Separate or Joint Financing? Optimal Conglomeration with Bankruptcy Costs

Albert Banal-Estañol* Marco Ottaviani†

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Abstract

This paper analyzes how to optimally group a multiple number of possibly heterogeneous risky projects into separately financed conglomerates. The projects grouped within the same conglomerate are financed jointly, but they are insulated from the projects that are financed in other conglomerates. The optimal conglomeration structure trades off the benefits of coinsurance and the costs of risk-contamination associated with joint financing. We characterize the resolution of this tradeoff depending on the distributional characteristics of project returns, the structure of bankruptcy costs, and the tax advantage of debt relative to equity. Our predictions shed light on the optimal scope of incorporation as well as on the role of financial intermediation.

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*Department of Economics and Business, Universitat Pompeu Fabra and Department of Economics, City University of London, Northampton Square, London EC1V 0HB, UK. Phone: +44–20–7040–4576. E-mail: a.banal-estanol@city.ac.uk.

†Department of Economics, Bocconi University, Via Roberto Sarfatti 25, 20136 Milan, Italy. Phone: +39–02–5836–3385. E-mail: marco.ottaviani@unibocconi.it.

1 Introduction

Which affiliates should a firm hold and finance as separately incorporated subsidiaries and which others as branches, whose liabilities represent claims on the parent institution? Should a financial institution structure itself as an integrated entity subject to a consolidated balance sheet, as in the case of a universal bank, or as a holding company where some of the divisions are separately capitalized? Similarly, in which firms should investors finance directly and for which firms should they make use of financial intermediaries?

This paper analyzes which risky projects (or groups of risky projects) should be financed jointly and which projects should be financed separately. The model builds on the classic costly state verification (CSV) setting in which the firm *privately observe* the ex post return realization. In the baseline specification the firm has no private funds available and so must obtain financing exclusively from outside investors that operate in a competitive financial market. Investors can observe the return realization by paying an ex post verification cost. As shown by Townsend (1979) and Gale and Hellwig (1985), in this CSV setting debt is the optimal financial arrangement. Thus, verification costs can be conveniently re-interpreted as bankruptcy costs.

When multiple projects are financed within the same corporation we naturally assume that only the *sum* of the returns of these projects is contractible. The optimal conglomeration structure is then the one that minimizes the total expected verification (or bankruptcy) costs. As we show, the optimal structure sometimes involves partial conglomeration, contrary to claims often made in the literature. For example, in his classic analysis of intermediation Diamond (1984, page 400) claims:

> "The per-entrepreneur cost of providing incentives to the intermediary is reduced as it contracts with more entrepreneurs with independently distributed projects. With independent and identically distributed projects, the per-entrepreneur cost, D_N, is a monotonically decreasing function of the number of entrepreneurs, N, because deadweight penalties are incurred when returns are in the extreme lower tail, and the probability of the

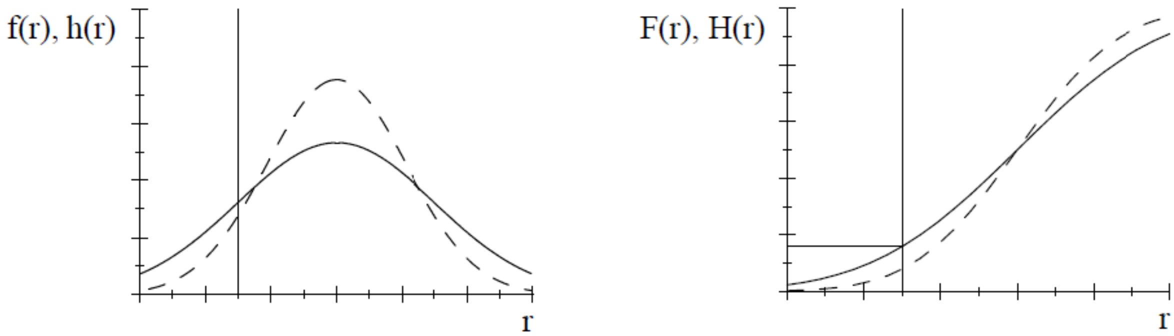

Figure 1: **Comparison of Bankruptcy Probabilities for an Exogenous Debt Obligation.** In the left panel, the continuous curve represents the probability density of returns when the projects are financed separately, while the dashed curve represents the density associated to the average return when the projects are financed jointly. In the right panel, the continuous curve represents the cumulative distribution of returns when the projects are financed separately, while the dashed curve represents the distribution associated to the average return when the projects are financed jointly.

average return across projects being in that tails is monotonically decreasing."

As we show, this monotonicity claim is not valid in general and only holds under additional assumptions that are not made by Diamond (1984). To understand this, consider a firm with two ex-ante identical projects with normal returns, as depicted in the solid curve in Figure 1. If the projects are financed separately, the probability of bankruptcy for each of them is equal to the area below the density of returns that lies to the left of the vertical line, corresponding to the outstanding debt obligation. When the projects are financed jointly, the relevant distribution is the dashed curve, representing the distribution of the average returns of the two projects. Note that the density of the average of two identical normals is more peaked around the mean than the density of the individual returns and therefore the distribution of the average is below the individual distribution for returns below the mean and above it for returns above the mean.

If the interest rate is held constant at the level depicted in the figure, the probability of default is reduced with joint financing. In addition, given that the probability of default is reduced by joint financing, creditors are forced to reduce the gross interest payment further, so that the default probability of joint financing in equilibrium is even lower. This is the logic of good conglomeration typically credited to Lewellen (1971) and used by Diamond (1984). The logic, however, is clearly reversed if the original gross interest is above the return at which the densities of returns (individual and average) peak and the distributions *cross*. In this case, the left tail of the distribution is actually higher under joint than separate financing, so that the probability of bankruptcy is increased under joint financing. Diamond (1984) disregards this possibility by implicitly assuming that the (endogenous) per-project repayment obligation with joint financing is always less than the one obtained with separate financing.[1]

Focusing on the case of two ex-ante identical projects, Banal-Estañol, Ottaviani, and Winton (2013) analyze the tradeoff between the coinsurance benefits and the risk-contamination costs of joint financing. This paper characterizes the tradeoff more generally when projects are heterogeneous and when there are multiple projects. We begin by showing that the coinsurance gains and the risk contamination losses may be present *simultaneously* when two binary heterogeneous projects are financed jointly. We characterize situations in which a first project either saves or drags down a second project, depending on whether the first project succeeds or fails. This situation arises when projects differ in their riskiness, measured by second-order stochastic dominance. We show that the relative profitability of separate financing increases in the difference of the riskiness of two projects. This theoretical prediction is in line with empirical findings by Gorton and Souleles (2005) that riskier originator banks are more likely to securitize.

We then characterize situations in which partial conglomeration of multiple projects into subgroups of intermediate size is optimal. By grouping subsets of projects into small conglomerates, some of the benefits of coinsurance can be obtained while also containing the costs of risk contamination. Nevertheless, using an argument based on the law of large numbers we can show that

[1] When illustrating this result as the number of entrepreneurs increases from one to two, Diamond (1984) implicitly assumes away bad conglomeration by focusing on the case in which the repayment obligation when using the intermediary (i.e., with joint financing) is less than twice the one obtained without the intermediary (with separate financing). His analysis, however, disregards the possibility of bad conglomeration. As we show in this paper, if the repayment rate with joint financing rate is above the crossing point, conglomeration is bad even when the intermediary can observe freely the entrepreneurs' returns.

joint financing dominates separate financing when the number of independent and identical projects becomes arbitrarily large. In the limit with an infinite number of projects, the risk-contamination effect vanishes and it becomes optimal to finance all the projects jointly. Taken together, our results say that joint financing becomes unambiguously better as the number of projects increases only in the limit case of many projects.

In our main specification, bankruptcy costs are proportional to the value of the assets under bankruptcy, as is often assumed in the theoretical and empirical literature. In a more general model with variable returns to scale in bankruptcy costs, we show that economies of scale (according to which per-project bankruptcy costs are lower when projects are financed jointly) favor joint financing, while diseconomies of scale favor separate financing. Nevertheless, our main results on the optimality of separate financing are robust to the introduction of mild (dis)economies of scale in bankruptcy costs. We also show that the logic of risk contamination still applies when bankruptcy costs depend on the number of projects that go bankrupt rather than on the value of assets under bankruptcy. In fact, separate financing is optimal for a larger set of parameters because it becomes easier to obtain joint financing, but only at a rate above the crossing point.

While for the bulk of our analysis we restrict financing to be obtained through debt, in an extension we allow for financing through outside equity in addition to debt.[2] As in the trade-off theory of capital structure, equity saves on bankruptcy costs but is subject to higher taxation. We show that if the incremental tax advantage of debt is sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether under either joint or separate financing. If the tax advantage is somewhat higher, joint financing becomes more profitable than in the baseline model, because equity financing makes it possible to obtain a repayment rate below the crossing point. Finally, if the tax advantage is sufficiently high, separate and joint financing are profitable in the same situations as in the main specification, because then no equity is used in either financing regime.

[2] As we discuss in the next section, the costly state verification literature shows that debt is the optimal contractual arrangement if returns are privately observed by the borrower and can be verified by creditors only by inducing bankruptcy and incurring the bankruptcy costs.

In our simple model with binary project returns, whenever separate financing is more profitable than joint financing, only debt financing is used. Equity is more expensive and is only used if it helps to obtain a repayment rate that decreases the probability of default, in which case joint financing is optimal. This dominance of debt in separate financing is consistent with the many empirical studies that find that a large proportion of funding in project finance is in the form of debt (see, e.g., Kleimeier and Megginson, 2000).

Finally, in the context of a version of the model with normal returns, we identify a simple sufficient condition for the optimality of separate financing. Our comparative statics predictions on the optimal scope of conglomeration are thus robust to a continuous specification of returns. Nevertheless, our baseline specification with binary returns allows us to investigate the role played by asymmetries in the distribution of returns as well as to reach a more thorough understanding and characterization of the determinants of the optimal scope of conglomeration.

