The Value of Public Information in Monopoly*

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Abstract

The logic of the linkage principle of Milgrom and Weber (1982) extends to price discrimination. A non-linear pricing monopolist who sells to a single buyer always prefers to commit to publicly reveal information affiliated to the valuation of the buyer.

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1. INTRODUCTION

Consider the standard nonlinear pricing monopoly problem (Mussa and Rosen (1978)). A monopolist offers a price-quantity menu to a single privately informed buyer with quasi-linear preferences. While the traditional framework takes the information available to the contracting parties as exogenously given, a seller can often control it. For example, the seller could rely on an outside certifier of quality, or structure the information system in such a way that data on past buyers become publicly available. Similar policies could be enacted in other monopoly problems, such as franchising or procurement. In this paper, we ask whether committing to reveal an additional signal increases the monopolist's expected profits.

This problem is closely related to the one solved by Milgrom and Weber (1982). They consider an auctioneer selling a single good to several asymmetrically informed bidders and they ask whether the seller gains by adopting a policy of revealing an additional public signal about the good. According to their so-called *linkage principle*, in an affiliated environment the expected revenue of the seller is increased by such a transparency policy. While Milgrom and Weber's seller operates with an exogenously given mechanism (a certain auction format), our monopolist can change the price-quantity schedule in response to the additional signal.

We show that the logic of the linkage principle extends to the monopoly problem. The monopolist's expected profits cannot decrease by committing to reveal a signal affiliated to the buyer's private signal. We prove this result for two scenarios according to what happens if the additional signal is not revealed.

In one scenario, the monopolist does not have access to the signal unless it is made public. The choice is between no information and public information. Revealing the public signal has two effects. First, for any fixed quantity vector the best deviation for each type of buyer is on average less attractive when the additional affiliated signal is revealed. The monopolist can then sell on average at higher prices. Second, the monopolist can further increase expected profits by conditioning the quantities offered on the realization of the public signal. As a corollary, we establish that no other policy of partial information disclosure is more profitable than full public revelation of an affiliated signal. Affiliation is crucial for the linkage principle to hold. We report simple examples in which affiliation fails and revealing a public signal hurts the monopolist.

In the other (perhaps more realistic) scenario, the alternative to public information revelation is for the monopolist to have private access to the same information, but in a nonverifiable form. For example, the seller may still have access to the information in absence of public certification. This is an informed principal problem and in equilibrium the buyer may infer (part of) the private information of the monopolist from the menu offered (see e.g. Judd and Riordan (1994)). Regardless of affiliation, it is shown that the monopolist would gain by committing to reveal directly the part of the information inferred by the buyer in the equilibrium and to "forget" the remaining information. By the corollary obtained for the first scenario, full public revelation is then shown to be the optimal policy.

Our results cover the case when the buyer's private signal and the public signal provide information also on the seller's cost of production or opportunity cost. We also allow the signals to provide information on the buyer's outside option.

The paper proceeds as follows. Section 2 defines the environment. Section 3 compares public to no information, and Section 4 public to private information. Section 5 discusses some applications and Section 6 concludes.

2. ENVIRONMENT

A monopolist wishes to sell to a single buyer. For notational simplicity, the supports of all random variables are taken to be finite. The payoff-relevant state of the world, unknown to both the buyer and the seller, is represented by the real random variable S with support $\mathcal{S}$. The private information of the buyer is represented by the real valued random variable T with support $\mathcal{T} = \{t_1, \ldots, t_n\}$, where without loss $t_1 < \ldots < t_n$. The additional non-verifiable signal Z has support $\mathcal{Z}$. While our results are valid when Z is multi-dimensional, for notational simplicity we will derive them for the uni-dimensional case. The random variables S, T, Z are assumed to be affiliated (cf. Milgrom and Weber (1982)):

$$\begin{aligned} &\Pr\left(\max\left\langle s', s''\right\rangle, \max\left\langle t', t''\right\rangle, \max\left\langle z', z''\right\rangle\right) \Pr\left(\min\left\langle s', s''\right\rangle, \min\left\langle t', t''\right\rangle, \min\left\langle z', z''\right\rangle\right) \\ &\qquad \geq \Pr(s', t', z') \Pr(s'', t'', z'') \end{aligned} \tag{1}$$

for any $s', s'' \in \mathcal{S}$, $t', t'' \in \mathcal{T}$ and $z', z'' \in \mathcal{Z}$.

