

The Economics of Advice*

Marco Ottaviani[†]

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Abstract

We formulate a simple model of advice which can be useful to guide the regulation of the financial retail industry. An informed agent (financial adviser) transmits information to an otherwise uninformed principal (investor) with uncertain degree of strategic sophistication. Incentives for truthful information disclosure and information acquisition are studied. The role of explicit monetary transfers is analyzed.

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[†]Department of Economics and ELSE, University College London, Gower Street, London WC1E 6BT, UK. Phone: +44-20-7679-5242. Fax: +44-20-7916-2775. E-mail: m.ottaviani@ucl.ac.uk. Web: <http://www.ucl.ac.uk/~uctpmao>.

1. Introduction

Independent financial advisers (IFA) have been playing an increasingly important role in the provision of information on financial services to small investors. Regulators of the financial retail industry actively monitor the quality of the services provided by such advisers.¹ This paper unifies and develops the economic theory of advice by building on the economics of communication and incentives. In order to account for the regulators' typical concern about unsophisticated investors, investors are allowed to have varying degree of strategic sophistication. The insights derived can be useful to guide the regulation of the financial retail industry.

In our simple model, an agent (adviser) possesses private information about the state of the world relevant for the decision to be taken by an otherwise uninformed principal (investor). The preferences of both agent and principal depend on the decision finally adopted (amount invested) as well as on the state of the world (optimal allocation for the investor). In the language of communication, the agent is the sender who transmits information to the principal (or receiver). In the special case where transfers are barred, this is the abstract setting considered by Holmström (1977/1984) for the case of full commitment by the principal, and Crawford and Sobel (1982) and Green and Stokey (1980a) for the case with no commitment.

We consider an adviser with both a professional and a partisan objective. Providing good advice improves the prospects of repeated custom and leads to favorable publicity. Abstracting from the partisan bias, the adviser is then assumed to act in the best interest of the customer.² We allow the adviser to have also a partisan objective. For example, IFAs are given explicit commissions by product providers, typically computed as a percentage of the funds invested. Furthermore, long-term relationships develop between advisers and product providers and preferential treatment is given to advisers who sell more of their product. This additional compensation, whether legal or illegal, introduces a partisan bias in the objective of an otherwise professional adviser.³

¹See Black (1997) for a legal analysis of the regulation of financial services in the UK. Chapter 4 overviews the recent development of the regulation of the retail sector for financial products.

²For simplicity, in this paper we adopt a reduced-form approach to modeling the professional objective. A purely professional adviser is assumed to minimize the mean square error of the recommendation given. Building on early work by Holmstrom (1982/1999), a recent literature on reputational cheap talk and signaling (cf. Scharfstein and Stein (1990), Prendergast and Stole (1996), Ottaviani and Sørensen (1999), and (2000) for the theory, and Chevalier and Ellison (1999) and Hong, Kubik and Solomon (2000) for the empirics) has argued that distortions might result even in a setting with a pure professional objective. Here, we choose instead to posit a "naive" professional objective, which dictates to a purely professional adviser to provide truthful information.

³The concerns of regulators for transfers from product providers to IFAs are testified by their close regulation. According to the SFA (Securities and Futures Authority) Rule Book permitted goods and services are: 1) research and advisory services, 2) portfolio valuation and analysis, 3) market price services,

For the purpose of illustrating the simple economics of this problem, we make specific but natural assumptions on the preferences and information of the parties. The preferences are quadratic and the state of the world has a uniform prior distribution. The agent either knows the state perfectly or has a noisy signal with quality parametrized in a simple way. These assumptions make the model algebraically tractable and allow us to derive explicit solutions in the different scenarios analyzed, so that the resulting welfare can be easily compared. In Section 7, we derive these preferences in a model where the investor with mean variance preferences solves a simple portfolio allocation problem with the help of an informed independent financial adviser. The robustness of our findings to different preferences and information structures is discussed in a companion paper (de Garidel-Thoron and Ottaviani (2000)).

We first review Crawford and Sobel's characterization of the communication (or "cheap-talk") equilibrium which results in the case where the principal has no commitment power. The principal makes the decision on the basis of the information contained in the recommendation given by the adviser. Due to the fact that the preferences of the principal and the adviser are different, the equilibrium cannot be fully revealing. If it were, the principal would always take her most preferred action, but then the agent would want to deviate by recommending something which is in his interest. As a result, some information must necessarily be destroyed. When there is enough common ground between principal and adviser (the preferences of principal and agent are not too different), some information can be successfully communicated in equilibrium.

The receiver of the communication game is required to have a great deal of strategic sophistication. A perfectly rational receiver who knows the communication game is never fooled in equilibrium into taking an action which is not to her best advantage *ex post*. It is natural to ask what would happen if the adviser thought that with positive chance the receiver is naive. A naive receiver is assumed to blindly follow the recommendation of the adviser. We show that the nature of the communication equilibrium changes drastically when the sender is naive even with arbitrarily small probability. The equilibrium is fully revealing and involves distortions. The partisan bias is not self-defeating any more. It is then in the interest of a product provider to provide commission-based incentives to advisers.

4) custodian services, 5) computer hardware associated with specialized computer software or research services, 6) dedicated telephone lines, 7) seminar fees (where the subject matter is relevant to the provision of investment services), 8) publications (where the subject matter is relevant to the provision of investment services). Non permitted goods and services are: 1) seminar fees not falling within permitted list, 2) subscriptions or publications not falling within the permitted list, 3) office administrative computer software, 4) computer hardware not associated with specialist software, 5) membership fees to professional organizations, 6) purchase or rental of office equipment or ancillary services, 7) employees salaries, 8) direct money payments. The rules set by the PIA (Personal Investment Authority) are similar but rather more extensive and detailed.

When the receiver is known to be naive for sure, communication is equivalent to delegation to the adviser. Under delegation, in any state the adviser makes the decision he likes the most in that state. All the information available is then used, but not to the best advantage of the principal. A recent paper by Dessein (1999) shows that delegation dominates in general cheap-talk communication when the (symmetric and convex) preferences of the agent are sufficiently congruent with those of the principal. In the uniform-quadratic example, the cheap-talk equilibrium (whenever informative) results in lower expected payoff for the principal than delegation.

This seemingly paradoxical result illustrates the power of commitment. Delegation is a simple and credible way for the principal to commit to take an action which is not ex-post optimal for her. More generally, Holmström (1977/1984) has characterized constrained delegation, the solution to the problem with full commitment: The optimal mechanism for the principal when utility is not transferable consists of choosing a subset of actions among which the agent is allowed to pick the most desired one. As the preferences of agent converge to those of the principal, the principal's gain from constraining the discretion of the agent becomes infinitesimal. Simple delegation is approximately optimal.

Within our model we can address the issue of incentives for information acquisition and truthful information disclosure. Furthermore, we consider the effect of allowing explicit monetary transfers between advisers and investor, which can be conditional on the adviser's recommendation. The role of regulation can also be analyzed.

The paper proceeds as follows. The uniform-quadratic model of advice with a possibly naive principal is formulated in Section 2 and analyzed in Section 3. The cheap talk equilibrium, which results when the principal is fully sophisticated, is reviewed in Section 3.1. Section 3.2 derives the delegation solution, which results when the investor commits to follow the recommendation of the adviser or is known to be naive. Cheap talk is compared to delegation in Section 3.3. Section 3.4 analyzes the communication equilibrium when the principal is possibly naive. Section 4 extends the model to the case where the adviser has noisy information, allowing us to endogenize the quality of the adviser's information. Section 5 considers the case where the direction of the bias of the expert is unknown to the principal. Section 6 introduces explicit transfers between adviser and principal and addresses the role of regulation. Section 7 applies our model to the problem of independent financial advice, where the principal is identified with an investor with mean-variance preferences. Section 8 concludes.

2. Model

A principal P has to make a decision with payoff which depends on a state of nature. P has imperfect information about the true state of nature. An adviser (or agent) A with possibly conflicting preferences has information about the state of nature. For now consider the case where the adviser knows the state of nature with certainty while P has no information. We consider the uniform/quadratic example to address a set of questions relevant for the economics of advice. This example and its variations are used extensively in the literature for illustrative purposes, starting with Holmström (1977) and Crawford and Sobel (1982). As shown in Section 7, these preferences can be derived from first principles in a natural model of investment and independent financial advice.

The action to be taken by the uninformed principal is denoted by a , while x denotes the state of the world. P 's prior belief about x is uniformly distributed on $[0, 1]$,

$$F(x) = x \text{ for } x \in [0, 1],$$

while A is perfectly informed about x . The preferences of both parties are quadratic:

$$U^P(a, x) = -(a - x)^2$$

$$U^A(a, x) = -(a - (x + b))^2$$

where $b \geq 0$ represents the bias in A 's preferences. A 's most preferred action given x is $a^A(x) = x + b$, while P 's most preferred action given x is $a^P(x) = x$.