The paper proceeds as follows. Section 2 formulates the model. Focusing on the baseline version of the model with two projects, Section 3 analyzes the conditions setting apart good from bad conglomeration and performs comparative statics with respect to the distribution of returns (mean, and variance) and the bankruptcy recovery rate. Generalizing the optimal conglomeration conditions to a setting with multiple projects, Section 4 characterizes situations in which partial conglomeration is profitable and demonstrates that joint financing is optimal when the number of independent projects is sufficiently large. Section 5 shows that our results are robust to different specifications of bankruptcy costs including economies of scale. Section 6 extends the analysis to a setting in which equity financing is available, albeit with a tax disadvantage relative to debt financing. Section 7 characterizes conditions for separate financing to result when projects' returns are normally distributed. Section 8 concludes. The Appendix collects the proofs.

2 Model

This section formulates the simplest possible model to analyze how multiple projects should be optimally financed in the presence of bankruptcy costs. In the rest of the paper we derive results for special cases of this baseline scenario.

A risk-neutral firm has access to n independent projects. Project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random payoff or *return* r^i with a binary distribution: the return is either low, $r^i = r_L^i > 0$, with probability $1 - p_i$, or high, $r^i = r_H^i > r_L^i$, with probability p_i. Each project has a positive net present value, $(1 - p_i)\, r_L^i + p_i r_H^i - 1 > 0$. The low return is insufficient to cover the initial investment outlay, $r_L^i < 1$.

Before raising external finance, the firm chooses how to group projects into corporations, or equivalently into separate nonrecourse loans. This means that investors in each corporation have access to the returns of all projects in that corporation, but they do not have access to the returns of the projects in the other corporations set up by the firm. Financing for each corporation can be obtained in a competitive credit market. For notational simplicity, we stipulate that the firm seeks financing only when expecting to obtain a *strictly* positive expected payoff.

Creditors are risk neutral and lend money through standard debt contracts. Without loss of generality, we normalize the risk-free interest rate to $r_f = 0$. Therefore, creditors expect to make zero expected profits. This is equivalent to assuming that each corporation makes a take-it-or-leave-it repayment offer to a single creditor for each loan j, promising to repay r_j^* at $t = 2$ for each unit borrowed at $t = 1$.[3] Thus r_j^* denotes the promised repayment per project. According to our accounting convention, this repayment rate comprises the amount borrowed as well as net interest.[4]

Creditors are repaid in full when the total realized return of the projects pledged is sufficient to cover the promised repayment. If instead the total realized return falls short of the repayment

[3] Thus, for the case in which each loan (or corporation) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

[4] The net interest rate i satisfies $1 + i = r_j^*$ and therefore the repayment obligation can be interpreted as the gross interest rate.

obligation, the corporation defaults and the ownership of the projects' realized returns is transferred to the creditor. Following default, the creditor is only able to recover a fraction $\gamma \in [0,1]$ of the realized returns r, so that the bankruptcy costs following default are equal to $B(r) = (1-\gamma)\, r$.[5] In Section 5, we show that our results hold robustly with a more general structure of bankruptcy costs, provided that economies or diseconomies of scale in bankruptcy are not too extreme.

For the baseline specification of the model we restrict external financing to be obtained through debt. Note that debt is the optimal contractual arrangement if we assume that returns are privately observed by the borrower and can be verified by creditors only at a cost. In the context of the classic analyses of the costly state verification model (see Townsend, 1979, Diamond, 1984, and Gale and Hellwig, 1985), the verification of returns can be interpreted as a costly bankruptcy process. In this context, they show that the optimal contract turns out to be the standard debt contract under which returns are observed if and only if the borrower cannot repay the loan in full. Once bankruptcy costs are re-interpreted as CSV verification costs, the optimal contractual agreement between the entrepreneur and the creditor is thus a debt contract. That is, if two projects are available, the optimal contracting strategy is either two separate debt contracts, each of which is backed by the returns of one project, or one debt contract, which is backed by the returns of the two projects. In Section 6, we extend the model to also allow for financing through tax-disadvantaged equity.

3 Two Projects

Consider first the case of two projects. Project i, $i = 1, 2$, yields (independent) returns r_H^i with probability p_i and r_L^i with probability $1 - p_i$. Without loss of generality, we assume that $r_H^1 + r_L^2 > r_L^1 + r_H^2$, interchanging the indices if necessary. Note that this is equivalent to $r_H^1 - r_L^1 > r_H^2 - r_L^2$, so that project 1 has a greater spread of possible outcomes than project 2. We have four levels of combined returns. In one of them, default is avoided if project 1 yields a high return and project 2

[5] For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2007).

a low return, whereas default is not avoided if the reverse occurs.

In Section 3.1 we proceed to examine the conditions for when the borrower is able to finance the two projects separately and jointly. In Section 3.2 we compare the profitability of separate and joint financing, when they are both feasible. In Section 3.3 we derive a set of comparative statics predictions for the occurrence of joint and separate financing. Finally, Section 3.4 characterizes situations in which the financing option with the lowest probability of bankruptcy is not optimal.

3.1 Financing Conditions

Consider the case of separate financing first. In order for the creditor to break even, the rate r_i' for each project i must satisfy $r_i' > 1 > r_L$, so that there is a positive probability that the loan is not repaid in full. Given that the credit market is competitive, creditors must make zero expected profits. Thus the repayment requested by the creditor, r_i', is such that the gross expected proceeds, $pr_i' + \gamma(1-p)r_L$, are equal to the initial investment outlay 1. As a result, project i can be financed (at r_i') if and only if

$$r_i' := \frac{1-(1-p_i)\gamma r_L^i}{p_i} < r_H^i. \tag{1}$$

In the case of joint financing, there are three possible rates and therefore three financing conditions. There exists r_m' such that bankruptcy can be avoided if one project's return is high and the other is low, $r_m' \leq (r_L^1 + r_H^2)/2$.[6] If projects are heterogeneous, there exists r_m'' such that bankruptcy can be avoided if project 1's return is high and project 2's is low but not viceversa, $r_m'' \leq (r_H^1 + r_L^2)/2$. Finally, there exists r_m''' such that bankruptcy cannot be avoided if any of the two projects' return is low, which can be obtained if and only if $r_m''' \leq (r_H^1 + r_H^2)/2$. The dotted and dashed lines in Figure **??** depict the cumulative distribution of returns of two heterogeneous projects, whereas the thick line depicts the distribution of the average returns of the two projects. The three possible types of rates correspond to the three flat parts of the average distribution.

[6] The precise expression is included in the Appendix, in the proof of the forthcoming Proposition 1.

3.2 Good and Bad Conglomeration

We now turn to the question of whether the borrower should finance the projects jointly or separately when both financing regimes are feasible. If a rate that avoids bankruptcy in both intermediate situations can be obtained, projects coinsure each other and should be financed jointly. If the firm can only obtain a rate that does not avoid bankruptcy in any of the intermediate situations, projects should be financed separately because they drag each other down. If bankruptcy can only be avoided for the more favorable intermediate situation, then both coinsurance and contamination effects are present at the same time. On the one hand, project 1, when it yields a high return, saves project 2 when project 2 yields a low return; on the other hand, project 1, when it yields a low return, contaminates project 2 when project 2 yields a high return. The optimality of separate or joint financing depends on whether the gains from coinsurance dominate the losses from risk contamination.

Proposition 1 (Separate v. joint financing with two projects) *When the borrower can finance two projects separately as well as jointly, there exist* r'_m, r''_m *and* r'''_m *such that*

(a) If $r'_m \leq (r^1_L + r^2_H)/2$, *it is optimal to finance the projects jointly to enjoy the coinsurance gains:* $(1-p_1)\,p_2(1-\gamma)r^1_L + p_1\,(1-p_2)\,(1-\gamma)r^2_L$.

(b) If $r''_m \leq (r^1_H + r^2_L)/2$ *but* $r'_m > (r^1_L + r^2_H)/2$, *it is optimal to finance the projects separately if and only if the risk-contamination losses dominate the coinsurance gains:* $(1-p_1)p_2(1-\gamma)r^2_H > p_1(1-p_2)(1-\gamma)r^2_L$.

(c) If $r'''_m \leq (r^1_H + r^2_H)/2$ *only is satisfied, it is optimal to finance the projects separately to avoid the risk-contamination losses:* $p_1(1-p_2)(1-\gamma)r^1_H + (1-p_1)p_2(1-\gamma)r^2_H$.

In the new case (b), the probability of default with joint financing is (i) increased by $(1-p_1)p_2$, because a successful project 2 is dragged down by a failing project 1, but (ii) decreased by $p_1(1-p_2)$, because a failing project 2 is saved by a successful project 1. Project 2, however, is saved when it yields a low return but it is dragged down following a high return. Thus, if project 1 has a chance of success that is no greater than that for project 2 ($p_1 \leq p_2$), the risk-contamination effect always dominates the coinsurance effect.

The trade-off between coinsurance and risk contamination in the new case (b) is depicted in Figure **??**. The risk-contamination losses, equal to $(1-p_1)p_2(1-\gamma)r_L^2$, are represented by the light gray area and correspond to the added bankruptcy costs on the high-return project 2 that is dragged down when project 1 has a low return. The coinsurance gains, equal to $p_1(1-p_2)(1-\gamma)r_H^2$, are represented by the gray area and correspond to reduced bankruptcy costs on the low-return project 2 that is saved when project 1 has a high return. For the numerical value used in the figure, it is more profitable to finance the projects separately because the risk-contamination losses are larger than the coinsurance gains.

3.3 Testable Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects (bankruptcy costs, mean, and variability). We show first separate financing becomes more attractive as bankruptcy costs increase.

Prediction 1 (Bankruptcy costs) *For higher bankruptcy costs (lower γ) then (a) both joint and separate financing can be obtained for a smaller region of parameters and (b) joint financing is optimal for a smaller region of the remaining parameters.*

Second, we show that separate financing becomes less attractive when projects have higher returns.

Prediction 2 (Mean) *If project 1 first-order stochastically dominates project 2, and in particular, $r_H^1 = r_H^2$ and $r_L^1 = r_L^2$ and $p_1 > p_2$, for a higher mean of any of the two projects (higher p_1 or p_2), the region of parameters for which joint financing is optimal increases.*

Third, we establish that more risk typically induces more separation when one project is a mean preserving spread of the other.

