Regret in Games

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Abstract

We develop a general approach to explore how regret influences strategic interaction and risky choice. Regret is captured by the payoff gap between what a player actually gets and what he believes he would have gotten had he chosen differently. *Ex post beliefs* are critical to that evaluation, and the modeling therefore draws on tools from psychological game theory. Predictions depend in novel ways on the information structure across end-nodes and assumptions regarding the precise nature of chance moves. Regret can have a powerful impact in a variety of economic settings including information acquisition, auctions, climate action, market entry, and delegation.

Introduction

- **Regret** is a negative feeling, generated when a person either finds out that – or comes to believe that – her choices are not *ex post* optimal.
- *How can regret play a role in decision making?* Regret is anticipated by the player before choice, influencing decision-making processes. (**Anticipated Regret**)

Regret in Economics

- Previous work is *decision theory* (Bell '82; Loomes & Sudgen '82; Quiggin '94; Sarver '08).
- But regret makes as much sense in games, and there is no general approach. Heterogenous works on specific games:
  - Auction (Engelbrecht-Wiggans '89; Filiz-Ozbay and Ozbay '07, Bergemann et al. '25).
  - Product differentiation (Syam, Krishnamurthy, Hess '08; Zou, Zhou, Jiang '20).
  - Specific static games (Cerrone, Feri, Neary '25).

What We Do: Regret in Games

In this paper:

- We develop a general approach to explore how regret influences strategic interaction and risky choice in simultaneous and sequential games and decision problems.
- We argue that to capture regret one needs to consider a form of belief-dependent utility ...

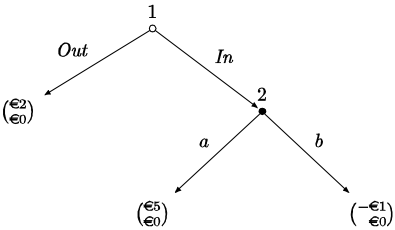
Decision Regret Theory (DRT)

DRT developed to explain decisions that deviate from the standard EU theory in certain experiments (e.g., Allais paradox).

- It focuses on settings where the DM always has **perfect feedback**: she observes *ex post* the realization of the lottery (or the state of nature).
- ⇒ With the benefit of hindsight, she understands whether she made a correct or wrong choice: regret is computed by directly comparing *ex post* the outcomes of different choices.
- Therefore, DRT *does not need* to define regret in terms of the player's *ex post beliefs* about
  1. counterfactuals,
  2. past unobserved choices of chance (or other co-players in games).

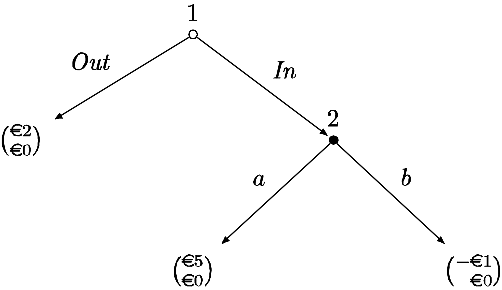
Why we need beliefs-based regret in games

- In games with sequential moves, players may be unable to fully learn *ex post* co-players' strategies (full descriptions of behavior at each decision node, including counterfactuals).
- **Example**: pl. 1 is uncertain about pl. 2, regret at end-node depends on comparison btw *benefit-of-hindsight-€optimal choice* and realized €payoff:



- If pl. 1 goes *Out*, her regret depends on belief $\alpha = \mathbb{P}_1(a|In)$ about counterfactual.

Beliefs-based regret and choice: example



- If pl. goes *Out*, her regret depends on belief $\alpha = \mathbb{P}_1(a|In)$: $r_1(Out, \alpha) = \max\{5\alpha - (1-\alpha), 2\} - 2 > 0$ iff $\alpha > \frac{1}{2}$.
  - If (In, a), *no* regret; if (In, b), regret$= 2 - (-1) = 3$.
  - Anticipated regret if *In*: $3(1-\alpha)$.
  - *Out* if $2 - \theta \cdot r_1(Out, \alpha) > 5\alpha - (1-\alpha) - \theta \cdot 3(1-\alpha)$ [θ=(linear) regret sensitivity].

