Surprised by the Parimutuel Odds?

By Marco Ottaviani and Peter Norman Sørensen*

This paper analyzes the role of private information in parimutuel (also known as pool betting) markets, a method commonly adopted to determine betting odds for horse races and other sporting events. According to the parimutuel system, the amount wagered on all outcomes is redistributed to the bets placed on the winning outcome. Given the frequent observation of the realized outcomes (which are not affected by the market process) and the absence of bookmakers (who could induce biases), parimutuel betting markets offer an ideal testbed for theories of information aggregation and market efficiency.

The efficient market hypothesis asserts that the fraction of money wagered by the market on an outcome is an unbiased estimate of the outcome's empirical frequency. Beginning with Richard M. Griffith (1949), empirical studies have established that market probabilities of favorites (i.e., outcomes with short odds) tend to underpredict their empirical probabilities; conversely, longshots are overbet and yield lower expected returns at the final odds.[1] The favorite-longshot bias (FLB) is perceived as a systematic deviation from the efficient market hypothesis.

In this paper, we argue that one should expect the FLB to result when a large number of privately informed bettors take simultaneous positions just before post time.[2] Our resolution of the FLB is based on the identification of the empirical probability of an outcome with the outcome's posterior probability derived by Bayes's rule to incorporate the information revealed by the bets placed in equilibrium. Using the equilibrium structure to compute the Bayesian posterior probability associated with any realized market probability, we show that the ex post realization of a high market probability indicates favorable information about the outcome's likelihood—and the opposite for longshots. The FLB is present because in a Bayes-Nash equilibrium bettors are not allowed to revise their positions to incorporate the surprise revealed by the final odds. The bias would instead be eliminated in a rational expectations equilibrium.

I. Model

To illustrate our argument, we focus on the simplest setting with a binary outcome, $k \in \{1, 2\}$. There are N ex ante identical bettors. To stress that our explanation does not rely on heterogeneity in prior beliefs, we make the conventional assumption that bettors share a common prior belief, $q \in (0, 1)$, that the realized outcome will be $k = 1$.[3]

*Ottaviani: Kellogg School of Management, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208-2013 (e-mail: m-ottaviani@northwestern.edu); Sørensen: Department of Economics, University of Copenhagen, Studiestræde 6, DK-1455 Copenhagen K, Denmark (e-mail: peter.sorensen@econ.ku.dk). We thank the reviewers for helpful comments.
[1] For surveys of the empirical literature see Richard H. Thaler and William T. Ziemba (1988), Donald B. Hausch and Ziemba (1995), and Bruno Jullien and Bernard Salanié (2008).

[2] In reality, racetrack bettors are allowed to commit money over time. The tote board displays provisional odds and updates them at discrete time intervals. Betting is then closed at post time when the race starts. Given that a substantial amount of money is wagered just before post time (National Thoroughbred Racing Association 2004), we focus on the last-minute simultaneous game.

[3] In a model with heterogeneous beliefs (but without private information), Mukhtar M. Ali (1977) shows that equilibrium market probabilities tend to be less extreme than the bettors' median belief. In a model with heterogeneous prior

Each bettor $i \in \{1, \ldots, N\}$ privately observes a signal leading to the private belief $p_i \in (0, 1)$ that outcome 1 will be realized. Conditional on k, signals (and therefore private beliefs) are independently and identically distributed across bettors. For convenience, we further assume that each bettor's private belief p is distributed according to a strictly increasing and continuous cumulative distribution G with full support over $(0, 1)$. By Bayes's rule, the private belief satisfies

$$(1) \qquad \frac{p}{1-p} = \frac{q}{1-q} \frac{g(p\,|\,1)}{g(p\,|\,2)}.$$

Given that the densities satisfy the strict monotone likelihood ratio property, the cumulative distributions are ranked by first-order stochastic dominance: $G(p\,|\,1) < G(p\,|\,2)$ for all $p \in (0, 1)$. Intuitively, higher Bayesian beliefs about outcome 1 are more likely to occur when 1 is the true outcome.[4]

On the basis of the private belief, each bettor decides the outcome on which to bet a fixed and indivisible amount, normalized to 1.[5] After the realization of outcome k, the total amount of money in the parimutuel pool is divided equally among those who bet on the winning outcome, k. Let b_k denote the total amount bet on k. If k is the winning outcome, then every unit bet on k receives the monetary payoff $(b_1 + b_2)/b_k$.