In the cheap-talk game, the principal listens to the agent's recommendation, then takes the ex-post optimal decision. Cheap-talk communication requires a great deal of strategic sophistication by the principal. Our model allows for unsophisticated, or "naive" principals, who follows the advice literally. Equivalently, a naive principal simply delegates the decision to the agent. Both sophisticated and naive principal coexist. The adviser is assumed not to know whether the principal is sophisticated or not. Let p denote the proportion of sophisticated principals. Alternatively, a principal is sophisticated with probability p .

3. Equilibrium

In this section we characterize the perfect Bayesian equilibria of this game. We begin by looking at two polar cases: (i) cheap talk communication, which results when $p = 1$, and (ii) delegation, when $p = 0$.

3.1. Cheap Talk ($p = 1$)

When P is sophisticated, P and A play the Crawford and Sobel cheap-talk game. In equilibrium, A reports coarse information, for example in the form of intervals of $[0, 1]$. First, it is instructive to show that there cannot be a fully revealing equilibrium. Otherwise, there would be an equilibrium invertible message strategy $m(x)$. The action taken by the receiver in such an equilibrium in response to message m would be $m^{-1}(m)$, i.e. action $m^{-1}(m(x)) = x$ is taken in state x . Then, the agent with type, say, $x = 0$ would deviate by sending $m(b)$ rather than $m(0)$, thereby achieving payoff $-(m^{-1}(m(b)) - b)^2 = 0$ rather than $-(m^{-1}(m(0)) - b)^2 = -b^2$. Full revelation cannot result in equilibrium.

Crawford and Sobel's characterization of the equilibrium (Section 4 of their paper) is now briefly summarized. Equilibria correspond to a partition

$$\{[x_0, x_1], \dots, [x_{i-1}, x_i], [x_i, x_{i+1}], \dots, [x_{n-1}, x_n]\}$$

of $[0, 1]$. The agent of type x_i must be indifferent between sending message $m_{i-1} = [x_{i-1}, x_i]$ and $m_i = [x_i, x_{i+1}]$, i.e.

$$-\left(\frac{x_{i-1} + x_i}{2} - (x_i + b)\right)^2 = -\left(\frac{x_i + x_{i+1}}{2} - (x_i + b)\right)^2$$

with $x_{i+1} > x_i > x_{i-1}$. This condition can only hold if $x_{i+1} - x_i = x_i - x_{i-1} + 4b$, a second-order difference equation. Setting $x_0 = 0$, the solutions to the difference equations parametrized by x_1 are $x_i = x_1 i + 2i(i-1)b$. Let $N(b)$ the largest positive integer i such that $2i(i-1)b < 1$, i.e. the smallest integer greater or equal to $(\sqrt{1 + 2/b} - 1)/2$. Finally, $x_n = 1$ determines $x_1 = (1 - 2n(n-1)b)/n$ so that $x_i = i/n + 2bi(i-n)$ or $x_i - x_{i-1} = i/n + 2b(2i-n-1)$. As computed by Crawford and Sobel in their equation (25), the expected payoff of the receiver is equal to the residual variance

$$V_{CT}^P = -\sum_{i=1}^n \int_{x_{i-1}}^{x_i} \left(\frac{x_{i-1} + x_i}{2} - x\right)^2 dx = -\frac{1}{12n} - \frac{b^2(n^2 - 1)}{3}. \quad (3.1)$$

This is increasing in the number n of elements of the partition. Equilibria are Pareto-ranked, according to the number n of intervals reported. According to the value of the agent's bias b , there exists an upper bound on n , say N . We focus on the most informative cheap-talk equilibrium, where $n = N$.

3.2. Delegation ($p = 0$)

When P is naive, he simply delegates the action to the agent. In this case, A chooses action $a^A(x) = x + b$. The resulting expected payoffs to A and P are $V_D^A = 0$ and

$$V_D^P = -b^2. \quad (3.2)$$

What is the optimal mechanism for the principal in this context without transferable utility and fully commitment power? As noticed by Holmström (1977), the optimal mechanism has the flavor of delegation. The optimal mechanism consists in constraining the agent to choose actions which belong to an optimally designed subset of all possible actions. This is Holmström’s delegation problem, i.e. the problem of a principal delegating limited decision-making authority to the agent. To contrast this to the simple unconstrained form of delegation considered above, we dub this “constrained delegation”. The delegation problem is a sequential game where

1. P designs a subset A^c of admissible actions (so as to maximize his own utility), and then delegates the decision to the agent,
2. A selects an action $a \in A^c$, conditional on the observed state x , so as to maximize his utility $U^A(a, x)$.

In the present quadratic example, P optimally chooses $A^c = [0, 1 - b]$. See Holmström’s (1977) section 2.3.2 on this. The payoff attained by P is $V_{CD}^P = -b^2 + \frac{4}{3}b^3$ for $b \in [0, 1/2]$, and $V_{CD}^P = -1/12$ otherwise. Clearly, $V_{CD}^P - V_D^P = \frac{4}{3}b^3 > 0$, so that constrained delegation strictly improves on simple delegation. Note, however, that to implement the constrained delegation outcome, P needs to be sophisticated and to be aware of the objective of the agent.

3.3. Comparison of Cheap Talk and Delegation

We are now ready to compare the principal’s payoff in the most informative cheap-talk equilibrium with that of delegation. From (3.1) and (3.2), we have

$$V_D^P - V_{CT}^P = \frac{1}{12N} + \frac{b^2(N^2 - 4)}{3},$$

where N is the smallest integer greater or equal to $(\sqrt{1 + 2/b} - 1)/2$. Then P prefers delegation to cheap talk whenever more than two distinct messages are sent, i.e. cheap talk is informative ($N \geq 2$):

Result 1 *In the uniform/quadratic model, unconstrained delegation gives higher expected payoff to the principal than cheap-talk, whenever the latter is informative.*

Intuitively, under delegation an ex-post suboptimal action for the principal is taken but all information is used. Under cheap-talk communication, the principal implements an optimal action ex-post given the information revealed by the agent, with the caveat that only part of the information is elicited (so as to make communication truthful and

credible). Moreover, the lengths of intervals of states of nature for which the agents sends the same message are of increasing size. This results in the presence of relatively large intervals, for which the loss from imprecise information is large.

The comparison of delegation with cheap talk communication is made more generally in a recent paper by Dessein (1999). Dessein shows that delegation dominates cheap-talk in a model with convex and symmetric preferences (where it is simple to generalize the characterization of the cheap talk equilibrium of the quadratic example) and under more general distributional assumptions, provided that the additive bias of the agent (adviser) is small enough. In a companion paper, we are currently investigating the comparison of delegation and communication in a number of other examples and under more general preferences and information structures. Furthermore, we allow for more general forms of communication (see e.g. Aumann (1987), Myerson (1986) and Forges (1986) for early developments of the theory, as well as Forges (1990) and Mitusch and Strausz (2000) for applications).

Result 1 shows that it is not possible to exhibit a nontrivial trade-off between cheap-talk and unconstrained delegation within the uniform-quadratic model. Notice that cheap talk is necessarily uninformative ($N = 1$) for $|b| \geq 1/4$, in which case the equilibrium payoff is equal to $-1/12$. One can always take the bias b to be extremely large so as to make simple delegation unattractive with respect to the *uninformative* cheap-talk equilibrium (which always exists, and provides a payoff constant with respect to b). Here, for $b > 1/(2\sqrt{3})$ the principal prefers to take the ex ante optimal action (also the cheap talk equilibrium) rather than delegating to the informed agent. Another consequence of this computation is that delegation can dominate uninformative cheap-talk (for $1/4 \leq |b| \leq 1/(2\sqrt{3})$ here), meaning that when objectives are so much conflicting that no information can be communicated, it might still be better to leave the decision to the agent rather than taking the ex-ante optimal action.

Remark 1 *As a corollary, delegation ex-ante Pareto dominates cheap talk. This is an immediate consequence of the previous result and the fact that delegation results in the highest possible payoff for the adviser ex-post state by state, and therefore also ex ante.*

Remark 2 *Constrained delegation dominates cheap talk regardless of whether it is informative. This is a more general result, since constrained delegation replicates the outcomes of the communication game where P can commit not to take an ex post optimal action given the agent's recommendation.*

Remark 3 *The formulas above imply that $V_{CT}^P \simeq -\frac{1}{6\sqrt{2}} \times \sqrt{b}$. Therefore, although V_{CT}^P , V_{CD}^P , and V_D^P converge to zero when b decreases to zero, both constrained and unconstrained delegation converge much faster than cheap-talk (see figure below). When b is*

small, V_{CD}^P , and V_D^P are very close: $V_{CD}^P - V_D^P = \frac{4}{3}b^3 \ll |V_D^P| = b^2$, whereas V_{CT}^P is infinitely smaller than V_{CD}^P or V_D^P . See figure 1 below.