Let $\mathcal{Q}$ be a finite set of nonnegative real numbers. For concreteness, $q \in \mathcal{Q}$ could be interpreted as quantity, but could also be seen as quality. Both buyer and seller have quasi-linear preferences. In state s, the total profit of the monopolist of providing quantity q for a non-linear price transfer p is $v(q, s) + p$. No assumption is made on the function v. The utility of the buyer is $u(q, s) - p$. Assume that u is strictly supermodular in q and s:

$$u(q'', s'') - u(q'', s') \geq u(q', s'') - u(q', s') \quad \forall s'' \geq s', \forall q'' \geq q',$$

with strict inequality whenever $s'' > s'$ and $q'' > q'$.

Our framework encompasses a number of monopoly markets. In a more familiar formulation of the non-linear monopoly pricing model, $u(q, s) = qs$ and $v(q, s) = -c(q)$. A buyer with private signal t_j has type $E[S|t_j]$ equal to the marginal willingness to pay for quantity. In the special case with $\mathcal{Q} = \{0, 1\}$, we have the classic model of monopoly pricing for a single unit, where the demand function at price $p = E[S|t_j]$ is equal to $\Pr(T \geq t_j)$.

More generally, $v(q,s)$ can depend on s to allow for common values. The buyer would then have private information on the opportunity value of the item for the seller. Clearly, the role of buyer and seller can be reversed with the appropriate modifications. For instance, our model applies to the problem of a (monopsonistic) buyer contracting with a seller who is (partially) informed on the value to the buyer, as in Akerlof's (1970) market for lemons.

The buyer's expected payoff, gross of the price paid, conditional on t_j and z is

$$U(q_i, t_j, z) = E_S\left[u(q_i, s)|t_j, z\right] = \sum_{s\in\mathcal{S}} \Pr(s|t_j, z)u(q_i, s)$$

when buying quantity q_i. Affiliation and supermodularity interplay nicely. By Milgrom's (1981) Proposition 1, supermodularity of u in q, s and affiliation of S and T , conditional on Z imply that

$$U(q,t,z) \text{ is strictly supermodular in } q,t \text{ for any } z. \tag{2}$$

Similarly,

$$U(q,t,z) \text{ is strictly supermodular in } q,z \text{ for any } t. \tag{3}$$

Timing of events is as follows. First, one of three possible information regimes is chosen. While the buyer always observes the private signal T, observation of Z depends on the information regime: (a) No Information: neither the monopolist nor the buyer observes Z; (b) Public Information: both parties observe Z; (c) Private Information: only the monopolist observes Z. Second, observation of the signals takes place, according to the information regime. Third, the monopolist proposes a menu of quantity-price pairs to the buyer. Fourth, the buyer selects a quantity-price pair within the menu offered by the seller or takes the outside option ($p = 0$, $q = 0$). The three games are denoted by $G(\emptyset, T)$, $G(Z, \{T, Z\})$, and $G(Z, T)$, corresponding respectively to regimes (a), (b), and (c). The associated maximum expected profits attainable ex ante by the monopolist are denoted by $\pi(\emptyset, T)$, $\pi(Z, \{T, Z\})$, and $\pi(Z, T)$.

3. PUBLIC INFORMATION VERSUS NO INFORMATION

This section compares public information to no information. The change in profits due to the addition of public information can be decomposed in two effects. First, holding constant the quantities sold to each type of buyer and optimizing only on price transfers, expected profits can either increase or decrease when the public signal is available. Second, the monopolist can increase expected profits by conditioning quantities on the realization of the public signal. Under affiliation, the first effect will be shown to be unambiguously positive, so that public information is more profitable than no information.