Our Model

Psychological utility function:

$$u_i(z, \alpha_i) = m_i(z) - \theta_i \cdot r_i(z, \alpha_i)$$

- $i \in I$, **player**/person, 0=chance, $I_0 = I \cup \{0\}$
- $z \in Z$ **terminal** histories/nodes
- $m_i : Z \to \mathbb{R}$ **monetary payoff** function of i
- $r_i(z, \alpha_i)$ belief-based **regret** given z; θ_i **sensitivity** to regret
- $h \in \bar{H}_i$ **information sets** of i (*including terminal information*)
- $\alpha_i = (\alpha_i(\cdot|h))_{h \in \bar{H}_i}$, with $\alpha_i(\cdot|h) \in \Delta\left(S_{I_0 \setminus \{i\}}(h)\right)$, system of (conditional) **1st-order beliefs** about info-dependent actions (strategies) of others (persons & chance), including *terminal beliefs*
- α_i is *consistent* with (known "objective") chance-probabilities and rules of conditional probabilities

Look into the regret term

$$u_i(z, \alpha_i) = m_i(z) - \theta_i \cdot r_i(z, \alpha_i)$$

- Suppose for simplicity that *monetary payoffs* are *observed*: $z', z'' \in h \in \bar{H}_i \Rightarrow m_i(z') = m_i(z'') = m_i(h)$
- Then, for $z \in h \subseteq Z$, $\quad r_i(z, \alpha_i) = r_i(h, \alpha_i) =$ f_i (**best** h-hindsight exp. monetary payoff $- m_i(h)$)
- **Best** h-hindsight exp. monetary payoff: $\max_{s_i \in S_i} \mathbb{E}_{s_i, \alpha_i}[m_i | h]$ depends on terminal belief $\alpha_i(\cdot | h)$
- $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ strictly *increasing* and *continuous* with $f_i(0) = 0$
- **Linear regret** if f_i is the identity: $f_i(x) = x$ for all $x \geq 0$
- **Intuition:** The more player i believes *ex post* that he could have been better off with different choices, the more he regrets the choice he has made.

One-person linear-regret games

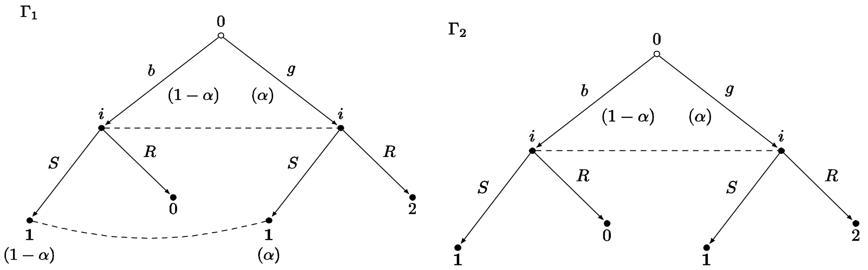
Focus for simplicity on

- $I_0 = \{i, 0\}$, i is the only personal player (Decision Maker)
- *observed* (monetary) *payoffs*
- *Linear* regret: $f_i(x) = x \Rightarrow$ for $z \in h \subseteq Z$ $(h \in \bar{H}_i)$

$$r_i(z, \alpha_i) = \max_{s_i \in S_i} \sum_{s_0 \in S_0(h)} m_i(\zeta(s_i, s_0)) \cdot \alpha_i(s_0|h) - m_i(h)$$

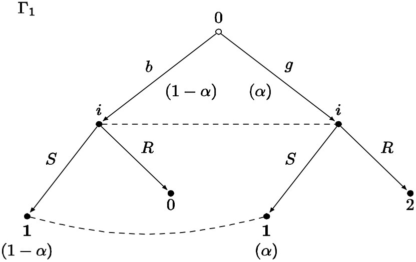
  where $\alpha_i(\cdot|h) \in \Delta(S_0(h))$ is the **ex post belief** about the "pure strategy/behavior of chance" (realizations of chance moves)
- Unlike DRT, we do *not* assume *perfect feedback*: terminal information sets h may contain more than one terminal node/history
- We illustrate some novel features of our belief-based approach

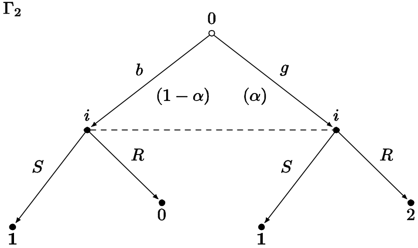
Feature I: Terminal information matters



Imperfect terminal information (only) in first game form (Γ_1) $\Rightarrow$ different regrets given (g, S)

Feature I: Why terminal information matters





- *Imperfect feedback* (Γ_1): $r_i\left((g,S),\alpha\right) = \max\left\{2\alpha, 1\right\} - 1 \leq 1$, *opposite optimal choices with and without regret aversion* if $\frac{1}{2} < \alpha < \frac{2}{3}$ and θ_i is sufficiently high.
- *Perfect feedback* (Γ_2): $r_i\left((g,S),\alpha\right) = 2 - 1 = 1$, and *same choice with and without regret aversion*.