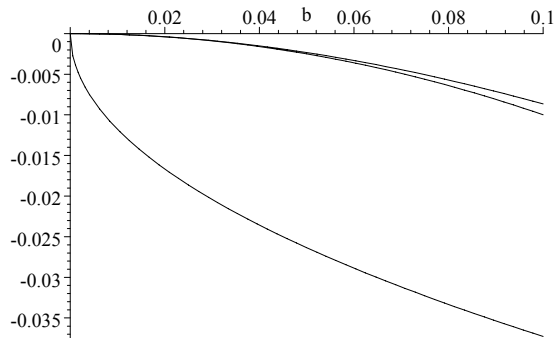


Figure 1: Constrained delegation, simple delegation and cheap-talk in the continuous uniform-quadratic example, for small values of b .

3.4. Communication with Possibly Naive Audience ($0 \leq p < 1$)

This section considers the more general case where P is sophisticated with probability p and naive with probability $(1 - p)$. The adviser A advises a population of principals of which a fraction $(1 - p)$ is naive, but the agent cannot tell whether the principal is sophisticated or naive when giving the recommendation. We will show that there exists a fully revealing equilibrium, whatever $p \in (0, 1)$.

3.4.1. Fully Revealing Equilibria

Result 2 For $p \in (0, 1)$, there exists a fully revealing equilibrium in which the agent reports message $m(x) = x + \frac{b}{1-p}$ given the state x .

Fully revealing equilibria are such that $m(\cdot)$ is invertible. Incentive compatibility for fully revealing equilibria is

$$m(x) \in \arg \max_m -p[m^{-1}(m) - (x + b)]^2 - (1 - p)[m - (x + b)]^2$$

for all $x \in [0, 1]$. The first order condition is

$$m'(x)[m(x) - (x + b)] = \frac{p}{1 - p}b, \quad (E)$$

for $x \in [0, 1]$. The second order condition is

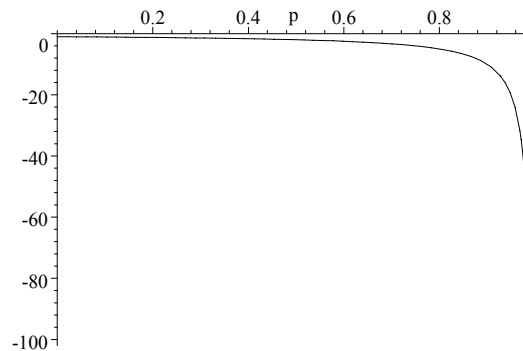
$$2(1 - p) + 2p\left(\frac{1}{m'(x)}\right)^2 + 2pb\frac{m''(x)}{(m'(x))^3} \geq 0, \quad (S)$$

for $x \in [0, 1]$. Note that $m(x) = x + \frac{b}{1-p}$ clearly satisfies the second order condition (S).

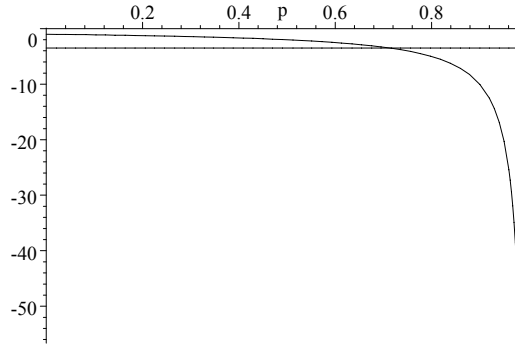
3.4.2. Principal's Payoff and Comparative Statics

In a fully revealing equilibrium, a sophisticated principal learns the true state x for sure, and subsequently takes an action $a = x$. As a result, her payoff is the highest possible, $V^{SP} = 0$. From now on, we focus on solution (i). It is straightforward to compute the payoff of a naive principal, $V^{NP} = -(\frac{b}{1-p})^2$. The expected welfare of principals is then $V^P = pV^{SP} + (1-p)V^{NP} = -\frac{b^2}{1-p}$. Both V^{NP} and V^P decrease with the agent's bias (b) and the probability that the principal is sophisticated (p).

The probability that the principal is sophisticated in the eyes of the agent can be interpreted as the proportion of sophisticated principal in the population. Notice that sophisticated principals benefit from the presence of naive principals. A sophisticated principal is able to glean all the information of the agent, and so obtains maximal utility (i.e. zero) rather than the cheap-talk payoff resulting in the absence of naive principals. Naive principals suffer instead from the presence of sophisticated principals: V^{NP} decreases in p . The expected welfare V^P follows the same pattern as V^{NP} : it is maximal (and equal to $-b^2$) when all agents are naive, decreases (to $-\infty$) when p increases, and is equal to $V_C^P < -b^2$ when $p = 1$.



Suppose now that the principal can choose to be naive or sophisticated. The best for her would be to commit to be naive (i.e., to commit to delegate the choice of action to the agent). But if the agent believed that P is naive with positive probability, then P would be better-off being sophisticated with probability one (and thereby obtaining maximal utility)! Therefore, if P cannot commit to be naive, the only equilibrium is one in which she chooses to be sophisticated with probability one. If there is a utility cost to be sophisticated, though, there might exist an equilibrium in which P chooses to be naive with probability $p \in (0, 1)$. For example, if becoming sophisticated costs $c > b^2$, there is an equilibrium where a fraction $p = 1 - b/\sqrt{c}$ of principals become sophisticated. The resulting expected payoff of the principal is $-b\sqrt{c}$.



Start from a situation where a proportion p of principals are sophisticated, the remaining $(1 - p)$ being naive. Consumer education (defined as making more principals sophisticated) would then decrease the principals' expected welfare, since V is a decreasing function of p : educated principals are better off, but those who remain naive are much worse off. We conjecture that the latter may not extend to the more natural (in the context of financial advice) case, partly explored below, where the action is restricted to be taken in $[0, 1]$.

When the agent's bias b increases, both V^{SP} and V^P decrease. The bias b in the agent's preferences may be seen as a bribe from a third layer of individuals (say, product providers).⁴ When all principals are sophisticated, the cheap-talk model predicts that any attempt to bias the agent is self-defeating. This is because the principal always takes an action which is ex post optimal for him, conditional on all the information inferred in equilibrium. The only effect of biasing the agent is to induce a reduction in the amount of information which can be used in the final decision. However, the present analysis shows that when a proportion $(1 - p)$ of principals are naive, biasing the agent may be rewarding. This is because there is no loss of information in the process of communication with a biased expert, while the action taken by naive principals (i.e. $a = x + \frac{b}{1-p}$) is increased. This provides a natural explanation of why a third layer of individuals may be willing to bias the adviser.

The agent's expected payoff is

$$-p(x - (x + b))^2 - (1 - p) \left(x + \frac{b}{1 - p} - (x + b) \right)^2 = -\frac{p}{1 - p} b^2.$$

⁴In the financial advice context of Section 7, it is shown that if agents have a professional objective and receive a commission proportional to the amount of wealth invested, they become biased as in the present model.

4. Noisy Information

It is natural to wonder whether the comparison of delegation and cheap talk depends on the assumption that the expert has perfect information. In this Section, we extend the model to account for noisy information by the expert. Austen-Smith (1993) provides the first analysis of a cheap-talk model with noisy information (and multiple experts). While in his paper the state and the signal is binary, we allow a continuous of states and signals.

We assume that the agent has perfect information with probability $t \in [0, 1]$, and a signal drawn from the prior with probability $1 - t$. The conditional distribution of the signal of the agent is then

$$F(s|x) = \begin{cases} (1-t)s & s \in [0, x) \\ t + (1-t)s & s \in [x, 1]. \end{cases}$$

with t parametrizing its (Blackwell) informativeness. In the uniform prior example the posterior expectation of x conditional on receiving the noisy signal s is

$$E[x|s] = ts + \int_0^1 (1-t)ydy = ts + \frac{1-t}{2}.$$

Notice that $E[x|s] \in [(1-t)/2, (1+t)/2]$.