Consider first the problem without public information, or equivalently with a completely uninformative public signal $Z = \emptyset$. Abusing notation, the buyer's expected payoff conditional on t_j is

$$U(q_i, t_j) = E_S[u(q_i, s)|t_j] = \sum_{s\in\mathcal{S}} \Pr(s|t_j)u(q_i, s),$$

where we have dropped the functional dependence on the uninformative realizations of $\emptyset$. The monopolist's maximal expected profit is

$$\pi(\emptyset, T) = \max_{\langle q,p\rangle} \sum_{s\in\mathcal{S}} \sum_{i=1}^{n} \Pr(t_i, s)(v(q_i, s) + p_i) \tag{4}$$

subject to the individual rationality and incentive compatibility constraints

$$\begin{aligned} U(q_j, t_j) - p_j &\geq U(0, t_j) \quad \forall j \\ U(q_j, t_j) - p_j &\geq U(q_k, t_j) - p_k \quad \forall j, \forall k. \end{aligned}$$

Each type t_j selects the contract q_j, p_j designed for that type. The individual rationality constraints are dealt with the convention that the menu of contracts offered by the monopolist must always include the null contract $q_0 \equiv p_0 \equiv 0$, which provides the outside option to the buyer. An allocation vector $q = (q_0, \ldots, q_n)$ is said to be *implementable* if there exists a transfer vector $p = (p_0, \ldots, p_n)$ such that $\langle q, p\rangle$ satisfies for all j and k

$$U(q_j, t_j) - p_j \geq U(q_k, t_j) - p_k. \tag{$IC_{i,k}$}$$

We now report the characterization of the solution of the monopolist problem, restating well-known results (e.g. Maskin and Riley (1984)) in our setting. Affiliation of T and S and supermodularity of $u(q, s)$ allow us to restrict attention to menus for which the local downward constraints are always binding:

Proposition 1 *Let $U(q, t)$ be strictly supermodular in q and t. Then: (i) q is implementable if and only if it is monotonic, $q_0 \leq \cdots \leq q_n$; (ii) Given an implementable q, at a profit maximizing price vector p, the local downward incentive compatibility constraints are binding,*

$$p_i = p_{i-1} + U(q_i, t_i) - U(q_{i-1}, t_i) \qquad \forall i, \tag{5}$$

with $p_0 \equiv 0$.

The monopolist's problem is not finite because price is a continuous variable. Nevertheless, Proposition 1 guarantees that the problem has a solution because each q yields a unique optimal price vector, and $\mathcal{Q}$ is finite.

With public information, the monopolist is allowed to offer a different menu of contracts $\langle q(z), p(z)\rangle = \langle q_i(z), p_i(z)\rangle_{i=0}^{n}$ depending on the realization z of the random variable Z. Consider the choice of the buyer who is offered such a price-quantity schedule in state

z. The expected payoff (conditional on z) from $q_i(z), p_i(z)$ for the buyer who observes realization t_j of the private signal T and z of the public signal Z, is $U(q_i(z), t_j, z) - p_i(z)$. Maximal profits are given by

$$\pi(Z, \{T, Z\}) = \max_{\langle q(z), p(z) \rangle} \sum_{s \in \mathcal{S}} \sum_{z \in \mathcal{Z}} \sum_{i=1}^{n} \Pr(t_i, z, s)(v(q_i(z), s) + p_i(z)) \qquad (6)$$

subject to $q(z)$ being implementable and $q_0(z) \equiv p_0(z) \equiv 0$.

Proposition 1 extends to the public information case for each z, because $U(q, t, z)$ is strictly supermodular in q and t for any z by (2). Our main result is:

Theorem 1 *If S, T, Z are affiliated random variables, the monopolist achieves higher expected profits by publicly revealing Z, $\pi(Z, \{T, Z\}) \geq \pi(\emptyset, T)$.*

The result is proven by showing that there is a suboptimal but feasible strategy which allows the monopolist to achieve higher expected profits once the additional affiliated signal is publicly revealed. Let $\langle \hat{q}, \hat{p} \rangle$ be a menu of contracts which solves the seller's problem with no public information. Because $\hat{q}$ is implementable with no information, the necessity part of Proposition 1 (i) implies that $\hat{q}$ is nondecreasing. Since $\hat{q}$ is nondecreasing and $U(q, t, z)$ is supermodular in q, t given any z, the sufficiency part of Proposition 1 (i) guarantees that $\hat{q}$ is implementable for any realization z of the public signal.