Irrelevance result

The following simple result gives a benchmark for the relevance *vs* irrelevance of regret aversion:

- **Proposition** *Assume* linear *regret. If there are essentially* simultaneous *moves and* perfect *feedback, regret aversion is* irrelevant*:*

$$\arg\max_{s_i \in S_i} \mathbb{E}_{\alpha_i, s_i}[u_i] = \arg\max_{s_i \in S_i} \mathbb{E}_{\alpha_i, s_i}[m_i].$$

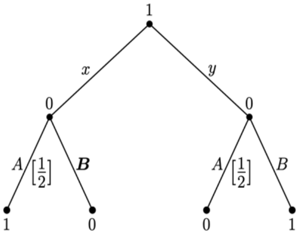
- *Same* result *as in DRT* under linear regret, because DRT assumes essentially simultaneous moves and perfect feedback.
- The result *generalizes* to n-person games ($|I| \geq 2$).

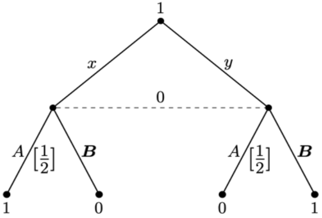
Information Avoidance

- Each (reduced) strategy s_i yields an *ex post* information partition $\mathcal{P}_i\left(s_i\right)$ of S_0 ($S_{-i} \times S_0$ in n-person games).
- **Proposition** Fix strategies $s_i', s_i'' \in S_i$ and a consistent belief system α_i so that $\mathbb{E}_{\alpha_i, s_i'}\left[m_i | h\right] \leq (<) \mathbb{E}_{\alpha_i, s_i''}\left[m_i | h\right]$. If $\mathcal{P}_i\left(s_i'\right)$ refines $\mathcal{P}_i\left(s_i''\right)$ then, under *linear regret*, $\mathbb{E}_{\alpha_i, s_i'}\left[u_i | h\right] \leq (<) \mathbb{E}_{\alpha_i, s_i''}\left[u_i | h\right]$.
- **Intuition** Finer ex post information allows a more fine-tuned evaluation of hindsight-maximized expected material payoffs, increasing anticipated regret.
  - *Extreme case*: if s_i^* yields no information about chance (and others) and maximizes expected material payoff, there is no regret, and s_i^* also maximizes expected psychological utility (ex ante and ex post beliefs coincide; see, e.g. *Out* with $\alpha < \frac{1}{2}$ in the first example).

Feature II: The information of Chance matters

- Unlike traditional DT-GT, a player's regret-related (counterfactual) reasoning requires introducing the concept of "information of Chance":





- If Chance is a *follower with perfect information* (left), regret after a bad outcome is moderate: the alternative choice would yield a "fresh new draw."
- Whereas, *if Chance's information is imperfect* (right), the outcome reveals Chance's choice/strategy (perfect correlation).

Strategic interaction: best replies

- In n-person games, first look at *sequential* best-reply correspondences $(\theta_i, \alpha_i) \mapsto BR_i^{\theta_i}(\alpha_i)$.
- (Technical) $BR_i^{\theta_i}(\alpha_i)$ is **upper-hemicontinuous** in (θ_i, α_i). Thus, for each belief system α_i and each $h \in H_i$, as $\theta_i \searrow 0$, we get a *selection* in $\arg\max_{s_i,\alpha_i} \mathbb{E}_{\alpha_i,s_i}[m_i|h]$.
- **Regret irrelevance**: The previous result generalizes. Furthermore, *without chance moves*, by a kind of law-of-iterated expectations argument, regret is *also irrelevant* for i *if* she holds *deterministic beliefs when active*: she is certain of feeling no regret IFF she maximizes $\mathbb{E}_{\alpha_i,s_i}[m_i|h]$.