4.1. Delegation

Under delegation the agent takes action $a(s, b) = E[x|s] + b$. The resulting expected payoff of the principal is

$$\begin{aligned} V_D^P(b) &= - \int_0^1 \left[t(a(x, b) - x)^2 + \int_0^1 (a(s, b) - x)^2 (1-t) ds \right] dx \\ &= - \int_0^1 \left(t \left((1-t) \left(\frac{1}{2} - x \right) + b \right)^2 + (1-t) \int_0^1 \left(ts + \frac{1-t}{2} + b - x \right)^2 ds \right) dx \\ &= - \frac{1}{12} (1-t^2) - b^2. \end{aligned}$$

and the agent

$$\begin{aligned} V_D^A(b) &= - \int_0^1 \left(t \left((1-t) \left(\frac{1}{2} - x \right) \right)^2 + (1-t) \int_0^1 \left(ts + \frac{1-t}{2} - x \right)^2 ds \right) dx \\ &= - \frac{1}{12} (1-t^2). \end{aligned}$$

Notice that the marginal benefit of information for the agent is $t/6 \geq 0$. In accordance with the more general finding of Radner and Stiglitz (1984), the marginal return of information (both private and social) is zero (and increasing) when the agent is uninformed.

Let $C(t)$ be the cost of acquiring information of quality t , with $C' > 0$ and $C'' > 0$. It is then immediate to see that if $C'''(t) > 0$, the level of information acquired by the agent is lower than the socially optimal level.

Example 1 Take $C(t) = -k(\ln(1-t) + t)$, with $k > 0$. Then $C(0) = 0$, $\lim_{t \rightarrow 1} C(t) = \infty$, and the marginal cost is

$$C'(t) = \frac{kt}{1-t} > 0,$$

with $C''(0) = 0$ and $\lim_{t \rightarrow 1^-} C''(t) = \infty$. Furthermore, it is immediate to verify that $C''(t) = k/(1-t)^2 > 0$ and $C'''(t) > 0$. The level of information acquired under delegation t^d is lower than the socially optimal level t^{so} :

$$t^d = \max\langle 1 - 12k, 0 \rangle \leq \max\langle 1 - 6k, 0 \rangle = t^{so}.$$

4.2. Cheap Talk

The posterior p.d.f of x conditional on receiving the message $m = [s_i, s_{i+1}]$ is

$$f(x | [s_i, s_{i+1}]) = \frac{\Pr([s_i, s_{i+1}] | x) f(x)}{\int_0^1 \Pr([s_i, s_{i+1}] | x) f(x) dx} = \begin{cases} 1-t & \text{for } x \notin [s_i, s_{i+1}] \\ (1-t) + \frac{t}{s_{i+1}-s_i} & \text{for } x \in [s_i, s_{i+1}], \end{cases} \quad (4.1)$$

by

$$\Pr([s_i, s_{i+1}] | x) = F(s_{i+1} | x) - F(s_i | x) = \begin{cases} (1-t)(s_{i+1} - s_i) & \text{if } x \notin [s_i, s_{i+1}] \\ t + (1-t)(s_{i+1} - s_i) & \text{if } x \in [s_i, s_{i+1}]. \end{cases}$$

The posterior distribution is

$$F(x | [s_i, s_{i+1}]) = \begin{cases} (1-t)x & \text{for } x \in [0, s_i] \\ t \frac{x-s_i}{s_{i+1}-s_i} + (1-t)x & \text{for } x \in [s_i, s_{i+1}] \\ t + (1-t)x & \text{for } x \in [s_{i+1}, 1]. \end{cases}$$

and the posterior mean is

$$\begin{aligned} E[x | [s_i, s_{i+1}]] &= \int_0^{s_i} x(1-t) dx + \int_{s_i}^{s_{i+1}} x \frac{t + (1-t)(s_{i+1} - s_i)}{s_{i+1} - s_i} dx + \int_{s_{i+1}}^1 x(1-t) dx \\ &= \frac{s_i^2}{2}(1-t) + \frac{t + (1-t)(s_{i+1} - s_i)}{s_{i+1} - s_i} \frac{s_{i+1}^2 - s_i^2}{2} + \frac{1 - s_{i+1}^2}{2}(1-t) \\ &= t \frac{s_{i+1} + s_i}{2} + \frac{1-t}{2}. \end{aligned}$$

The utility of the agent with signal s when sending signal $[s_i, s_{i+1}]$ is then

$$- \int_0^1 \left(t \frac{s_{i+1} + s_i}{2} + \frac{1-t}{2} - (x+b) \right)^2 (1-t) dx - \left(t \frac{s_{i+1} + s_i}{2} + \frac{1-t}{2} - (s+b) \right)^2 t$$

so that the condition of indifference for agent of type s_i between messages $[s_{i-1}, s_i]$ and $[s_i, s_i]$ is

$$s_{i-1} (2ts_i - ts_{i-1} + 4b) = s_{i+1} (2ts_i - ts_{i+1} + 4b).$$

solved by

$$s_{i+1} - s_i = s_i - s_{i-1} + \frac{4b}{t}.$$

The cheap talk equilibrium has the same structure as in the case the agent is perfectly informed. Messages are of increasing length, with $4b/t$ replacing $4b$ as the increase in step size. There exists an informative equilibrium for $t \geq 4b$.

Notice that the expected payoff of the agent of type s when sending message $[s_i, s_{i+1}]$ is

$$\begin{aligned} & - \int_0^1 \left(\frac{s_i + s_{i+1}}{2} - (x + b) \right)^2 (1-t) dx - \left(\frac{s_i + s_{i+1}}{2} - (s + b) \right)^2 t \\ & = \left(\frac{1}{3} - \frac{s_i + s_{i+1}}{2} + b + \left(\frac{s_i + s_{i+1}}{2} - b \right)^2 \right) (1-t) - \left(\left(\frac{s_i + s_{i+1}}{2} - b \right) - s \right)^2 t \end{aligned}$$

4.3. Comparison of Delegation and Cheap Talk

The expected payoff of the principal in cheap talk (as computed in the Appendix) is

$$V_{CT}^P(b, t) = -(1-t) \frac{1}{6} - (2t-1) \left(\frac{1}{12N^2} + \frac{b^2(N^2-1)}{3t^2} \right) \quad (4.2)$$

where the number of equilibrium messages $N(b, t)$ is the smallest integer greater or equal to

$$-\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2t}{b}}. \quad (4.3)$$

The difference of the expected payoff of the principal under cheap talk and delegation is

$$V_{CT}^P - V_D^P = -(1-t) \frac{1}{6} + (1-2t) \left(\frac{1}{12N^2} + \frac{b^2(N^2-1)}{3t^2} \right) + \frac{1}{12} (1-t^2) + b^2.$$

For b small and $4b \ll t$, N can be approximated by (4.3), and $V_{CT}^P - V_D^P$ is then seen to be maximized at $t = 1$. For t slightly below $4b$, $N = 2$ and $V_{CT}^P - V_D^P$ is approximately $-\frac{1}{3}b^2$. Furthermore, $V_{CT}^P - V_D^P$ with $N = 2$ is a decreasing function of t at $t = 4b$ for $b \leq 1/4$.

In conclusion,

Result 3 *With noisy information the principal prefers delegation to cheap talk whenever $N \geq 2$, i.e. whenever cheap talk is informative.*

4.4. Communication with Naive Audience and Noisy Information ($0 \leq p < 1$, $0 \leq t \leq 1$)

We now characterize the equilibrium with possibly naive audience and noisy information. According to Crawford and Sobel (1982): “Signaling models typically have exogenously given differential signaling costs, which allow the existence of equilibria in which agents are perfectly sorted. Our model has no such costs. But the receiver’s equilibrium choice of action rule generally creates endogenous signaling costs, which allow equilibria with partial sorting.” Once the model is modified by introducing a small probability of a naive audience, the sender’s message has direct effect on her payoff. It is not surprising that complete sorting can then result. Conceptually, our model of not-so-cheap talk is related to the another modification of Crawford and Sobel’s model investigated recently by Austen-Smith and Banks (1999). While Austen-Smith and Banks allow the agent to send a costless as well as a costly signal, our agent can only send a single message which is costly with some (possibly small) probability.