Next, consider the case with public information and suppose that the monopolist continues to offer the quantity vector which was optimal in the absence of information, while appropriately modifying the prices in response to the realization of the public information. For each z the seller offers menu $\langle \hat{q}, \tilde{p}(z) \rangle$, where $\tilde{p}(z)$ is defined by

$$\tilde{p}_i(z) = \tilde{p}_{i-1}(z) + U(\hat{q}_i, t_i, z) - U(\hat{q}_{i-1}, t_i, z) \qquad \forall i \qquad (7)$$

with $\tilde{p}_0(z) \equiv 0$. Notice that $\langle \hat{q}, \tilde{p}(z) \rangle$ is the solution to the monopolist problem when constrained to keep offering quantity $\hat{q}$.

The following statistical property will be useful to show that this possibly suboptimal menu results in higher expected profits under the affiliation assumption:

Lemma 1 *Take any $q_{j-1} \leq q_j$. The expected marginal utility of type t_j from buying q_j rather than q_{j-1} for all $j \leq i$ in the absence of public information is (weakly) lower than its expectation with respect to the affiliated signal Z conditional on information t_i:*

$$\sum_{z \in \mathcal{Z}} \Pr(z|t_i)(U(q_j, t_j, z) - U(q_{j-1}, t_j, z)) \geq U(q_j, t_j) - U(q_{j-1}, t_j) \quad \forall j \leq i. \qquad (8)$$

Proof. By supermodularity (3), $U(q_j, t_j, z) - U(q_{j-1}, t_j, z)$ is non-decreasing in z. Affiliation of T and Z implies that $Z|t_i$ first-order stochastically dominates $Z|t_j$ if $i \geq j$. Then

$$\sum_{z\in\mathcal{Z}} \Pr(z|t_i)\left(U(q_j, t_j, z) - U(q_{j-1}, t_j, z)\right) \geq \sum_{z\in\mathcal{Z}} \Pr(z|t_j)\left(U(q_j, t_j, z) - U(q_{j-1}, t_j, z)\right) \quad (9)$$

for all $j \leq i$. Using the definition of U, the right-hand side of (9) becomes

$$\sum_{s\in\mathcal{S}} \sum_{z\in\mathcal{Z}} \Pr(z|t_j) \Pr(s|t_j, z)(u(q_j, s) - u(q_{j-1}, s) = U(q_j, t_j) - U(q_{j-1}, t_j).$$

The result follows. □

To interpret this result, suppose that all the different quantities were sold at the same price. Consider a *local downward deviation* for the buyer with type t_j. Type t_j's utility loss when buying the quantity designed for the type immediately below is equal to $U(q_j, t_j, z) - U(q_{j-1}, t_j, z)$ conditional on z. First, type t_j perceives the same expected cost for this deviation in the presence or absence of public information. Second, in the eyes of a higher type $t_i \geq t_j$ the expected cost of such local deviation by type t_j is higher with public information than without.

Applying Lemma 1 to $\hat{q}$ and substituting (5) and (7) into (8) we obtain

$$\sum_{z\in\mathcal{Z}} \Pr(z|t_i)\left(\tilde{p}_j(z) - \tilde{p}_{j-1}(z)\right) \geq \hat{p}_j - \hat{p}_{j-1} \quad \forall j \leq i. \quad (10)$$

Now, fix i and sum (10) from $j = 1$ to $j = i$. As $\tilde{p}_0(z) = \hat{p}_0 = 0$, we conclude that, once Z is revealed publicly, the monopolist can charge on average a higher price to each type for selling the same quantity:

$$\sum_{z\in\mathcal{Z}} \Pr(z|t_i)\, \tilde{p}_i(z) \geq \hat{p}_i \quad \forall i. \quad (11)$$