All bettors are risk neutral and maximize individually their expected monetary payoff, conditional on the information available when betting.[6] A bettor's strategy maps every private belief into one of the two possible bets. In equilibrium, every bettor correctly conjectures the opponents' strategies and plays a best response to this conjecture. Given that by construction the game is symmetric with respect to the players, we restrict attention to symmetric equilibria, in which all bettors use the same strategy.[7]

II. Equilibrium

Given the opponents' identical strategy, each bettor can calculate $c(l\,|\,k)$, the conditional chance that an opponent bets on outcome l when k is the winning outcome. A bettor's payoff conditional on winning is random, because opponents' signals and bets are uncertain. We consider the game in the limit as the number of players grows, $N \to \infty$, where the law of large numbers guarantees that the uncertainty over the aggregate distribution of signals (and therefore of bets) vanishes. In

beliefs *and* private information, Ottaviani and Sørensen (2005) formulate a complementary explanation for under-reaction based on the interaction of these two ingredients. That explanation is valid even when traders are allowed to revise their positions in response to the private information revealed by the odds, as in a fully revealing rational expectations equilibrium. In contrast, the explanation proposed here crucially relies on the fact that bettors do not know the final odds in a Bayes-Nash equilibrium. See the discussion in Section IV.

[4] Thus, we depart from Ali's (1977) formulation by making explicit the dependence of the distribution of beliefs on the true outcome.

[5] Ottaviani and Sørensen (forthcoming) endogenize the participation decision by allowing bettors to derive recreational utility when partaking in the market. Bettors with more extreme posterior beliefs are the last ones to be driven out of the market. As the recreational utility is reduced and the no-trade outcome is approached, the main features of the outcome achieved are similar to those obtained in the current formulation with forced participation when the number of bettors becomes arbitrarily large.

[6] Our model assumes risk neutrality to avoid confounding our information-based explanation with biases induced by risk preferences. The FLB is also consistent with risk-loving preferences (see Martin Weitzman 1965). As we argue in Section IV, our theory can rationalize not only the FLB but also the informativeness of the changes in odds close to post time.

[7] In a setting with binary signals, Frédéric Koessler, Charles Noussair, and Anthony Ziegelmeyer (2008) note that asymmetric equilibria may also exist when the number of bettors, N, is small. Here, we instead focus on the limit as $N \to \infty$. In addition, equilibrium analysis in our model is simplified by positing continuously distributed signals.

this limit, a fraction $c(k|k)$ of bets is on the winner when the outcome is k, and thus the expected payment to each bet on k is $1/c(k|k)$.

By risk neutrality, a bettor with private belief p expects to obtain a payoff equal to $p/c(1|1) - 1$ when betting on outcome 1, and $(1-p)/c(2|2) - 1$ when betting on outcome 2. Let $\overline{p} \in (0,1)$ be the unique solution to $\overline{p}/c(1|1) = (1-\overline{p})/c(2|2)$. A bettor with threshold belief $\overline{p}$ is indifferent between betting on either of the two outcomes. Symmetric equilibrium requires $\overline{p} = \hat{p}$, where $\hat{p}$ is the threshold belief conjectured for each opponent. Since $c(1|1) = 1 - G(\hat{p}|1)$ and $c(2|2) = G(\hat{p}|2)$, the equilibrium threshold belief, $\hat{p}$, is the unique solution to

$$(2) \qquad \frac{p}{1-p} = \frac{c(1|1)}{c(2|2)} = \frac{1 - G(p|1)}{G(p|2)}.$$

Uniqueness follows from the fact that the left-hand side is strictly increasing in p, while the right-hand side is nonincreasing.