An agent of type s sends message $m(s)$. A fully separating equilibrium is constructed with a message strategy m differentiable and strictly increasing, $m' > 0$. In such an equilibrium, the sophisticated principal learns at equilibrium the true state by inverting the message strategy and can therefore take the optimal action,

$$a(s) = \arg \max_a \int_0^1 (a - x)^2 dF(x|s) \quad (4.4)$$

in each state $s = m^{-1}(m(s))$, where

$$F(x|s) = \begin{cases} (1-t)x & x \in [0, s] \\ t + (1-t)x & x \in [s, 1] \end{cases}$$

is the posterior distribution of the state upon observation of signal s . With the quadratic loss function (4.4), the optimal action is equal to the posterior mean on the state

$$a(s) = E[x|s] = ts + \int_0^1 (1-t)ydy = ts + \frac{1-t}{2}. \quad (4.5)$$

The expected payoff of an agent of type s when reporting message m in such an equilibrium is

$$U^A(m, s) = - \int_0^1 \left(p(a(m^{-1}(m)) - (x+b))^2 + (1-p)(m - (x+b))^2 \right) dF(x|s).$$

The FOC for the agent’s maximization is

$$\begin{aligned} & -p \left(t \left(tm^{-1}(m) + \frac{1-t}{2} - (s+b) \right) + (1-t) \int_0^1 \left(tm^{-1}(m) + \frac{1-t}{2} - (x+b) \right) dx \right) \\ & = \frac{1}{\frac{\partial a(m^{-1}(m))}{\partial m}} (1-p) \left(t(m - (s+b)) + (1-t) \int_0^1 (m - (x+b)) dx \right). \end{aligned}$$

Notice that

$$\frac{\partial a(m^{-1}(m))}{\partial m} = \frac{a'(m^{-1}(m))}{m'[m^{-1}(m)]},$$

$m^{-1}(m(s)) = s$, and

$$\begin{aligned} & t \left(ts + \frac{1-t}{2} - (s+b) \right) + (1-t) \int_0^1 \left(ts + \frac{1-t}{2} - (x+b) \right) dx \\ &= t \left((1-t) \left(\frac{1}{2} - s \right) - b \right) + (1-t) \left(t \left(s - \frac{1}{2} \right) - b \right) = -b, \end{aligned}$$

so that the differential equation defining the equilibrium is

$$pbt = m'(s)(1-p) \left(m(s) - \left(ts + (1-t) \frac{1}{2} \right) - b \right)$$

since $a' = t$. Substituting the linear guess $m(s) = \alpha s + \beta$, one obtains

$$pbt = \alpha(1-p) \left(\alpha s + \beta - \left(ts + (1-t) \frac{1}{2} \right) - b \right)$$

solved by

$$\alpha(t) = t, \tag{4.6}$$

$$\beta(p, t, b) = \frac{b}{1-p} + (1-t) \frac{1}{2}. \tag{4.7}$$

While the bias b has no impact on the slope (α) of the equilibrium recommendation, it has a more than one-to-one impact on the intercept (β): an increase in b results in a more than unitary increase in β , since $\partial\beta(p, t, b)/\partial b = 1/(1-p) > 1$. Notice that $\alpha(1) = 1$ and $\beta(p, 1, b) = b/(1-p)$ as expected.

The expected payoff of the principal is

$$V_C^P(p, t) = -\frac{1-t^2}{12} - \frac{b^2}{1-p}, \tag{4.8}$$

as computed in the Appendix. The expected payoff of the agent is

$$\begin{aligned} V_C^A(p, t) &= -p \left(\frac{1}{12} (1-t^2) + b^2 \right) - (1-p) \left(\frac{1}{12} (1-t^2) + \frac{p^2 b^2}{(1-p)^2} \right) \\ &= -\frac{1}{12} (1-t^2) - \frac{p}{1-p} b^2. \end{aligned}$$

4.4.1. Information Acquisition

Consider the following covert information acquisition game. The agent secretly acquires information of quality t and then reports to the principal his recommendation. For t to be the equilibrium level of information acquired by the agent, it must be the case that the agent does not obtain a higher expected payoff by deviating to a different quality of information.

In order to compute the best possible payoff following a deviation to \hat{t} from a putative equilibrium level t . The FOC for the optimal message once information of quality \hat{t} has been acquired is

$$\begin{aligned} & -p \left(\hat{t} \left(a \left(\frac{\hat{m} - \beta}{\alpha} \right) - (s + b) \right) + (1 - \hat{t}) \int_0^1 \left(a \left(\frac{\hat{m} - \beta}{\alpha} \right) - (x + b) \right) dx \right) \\ &= \frac{1}{\frac{\partial}{\partial M} a \left(\frac{\hat{m} - \beta}{\alpha} \right)} (1 - p) \left(\hat{t} (\hat{m} - (s + b)) + (1 - \hat{t}) \int_0^1 (\hat{m} - (x + b)) dx \right) \end{aligned}$$

where

$$a \left(\frac{\hat{m} - \beta}{\alpha} \right) = t \frac{\hat{m} - \beta}{\alpha} + \frac{1 - t}{2}$$

is the action taken by the sophisticated agent when receiving message M . Notice that in equilibrium, i.e. for $\hat{t} = t$, $\hat{m} = m$ so that

$$a \left(\frac{m - \beta}{\alpha} \right) = t \frac{m - \beta}{\alpha} + \frac{1 - t}{2} = ts + \frac{1 - t}{2},$$

as in (4.5).

Substituting

$$\frac{\partial}{\partial \hat{m}} a \left(\frac{\hat{m} - \beta}{\alpha} \right) = \frac{t}{\alpha}$$

and the linear guess for the optimal deviation message strategy $\hat{m}(s) = \hat{\alpha}s + \hat{\beta}$, the coefficients $\hat{\alpha}$ and $\hat{\beta}$ must solve

$$\begin{aligned} & -pt \left(\left(t \frac{\hat{\alpha}s}{\alpha} + \frac{1 - t}{2} - b - t \frac{\beta - \hat{\beta}}{\alpha} \right) - \left(\hat{t}s + (1 - \hat{t}) \frac{1}{2} \right) \right) \\ &= \alpha(1 - p) \left(\hat{\alpha}s + \hat{\beta} - b - \left(\hat{t}s + (1 - \hat{t}) \frac{1}{2} \right) \right) \end{aligned}$$

for all s , so that

$$\begin{aligned} \hat{\alpha} &= \frac{\hat{t}}{t} \alpha = \hat{t} \\ \hat{\beta} &= \frac{b}{1 - p} + (1 - \hat{t}) \frac{1}{2}. \end{aligned}$$

As expected, $\hat{\alpha} = \alpha$ and $\hat{\beta} = \beta$ for $\hat{t} = t$. Furthermore, $\hat{\beta} - \beta = (t - \hat{t})/2$.

We can now compute the expected payoff of the agent when deviating from t to \hat{t} :

$$\begin{aligned} & -p \int_0^1 \left(\hat{t} \left(a \left(\frac{\hat{\alpha}x + \hat{\beta} - \beta}{\alpha} \right) - (x + b) \right)^2 + (1 - \hat{t}) \int_0^1 \left(a \left(\frac{\hat{\alpha}s + \hat{\beta} - \beta}{\alpha} \right) - (x + b) \right)^2 ds \right) dx \\ & - (1 - p) \int_0^1 \left(\hat{t} \left(\hat{\alpha}x + \hat{\beta} - (x + b) \right)^2 + (1 - \hat{t}) \int_0^1 \left(\hat{\alpha}s + \hat{\beta} - (x + b) \right)^2 ds \right) dx \end{aligned}$$

Notice that

$$\frac{\hat{\alpha}x}{\alpha} + \frac{\hat{\beta} - \beta}{\alpha} = \frac{1}{t} \left(\hat{t}x + (t - \hat{t}) \frac{1}{2} \right)$$

so that

$$a \left(\frac{\hat{\alpha}x + \hat{\beta} - \beta}{\alpha} \right) = \hat{t}x + (t - \hat{t}) \frac{1}{2} + \frac{(1 - t)}{2} = \hat{t}x + \frac{(1 - \hat{t})}{2}.$$

We conclude that in the uniform-quadratic example the level of information acquired in the game with covert information acquisition is the same as that acquired in the game with overt information acquisition. The professional objective (quadratic loss function of the adviser) solves automatically the moral hazard problem in information acquisition. We do not expect this result to generalize beyond the uniform-quadratic specification.

5. Uncertain Bias

This Section considers briefly the case where the bias of the agent is unknown to the principal. In this case, the agent's type has two dimensions. Morgan and Stocken (2000) provide the first analysis of the cheap-talk equilibrium in the uniform-quadratic model also adopted in the present paper. They look at the case where the agent's bias b is either positive $b = \beta > 0$ or zero. In this case, they show that there cannot be a subset $[\underline{x}, \bar{x}] \subseteq [0, 1]$ of the support of the state with $\bar{x} > \beta$ where the principal's response is continuous in the state. The equilibrium cannot be fully revealing when the agent has a strictly positive (or negative) expected bias. We conjecture that Dessein's finding of the limit domination of simple delegation over cheap talk to be robust to the introduction of uncertainty around a positive bias.

Often the principal does not know the direction of the bias of the agent. We now consider the case where the agent is unbiased in expectation and we show that the cheap-talk equilibrium dominates simple delegation. This result is easily understood once we notice that the cheap talk equilibrium achieves the constrained delegation solution.