According to the characterization of the binding constraints, the price charged to each type t_i is equal to the sum of the marginal utilities for the quantities designed for all the inframarginal types $t_j \leq t_i$. Lemma 1 then guarantees that higher prices are incentive compatible because the expected marginal utilities for quantity of all such inframarginal types are higher with public information than without, when evaluated with the more favorable signal t_i. We are now ready for the:

Proof of Theorem 1. By (4),

$$\pi(\emptyset, T) = \sum_{s\in\mathcal{S}} \sum_{i=1}^{n} \Pr(t_i, s)(v(\hat{q}_i, s) + \hat{p}_i)$$

and

$$\pi(Z, \{T, Z\}) \geq \sum_{s\in\mathcal{S}} \sum_{z\in\mathcal{Z}} \sum_{i=1}^{n} \Pr(t_i, z, s)(v(\hat{q}_i, s) + \tilde{p}_i(z))$$

Because the cost to the seller is the same in both cases

$$\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{i=1}^{n}\Pr(t_i,z,s)v(\hat{q}_i,s)=\sum_{s\in\mathcal{S}}\sum_{i=1}^{n}\Pr(t_i,s)v(\hat{q}_i,s),$$

inequality $\pi\left(Z,\{T,Z\}\right)\geq\pi\left(\emptyset,T\right)$ reduces to

$$\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{i=1}^{n}\Pr(t_i,z,s)\tilde{p}_i\left(z\right)\geq\sum_{s\in\mathcal{S}}\sum_{i=1}^{n}\Pr(t_i,s)\hat{p}_i$$

or,

$$\sum_{i=1}^{n}\Pr(t_i)\sum_{z\in\mathcal{Z}}\Pr\left(z|t_i\right)\tilde{p}_i\left(z\right)\geq\sum_{i=1}^{n}\Pr(t_i)\hat{p}_i$$

which is guaranteed by (11). □

If the monopolist does not alter the quantity vector, the expected value of the social welfare $u(q,s)+v(q,s)$ remains constant. At this allocation, public revelation of an affiliated signal allows the monopolist to increase profits by reducing the informational rent of the buyer.

Would any other policy whereby revelation takes place only in some circumstances be preferable to public information? Our result can be strengthened by showing that the monopolist cannot do better with any other policy of partial information disclosure. Any such policy corresponds to revelation of a garbling W of Z, i.e. $\Pr\left(w|s,t,z\right)=\Pr\left(w|z\right)$. Then Z is sufficient for (Z,W), i.e. the conditional distribution of S and T given Z and W is identical to the conditional distribution of S and T given Z only:

$$\Pr(s,t|z,w)=\Pr(s,t|z). \tag{12}$$

We establish the following simple result on affiliation:

Lemma 2 *Assume that S, T, and Z are affiliated random variables, and that W is a garbling of Z. Then S, T, and Z are affiliated conditional on W.*

Proof. We have

$$\Pr(s,t,z|w)=\Pr(s,t|z,w)\Pr(z|w)=\Pr(s,t|z)\Pr(z|w)=\frac{\Pr(s,t,z)\Pr(z|w)}{\Pr(z)}, \tag{13}$$

where the second equality is due to the sufficiency property (12). Next, substitute (13) in the definition (1) of affiliation of S, T, and Z conditional on W, and notice that

$$\frac{\Pr(\max\langle z',z''\rangle|w)}{\Pr\left(\max\langle z',z''\rangle\right)}\frac{\Pr(\min\langle z',z''\rangle|w)}{\Pr\left(\min\langle z',z''\rangle\right)}=\frac{\Pr(z'|w)}{\Pr\left(z'\right)}\frac{\Pr(z''|w)}{\Pr\left(z''\right)}. \tag{14}$$

The result then follows from the assumption that S, T, and Z are affiliated. □

Theorem 1 can be applied repeatedly, once part W of the information contained in Z has become public. Regardless of the information W which has already become public, making public the remaining information contained in Z cannot hurt the seller. As this holds for any possible realization, it holds also ex ante:

Theorem 2 *If S, T, Z are affiliated random variables, the monopolist achieves higher expected profits by publicly revealing Z than its garbling W, $\pi(Z, \{T, Z\}) \geq \pi(W, \{T, W\})$.*