Prediction 3 (Mean-preserving spread) *If project 2 second-order stochastically dominates project 1 so that* $p_1 = p_2$ *and* $r_H^1 = r_H^2 + \varepsilon$ *and* $r_L^1 = r_L^2 - \frac{p_1}{1-p_1}\varepsilon$ *for* $\varepsilon > 0$*, a higher spread of the risky project (higher* ε*) leads to a decrease in the region of parameters for which joint financing is optimal.*

As explained after Proposition 1, if the probabilities of success are the same joint financing is optimal only if r'_m can be obtained. This condition becomes more stringent as the spread of the risky project increases. Indeed, the less favorable intermediate return $(r_L^1 + r_H^2)$ decreases in the spread of project 1 and the repayment rate (r'_m) increases, as the creditor recovers less in the event of bankruptcy (when both projects yield low returns). In addition, it is easier to finance the projects separately as the increase in the high realization of the return is not fully compensated by the increase in the repayment rate (r_i).

Gorton and Souleles (2005) and Bannier and Hansel (2008) provide evidence that riskier originator banks are more likely to securitize their loans, consistent with our prediction that separate financing is more attractive when the risky project (the bank) is riskier.[7] Similarly, Mills and Newberry (2005) find that nonfinancial firms with greater credit risks are more prone to use off-balance sheet debt.

3.4 Managerial Implications

In this section, we characterize situations in which the financing option with the lowest probability of bankruptcy is not optimal.

Proposition 2 (Separate financing with higher bankruptcy probability) *Separate financing is optimal even though it results in a higher probability of bankruptcy if and only if (i) the risk-contamination losses dominate the coinsurance gains in case (b) of Proposition 1, i.e.,* $(1-p_1)p_2 r_H^2 > p_1(1-p_2) r_L^2$*, but (ii) the probability of dragging down the second project is lower than the probability of saving it, i.e.,* $(1-p_1)p_2 < p_1(1-p_2)$*.*

[7] Of course, these findings are consistent with other explanations. For example, riskier banks may have a higher shadow cost of equity capital, which may make securitization more attractive as a means of conserving costly capital. Also, riskier banks may be more prone to risk-shifting behavior, making it more attractive to shield assets from this through securitization (Kahn and Winton, 2004).

Notice first that if the levels of bankruptcy costs are "small", so that the borrower can finance the two projects jointly at a rate r'_m (case (a) of Proposition 1), then joint financing results in lower probability of bankruptcy than with separate financing ($(1-p_1)(1-p_2)$ as compared to $1-p_1$ and $1-p_2$) and in lower inefficiency losses. If the levels of bankruptcy costs are, instead, "large", so that the borrower can finance the two projects jointly only at a rate r'''_m (case (c) of Proposition 1), then joint financing results in higher probability of bankruptcy ($1-p_1p_2$ as compared to $1-p_1$ and $1-p_2$) and in higher inefficiency losses. In both cases, it is optimal to finance the option (joint or separate) with the lowest probability of bankruptcy.

Suppose now that the levels of bankruptcy costs are "intermediate" so that the borrower can finance the two projects jointly at a rate r''_m but not at a rate r'_m, so that we are in case (b) of Proposition 1. In this case, (i) if project 1 yields a low return, it drags down project 2 if project 2 has a high return (whereas project 2 would have stayed afloat with separate financing) and, at the same time, (ii) if project 1 yields a high return, it saves project 2 if project 2 has a low return (whereas project 2 would have defaulted with separate financing). As shown in Proposition 1, projects should be financed separately if the expected benefits from coinsuring project 2 are dominated by the expected losses from risk-contaminating it. Proposition 2 highlights that the risk-contamination losses can be greater even if the probability of saving project 2 is higher than the probability of contaminating it ($p_1(1-p_2) > p_2(1-p_1)$), given that the losses from dragging down the second project are greater than the gains from saving it ($r^2_H > r^2_L$). This situation is likely to occur if (i) the probability of success of the first project is slightly higher than that of the second project ($p_1 > p_2$), and (ii) the difference in realized returns of the second project is large ($r^2_H >> r^2_L$).

Figure **??** is an example in point. Provided that the joint financing rate is r''_m, the risk-contamination losses, represented by the light gray area, dominate the coinsurance gains, represented by the gray area, and therefore it is more profitable to finance the projects separately, even if the probability of risk-contamination (height of the light gray area) is smaller than the probability of coinsurance (height of the gray area). The borrower might then feel tempted to finance the projects jointly, but this is suboptimal. In this case, a lower probability of bankruptcy associated with joint financing is deceptively attractive.

4 Multiple Projects

In this section, we consider a borrower with access to a general number of identical projects with independent returns. In Section 4.1, we characterize the size (and the number) of the groups that it is optimal to finance jointly, thereby identifying conditions for partial conglomeration. In Section 4.2, we show that if the number of independent projects is sufficiently large, it becomes possible and optimal for the borrower to finance all of them jointly, so that full conglomeration results.

Consider a group with k identically and independently distributed projects. Each project i yields a low return $r_L^i \equiv r_L$ with probability $1-p_i \equiv 1-p$ and a high return $r_H^i \equiv r_H > r_L$ with probability $p_i \equiv p$. Generalizing our baseline analysis for a group with two projects, the per-project repayment rates depend on the number of projects with high return m ($1 \leq m \leq k$) that are necessary to avoid bankruptcy,

$$r_k(m) := \frac{1-\gamma\left[\sum_{s=0}^{m-1} h(s)\frac{sr_H+(k-s)r_L}{k}\right]}{1-H(m-1)}, \tag{2}$$

where $h(s)$ is the probability that s out of the k projects yield a high return,

$$h(s) := \binom{k}{s} p^s (1-p)^{k-s} \text{ for } s = 0, 1, .., k, \tag{3}$$

and $H(s)$ is the corresponding probability distribution, $H(s) := \sum_{t=0}^{s} h(t)$.[8] As before, the equilibrium repayment rate is the one which requires the minimum number of high returns, i.e. $r_k^* = r_k(m')$ where m' is the lowest m that satisfies $r_k(m) < [(m-1)r_L + r_H]/m$.

4.1 Partial Conglomeration

To simplify the comparison in this section, assume that the firm can only form groups of symmetric sizes. Then, the number of available projects n is such that $n = 2^z$ for some $z \in \mathbb{N}$. In this context,

[8] For notational convenience we define here the density and distribution functions at the number of projects with high return s rather than at the corresponding return, $\widehat{r}(s) := [sr_H + (k-s)r_L]/k$.

the firm should choose the size of the group k, where $k = 2^w$ for $w = 0, 1, .., z$. If k is the size of the groups then n/k is the number of groups.

Proposition 3 (Partial conglomeration) *Suppose that there are n projects that can be financed in symmetric groups. If the probability of high return is sufficiently small, $p \leq p^*$, then it is optimal to finance the projects in groups of size k^*, where k^* is the largest k that satisfies $r_k^* < [(k-1)r_L + r_H]/k$.*

First, if a rate that satisfies $r_{k^*}^* < [(k^* - 1)r_L + r_H]/k^*$ can be obtained by financing the projects in groups of size k^*, then it is better to finance the projects in groups of size k^* rather than in smaller groups. In this case, a single high return and $k^* - 1$ low returns allow all the projects in the group to stay afloat, so that a single project coinsures the rest of the group. Groups of smaller size cannot be better because one high-return project would save, at most, only the low-return projects of the smaller group. If all the projects in the other group(s) yield a low return, they will go bankrupt and the bankruptcy losses would be higher.

Second, if p is small, forming a group of size k^* also dominates forming groups of larger size k' in which $r_{k'}^* > ((k' - 1)r_L + r_H)/k'$. In this case, if k' projects are financed jointly and $k' - 1$ low returns are realized, risk-contamination would result. Instead, if the projects had been financed in smaller groups of size k^*, the group with a single high-return realization would have been saved. To illustrate, consider the case with four projects ($n = 4$), with $(3r_L + r_H)/4 < r_4^* < (r_L + r_H)/2$ and $r_2^* < (r_L + r_H)/2$. In this case, if all four projects are financed jointly, three low returns risk-contaminate the fourth, while two high returns coinsure the other two projects. If the projects are financed in two groups composed of two projects each, a high return in one project coinsures the other project in the same group. The advantage of financing projects in two partial conglomerates with two projects each is that in the event of three low returns, one of the partial conglomerates is saved through coinsurance, while risk contamination is contained. The disadvantage is that if one group yields two low returns and the other group yields two high returns, it would have been possible to save the two projects with low returns through coinsurance if all projects had been financed jointly in a full conglomerate. If p is small (below $p^* = 2/3$ if $n = 4$), the first effect dominates and it is optimal to finance projects in groups of two.

Overall, this proposition generalizes the intuition obtained from the two-project model to the case of multiple projects for p small ($p < p^*$). As in the two-project case, projects should be financed in small groups if, when financing in groups of larger sizes, we cannot obtain rates that would make a successful project save the rest. Higher bankruptcy costs, for example, makes funding of groups of smaller size more likely to be optimal because it is more difficult to get rates that make one project save the rest in larger groups, extending the logic of Prediction 1. Following the same reasoning, higher probability of high return (as long as $p < p^*$) makes funding of groups of larger size more likely to be optimal, as in Prediction 2.

The specific circumstances that this result requires—probability of success not too high, high return large enough to "rescue" the other projects if their returns are low—bears some resemblance to the case of venture capital funds. These funds are limited partnerships that typically target firms with a small chance of very high returns and a large probability of failure, and are funded with convertible preferred equity from limited partners (see Sahlman, 1990, and Fenn, Liang, and Prowse, 1995). Although failure to pay dividends on the preferred equity does not cause bankruptcy per se, it does hurt the reputation of the fund manager. (Note that the manager may run several funds at any one time.) Taking this as a generalized cost of "default," Proposition 3 suggests that each venture capital fund should be limited enough that one success can balance out failures in the rest of its portfolio.

If p is large ($p > p^*$), it might be optimal to form groups of larger sizes even if, in such groups, one cannot obtain a rate that makes a successful project save the rest.[9] That is again consistent with Prediction 2: an increase in the probability of high return favors larger groups. But that makes it difficult to state a necessary and sufficient condition on group formation for a given p. Still, for the

[9] In the case of four projects, if $p > p^* = 2/3$, the second effect described above dominates the first and it is optimal to finance all four projects jointly.

case in which projects are symmetric ($p = 1/2$), we can state a sufficient condition on full separation and full conglomeration, thus expanding the results of Proposition 3 for the two extreme group sizes. These sufficient conditions become necessary and sufficient if $n = 2$.

