- It cannot be a sequential equilibrium in our model.

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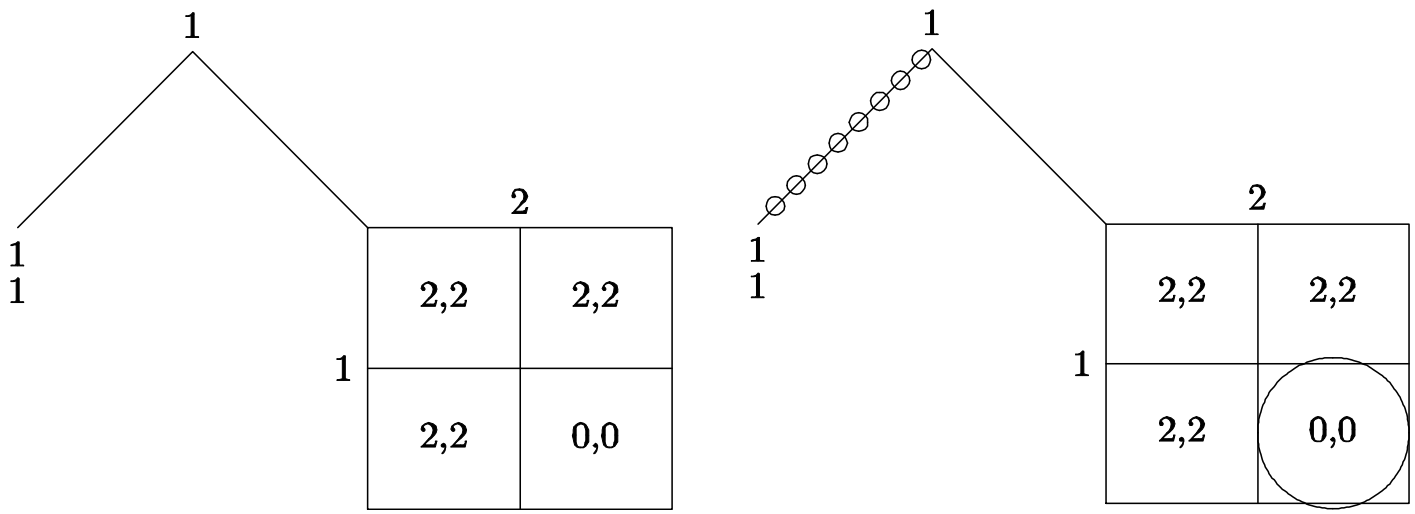

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References

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