Proof. Lemma 2 guarantees that S, T, and Z are affiliated conditionally on any realization w of the garbling W. For any such realization, Theorem 1 applies, so that committing to reveal Z is profitable. This is also true taking expectation over W. Therefore, publicly revealing both W and Z is more profitable than revealing only W, $\pi(\{Z, W\}, \{T, Z, W\}) \geq \pi(W, \{T, W\})$. Finally, $\pi(\{Z, W\}, \{T, Z, W\}) = \pi(Z, \{T, Z\})$ because revealing both W and Z is equivalent to revealing only Z sufficient for (Z, W). $\square$

3.1 Welfare of the Buyer and Social Welfare

Revelation of affiliated public information has an ambiguous effect on the expected payoff of the buyer. Clearly, when a perfectly informative signal is revealed publicly, the buyer is necessarily (weakly) worse off, being deprived of all informational rent. When the quantity vector is held fixed, public information results in a reduction of the rent of each type of buyer. Nevertheless, the buyer may benefit from the introduction of affiliated information, once the quantity vector offered is optimally re-adjusted by the monopolist in response to the affiliated public signal. For example, suppose the monopolist can sell zero or one unit at no cost and the buyer's utility has a multiplicative structure: $\mathcal{Q} = \{0, 1\}$, $v(q, s) \equiv 0$, $u(q, s) = qs$. There are three a-priori equally likely states: $s_1 = 0$, $s_2 = 9$, and $s_3 = 10$. Signals Z and T are simple binary partitions of the state space: $\Pr(t_1|s_1) = \Pr(t_1|s_2) = 1$ and $\Pr(t_2|s_3) = 1$, and $\Pr(z_1|s_1) = 1$ and $\Pr(z_2|s_2) = \Pr(z_2|s_3) = 1$ (with all other conditional probabilities equal to zero). Affiliation is easily checked. Without public information, the monopolist maximizes profits by setting $p = 10$ and hence selling only to t_2. With public information, the optimal prices conditional on z are $p(z_1) = 0$ and $p(z_2) = 9$. The buyer has zero rent in the no information case but positive rent when Z is publicly revealed.

Similarly, the effect of affiliated public information on the expected value of the sum of the payoffs of the buyer and the seller is ambiguous. Clearly, a perfectly informative public signal cannot decrease total welfare. However, a partially informative signal may decrease it by inducing the seller to distort more the allocation, in order to extract more rent from the buyer.

3.3 When Affiliation Fails

Affiliation between S, T, and Z implies three pairwise conditional affiliations. To prove our results, we have used all three. Affiliation of S and T given any z guarantees that $U(\hat{q}_j, t, z) - U(\hat{q}_{j-1}, t, z)$ is a nondecreasing function of t for given z, a fact used in Proposition 1. The other two pairwise affiliations are essential for the proof of Lemma 1. It can be shown by example that if any of these three conditions is violated, the monopolist may lose from committing to reveal public information.

We provide a counterexample to Theorem 1 when affiliation of Z and S is relaxed. The monopolist can sell zero or one unit at no cost and the buyer's utility has a simple multiplicative structure: $\mathcal{Q} = \{0, 1\}$, $v(q, s) \equiv 0$, $u(q, s) = qs$. There are three equally likely states, $\{s_1 = 10, s_2 = 11, s_3 = 12\}$, a binary private signal T affiliated to S with $\Pr(t_1|s_1) = \Pr(t_1|s_2) = 1$ and $\Pr(t_2|s_3) = 1$, and a binary public signal Z not affiliated to S with $\Pr(z_1|s_1) = \Pr(z_1|s_3) = 1$ and $\Pr(z_2|s_2) = 1$. Furthermore, Z and T are deterministic conditionally on S, and therefore affiliated conditionally on S. All the affiliation conditions used in the proof of Theorem 1 are satisfied, other than affiliation of Z and S conditional on some t. With no public information, the monopolist optimally offers $q_1 = q_2 = 1$ at $p_1 = p_2 = E[S|t_1] = 21/2$. With public information the expected profit if $q_1 = q_2 = 1$ (which is clearly always optimal) is equal to $E[S|t_1, z_1] = 10$ with probability $2/3$ and $E[S|t_2, z_2] = 11$ with probability $1/3$. Public information decreases expected profits to $31/3 < 21/2$.