Proposition 4 (Joint v. separate financing of multiple projects) *If there are n symmetric projects which have to be financed in symmetric groups, then they should all be financed jointly if $r_n^* < [(n-1)r_L + r_H]/n$ and all separately if $r_k^* > (r_H + r_L)/2$ for any $k \geq 2$.*

Proposition 3 shows that all projects should be financed jointly if, with a full conglomerate, it is possible to obtain a rate such that a single high-return project coinsures all the other projects. At the other extreme, it shows that if, in any group of projects, it is not possible to obtain a rate such that a high-return project coinsures the rest of the group, it is better to finance all projects separately. Proposition 4 expands the set of cases for which full separation is optimal for $p = 1/2$. It exploits the fact the distribution functions of the average returns of any group of symmetric projects cross the distribution of the single return at the mean and, as a result, the probability of bankruptcy of groups with repayment rates above the mean is higher than the probability of bankruptcy of stand-alone projects. Hence, full separation is optimal if, for any group of projects, it is not possible to obtain a rate below the mean return, or equivalently, a rate such that half of the projects save the other half. For example, this occurs for a larger set of parameters when bankruptcy costs are high or when the mean return is low.

4.2 Large Number of Projects

As the analysis of our baseline model shows, the set of parameters for which joint financing is optimal does not necessarily increase with the number of projects. This result stands in contrast with claims often made in the literature; for example, see the discussion on page 400 and footnote 3 in Diamond (1984). Compared to our model, Diamond (1984) adds an intermediary who contracts with several entrepreneurs to achieve joint financing; in his model this intermediary can observe the entrepreneurs' returns only by paying a cost. Joint financing in our model can be seen as a special case of Diamond's

(1984) model with an intermediary who can costlessly observe the entrepreneurs' returns. In the last paragraph on page 400 reported above in the introduction, Diamond (1984) claims informally that the expected bankruptcy costs associated to intermediary financing decrease monotonically with the number of projects. When illustrating this result as the number of projects increases from one to two, in footnote 3 Diamond (1984) implicitly assumes away bad conglomeration by focusing on the case in which the repayment obligation when using the intermediary (i.e., with joint financing) is less than twice the one obtained without the intermediary (with separate financing). This argument is incomplete because it disregards the possibility of bad conglomeration. If the repayment rate with joint financing rate is above the crossing point, conglomeration is bad even when the intermediary can observe freely the entrepreneurs' returns.

Thus, there is a meaningful tradeoff between joint and separate financing without need of handicapping joint financing through the monitoring cost associated to intermediation. Nevertheless, using an argument based on the law of large numbers we can show that joint financing dominates separate financing when the number of independent and identical projects increases to infinity.

Proposition 5 (Many projects) *There exists n' and $q \in (0, p)$ such that when the number of projects satisfies $n > n'$, joint financing of all projects can be obtained at a repayment rate that avoids bankruptcy when nq projects have high returns. The resulting per-project return approaches the net present expected value of each project as n grows.*

If the number of independent projects is sufficiently large, it always becomes possible for the borrower to finance all the projects jointly. This result exploits the law of large numbers. Namely, as the number of projects n increases, the probability that the average number of projects with high returns differs from p, the probability of a high return, by more than a small amount ε tends to zero. We can then construct a rate offer to finance all projects jointly that is acceptable to the creditors. The borrower's returns when financing all projects jointly is then arbitrarily close to the first-best as the number of projects increases. Therefore, when the number of projects is large, financing all the projects jointly is approximately optimal for the borrower because the resulting payoff is close to the highest possible level.

Prediction 4 (Full conglomeration) *If there is a large number of independent projects, it is optimal to finance all of them jointly.*

In practice, however, there is an important caveat to this result: for any given firm, projects are likely to be generally positively correlated, due to common shocks to the firm's industry or the general economy. As we have seen, such correlation can reverse the optimality of full joint financing.

5 Structure of Verification/Bankruptcy Costs

In line with most of the theoretical and empirical literature, in our previous specifications ex post verification (or, equivalently, bankruptcy) costs are proportional to realized returns, with $B(r) = (1-\gamma)r$. Note that this baseline specification entails constant returns to scale: $B(tr) = (1-\gamma)tr = tB(r)$ for any $t > 0$. To investigate the robustness of our results to the structure of the bankruptcy costs, this section consider a general specification that allows for economies or diseconomies of scale in bankruptcy. We retain the feature that bankruptcy costs are larger for higher levels of realized returns, so that $B(r)$ is increasing in r.

As is intuitive, economies of scale in bankruptcy favor joint financing whereas diseconomies of scale favor separation. As demonstrated in the next result, if economies of scale are sufficiently strong, so that $B(2r_L) - 2B(r_L)$ is negative enough, then joint financing is optimal. Separate financing is optimal if, instead, there are sufficiently strong diseconomies of scale, so that $B(2r_L) - 2B(r_L)$ is positive enough. In the intermediate case, which includes constant returns to scale as well as weak economies and diseconomies of scale, separation is optimal if the rate that avoids intermediate bankruptcy cannot be obtained.

Proposition 6 (Scale economies in bankruptcy costs) *With a general structure of bankruptcy costs, there exist thresholds $\underline{S} < 0$ and $\overline{S} > 0$ such that* *(i) If $B(2r_L) - 2B(r_L) < \underline{S}$, joint financing is always optimal;*

(ii) If $\underline{S} < B(2r_L) - 2B(r_L) < \overline{S}$, separate financing is optimal if and only if

$$\frac{1 - (1-p)^2 \left[r_L - B(2r_L)/2\right]}{1-(1-p)^2} > \frac{r_H + r_L}{2};$$

(iii) If $B(2r_L) - 2B(r_L) > \overline{S}$, separate financing is always optimal.

To further characterize the thresholds independently of the level of returns, consider bankruptcy costs given by $B(r) \equiv (1-\gamma)r + \alpha(r - r_L)r$. This specification allows for economies ($\alpha < 0$) and diseconomies of scale ($\alpha > 0$) and includes our baseline case with constant returns to scale as a special case ($\alpha = 0$). Following the procedure set out in the previous proposition, if $\alpha < \underline{\alpha} := -(1-\gamma)pr_H / \left[(1-p)r_L^2 + pr_H^2 + pr_L r_H\right]$, joint financing is optimal (case (i)); if $\underline{\alpha} < \alpha < \overline{\alpha}$, separate financing is optimal if and only if the rate that avoids intermediate bankruptcy cannot be obtained (case (ii)); and if $\alpha > \overline{\alpha} := p(1-\gamma)/\left[(1-p)r_L\right]$, separate financing is optimal (case (iii)).

An alternative specification with constant returns to scale consists in assuming a fixed per-project bankruptcy cost b ($< r_L$), so that $B(r) = b$ for $r = r_H, r_L$ and $B(r) = 2b$ for $r = 2r_H, r_H + r_L, 2r_L$. Thus, we have $B(2r_L) - 2B(r_L) = 0$, so that case (ii) always results. In addition, it can be shown that with per project bankruptcy costs separate financing is optimal for a relatively larger set of parameters than in our baseline case with proportional bankruptcy costs.[10] Next, if there is a fixed recovery rate per project w ($< r_L$), case (ii) also results.

In sum, joint financing is optimal if there are significant economies of scale in bankruptcy, while separate financing is optimal if there are sufficiently strong diseconomies of scale. For weaker economies or diseconomies of scale, as well as for several specifications with constant returns of

[10] To show this, set per-project bankruptcy costs at the same level as the proportional losses of the low-return project, $b = (1-\gamma)r_L$. Then, the proceeds from a bankrupt high-return project are relatively higher in the per-project case compared to the proportional case. The rate for separate financing r_i^* and the rate for joint financing that avoids intermediate bankruptcy r_m^* are the same as in the proportional bankruptcy costs case, so the financing conditions do not change. However, the rate at which projects can be financed without avoiding intermediate bankruptcy r_m^{**} is now lower and it becomes easier to satisfy the condition. As a result, it becomes easier to obtain joint financing, but only at the rate for which intermediate bankruptcy occurs. Therefore, when both separate and joint financing are feasible, separate financing is optimal for a relatively larger set of parameters.

scale in bankruptcy, separate financing is optimal as long as intermediate bankruptcy cannot be avoided. Higher bankruptcy costs then favor separate financing more generally, as in our baseline specification.

6 Debt, Equity, and Taxes

We now extend the two-project symmetric case to allow the firm to use outside equity, as well as debt, to finance part of the initial investment. As in the standard trade-off theory of capital structure, equity payments are subject to corporate taxation, whereas debt payments are tax deductible and are therefore exempt from taxes. Our framework is isomorphic to other frictional costs linked to equity financing, such as higher underwriting fees, negative signaling costs, or agency costs of excess equity.

Model Extension. Financing for each corporation can be obtained in competitive credit and equity markets. As in the basic model, the availability of a competitive credit market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single creditor. Corporation j, consisting of n_j projects, promises to repay $n_j r'_j$ at $t = 2$ in exchange of $n_j D_j$ at $t = 1$. Thus, the promised per-project repayment r'_j now depends on the part of the initial investment outlay of each project that is financed through debt, $D_j \leq 1$.

A competitive equity market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single outside equity investor. We denote the fraction of the equity sold by corporation j as α_j, and the equity value of the corporation, if it consists of n_j projects, as $n_j E_j$. For all the projects to be financed, the sum of debt and equity financing per-project must cover the initial investment outlay of each project, $D_j + \alpha_j E_j = 1$. We also assume that, while debt payments are tax deductible and therefore exempt from taxes, equity payments are subject to a corporate tax of τ, which captures the tax disadvantage (or other net costs) of equity relative to debt.[11]

[11] Leland (2007) makes the more realistic assumption that only interest expenses are tax deductible. This, however, creates an endogeneity problem. When interest only is deductible, the fraction of debt service attributed to interest

Financing Conditions. For the case of separate financing, we now need to distinguish two cases, because there are situations in which it is possible to obtain a rate r_i' that avoids bankruptcy altogether, $r_i' \leq r_L$, by selling a fraction α of the corporation. If this rate exists, it should satisfy

$$\alpha(1-\tau)\left[p\left(r_H - r_i'\right) + (1-p)\left(r_L - r_i'\right)\right] = \alpha E_i \quad \text{and} \quad r_i' = D_i.$$

Since there is no bankruptcy, the net interest rate is zero and the principal is equal to the debt value. Substituting into the total financing condition, $D_i + \alpha E_i = 1$, this rate can be obtained if and only if

$$r_i'(\alpha) := \frac{1 - \alpha(1-\tau)\left[pr_H + (1-p)r_L\right]}{1-\alpha(1-\tau)} \leq r_L. \tag{4}$$

If the firm uses no equity ($\alpha = 0$), then $r_i' = 1$ and the condition is never satisfied ($r_L < 1$), as in the baseline debt-only case. But, as more equity is offered, the debt repayment is lower ($r_i'(\alpha)$ is decreasing) and, if taxes are low, the condition can be satisfied. Equity, however, is costly because of taxes. It is optimal for the firm to sell the lowest equity stake α_i' satisfying condition (4), $r_i'(\alpha_i') = r_L$. Still, if taxes are high enough, it is not possible to obtain this rate, not even by selling all the equity.