First, consider the payoff achieved with simple delegation. Let b be distributed according to $f(b)$ with support B . Then

$$V_D^P(b) = - \int_B \left(\int_0^1 (a_D(x, b) - x)^2 \right) f(b) db$$

where $a_D(x, b) = a^A(x, b) = x + b$ under simple delegation. The expected payoff of the principal is $V_D^P(b) = -E[b^2]$. For the rest of this Section, consider the case where the bias has a binary distribution symmetric around 0, $\Pr(\tilde{b} = b) = \Pr(\tilde{b} = -b) = 1/2$, so that $E[\tilde{b}] = 0$, $Var[\tilde{b}] = b^2$, and $E[\tilde{b}^2] = b^2$. Simple delegation then results in payoff $V_D^P(b) = -b^2$.

Second, we characterize the constrained delegation solution. As shown by Armstrong (1994), in this case the principal should optimally constrain the agent to pick an action in the set $[c, d] \subseteq [0, 1]$. Then the agent would take action

$$a_{CD}(x, b) = \begin{cases} c & \text{if } x + b \leq c \\ x + b & \text{if } c \leq x + b \leq d \\ d & \text{if } d \leq x + b \end{cases},$$

so that

$$V_{CD}^P(b) = - \int_B \left(\int_0^1 (a_{CD}(x, b) - x)^2 dx \right) f(b) db$$

where

$$\begin{aligned} \int_0^1 (a_{CD}(x, b) - x)^2 dx &= \int_0^{c-b} (c - x)^2 dx + \int_{c-b}^{d-b} (b)^2 dx + \int_{d-b}^1 (d - x)^2 dx \\ &= \frac{1}{3}c^3 + (d - c)b^2 - \frac{1}{3}d^3 + d^2 - d + \frac{1}{3} \end{aligned}$$

Finally, it is immediate to see that for $b \leq 1/2$ the optimal bounds on discretion are $c = b$ and $d = 1 - b$, where

$$V_{CD}^P(b) = -b^2 + \frac{4}{3}b^3.$$

Finally, there exists a cheap-talk equilibrium which implements exactly this constrained delegation solution. In this equilibrium, the agent with bias b reports

$$m_b(x) = \begin{cases} x + b & \text{for } x \in [0, 1 - 2b] \\ 1 - b & \text{for } x \in [1 - 2b, 1] \end{cases}$$

and the one with bias $-b$ reports

$$m_{-b}(x) = \begin{cases} b & \text{for } x \in [0, 2b] \\ x - b & \text{for } x \in [2b, 1]. \end{cases}$$

The principal's action is

$$a(m) = \begin{cases} b & \text{for } m < b \\ m & \text{for } m \in [b, 1 - b] \\ 1 - b & \text{for } m > b. \end{cases}$$

It is immediate to verify that this is an equilibrium for $b \leq 1/2$.

In this example the constrained delegation outcome can be achieved as a cheap-talk equilibrium. Therefore, cheap talk dominates simple delegation even as b goes to zero. Notice that this model is similar in spirit to Aghion and Tirole (1997). Communication is quite good in this setting, when the quality of the agent's information is taken as given. Yet, delegation might give better incentives for information acquisition to the agent, as informally argued by Holmström (1977). This is confirmed in Aghion and Tirole's model where also the principal has private information.

6. Explicit Incentives

This section enriches the basic model of advice, by allowing the principal to reward the agent through monetary transfers. Even if monetary transfers are typically made to experts based on their recommendations, the only paper which look at communication in settings with transferable utility is Baron (1999). Even if his paper is cast in the political science setting of Gilligan and Krehbiel (1987) and (1989), his model and findings are rather similar to ours. In an economic setting like ours, it is even more natural to use transfers to give incentives to the agent. The effects of allowing statement contingent transfers can be more easily understood within our simpler and less structured model.

The agent is perfectly informed and the principal is sophisticated with probability $p \in [0, 1]$. Preferences are quadratic with respect to the decision and linear in transfers

$$U^P(a, x, T) = -(a - x)^2 - T$$

and

$$U^A(a, x, T) = -(a - (x + b))^2 + T,$$

where T denotes the monetary transfer from P to A .

We first characterize the best transfer scheme $T(\cdot)$ from the point of view of P , in the polar cases of pure cheap-talk and delegation. Both cases impose additional restrictions with respect to the general Principal-Agent problem in which P chooses the action and transfer schemes $a(\cdot)$ and $T(\cdot)$ to maximize his utility subject to incentive compatibility, individual rationality, and possibly limited liability constraints. This general Principal-Agent problem is the constrained delegation problem with explicit transfers. Under pure cheap-talk, the action scheme must be such that actions are ex-post optimal for P given

his beliefs about x , while under delegation, the incentive compatibility constraint is more stringent: given x , the agent must prefer $a(x)$ to any feasible action (not just to any action $a(x')$ lying in the image of $a(\cdot)$). Therefore, it is still the case that constrained delegation dominates both simple delegation and cheap-talk, albeit for different reasons. Thus we should expect no general ranking between simple delegation and cheap-talk in general. However, it will be shown that in the uniform-quadratic model of this paper, delegation always dominates fully-revealing cheap-talk. It would be interesting to know how close simple delegation (with transfers) is from the constrained delegation solution (with transfers).

6.1. Pure Cheap Talk ($p = 1$)

The sophisticated principal rewards the agent by using a monetary transfer scheme $T(\cdot)$ conditional on the message reported by A . The timing of the game is as follows:

1. P designs a message space M and a transfer scheme $T(\cdot)$;
2. A decides whether to reject (in which case no communication takes place, and P takes his ex ante most preferred action) or accept the offer. In the latter case, a communication game is played according to the following two steps:
3. A observes x and reports a message m ;
4. P observes m , implements a transfer $T(m)$ and picks an action a .

The utilities of P and A given state x , message m and action a are then:

$$U^P(a, x, m) = -(a - x)^2 - T(m)$$

and

$$U^A(a, x, m) = -(a - (x + b))^2 + T(m)$$

6.1.1. Fully Revealing Cheap-talk Equilibria

We characterize transfer schemes such that fully revealing cheap-talk equilibria exist.

Result 4 *For a fully revealing equilibrium of the cheap-talk game to exist, the transfer scheme $T(\cdot)$ must be linear as a function of the state x : $T(x) = -2bx + \lambda$, where λ is a real number.*

Proof: Consider a transfer function $T(\cdot)$ for which there exists a fully revealing equilibrium of the communication game. The incentive compatibility constraint of the agent is

$$m(x) \in \arg \max_m \{-[m^{-1}(m) - (x + b)]^2 + T(m)\} \quad \forall x \in [0, 1]$$

The first-order condition for this program is

$$m'(x)T'(m(x)) = -2b, \quad \forall x \in [0, 1]$$

or $(T \circ m)'(x) = -2b$. This shows that T must be a linear function of x : $T(x) = -2bx + \lambda$, with $\lambda \in \mathfrak{R}$. The second-order condition is then also satisfied. \square

Note that $m(\cdot)$ can be any strictly increasing function: the words themselves do not matter, provided that parties are able to decode. In particular, truth-telling ($m(x) = x$) is a possibility. The constant λ then obtains from an individual rationality constraint. We consider two cases. One where limited liability (i.e. $T \geq 0$) is the relevant constraint, and one with unlimited liability and an individual rationality constraint. We concentrate on fully revealing equilibria, thus linear transfers as above.

6.1.2. Individual Rationality

First, consider the case where the agent has the option to refuse the contract before observing x . In this case, P would implement his ex ante optimal action, that is $1/2$, and A would obtain utility $\bar{U}^A = -\int_0^1 (1/2 - x - b)^2 dx = -b^2 - 1/12$. The individual rationality constraint is

$$-b^2 + \int_0^1 (\lambda - 2bx) dx \geq -b^2 - \frac{1}{12},$$

or

$$\lambda \geq b - \frac{1}{12}.$$

Second, consider the situation where the agent may opt out of the contract after observing x . In this case, the individual rationality constraint has to be satisfied state by state:

$$-(b)^2 + (\lambda - 2bx) \geq -\left(\frac{1}{2} - x - b\right)^2$$

or $\lambda \geq b - 1/4 + x(1 - x)$, for all $x \in [0, 1]$, that is

$$\lambda \geq b.$$

Which individual rationality constraint should be considered depends on the situation analyzed. In what follows, we focus on the first individual rationality constraint.

6.1.3. Unlimited Liability

When the transfer T can take negative values, it is only constrained by individual rationality. To maximize his own utility, P would then choose $\lambda = b - 1/12$, thereby achieving utility

$$V_{CT-UL}^P(b) = \frac{1}{12}$$

This high level of utility for P is achieved by threatening the agent to take the ex ante optimal action $1/2$, so that A is ready to pay (i.e. $T \leq 0$) in order to avoid that situation.