4. PUBLIC VERSUS PRIVATE INFORMATION

In this section we compare maximum expected profits achieved in $G(Z, \{T, Z\})$ and $G(Z, T)$. In $G(Z, T)$ the monopolist has private information Z when offering the menu to the buyer. In an equilibrium of $G(Z, T)$, the buyer may infer part of the monopolist's information from observing the menu offered. We show that the monopolist is better off by committing to reveal directly the information inferred by the buyer in equilibrium and to forget the remaining information.

In order to define a perfect Bayesian equilibrium of $G(Z, T)$, we now introduce strategies and beliefs. Let $\mu(M|z)$ be the probability that the monopolist offers the menu of contracts M when observing z. A menu M is a collection of quantity-price pairs (q, p) with $q \in \mathcal{Q}$ and $p \in [0, \infty)$, containing the null contract $(0, 0)$. Let $\mathcal{M}$ be the collection of all possible M's. Let $\sigma((q, p)|M, t)$ be the probability that the buyer who observes t and is offered menu M selects the quantity-price pair (q, p) within M. To lighten the notational burden, we restrict the monopolist to randomize between only a finite number of menus for each realization of z, and to offer menus containing only a finite number of price-quantity pairs.[1] Given observation of the menu M and private signal t, let $\beta(z|M, t)$ be the buyer's

[1] Formally, let K and L be two positive natural numbers. A menu M is defined as a collection of less than K quantity-price pairs (q, p). The mixed strategy μ of the monopolist must be such that, for each z, at most L menus are played with positive probability. As it is clear from the proofs, our results do not depend on this finiteness assumption.

belief that the monopolist has observed signal z. Similarly to the definition of U, let $V(q,t,z) = \sum_{s\in\mathcal{S}} \Pr(s|t,z) v(q,s)$.

A *perfect Bayesian equilibrium (PBE)* e of $G(Z,T)$ is a triple $(\mu^*, \sigma^*, \beta^*)$ satisfying:
(i) monopolist's best reply

$$\sum_{M\in\mathcal{M}^*(z)} \sum_{t\in\mathcal{T}} \sum_{(q,p)\in M} \mu^*(M|z) \Pr(t|z) \sigma^*((q,p)|M,t)[V(q,t,z)+p] \tag{15}$$
$$\geq \sum_{t\in\mathcal{T}} \sum_{(q,p)\in M'} \Pr(t|z) \sigma^*((q,p)|M',t)[V(q,t,z)+p] \quad \forall z\in\mathcal{Z}, \forall M'\in\mathcal{M}$$

(ii) buyer's best-reply

$$\sum_{z\in\mathcal{Z}} \sum_{(q,p)\in M} \beta^*(z|M,t)\sigma^*((q,p)|M,t)[U(q,t,z)-p] \tag{16}$$
$$\geq \sum_{z\in\mathcal{Z}} \beta^*(z|M,t)[U(q',t,z)-p'] \quad \forall t\in\mathcal{T}, \forall M\in\mathcal{M}, \forall (q',p')\in M,$$

(iii) consistency of buyer's equilibrium beliefs

$$\beta^*(z|M,t) = \frac{\mu^*(M|z)\Pr(z|t)}{\sum_{\tilde{z}\in\mathcal{Z}} \mu^*(M|\tilde{z})\Pr(\tilde{z}|t)} \quad \forall z\in\mathcal{Z}, \forall M\in\mathcal{M}^*(z), \forall t\in\mathcal{T}, \tag{17}$$

where $\mathcal{M}^*(z) = \{M \in \mathcal{M} | \mu^*(M|z) > 0\}$. We defined the monopolist's best reply in pure strategies and buyer's best reply in mixed strategies for notational convenience. Let $\pi_e(Z,T)$ be the expected profit of the monopolist in equilibrium e, and $\pi(Z,T) = \sup_e \pi_e(Z,T)$.