Following the same procedure, a rate such that $r_i'' < r_H$ can be obtained if and only if

$$r_i''(\alpha) := \frac{1 - \alpha(1-\tau)pr_H - (1-p)\gamma r_L}{\left[1-\alpha(1-\tau)\right]p} \leq r_H, \tag{5}$$

which generalizes condition (1) of the baseline setup to $\alpha > 0$, as $r_i''(0) = r_i^* \leq r_H$. This condition is satisfied precisely as long as condition (1) is satisfied, independently of the level of equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, no equity at all is sold in the optimum, $\alpha_i'' = 0$. In this case, equity does not help in reducing the probability of bankruptcy.

The following proposition characterizes which of these two rates is optimally chosen when they are both available.

Proposition 7 (Equity and taxes: separate financing) *Suppose that both rates r_i' and r_i'' are*

payments depends on the value of the debt, which in turn depends on the fraction of debt service attributed to interest payments. Instead of relying in numerical techniques to find debt values and optimal leverage, we follow Kale, Noe, and Ramirez (1991) and assume that both interest and principle are tax deductible. We also assume away personal taxes.

available. There exists $\overline{\tau}_i$ such that the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha_i' > 0$ if $\tau \leq \overline{\tau}_i$, and r_i^* and $\alpha = 0$ if $\tau > \overline{\tau}_i$.

If taxes are sufficiently high, the projects are financed at the same rate as in the baseline case without equity. Moreover, it is then optimal to finance the projects entirely with debt. When taxes are lower, however, it becomes optimal to finance the projects at a rate that avoids bankruptcy altogether ($r_i'(\alpha_i') = r_L$) by selling a positive amount of equity, $\alpha_i' > 0$.

For the case of joint financing, there are three potential rates. The first rate, which avoids bankruptcy altogether, $r_m' \leq r_L$, is the same as (and can be obtained under the same circumstances as) the rate resulting with separate financing, $r_m' = r_i'$. Indeed, if bankruptcy can be avoided, then the corporate structure does not matter.

Second, a rate that avoids bankruptcy if one realized return is high and the other is low can be obtained if and only if

$$r_m''(\alpha) := \frac{1 - \alpha(1-\tau)\left[p^2 r_H + 2p(1-p)\frac{r_H + r_L}{2}\right] - (1-p)^2\gamma r_L}{[1-\alpha(1-\tau)]\left[1-(1-p)^2\right]} \leq \frac{r_H + r_L}{2}, \tag{6}$$

which is again a generalization for $\alpha \geq 0$ of the financing condition of the baseline case, for the case of ex-ante symmetric projects, $r_m''(0) = r_m^* \leq (r_H + r_L)/2$, where

$$r_m^* := \frac{1 - \gamma\,(1-p)^2\, r_L}{1-(1-p)^2} \leq \frac{r_H + r_L}{2}. \tag{7}$$

Again, it is optimal for the firm to choose the minimum amount of equity that satisfies condition (6). If condition (7) is satisfied, the firm does not need to sell any equity at all, $\alpha_m'' = 0$. If condition (7) is not satisfied, this rate can still be obtained, however, by selling some equity.

Third, a rate that avoids bankruptcy only if both realized returns are high can be obtained as long as

$$r_m'''(\alpha) := \frac{1 - (1-p)^2\gamma r_L - 2p(1-p)\gamma\frac{r_H + r_L}{2} - \alpha(1-\tau)p^2 r_H}{[1-\alpha(1-\tau)]\,p^2} \leq r_H, \tag{8}$$

which again, generalizes the condition of the baseline symmetric case for $\alpha \geq 0$, i.e. $r_m'''(0) = r_m^{**} \leq r_H$, where

$$r_m^{**} := \frac{1 - \gamma (1-p)(pr_H + r_L)}{p^2} \leq r_H. \tag{9}$$

As in the highest rate for separate financing (8) as long as (9) is satisfied, independently of the equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, the resulting level is $\alpha_m''' = 0$.

Proposition 8 (Equity and taxes: joint financing) *Suppose more than one rate (r_m', r_m'', r_m''') is available. There exist $\overline{\tau}_m^a$, $\overline{\tau}_m^b$, and $\overline{\tau}_m^c$ such that:*

(i) If condition (7) is satisfied, the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha_m' > 0$ if $\tau \leq \overline{\tau}_m^a$, and r_m^* and $\alpha = 0$ if $\tau > \overline{\tau}_m^a$.

(ii) If condition (7) is not satisfied, the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha_m' > 0$ if $\tau \leq \overline{\tau}_m^b$, $\frac{r_H + r_L}{2}$ and $\alpha_m'' > 0$ if $\overline{\tau}_m^b < \tau \leq \overline{\tau}_m^c$, and r_m^{**} and $\alpha = 0$ if $\tau > \overline{\tau}_m^c$.

Good and Bad Conglomeration. The profitability of joint financing depends on the cases identified in Proposition 8. In case (i), joint financing is always profitable at least weakly. This case is equivalent to the case of good conglomeration in the baseline model. The condition is exactly the same as the condition enabling the firm to obtain r_m^* in the baseline specification. In case (ii), conglomeration is bad in the baseline model. And, if taxes are sufficiently high, conglomeration is still bad here. If taxes are lower, however, financing with equity allows the firm to finance the projects with rates that avoid bankruptcy in the case with intermediate returns and even with rates that avoid bankruptcy altogether.

Proposition 9 (Equity and taxes: joint v. separate financing) *When both separate and joint financing are feasible:*

(i) If condition (7) is satisfied, both financing regimes are equally profitable if $\tau \leq \overline{\tau}_m^a$, whereas joint financing dominates if $\tau > \overline{\tau}_m^a$.

(ii) If condition (7) is not satisfied, both financing regimes are equally profitable if $\tau \leq \overline{\tau}_m^b$, joint financing dominates if $\overline{\tau}_m^b < \tau \leq \overline{\tau}_i$, and separate financing dominates if $\tau > \overline{\tau}_i$.

In sum, if taxes or other equity costs are sufficiently high, only debt is used and the same situation analyzed in the baseline model arises. That is, joint financing is profitable in case (i) and separate financing is profitable in case (ii). The condition setting apart joint and separate financing is exactly the same as in the baseline model without equity. If taxes are intermediate, joint financing can be profitable in cases in which it is not profitable in the baseline model with only debt (case ii). This is because, by financing jointly and using equity, it becomes possible to obtain a rate that avoids intermediate bankruptcy or bankruptcy altogether. Finally, if taxes are sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether with joint as well as with separate financing.

The exclusive use of debt in separate finance is consistent with the many empirical studies that find that a disproportionate proportion of funding in project finance is in the form of debt. Kleimeier and Megginson (2000), for example, find that projects funded with project finance loans have an average loan-to-project value ratio of 67%. Esty (2003) shows that the average (respectively median) project company has a book value debt-to-total capitalization ratio of 70% (respectively 70%) compared to 33.1% (respectively 30.5%) for similar-sized firms. Our result is also consistent with the almost exclusive use of debt financing in securitization structures, where little if any external equity is issued.

7 Normal Returns

This section analyzes the model when returns are normally distributed rather than binary. We show that our results on bad conglomeration and the main comparative statics predictions are robust to continuous distributions. As part of the analysis, we also provide an easy-to-verify sufficient condition for the optimality of separate financing.

7.1 Model Extension

A firm has access to n symmetric, normally distributed projects, $r_i \sim N(\mu, \sigma^2)$ for $i = 1, ..., n$, with symmetric correlation coefficient ρ. As in the binary case, the distribution function of the average returns lies below the distribution of a single return until a unique crossing point (here equal to the mean because of symmetry), after which the ordering is reserved. Indeed, the average of two normal random variables is also normal with a density that is more peaked around the mean than the original normal density. To retain analytical tractability, we assume (i) that there is a fixed per-project recovery rate w ($w < 1 < \mu$);[12] (ii) that the firm can only form symmetric groups of projects (and therefore $n = 2^z$ for some $z \in \mathbb{N}$), as in Section 4.1; and (iii) that projects need to be financed exclusively with debt.

7.2 Financing Conditions

As in Section 4.1, the firm should choose the size of the groups k, where $k = 2^w$ for $w = 0, 1, .., z$. The *per project* repayment requested by a creditor in a competitive market to finance a group of size k, r_k^*, is defined by

$$kr_k^* \left[1 - G\left(kr_k^*\right)\right] + wkG\left(kr_k^*\right) = k, \tag{10}$$

where G is the distribution function of the sum of k normal random variables. Noting that the distribution of the sum computed at kr is

$$G\left(kr\right) = \Pr\left(r_1 + ... + r_k \leq kr\right) = \Pr\left(\frac{r_1 + ... + r_k}{k} \leq r\right) =: H(r), \tag{11}$$

where H is the distribution of the average of $r_1, ..., r_k$, this condition is equivalent to

$$r_k^* \left[1 - H\left(r_k^*\right)\right] + wH\left(r_k^*\right) = 1. \tag{12}$$

[12] Given that returns are normally distributed, with positive probability there realized return is lower than the recovery rate. For simplicity, we disregard this problem, given that the probability of these realizations can be made arbitrarily small with an appropriate choice of parameters. Alternatively, the proof of Proposition 10 holds for the general class of log-concave symmetric distributions, which allow for positive support and recovery rates below the support. The key property driving the result is that the density of the average of n random variables is more peaked around the mean compared to the original density. As shown by Proschan (1965), this property holds generally for log-concave symmetric distributions.