III. Surprise

To investigate market efficiency, empiricists typically group observations into classes according to their market probabilities, where the *market probability* of outcome 1 is equal to the fraction of money bet on that outcome, $\pi = b_1/N$. For each observation class, empiricists then compute the associated *empirical probability* as the fraction of races that are won by the horses in the class. When comparing market and empirical probabilities, empiricists typically find a systematic difference between these probabilities: when the market probability is large, it is still smaller than the corresponding empirical probability. That is, a favorite is more likely to win than indicated by the market probability. Conversely, market probabilities of longshots overpredict on average their empirical probabilities computed from race outcomes.

Our explanation for the FLB relies on the fact that realized market probabilities contain information about the chance of different outcomes. Applying Bayes's rule, for any realized market probability we can compute the corresponding *posterior probability* belief, β, that outcome 1 will be realized, incorporating the information contained by the realization of this market probability in equilibrium. The law of large numbers guarantees that the empirical frequency of outcome 1 across a large sample of outcomes is approximately equal to this posterior probability, β. This posterior probability incorporates the information revealed in the betting distribution, and thus correctly estimates the outcome's empirical probability.

When exactly b_1 out of N bets are placed on 1, Bayes's rule yields the posterior probability

$$(3)\,\beta = \frac{q\,\mathrm{Pr}\,(\text{bets}\,|\,1\text{ true})}{\mathrm{Pr}(\text{bets})} = \frac{qc(1|1)^{b_1}[1 - c(1|1)]^{N-b_1}}{qc(1|1)^{b_1}[1 - c(1|1)]^{N-b_1} + (1-q)[1 - c(2|2)]^{b_1}c(2|2)^{N-b_1}},$$

using the binomial distribution of bets. We are now ready to compare any given market probability $\pi = b_1/N$ with the associated posterior probability β:

PROPOSITION 1: *In the limit, as the number of bettors becomes arbitrarily large, $N \to \infty$, for market probabilities*

$$(4) \qquad \pi > \pi^* = \frac{\log\big((1 - c(1|1))/c(2|2)\big)}{\log\big((1 - c(1|1))/c(2|2)\big) + \log\big((1 - c(2|2))/c(1|1)\big)}$$

(*respectively*, $\pi < \pi^*$), *the posterior probability β revealed by the bets is* 1 (*respectively*, 0).

PROOF.

Using $\pi = b_1/N$, rewrite (3) as

$$(5) \qquad \frac{\beta}{1 - \beta} = \frac{q}{1 - q} \left(\frac{c(1 \,|\, 1)^{\pi} [1 - c(1 \,|\, 1)]^{(1 - \pi)}}{[1 - c(2 \,|\, 2)]^{\pi} c(2 \,|\, 2)^{(1 - \pi)}} \right)^N.$$

Consider the limit as N goes to infinity, when the realized bets contain full information about the outcome, as they are based on an increasing number of i.i.d. signals. Thus, the right-hand side of (5) tends to zero or infinity, depending on whether the realized π is below or above the *switching market probability π^**, as defined in (4). Because market bets perfectly reveal the outcome, the posterior probability β converges to either zero or one. The result then follows immediately from (5).

According to this result, the FLB results from the surprise generated by the realization of the market probability. For high (or low) market probabilities, the posterior, and thus empirical, probability is higher (or lower) than the market probability. More precisely, whether an outcome is the ex ante favorite or longshot, when sufficiently many (or few) bettors choose it, the outcome is revealed to be even more (less) likely than indicated by the market probability.

Intuitively, the observation of one more bet on outcome 1 is good news for outcome 1 because $c(1 \,|\, 1) = 1 - G(p \,|\, 1) > 1 - G(p \,|\, 2) = 1 - c(2 \,|\, 2)$. When the fraction of bets placed on outcome 1 is exactly equal to π^*, this piece of news is exactly neutralized by the fact that fraction $1 - \pi^*$ bet on outcome 2, and the posterior probability is equal to the prior, $\beta = q$. When the realized market probability π is above the switching level π^*, favorable news outweighs unfavorable news for outcome 1—and with a population of infinite size, this realization reveals that outcome 1 is true with probability 1.