Strategic interaction: solution concepts

- Results above apply to **sequential equilibrium** (Kreps & Wilson 1982, B&D 2009), and all best-reply-based solution concepts, including those *with interesting foundations*, e.g., versions of rationalizability and self-confirming equilibrium.
- Under *complete information* [CK of $(\theta_i, f_i)_{i \in I}$], by upper-hemicontinuity, as $\max_{i \in I} \theta_i \searrow 0$, we get *equilibrium selection in the material-payoff game* ($\theta_i = 0$ for all $i \in I$).
- **Results about regret (ir)relevance and information avoidance** extend to *n*-person games.
- **Interesting applications**:
  - Auctions (see below).
  - Climate action.
  - Market entry.
  - Delegation.

Auctions

- Consider (finite, discretized) first-price auctions with *independent, private values* (IPP). For simplicity, assume two bidders $i \in \{1, 2\}$ (with ties broken in favor of 1).
- For standard expected payoff maximization, terminal information does not matter. With anticipated regret aversion, instead, *Hiding Losing Bids yields more aggressive bidding compared to Full Revelation.*
- This is consistent with claims in the behavioral literature on auctions (Engelbrecht-Wiggans '89, Filiz-Ozbay & Ozbay (2007 Bergemann et al. '25 and refs therein) based on ad hoc assumptions about regret.

The advantage of hiding losing bids

- *Hiding Losing Bids (HL) yields more aggressive bidding compared to Full Revelation (FR).*
- **Intuition** Consider bidder 1 with valuation v_1 and regret parameter $\theta_1 > 0$. Fix (marginal) belief $\hat{\alpha}_1$ (e.g., standard eq.) about competitor's bid (IPP).
  - Obtain associated games $\hat{G}^{\mathrm{HL}}_{v_1,\theta_1}$ and $\hat{G}^{\mathrm{FR}}_{v_1,\theta_1}$ with chance and essentially simultaneous moves. $\hat{G}^{\mathrm{FR}}_{v_1,\theta_1}$ also has perfect feedback, the expected material payoff maximizing bid $b^{\mathrm{FR}}_{v_1,0}$ is also optimal: $b^{\mathrm{FR}}_{v_1,0} = b^{\mathrm{FR}}_{v_1,\theta_1}$ (*irrelevance result*).
  - In $\hat{G}^{\mathrm{HL}}_{v_1,\theta_1}$, $b_1 < b^{\mathrm{FR}}_{v_1,0}$ cannot be optimal, because it is more revealing and gives less payoff in expectation (*info. avoidance result*). Thus, optimal bid in $\hat{G}^{\mathrm{HL}}_{v_1,\theta_1}$ is weakly higher than in $\hat{G}^{\mathrm{FR}}_{v_1,\theta_1}$: $b^{\mathrm{FR}}_{v_1,\theta_1} \geq b^{\mathrm{FR}}_{v_1,0} = b^{\mathrm{FR}}_{v_1,\theta_1}$ (inequality typically strict).

THANK YOU!

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Proof of linear-regret irrelevance

- By essentially *simultaneous moves* and *perfect feedback*, Chance realization is observed ex post:
  - For every $(s_i', s_0) \in S_i \times S_0$, $\bar{H}(s_i', s_0) = \{(s_i', s_0)\}$ and regret
$$r_i\left(\{(s_i', s_0)\}, \alpha_i\right) = \max_{s_i \in S_i} m_i(s_i, s_0) - m_i(s_i', s_0),$$
    is belief-independent.
  - Thus,
$$\begin{aligned} u_i(s_i', s_0, \alpha_i) &= m_i(s_i', s_0) - \theta_i \left(\max_{s_i \in S_i} m_i(s_i, s_0) - m_i(s_i', s_0) \right) \\ &= (1+\theta_i) m_i(s_i', s_0) - \theta_i \max_{s_i \in S_i} m_i(s_i, s_0) \end{aligned}$$
    and
$$\arg\max_{s_i' \in S_i} \sum_{s_0 \in S_0} \alpha_i(s_0|\varnothing) u_i(s_i', s_0, \alpha_i) = \arg\max_{s_i' \in S_i} \sum_{s_0 \in S_0} \alpha_i(s_0|\varnothing) m_i(s_i', s_0).$$