The firm's per-project payoff is then

$$\int_{r_k^*}^{+\infty} \frac{r_1 + ... + r_k}{k} dH - r_k^* \left[1 - H(r_k^*)\right] = \int_{r_k^*}^{+\infty} \left(\frac{r_1 + ... + r_k}{k} - r_k^* \right) dH. \tag{13}$$

Given that this payoff is a decreasing function of r_k^*, it is optimal for the firm to select the lowest r_k^* at which condition (12) is satisfied, if such a r_k^* exists. Financing is obtained in such a case. Figure **??** represents the mean-variance parameters allowing projects to be financed separately ($k = 1$) and in groups of two ($k = 2$).

7.3 Good and Bad Conglomeration

We now turn to the question of when is it optimal to finance the projects separately when there are multiple options available.

Proposition 10 (Optimality of separate financing) *If it is feasible to finance separately n normally distributed projects with mean μ and standard deviation σ, separate financing is optimal if*

$$\mu + w < 2 \text{ and } \mu - w < \sigma \sqrt{\left[1 + \rho(n-1)\right] \pi / 2n}. \tag{14}$$

These conditions identify the region of parameters for which separate financing is optimal in Figure **??**. We obtain the same comparative statics as in the baseline model. Separation holds for a larger region of parameters if the mean returns are low (Prediction 2) and if the variance is high (Prediction 3). Indeed, it is more difficult to satisfy both conditions in (14) if μ increases, and it is easier to satisfy the second condition if σ increases. In addition, when the coefficient of correlation increases the region for which separate financing is optimal increases. Similar to the binary case in Section 4.2, when the number of projects increases the region for which separation is optimal shrinks. In the limit, if there is a large number of independent projects the second condition is never satisfied, in accordance with Prediction 5.

Similarly, an increase in the recovery rate favors the optimality of joint financing (Prediction 1). To see this, consider the mean-variance parameter combinations for which joint and separate financing are both feasible for two levels of recovery rates, $w = w_1$ and $w = w_2$ where $w_1 < w_2$.

Then, the region for which separate financing is optimal is smaller for $w = w_2$ than for $w = w_1$. Indeed, an increase from w_1 to w_2 makes it more difficult for the first condition in (14) to be satisfied, thereby shrinking the region in which separate financing is optimal. Even though it becomes easier to satisfy the second condition, the new parameter values for which separate financing is optimal belongs to a region in which it is not feasible to finance the projects.

8 Conclusion

We show that the benefits of coinsurance and the costs of risk-contamination associated with joint financing depend on the distributional characteristics of the returns of each project, as well as on the number of projects available, the structure of the bankruptcy cost, and the tax advantage of debt relative to equity. We derive the following predictions: (1) An increase in the bankruptcy costs favors separate financing. (2) An increase in the probability of a high return of any of the two projects favors joint financing. (3) An increase in the riskiness of the projects favors separate financing. (4) Partial conglomeration with groups of intermediate size can be optimal in the presence of multiple projects. (5) Joint financing of a sufficiently large number of independent projects is always preferred. (6) The presence of tax-disadvantaged outside equity favors joint financing. (7) If separation is optimal, no tax-disadvantaged outside equity shall be used.

We show that there is a meaningful trade-off between joint and separate financing without need of handicapping joint financing through the monitoring cost associated to intermediation. In contrast with claims often made in the literature, the set of parameters for which joint financing is optimal does not necessarily increase with the number of projects. Compared to our model, Diamond (1984) adds an intermediary who contracts with several entrepreneurs to achieve joint financing; in his model this intermediary can observe the entrepreneurs' returns only by paying a cost. Joint financing in our model can be seen as a special case of Diamond's (1984) model with an intermediary who can costlessly observe the entrepreneurs' returns.

Appendix: Proofs

Proof of Proposition 1: We first derive the financing conditions for the joint financing case. There are three cases in which joint financing is feasible depending on whether bankruptcy can be avoided in both cases with intermediate returns, or only when project 1 yields a high return and project 2 yields a low return, or in neither case. If bankruptcy can be avoided in both cases with intermediate returns, competition in the credit market results in

$$[1-(1-p_1)(1-p_2)]\,2r'_m + (1-p_1)(1-p_2)\,\gamma\left(r_L^1 + r_L^2\right) - 2 = 0, \tag{15}$$

so that this case is possible if and only if

$$r'_m := \frac{1-(1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{1-(1-p_1)(1-p_2)} < \frac{r_L^1 + r_H^2}{2}. \tag{16}$$

If bankruptcy can be avoided with high intermediate returns but not with low intermediate returns, then

$$p_1p_2 2r''_m + p_1(1-p_2)2r''_m + (1-p_1)p_2\gamma\left(r_L^1 + r_H^2\right) + (1-p_1)(1-p_2)\,\gamma\left(r_L^1 + r_L^2\right) - 2 = 0, \tag{17}$$

and therefore this case is possible if and only if

$$\frac{r_L^1 + r_H^2}{2} < r''_m := \frac{1-(1-p_1)p_2\gamma\frac{r_L^1+r_H^2}{2} - (1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{p_1} < \frac{r_H^1 + r_L^2}{2}. \tag{18}$$

If bankruptcy cannot be avoided with either intermediate returns, then

$$p_1p_2 2r'''_m + p_1(1-p_2)\gamma\left(r_H^1 + r_L^2\right) + (1-p_1)p_2\gamma\left(r_L^1 + r_H^2\right) + (1-p_1)(1-p_2)\,\gamma\left(r_L^1 + r_L^2\right) - 2 = 0, \tag{19}$$

and therefore this is possible if and only if

$$\frac{r_H^1 + r_L^2}{2} < r'''_m < \frac{r_H^1 + r_H^2}{2}, \tag{20}$$

where

$$r'''_m := \frac{1-p_1(1-p_2)\gamma\frac{r_H^1+r_L^2}{2} - p_2(1-p_1)\gamma\frac{r_L^1+r_H^2}{2} - (1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{p_1p_2}.$$

Again, since the borrower obtains all the ex post net present value, rate r'_m is preferred to r''_m and

r''_m is preferred to r'''_m. To complete the proof we only need to show that the lower bound conditions for r''_m and r'''_m are irrelevant. From (15) and (17), and rearranging, we have

$$p_1(r'_m - r''_m) = p_2(1-p_1)\left[\gamma\left(\frac{r^1_L + r^2_H}{2}\right) - r'_m\right],$$

and therefore if $r'_m > \frac{r^1_L+r^2_H}{2}$ then the right-hand side is negative. As a consequence, we have $r''_m > r'_m > \frac{r^1_L+r^2_H}{2}$. Similarly, from (17) and (19) and rearranging, we have

$$p_2\left(r''_m - r'''_m\right) = (1-p_2)\left[\gamma\left(\frac{r^1_H + r^2_L}{2}\right) - r''_m\right]$$

and therefore if $r''_m > \frac{r^1_H+r^2_L}{2}$ then the right-hand side is negative. As a consequence, we have $r'''_m > r''_m > \frac{r^1_H+r^2_L}{2}$.

We now turn to the choice between joint and separate financing. Substituting r'_m in the right-hand side of (15) and subtracting the ex post net present value of financing the two projects separately, we have

$$p_2\left(1-p_1\right)\left(1-\gamma\right)r^1_L + p_1\left(1-p_2\right)\left(1-\gamma\right)r^2_L \ (>0).$$

Similarly, substituting r''_m in the right-hand side of (17) and subtracting again the ex post net present value of financing the two projects separately from this, we obtain

$$-(1-p_1)p_2(1-\gamma)r^2_H + p_1(1-p_2)(1-\gamma)r^2_L,$$

which can be positive or negative. Lastly, substituting r'''_m in the right-hand side of (19) and subtracting the ex post net present value of financing the two projects separately from this, we have

$$-p_1(1-p_2)(1-\gamma)r^1_H - p_2(1-p_1)(1-\gamma)r^2_H \ (<0),$$

as desired. Q.E.D.

Proof of Prediction 1: The statements follow from the fact that the derivatives with respect to γ of r'_m, r''_m and r'''_m, defined in (16), (18), and (20), are negative. Q.E.D.

Proof of Prediction 2: From the proof of Proposition *1*, if $r^1_L = r^2_L$ and $r^1_L = r^2_L$, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r'_m can be obtained. The statement follows from the fact that the derivatives of the left-hand of (16) with respect to p_1 and p_2 are negative. Q.E.D.

Proof of Prediction 3: Given that one project is obtained from an elementary increase in risk from the other and returns should still be binary, we must have that $p_1 = p_2 \equiv p$. Letting ε be such that $r^1_H = r^2_H + \varepsilon$, we have $r^1_L = r^2_L - \frac{p}{1-p}\varepsilon$. Indeed, $p(r^2_H + \varepsilon) + (1-p)r^1_L = pr^2_H + (1-p)r^2_L$. We can also check that $r^1_L + r^2_H = r^2_L - \frac{p}{1-p}\varepsilon + r^2_H < r^2_L + \varepsilon + r^2_H = r^1_H + r^2_L$.

As shown in the previous proposition, given that the probabilities of success are equal, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r'_m can be obtained. Moreover, the region for which joint financing is optimal shrinks as the repayment rate r'_m is more difficult to obtain if ε increases. Indeed, the left-hand side of condition (16) decreases in ε and the repayment rate (the right-hand side) increases in ε.

On the other hand, the region for which separate financing is possible expands if ε increases. Indeed, the derivative of the left-hand side of condition (1) is equal to γ whereas the right-hand side is equal to 1. Hence, this condition is more easily satisfied as ε increases. Q.E.D.

Proof of Proposition 2: Clearly, from Proposition 1, if statements (i) and (ii) are satisfied, separation is optimal. The probability of default of project 1 is the same in both financing regimes. With separate financing, the probability of default of project 2 is (i) reduced by $(1-p_1)p_2$, as a successful project 2 would not be dragged down if project 1 fails, but (ii) increased by $p_1(1-p_2)$, as a failing project 2 would not be saved if project 1 is successful. Given that, according to (iii), $p_1 > p_2$, we have that $p_1(1-p_2) > (1-p_1)p_2$. As a result, the probability of default with separate financing is higher. Q.E.D.