6.1.4. Limited Liability

If on the contrary transfers from P to A must be positive, this limited liability constraint is $\lambda \geq 2b$, which implies the individual rationality constraint. The cheapest transfer scheme for P is then obtained for $\lambda = 2b$, that is:

$$T(x) = 2b(1 - x)$$

The corresponding utility achieved by P is thus: $V_{CT-LL}^P(b) = 0 - \int_0^1 2b(1 - x)dx$, or

$$V_{CT-LL}^P(b) = -b$$

6.2. Delegation ($p = 0$)

In the delegation case, the principal (or, perhaps rather, the regulator) sets a transfer function $T(a)$ from P to A , which specifies how much is to be paid to the agent if action a is implemented. For simplicity, we concentrate on linear transfer schemes: $T(a) = \alpha a + \beta$.

6.2.1. Agent's Behavior

Given this transfer scheme, the agent's problem is

$$\max_{a \in \mathfrak{R}} \{-(a - (x + b))^2 + \alpha a + \beta\},$$

solved by

$$a^A(x) = x + b + \frac{\alpha}{2}$$

As above, we distinguish two cases, limited and unlimited liability.

Unlimited Liability The individual rationality constraint (which holds with equality, to maximize P 's utility) is

$$\int_0^1 \left[T(a^A(x)) - \left(\frac{\alpha}{2}\right)^2 \right] dx \geq \bar{U}^A = -b^2 - \frac{1}{12}.$$

This and the maximization of P 's utility yield $\beta = -\frac{1}{12} - b^2 - \frac{\alpha}{2} - \alpha b - \frac{\alpha^2}{4}$. Thus P 's utility is $V_{D-UL}^P(\alpha, b) = 1/12 - \alpha b - \alpha^2/2$, maximized by $\alpha = -b$ (in which case $\beta = -1/12 + b/2 - b^2/4$). Finally, P achieves utility

$$V_{D-IR}^P(b) = \frac{1}{12} + \frac{b^2}{2}.$$

We conclude:

Result 5 *With transferable utility and unlimited liability, delegation dominates (fully revealing) cheap talk.*

Limited Liability Limited liability imposes that for all values of $x \in [0, 1]$, $T(a^A(x)) \geq 0$. If $\alpha \geq 0$, this condition and the maximization of P 's utility imply that $T(a^A(0)) = 0$, or $\beta = -\alpha(b + \alpha/2)$. If $\alpha \leq 0$, they imply $T(a^A(1)) = 0$, or $\beta = -\alpha(1 + b + \alpha/2)$. The utility of P is then

$$V_{D-LL}^P(\alpha, \beta, b) = -\left(b + \frac{\alpha}{2}\right)^2 - \beta - \alpha\left(b + \frac{\alpha}{2} + \frac{1}{2}\right)$$

For $\alpha \geq 0$, this expression becomes $V_{D-LL}^P(\alpha, b) = -b^2 - \alpha^2/4 - b\alpha - \alpha/2$, which is maximized for $\alpha = 0$, leading to utility $V_{D-LL}^P(b) = -b^2$. When instead $\alpha \leq 0$, the expression becomes $V_{D-LL}^P(\alpha, b) = -b^2 - \alpha^2/4 - b\alpha + \alpha/2$, which is maximized for $\alpha = 1 - 2b$ when $b \geq 1/2$, and for $\alpha = 0$ when $b < 1/2$.

Result 6 *The utility achieved by P is*

$$V_{D-LL}^P(b) = \begin{cases} -b^2 & \text{for } b < \frac{1}{2} \\ \frac{1}{4} - b & \text{for } b \geq \frac{1}{2}. \end{cases}$$

Corollary 1 *With transferable utility and limited liability, delegation dominates (fully revealing) cheap-talk communication.*

Within the uniform-quadratic specification of this paper, allowing for transfers does not modify the fact that delegation dominates cheap-talk communication. However, transfers do matter since they allow P to achieve a much higher level of utility, in both the delegation and cheap-talk cases. It can also be shown that with unlimited liability, if individual rationality has to hold type by type (i.e., for each x), then cheap-talk and delegation allow P to achieve the same level of utility $V^P = 0$. It is to be noted, too, that under delegation, if the principal is naive enough not to be able to build a transfer scheme, we are back to delegation without transfers, which may be dominated by cheap-talk (with transfers), at least in the unlimited liability case.

6.3. Sophisticated and Naive Principals ($p \in (0, 1)$)

We now consider the more general optimal transfer problem where P is sophisticated with probability p . Given $T(\cdot)$, the agent observes the state x and makes a recommendation $m(x)$. The naive principal would then follow the recommendation $a^{NP}(x) = m(x)$ and pay correspondingly $T(m(x))$, while the sophisticated principal updates his prior (given the agent's equilibrium strategy), takes the ex-post optimal action $a^{SP}(x)$ and pays the corresponding transfer $T(a^{SP}(x))$.

Consider fully revealing equilibria, where a sophisticated principal infers x from $m(x)$ with linear transfer scheme

$$T(m) = \alpha m + \beta.$$

A sophisticated principal would then infer the value of x from $m(x)$ and take action $a^{SP}(x)$ so as to maximize $\{-(a-x)^2 - \alpha a - \beta\}$. In a fully revealing equilibrium,

$$a^{SP}(x) = x - \frac{\alpha}{2}.$$

The incentive compatibility constraint for the agent is then

$$m(x) \in \arg \max \{ -p[(m^{-1}(m) - x - b)^2 + \alpha a^{SP}(m^{-1}(m)) + \beta] \\ - (1-p)[(m - x - b)^2 + \alpha m + \beta] \}$$

$\forall x \in [0, 1]$. The first-order condition is

$$p \frac{b - \frac{\alpha}{2}}{m'(x)} = (1-p)[m(x) - x - b - \frac{\alpha}{2}].$$

This differential equation is solved as in Lemma ???. In particular, it admits a linear solution

$$m(x) = x + \frac{1}{1-p} \left(b + \frac{\alpha}{2} \right),$$

which satisfies also the second-order condition. Notice the possible role of the transfer in reducing (if $\alpha < 0$) the discrepancy between the true state and the recommendation of the agent. As in the special cases of pure cheap-talk and delegation, the optimal transfer is then obtained by maximizing the objective function of the social planner subject to the relevant limited liability or individual rationality constraints.

7. Independent Financial Advice

We now illustrate how the quadratic preferences can be derived from first principles in a model of independent financial advice. There is an investor (or principal P), an independent financial adviser (or agent A), and a provider of a financial product (PP).

The investor has initial wealth normalized to 1, and mean-variance preferences $U^P(\tilde{w}) = \mathbf{E}[\tilde{w}] - \frac{1}{2}\gamma V(\tilde{w})$, where w is wealth.⁵ The parameter γ represents P 's attitude towards risk. There is a risk-free asset with return normalized to one and a risky asset with return \tilde{r} , with mean μ and variance σ^2 . The investor has to decide what fraction a of wealth to invest in the risky asset. The product provider charges directly to the investor a managing commission ca , proportional to the amount invested a . The financial adviser charges $f + da$ to the investor, i.e. a fixed fee f and a percentage (possibly negative) commission d . The product provider gives a sale commission of ba to the adviser. The wealth of the investor when investing a in the risky asset is then

$$\tilde{w} = (1 - a - ca - f - da) + a\tilde{r}$$

Given μ and σ^2 , the optimal action x for the investor solves

$$x = \arg \max_a [(1 - a - ca - f - da) + a\mu] - \frac{1}{2}\gamma a^2 \sigma^2$$

An investor who knows all the parameters of the problem would therefore invest a fraction

$$x = \frac{\mu - (1 + c + d)}{\gamma \sigma^2}$$

in the risky asset. The optimal action x is also referred to as the “state of the world”.

We are interested in situations where the investor does not possess all the relevant information to make the optimal decision. For example, both μ and σ^2 may be unknown to P . The investor consults a financial adviser who has superior information about these parameters. In order to obtain a simple one-dimensional problem, we consider cases where only the mean is unknown, assuming that $\gamma = \sigma^2 = 1$. In this case, P 's utility can be written as a function of the action a and of the optimal action $x = \mu - (1 + c + d)$ as

$$U^P(a, x) = -\frac{1}{2}(a - x)^2 + 1 - f + \frac{1}{2}x^2. \quad (7.1)$$

Appropriate choice of the support of μ amounts to assuming that x is uniformly distributed in the interval $[0, 1]$. More generally, a different distribution of the asset's mean (or variance) determines a different distribution of x . The model could then be analyzed with the same method.

Consider next the preferences of the independent adviser who has both a professional and a partisan objective. On the one hand, professional ethics and incentives dictate the adviser to provide the best advice to the investor. We model the professional advice by assuming that the net utility of the investor enters the utility of the advisor with a weight

⁵For example, assume exponential utility (for P) and normally distributed returns.