The FLB predicted in this simple setting is clearly extreme. With an infinite number of bettors with i.i.d. signals, the information contained in the market probability fully reveals outcome k. Thus, the posterior β formed after aggregating the information of all the individual bets is equal to either 0 or 1. Posterior probabilities would be bounded (and the FLB less extreme than here) in more realistic specifications of the model that allow for conditionally dependent signals across bettors or, equivalently, an unpredictable component in the outcome realization.

IV. Discussion

This paper contributes to the literature by linking the FLB in parimutuel markets to the surprise generated by the information contained in last-minute movements of market odds. In contrast, informed bettors share the same information in William Hurley and Lawrence McDonough's (1995) limited arbitrage model, and therefore are not surprised. For odds set by bookmakers dealing with privately informed bettors, Hyun Song Shin (1991) derives the FLB as an ex ante phenomenon across outcomes with asymmetric prior probabilities. Despite a commonality of assumptions, Shin's explanation for fixed odds markets is fundamentally different from our ex post explanation for parimutuel markets.[8] Overall, the FLB can be compatible with a weak form of the efficient market hypothesis, whether odds are set by bookmakers or the parimutuel system.

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[8] In ongoing research, we are developing a methodology for comparing the effect of information on the FLB across parimutuel and fixed-odds betting markets.

We conclude by elaborating on the role of three key features of our model. First, we focus on analyzing a simultaneous move game. Indeed, waiting until the last minute to bet may allow bettors to conceal their private information and use movements in the provisional odds to infer the information of others—this incentive rationalizes the observed rush of activity at post time. Given that late bets are likely to be made by informed bettors, changes in odds close to post time should be informative about race outcomes, as verified empirically by Peter Asch, Burton G. Malkiel, and Richard E. Quandt (1982).[9] Thus, our theory based on private information is compatible with evidence on both the timing of bets and the information content of changes in odds near post time. We see this as an important advantage over alternative theories, such as those based on risk preferences.[10]

Second, the FLB arises in our game-theoretic model (as well as at the real-world racetrack) because bettors are not allowed to condition their behavior on the final odds. Indeed, bettors in parimutuel markets take positions before observing the final realization of the market odds. As shown above, our Bayes-Nash equilibrium does not converge to a rational expectations equilibrium as the number of players increase.[11] The FLB would instead be eliminated if rational bettors were allowed to revise their positions after the market is closed, as in a rational expectations equilibrium. Note that in our Bayes-Nash equilibrium the FLB arises even though bettors anticipate that the realized bets will be correlated with the likelihood of the outcomes and adjust their behavior in anticipation of this correlation.[12]

Third, we focus here on the limit with an infinite number of bettors who are forced to participate—for the FLB to result, the number of bettors must be sufficiently large. In a more general version of this model, Ottaviani and Sørensen (forthcoming) analyze how the sign and extent of the FLB depend on the number of competing bettors, their willingness to participate, the quality of their private information, the number of outcomes, the divisibility of bets, and asymmetries in prior probabilities. Analyzing the impact of these variables on the signal to noise ratio present in equilibrium delivers a number of testable comparative statics predictions broadly in line with evidence from parimutuel markets (from horse racing to Lotto).

REFERENCES

Ali, Mukhtar M. 1977. "Probability and Utility Estimates for Racetrack Bettors." *Journal of Political Economy*, 85(4): 803–15.

Asch, Peter, Burton G. Malkiel, and Richard E. Quandt. 1982. "Racetrack Betting and Informed Behavior." *Journal of Financial Economics*, 10(2): 187–94.

Griffith, Richard M. 1949. "Odds Adjustment by American Horse-Race Bettors." *American Journal of Psychology*, 62(2): 290–94.