Proof of Proposition 3*:* We first show that if $r^*_k < [(k-1)r_L + r_H]/k$ then it is better to form groups of k projects rather than smaller groups. If this condition is satisfied, the per-project expected bankruptcy losses are given by $(1-p)^k \gamma r_L$. In groups of smaller size, $m < k$, the minimum loss is given by $(1-p)^m \gamma r_L$, which is larger.

We now show that if $r_k^* < [(k-1)r_L + r_H]/k$ and $r_m^* > [(m-1)r_L + r_H]/m$ for $m > k$ then it is better to form groups of size k rather than groups of larger size. Indeed, the bankruptcy losses of a group of size $m > k$, are greater or equal to $(1-p)^m \gamma r_L + m(1-p)^{m-1} p\gamma[(m-1)r_L + r_H)]/m$ which is in turn greater than $\left[(1-p)^m + m(1-p)^{m-1}p\right] \gamma r_L$. Then, subtracting the per-project bankruptcy costs of a group of size k, the difference is given by $(1-p)^k w(p)$ where $w(p) := (1-p)^{m-k-1}(1+mp)-1$. But, $w(0) = 0$, $w(1) < 0$ and $w'(p) := (1-p)^{m-k-2}\left[-(m-k)mp + k + 1\right]$. Given that $w'(0) > 0$ and $w'(1) \leq 0$ there exists a unique $0 < p' \leq 1$ such that $w'(p) > 0$ if $p < p'$ and $w'(p) < 0$ if $p > p'$. Therefore, there exists a unique $0 < p^* < 1$, such that $w(p) > 0$ if $p < p^*$ and $w(p) < 0$ if $p > p^*$. As a result, the per-project bankruptcy costs for a group of m projects are larger than for a group of k projects if $p < p^*$. Q.E.D.

Proof of Proposition 4: The first statement follows from the same argument as in the first part of the proof of Proposition 3.

With respect to the second statement, note that for any discrete symmetric distribution $H((r_H + r_L)/2 - \varepsilon) = 1 - H((r_H + r_L)/2 + \varepsilon)$ for any $\varepsilon > 0$, where the density and distribution functions are defined at the average returns (rather than at the number of projects with high return s as in the text). Given that $H(\cdot)$ is increasing and $h((r_H + r_L)/2) > 0$, we have that $H((r_H + r_L)/2 - \varepsilon) < H((r_H + r_L)/2 + \varepsilon)$ and substituting $H((r_H + r_L)/2 - \varepsilon) < 1/2$ and therefore $H((r_H + r_L)/2 + \varepsilon) > 1/2$. Hence, we have that $H(r) > 1/2$ for any $(r_H + r_L)/2 < r \leq r_H$ and therefore the distribution of the average return of any group of projects for $(r_H + r_L)/2 < r \leq r_H$ is above that of the returns of the projects financed separately, which is equal to $1/2$ for $r_L \leq r < r_H$.

Now, if $r_k^* > (r_H + r_L)/2$ for any group of k projects, then we have that the per project bankruptcy losses is greater than $H((r_H + r_L)/2 + \varepsilon)\gamma r_L$, which is in turn greater than the losses in the case of separate projects, $(1/2)\gamma r_L$. Q.E.D.

Proof of Proposition 5: First statement. Define $g(\delta) := \delta r_H + (1-\delta)\, r_L$. We have that $g(p) > 1$ because of the positive net present value condition, and trivially $g(0) = r_L < 1$ and $g'(\delta) > 0$. Then there exists a unique $\delta^* \in (0,p)$ such that $g(\delta^*) = 1$. For a fixed rational number ε (small) define $q := \delta^* + \varepsilon$. Clearly, $qr_H + (1-q)\, r_L > 1$

Take any number of projects n such that nq is an integer number. Suppose that we were to finance all these n projects jointly at an interest rate that avoids bankruptcy when at least nq of them have high returns. This is possible if and only if the per-project repayment satisfies

$$r_n^* \le qr_H + (1-q)\, r_L.$$

Given that the returns recovered in the event of bankruptcy are positive, we have that the equilibrium repayment rate in (2) satisfies

$$r_n^* \le \frac{1}{1 - H(nq-1)} < \frac{1}{1 - H(nq)}.$$

From the law of large numbers we have that $H(nq)$ tends to 0 as n grows large (remembering that $q < p$). Therefore r_n^* is bounded above by a number that is arbitrarily close to 1. Given that $qr_H + (1-q)\, r_L > 1$, there exists n' such that for all $n > n'$ then r_n^* is such that

$$r_n^* \le qr_H + (1-q)\, r_L,$$

as was to be shown.

Second statement: From the loan described above, the borrower obtains a per-project gross profit

$$\pi_n = \gamma \sum\nolimits_{k=0}^{nq-1} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L\right] + \sum\nolimits_{k=nq}^{n} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L\right].$$

Fix a small rational number ε and an integer n such that $n(p-\varepsilon)$ and $n(p+\varepsilon)$ are integer numbers. Then, given that $q < p - \varepsilon$, and that all terms in the first and in the second sum are positive, we have that

$$\pi_n \ge \sum\nolimits_{k=n(p-\varepsilon)}^{n(p+\varepsilon)} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L\right].$$

Given that the terms in the second factor in the sum are larger for larger k, the sum is reduced by replacing the summand of a given k by that of $n(p-\varepsilon)$, the smallest term. Then, rearranging, we obtain

$$\pi_n \geq [(p-\varepsilon)r_H + [1-(p-\varepsilon)]\, r_L]\, [H\,(n(p+\varepsilon)) - H\,(n(p-\varepsilon))]\,.$$

From the law of large numbers, $H\,[n(p+\varepsilon)] - H\,[n(p-\varepsilon)]$ tends to 1 as n grows. Indeed from Chebyshev's inequality we know that

$$H\,(n(p+\varepsilon)) - H\,(n(p-\varepsilon)) \geq 1 - \frac{(p+\varepsilon)(1-p)}{n\varepsilon^2} - \frac{(1-p+\varepsilon)p}{n\varepsilon^2} = 1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}$$

and therefore

$$\pi_n \geq [pr_H + (1-p)\, r_L - \varepsilon(r_H - r_L)] \left(1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}\right).$$

That is for n large, the gross per-project profit differs from the (gross) present value of each project by an amount that is arbitrarily small, $\varepsilon(r_H - r_L)$. Similarly,

$$\frac{\pi_n}{\pi^*} \geq \left(1 - \frac{\varepsilon(r_H - r_L)}{pr_H + (1-p)\, r_L}\right) \left(1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}\right)$$

where π^* is equal to first-best gross profits, $\pi^* = pr_H + (1-p)\, r_L$. Q.E.D.

Proof of Prediction 4: The proof follows directly from Proposition 5. Q.E.D.

Proof of Proposition 6: With separate financing the rate satisfies $pr_i^* + (1-p)\,[r_L - B(r_L)] = 1$ and therefore the condition is

$$r_i^* := \frac{1-(1-p)\,[r_L - B(r_L)]}{1-(1-p)} < r_H,$$

and the per-project net present value is $pr_H + (1-p)r_L - (1-p)B(r_L) - 1$. Similarly, the condition for obtaining a rate for joint financing that saves both projects when one has low return is given by

$$r_m^* := \frac{1-(1-p)^2\,[r_L - B(2r_L)/2]}{1-(1-p)^2} < \frac{r_H + r_L}{2}, \tag{21}$$

and the net present value is $pr_H + (1-p)r_L - (1-p)^2 B(2r_L)/2 - 1$. Finally, the rate for joint financing that saves both projects only when both give high returns is given by

$$r_m^{**} = \frac{1-(1-p)\,(p\,[r_L + r_H - B(r_L + r_H)] - (1-p)\,[r_L - B(2r_L)/2])}{p^2} < r_H.$$

and the per-project net present value is $pr_H + (1-p)r_L - (1-p)[pB(r_L + r_H) + (1-p)B(2r_L)/2] - 1$.

As in the baseline case, if it is possible to choose, the second rate is better than the third, as the net present value is larger. Separate financing is therefore optimal if the first rate (and therefore also the second) is better than the second, that is if $(1-p)B(2r_L)/2 > B(r_L)$ or equivalently

$$B(2r_L) - 2B(r_L) > \frac{2p}{1-p} B(r_L) \equiv \overline{S}.$$

Similarly, joint financing is optimal if the third rate (and therefore also the second) is better than the first, that is if $pB(r_L + r_H) + (1-p)B(2r_L)/2 < B(r_L)$, or equivalently

$$B(2r_L) - 2B(r_L) < -\frac{2p}{1-p} \left[B(r_L + r_H) - B(r_L) \right] \equiv \underline{S}.$$

Finally, if the first rate is better than the third but worse than the second we have, as in our baseline case, that joint financing is optimal if and only if the second rate can be obtained in joint financing, i.e. if condition (21) is satisfied. Q.E.D.

Proof of Proposition 7: We proceed by computing the payoff obtained when using each of the two rates and then we compare the payoffs. If the firm uses $r_i'(\alpha)$ (specified in (4)), the firm obtains, substituting into $(1-\alpha)E_i$,

$$\frac{(1-\alpha)(1-\tau)}{\tau + (1-\alpha)(1-\tau)} \left[pr_H + (1-p)r_L - 1 \right]. \tag{22}$$

This payoff is decreasing in α, as the firm obtains a fraction of the net present value that corresponds to the (after-tax) equity holding; the remaining part is retained by the government through taxes. Therefore the firm should use the smallest level of equity possible. But, as explained in the text, the firm should use a positive level of equity to satisfy condition (4). Optimally, we have

$$\alpha_i' := \frac{(1-r_L)}{(1-\tau)\left[pr_H + (1-p)r_L - 1 + (1-r_L) \right]}.$$

Provided that $\alpha_i' \leq 1$ ($r_i'(\alpha)$ can be obtained), the firm obtains, substituting into (22),

$$\left[pr_H + (1-p)r_L - 1 \right] - \tau p \left(r_H - r_L \right). \tag{23}$$

As argued in the text, if the firm uses $r_i''(\alpha)$ (specified in (5)), the optimal amount of equity is $\alpha_i'' = 0$. The borrower then obtains

$$(1-\tau)\left[pr_H + (1-p)\gamma r_L - 1\right]. \tag{24}$$

Comparing the payoffs in each case, (23) and (24), it is optimal for the firm to choose the first over the second rate if and only

$$\tau < \overline{\tau}_i := 1 - \frac{(1-r_L)}{(1-\gamma)(1-p)r_L + (1-r_L)},$$

that is if bankruptcy costs $(1-\gamma)$ are high enough and/or taxes are small. Q.E.D.