λ . The adviser also cares about the monetary transfers received from the investor and the product provider

$$U^A(a, x) = \lambda U^P(a, x) + (f + da) + ba.$$

Substitution of (7.1) gives

$$U^A(a, x) = -\frac{\lambda}{2} \left(a - \left(x + \frac{d+b}{\lambda} \right) \right)^2 + \lambda \left(\frac{1}{2} \left(\frac{d+b}{\lambda} \right)^2 + x \left(\frac{d+b}{\lambda} \right) + 1 - f + \frac{f}{\lambda} + \frac{x^2}{2} \right).$$

for $\lambda \neq 0$. In this setting, the bias $(d+b)/\lambda$ of the agent is explicitly derived from the transfers received. The model can easily be extended to endogenize the partisan bias of the adviser via the optimal choice of the commission b by the product provider. Notice that there is no interaction between a and x in the second addend of the expressions of the preferences of the agent. The analysis of the cheap talk and communication equilibrium in this model then replicates the one of the quadratic model analyzed in this paper.

8. Conclusion

Our investigation on the economics of advice is motivated by the need to have a good economic model of financial advice. The insights gained in this exercise have implications also for other applications of models of cheap talk communication. Cheap-talk models of communications have been rather influential in political science and political economy (see e.g. Austen-Smith and Riker (1987), Gilligan and Krehbiel (1987), (1989), (1990), Austen-Smith (1990), (1992), (1993), and Banks (1991) for an early overview). For example, models of cheap talk communications have been used to compare the relative desirability of open and closed rules in legislative committees. According to the open rule, the floor votes between a status quo and an alternative policy selected by the floor after listening to the recommendation of an expert legislative committee. The open rule is therefore analogous to cheap talk. According to the closed rule, the expert committee proposes directly the alternative policy to the floor for voting against the status quo. The choice of the alternative is therefore delegated to the informed party.

Finally, it is natural to ask what would happen if more than one adviser were consulted. If each adviser is perfectly informed, the only advantage from consulting more than one is to discipline the others who would change strategically their behavior. Clearly, if perfect revelation results with one (perfectly informed) adviser there is no reason to consult another adviser. Krishna and Morgan (1998) show that when perfectly informed experts are consulted sequentially, full revelation is not the typical outcome. Battaglini (1999) shows that full revelation is instead the generic outcome (in a multi-dimensional environment) when the consultation is simultaneous. It is natural to conjecture that the pure cheap-talk equilibrium would be coarse when consulting multiple experts with noisy information (e.g.

with the information structure we have specified in the Section 4). Simultaneous consultation of more than one imperfectly informed expert allows the decision maker to have access to more information not only because of the addition of new information, but also because competition among experts disciplines them to reveal more information.

9. Appendix

9.1. Derivation of the Expected Payoff of the Principal with Cheap Talk (4.2)

The expected payoff of the principal with cheap talk is equal to

$$\begin{aligned} & \sum_{i=1}^N \int_{s_{i-1}}^{s_i} \left(\frac{1}{3} - A + A^2 \right) (1-t) ds - t \sum_{i=1}^N \int_{s_{i-1}}^{s_i} (A-s)^2 ds \\ &= (1-t) \sum_{i=1}^N \left(\frac{1}{3} - A + A^2 \right) (s_i - s_{i-1}) + \\ & \quad t \sum_{i=1}^N \left(\frac{1}{3} (s_i^3 - s_{i-1}^3) - A (s_i^2 - s_{i-1}^2) + A^2 (s_i - s_{i-1}) \right). \end{aligned}$$

where $A = (s_i + s_{i+1})/2$.

To compute the first addend, notice that

$$\begin{aligned} & \left(\frac{1}{3} - \frac{s_i + s_{i-1}}{2} + \left(\frac{s_i + s_{i-1}}{2} \right)^2 \right) (s_i - s_{i-1}) \\ &= \frac{1}{12} (4(s_i - s_{i-1}) - 6(s_i^2 - s_{i-1}^2) + 3(s_i + s_{i-1})^2 (s_i - s_{i-1})) \end{aligned}$$

where

$$\begin{aligned} s_i &= \frac{i}{N} + \frac{2bi(i-N)}{t} \\ s_{i-1} &= \frac{i-1}{N} + \frac{2b(i-1)(i-1-N)}{t} \end{aligned}$$

we have

$$s_j - s_{j-1} = \frac{1}{N} + \frac{2b(2i-1-N)}{t}$$

and similarly

$$s_j + s_{j-1} = \frac{2j-1}{N} + \frac{2b((2j-1)(j-N) - (j-1))}{t}$$

It is useful to compute

$$\sum_{j=1}^N (s_j - s_{j-1}) = \sum_{j=1}^N (s_j^2 - s_{j-1}^2) = 1,$$

$$\sum_{i=1}^N (s_j + s_{j-1})^2 (s_j - s_{j-1}) = \frac{1}{3} \left(\frac{4(1 + b^2(1 - N^2))}{t^2} - \frac{1}{N^2} \right).$$

$$\sum_{j=1}^N (s_j^3 - s_{j-1}^3) = \sum_{j=1}^N \left(\frac{1}{N} + \frac{2b(2j - N - 1)}{t} \right)^3 = \frac{1}{N^2} + \frac{4b^2(N^2 - 1)}{t^2}$$

so that the first addend is then

$$(1 - t) \left(\frac{1}{12} \left(2 - \frac{1}{N^2} \right) + \frac{b^2(1 - N^2)}{3t^2} \right). \quad (9.1)$$

The second addend is the usual one, which can be shown to be equal to

$$t \left(\frac{1}{12N^2} + \frac{b^2(N^2 - 1)}{3t^2} \right), \quad (9.2)$$

as in (3.1).

Summing (9.1) and (9.2), the expected payoff of the principal is

$$V_{CT}^P(b, t) = -(1 - t) \left(\frac{1}{12} \left(2 - \frac{1}{N^2} \right) + \frac{b^2(1 - N^2)}{3t^2} \right) - t \left(\frac{1}{12N^2} + \frac{b^2(N^2 - 1)}{3t^2} \right).$$

which is equal to (4.2).

9.2. Derivation of the Expected Payoff of the Principal with Communication (4.8)

The expected payoff of the sophisticated principal is

$$- \int_0^1 \left[\int_0^1 (a(s) - x)^2 dF(s|x) \right] dF(x), \quad (9.3)$$

which can be rewritten as

$$\begin{aligned} & - \int_0^1 \left[t(a(x) - x)^2 + \int_0^1 (a(s) - x)^2 (1 - t) ds \right] dx \\ &= - \int_0^1 \left[t \left(tx + \frac{1-t}{2} - x \right)^2 + \int_0^1 \left(ts + \frac{1-t}{2} - x \right)^2 (1-t) ds \right] dx \\ &= - \frac{1}{12} (1 - t^2). \end{aligned}$$

Similarly, the expected payoff of the naive principal is

$$- \int_0^1 \left(t(m(x) - x)^2 + \int_0^1 (m(s) - x)^2 (1 - t) ds \right) dx$$

with $m(s) = \alpha s + \beta$, i.e.

$$-t \int_0^1 (\alpha x + \beta - x)^2 dx - (1-t) \int_0^1 \left(\int_0^1 (\alpha s + \beta - x)^2 ds \right) dx$$

Using

$$\int_0^1 (Ax + B)^2 dx = \frac{1}{3}A^2 + AB + B^2,$$

the first addend is computed to be

$$-t \int_0^1 ((\alpha - 1)x + \beta)^2 dx = -t \left(\frac{1}{3}(\alpha - 1)^2 + (\alpha - 1)\beta + \beta^2 \right)$$

Similarly, the inner integral in the second addend is

$$\int_0^1 (\alpha s + \beta - x)^2 ds = x^2 - \underbrace{(\alpha + 2\beta)x}_D + \underbrace{\frac{\alpha^2}{3} + \alpha\beta + \beta^2}_E$$

Noticing that

$$\int_0^1 (Cx^2 + Dx + E) dx = \frac{1}{3}C + \frac{1}{2}D + E,$$

the outer integral is then

$$\int_0^1 \left(\int_0^1 (\alpha s + \beta - x)^2 ds \right) dx = \frac{1}{3} - \frac{1}{2}(\alpha + 2\beta) + \left(\frac{\alpha^2}{3} + \alpha\beta + \beta^2 \right)$$

The expected payoff of the naive principal is then

$$= \frac{\alpha t}{6} - \left(-\frac{\alpha}{2} + \frac{1}{3} + \frac{\alpha^2}{3} + (\alpha - 1)\beta + \beta^2 \right) \quad (9.4)$$

After substituting the values of α and β from (4.6) and (4.7) this is

$$\frac{1}{12} (1 - t^2) + \frac{b^2}{(1 - p)^2},$$

so that the expected payoff of the principal is (4.8).

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