Hausch, Donald B., and William T. Ziemba. 1995. "Efficiency of Sports and Lottery Betting Markets." In *Handbook of Operations Research and Management Science*. Vol. 9, ed. Robert A. Jarrow, Vojislav Maksimovic, and William T. Ziemba, 545–80. Amsterdam: Elsevier.

Hurley, William, and Lawrence McDonough. 1995. "A Note on the Hayek Hypothesis and the Favorite-Longshot Bias in Parimutuel Betting." *American Economic Review*, 85(4): 949–55.

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[9] In their analysis of the strategic incentives to time parimutuel bets, Ottaviani and Sørensen (2006) identify an additional incentive in parimutuel markets. Given that parimutuel betting is a quantity competition game, bettors have an incentive to move early to capture a good market share of profitable bets. This second incentive is consistent with the observation of early bets based on public information. The non-trivial dynamics observed in parimutuel markets may be rationalized by the presence of these two countervailing forces.

[10] See Ottaviani and Sørensen (2008) for an overview of the main alternative theories proposed to explain the FLB.

[11] In Philip J. Reny and Motty Perry's (2006) double auction model, instead, players are allowed to choose a reservation price, and thus have a less constrained strategy space than in our model. As the number of players increases, the equilibrium in their double auction converges to a rational expectations equilibrium.

[12] In contrast, Jan Potters and Jörgen Wit (1996) posit that bettors adjust their positions at the final market odds (as in a rational expectations equilibrium), but that bettors ignore the information contained in the bets.

Jullien, Bruno, and Bernard Salanié. 2008. "Empirical Evidence on the Preferences of Racetrack Bettors." In *Handbook of Sports and Lottery Markets*, ed. William T. Ziemba and Donald B. Hausch, 27–49. Amsterdam: Elsevier.

Koessler, Frédéric, Charles Noussair, and Anthony Ziegelmyer. 2008. "Parimutuel Betting under Asymmetric Information." *Journal of Mathematical Economics*, 44(7–8): 733–44.

National Thoroughbred Racing Association: Wagering Systems Task Force. 2004. *Declining Purses and Track Commissions in Thoroughbred Racing: Causes and Solutions.* http://www.hbpa.org/resources/Ntrataskreportsep04.pdf.

Ottaviani, Marco, and Peter N. Sørensen. 2005. "Aggregation of Information and Beliefs: Asset Pricing Lessons from Prediction Markets." http://www.econ.ku.dk/sorensen/Papers/predictionmarkets.pdf.

Ottaviani, Marco, and Peter N. Sørensen. 2006. "The Timing of Parimutuel Bets." http://www.econ.ku.dk/sorensen/Papers/TheTimingofParimutuelBets.pdf.

Ottaviani, Marco, and Peter N. Sørensen. 2008. "The Favorite-Longshot Bias: An Overview of the Main Explanations." In *Handbook of Sports and Lottery Markets*, ed. William T. Ziemba and Donald B. Hausch, 83–101. Amsterdam: Elsevier.

Ottaviani, Marco, and Peter N. Sørensen. Forthcoming. "Noise, Information, and the Favorite-Longshot Bias in Parimutuel Predictions." *American Economic Journal: Microeconomics*.

Potters, Jan, and Jörgen Wit. 1996. "Bets and Bids: Favorite-Longshot Bias and the Winner's Curse." Tilburg University Center for Economic Research Discussion Paper 4.

Reny, Philip J., and Motty Perry. 2006. "Toward a Strategic Foundation for Rational Expectations Equilibrium." *Econometrica*, 74(5): 1231–69.

Shin, Hyun Song. 1991. "Optimal Betting Odds against Insider Traders." *Economic Journal*, 101(408): 1179–85.

Thaler, Richard H., and William T. Ziemba. 1988. "Parimutuel Betting Markets: Racetracks and Lotteries." *Journal of Economic Perspectives*, 2(2): 161–74.

Weitzman, Martin. 1965. "Utility Analysis and Group Behavior: An Empirical Analysis." *Journal of Political Economy*, 73(1): 18–26.