Proof of Proposition 8: Following the same procedure as in the proof of Proposition 7, we first compute the per-project payoff of the firm when using each of the three types of rate and then we compare these payoffs. If the projects are financed at a rate that avoids bankruptcy altogether, $r_m'(\alpha)$, which is equal to $r_i'(\alpha)$ (as specified in (4)), the firm obtains the same payoff as in the case of separate financing, equal to (23). As in the case of separate financing, the firm needs to use a positive level of equity to obtain this rate, and therefore uses the minimum amount $\alpha_m' > 0$ such that $r_m'(\alpha_m') = r_L$.

If projects are financed at a rate $r_m''(\alpha)$ (as specified in (6)), the firm obtains

$$(1-\tau)\left[p^2 r_H + 2p(1-p)\left(\frac{r_H + r_L}{2}\right) + (1-p)^2\gamma r_L - 1\right] \tag{25}$$

if condition (7) is satisfied and

$$(1-\tau)\left(p^2 r_H - p^2\frac{r_H + r_L}{2}\right) + \left[1-(1-p)^2\right]\frac{r_H + r_L}{2} + (1-p)^2\gamma r_L - 1 \tag{26}$$

if condition (7) is not satisfied. If condition (7) is satisfied, the firm does not need to use any equity to obtain r_m'', and therefore $\alpha_m'' = 0$ and $r_m''(0) = r_m^*$. If condition (7) is not satisfied, the firm needs to use a positive level of equity to obtain r_m'', and therefore uses the minimum amount $\alpha_m'' > 0$ such that $r_m''(\alpha_m'') = \frac{r_H + r_L}{2}$.

Finally, if projects are financed at a rate $r'''_m(\alpha)$ (specified in (8)), the firm obtains

$$(1-\tau)\left[p^2 r_H + 2p(1-p)\gamma\frac{r_H+r_L}{2} + (1-p)^2\gamma r_L - 1\right], \tag{27}$$

and no equity is used, $\alpha'''_m = 0$, as it does not help to reduce the probability of bankruptcy.

We now compare the payoffs in each case. Suppose first that condition (7) is satisfied (part (i) in the statement of the proposition). Then, the payoff when using r''_m is given by (25). It can be easily checked that this is always greater than the payoff that can be obtained when using r'''_m, (27). Comparing the payoffs when using r'_m with those of using r''_m, r'_m is optimal if and only if

$$\tau < \overline{\tau}^a_m := 1 - \frac{(1-r_L)}{(1-r_L)+(1-p)^2(1-\gamma)r_L}.$$

Suppose second that the condition (7) is not satisfied (part (ii) in the statement of the proposition). Then, the payoff when using r''_m is given by (26). In this case, r'_m is preferred to r''_m as long as

$$\tau < \overline{\tau}^b_m := 1 - \frac{\left[1-(1-p)^2\right]\frac{r_H+r_L}{2} + (1-p)^2\gamma r_L - r_L}{(1-\frac{p}{2})p\left[r_H - r_L\right]},$$

r''_m is preferred to r'''_m as long as

$$\tau < \overline{\tau}^c_m := 1 - \frac{1-\left[1-(1-p)^2\right]\frac{r_H+r_L}{2} - (1-p)^2\gamma r_L}{1-p^2\frac{r_H+r_L}{2} - 2p(1-p)\gamma\frac{r_H+r_L}{2} - (1-p)^2\gamma r_L},$$

and r'_m is preferred to r'''_m as long as

$$\tau < \overline{\tau}^d_m := 1 - \frac{(1-r_L)}{(1-\gamma)\left[2p(1-p)\frac{r_H+r_L}{2} + (1-p)^2 r_L\right] + (1-r_L)}.$$

It can be easily shown that the order of these cutoffs is given by $\overline{\tau}^b_m < \overline{\tau}^d_m < \overline{\tau}^c_m$. Therefore, we have the optimal choices claimed in the text. Q.E.D.

Proof of Proposition 9: In this proof, we need to compare the payoffs of joint and separate financing. Suppose first that condition (7) is satisfied (part (i) in the statement of the proposition). If r'_m is used for joint financing ($\tau < \overline{\tau}^a_m$), then the payoff with joint and separate financing are the same. If r''_m is used ($\tau > \overline{\tau}^a_m$), then the payoff with joint financing is larger than the payoff of separate financing.

Suppose now that condition (7) is not satisfied (part (ii) in the statement of the proposition). Comparing the cutoffs for joint and separate financing, it is easy to show that $\overline{\tau}_m^b < \overline{\tau}_i < \overline{\tau}_m^c$. Then, we can compare the payoffs under joint and separate financing. First, when r_m' is optimal with joint financing ($\tau < \overline{\tau}_m^b$), the payoffs under joint and separate financing are the same. When r_m'' is optimal with joint financing ($\overline{\tau}_m^b < \tau < \overline{\tau}_m^c$), it is straightforward to check that the payoffs are higher under joint financing if r_i' is obtained with separate financing, i.e. $\tau < \overline{\tau}_i$, but are lower if r_i'' is obtained with separate financing, i.e. $\tau > \overline{\tau}_i$. Finally, when r_m''' is optimal with joint financing ($\tau > \overline{\tau}_m^c$), joint financing yields a lower payoff than separate financing (under separate financing r_i'' would be optimal because $\overline{\tau}_i < \overline{\tau}_m^c$). As in the baseline model, the bankruptcy costs are higher under joint financing because of the risk-contamination effect.   Q.E.D.

Proof of Proposition 10: Consider two symmetric groups of $n/2$ normally distributed projects with mean μ and variance σ^2. The average distribution of returns of each group of $n/2$ projects, denoted as $F(r)$, is a normal distribution with mean μ and variance $[1+\rho(n/2-1)]\sigma^2/(n/2)$. The average distribution of the total set of n projects, denoted as $H(r)$, is a normal distribution with mean μ and variance $[1+\rho(n-1)]\sigma^2/n$. The two distributions cross at $r=\mu$ and the second distribution is more peaked around $r=\mu$ than the first. Thus we have

$$F(r) \gtreqless H(r) \Leftrightarrow r \lesseqgtr \mu \tag{28}$$

and as a result, for $r > w$,

$$(r-w)\left[1-F(r)\right]+w \lesseqgtr (r-w)\left[1-H(r)\right]+w \Leftrightarrow r \lesseqgtr \mu, \tag{29}$$

and rearranging

$$r\left[1-F(r)\right]+wF(r) \lesseqgtr r\left[1-H(r)\right]+wH(r) \Leftrightarrow r \lesseqgtr \mu. \tag{30}$$

Note first that the equilibrium repayment rates for each of the two groups separately (r_i^*) and jointly (r_n^*) satisfy r_i^*,$r_n^* > w$. Indeed, if $r_n^* < w$, the creditor's profits would be $r_n^*\left[1-H(r_n^*)\right]+wH(r_n^*) < w$. Given that by assumption $w<1$, the creditor would not be able to recover the initial investment. Applying the same reasoning, we conclude also that $r_i^* < w$ cannot hold. From now on, we thus restrict to repayment rates r_i^*,$r_n^* > w$.

If r_n^*, the lowest r such that $r\left[1-H\left(r\right)\right]+wH\left(r\right)=1$, is such that $r_n^* < \mu$, then $r_n^* < r_i^*$. Indeed, even though r_i^* exists by assumption, it is not possible that $r_i^* < r_n^*$ because, by (30) and single-peakedness of the profit function, we have that for $r < r_n^*$, $r\left[1-F(r)\right]+wF(r) < r\left[1-H\left(r\right)\right]+wH\left(r\right) < r_n^*\left[1-H\left(r_n^*\right)\right]+wH\left(r_n^*\right)=1$. As a result, from (28) and monotonicity of F, we conclude that the probability of bankruptcy is lower with joint financing, $H\left(r_n^*\right) < F(r_n^*) < F(r_i^*)$.

On the other hand, if r_n^* is such that $r_n^* > \mu$, then $r_n^* > r_i^*$. Indeed, given that the creditor's proceeds at $r=w$ are equal to $w<1$ and they are higher than 1 at $r=r_n^*$, as $r_n^*\left[1-F(r_n^*)\right]+wF\left(r_n^*\right) > r_n^*\left[1-H\left(r_n^*\right)\right]+wH\left(r_n^*\right)=1$, by the intermediate value theorem there exists some $r_i^* < r_n^*$ at which $r_i^*\left[1-F\left(r_i^*\right)\right]+wH\left(r_i^*\right)=1$. As a result, from (28) and monotonicity of H, we have that the probability of bankruptcy is lower with separate financing, $F(r_i^*) < H\left(r_i^*\right) < H(r_n^*)$. Since $F(r) < H(r)$ for $r > \mu$, the net surplus of the borrower is then

$$\int_0^{r_n^*}\left[1-H(x)\right]dx < \int_0^{r_n^*}\left[1-F(x)\right]dx < \int_0^{r_i^*}\left[1-F(x)\right]dx.$$

Therefore financing the two groups separately is optimal.

By single-peakedness, r_n^* is such that $r_n^* > \mu$ if and only if the following two conditions hold

$$r\left[1-H\left(r\right)\right]+wH\left(r\right)\big|_{r=\mu} < 1 \text{ and } \left.\frac{\partial}{\partial r}\left(r\left[1-H\left(r\right)\right]+wH\left(r\right)\right)\right|_{r=\mu} > 0,$$

which are equivalent to

$$\mu + w < 2 \text{ and } (\mu - w)h(\mu) < \frac{1}{2}.$$

We then obtain conditions (14) by substituting for the density h of a normal distribution with mean μ and variance $\left[1+\rho(n-1)\right]\sigma^2/n$.

Note that if this condition is satisfied for n then it is better to finance each half of the n available projects separately rather than all the n projects jointly. Now, if this condition is satisfied for n then it is also be satisfied for $n/2$. As a result, it is optimal to finance each half of the $n/2$ projects separately rather than jointly. Iterating this reasoning, we conclude that it is optimal to finance all projects separately, as claimed. Q.E.D.

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