The information signaled in equilibrium e by the monopolist's choice of menu can be represented by the signal W_e, constructed as follows. Let $\mathcal{M}^* = \bigcup_{z\in\mathcal{Z}} \mathcal{M}^*(z)$ be the (finite) set of menus offered with positive probability in e. Assign to each element of $\mathcal{M}^*$ a different index $w \in \mathcal{W}_e$, where $\mathcal{W}_e$ is a set with the same cardinality as $\mathcal{M}^*$. Hence, $M^*(w)$ denotes a menu of contracts which is chosen in equilibrium with positive probability and is indexed with w. The random variable W_e is defined to have support $\mathcal{W}_e$ and conditional probability

$$\Pr(W_e = w|t,s,z) = \mu^*(M^*(w)|z) \quad \forall z, \forall w, \forall s, \forall t.$$

Clearly, W_e is a garbling of Z: $\Pr(w|t,s,z) = \Pr(w|z)$.

Given a PBE e of $G(Z,T)$, construct the game $G(W_e, \{T, W_e\})$ where the monopolist observes W_e and the buyer both T and W_e. In order to find the maximum expected profits of the monopolist in this problem, it is convenient to think of the monopolist as choosing also the agent's strategy, subject to it being a best response. A pure strategy for a monopolist is the choice of a (finite) menu $\nu(w) \in \mathcal{M}$ for each realization of signal w. A mixed strategy for the buyer assigns probability $\tau((q,p)|M,t,w)$ of selecting the price-quantity pair (q, p) from the offered menu M, given observation of t and w. The monopolist's problem is

$$\pi(W_e, \{T, W_e\}) = \max_{\nu,\tau} \sum_{w \in \mathcal{W}_e} \sum_{t \in \mathcal{T}} \sum_{(q,p) \in \nu(w)} \Pr(t, w)\, \tau((q,p)|\nu(w), t, w)[V(q,t,w) + p] \quad (18)$$

subject to buyer's best response

$$\sum_{(q,p) \in M} \tau((q,p)|M,t,w)[U(q,t,w) - p] \geq U(q,t,w) - p \qquad \forall t, \forall w, \forall M, \forall (q,p) \in M. \quad (19)$$

As seen in the previous section, this problem can be reduced to a finite problem and thus has a solution.

The monopolist's profit are higher in the new game $G(W_e, \{T, W_e\})$ constructed departing from equilibrium e. In a PBE e of the original game $G(Z, T)$, the monopolist cannot "fool" the buyer when offering the equilibrium menu $M^*(w)$, because the buyer's beliefs are correct in equilibrium. Hence, in the new game $G(W_e, \{T, W_e\})$, for each realization w the monopolist can induce the same response from the buyer by offering the same menu $M^*(w)$. The monopolist does at least as well in the new game. This simple intuition is formalized in the following result, which to the best of our knowledge is new:

Theorem 3 *Let e be a PBE of $G(Z, T)$ resulting in expected profits $\pi_e(Z, T)$ and W_e the corresponding implicit signal revealed. The monopolist achieves higher expected profits by revealing W_e directly and destroying the remaining information: $\pi(W_e, \{T, W_e\}) \geq \pi_e(Z, T)$.*

Proof. Let (ν°, τ°) be a solution of the monopolist problem (18). Define $(\hat{\nu}, \hat{\tau})$ as:

$$\hat{\nu}(w) = M^*(w) \quad \forall w$$

$$\hat{\tau}((q,p)|M,t,w) = \begin{cases} \sigma^*((q,p)|M,t) & \text{if } M = M^*(w) \\ \tau^\circ((q,p)|M,t,w) & \text{otherwise} \end{cases} \qquad \forall w, \forall t, \forall M,$$

and denote the resulting expected profits of the monopolist by $\hat{\pi}$. First, it can be easily verified that $\pi_e(Z, T) = \hat{\pi}$. Second, we show that $\hat{\pi} \leq \pi(W_e, \{T, W_e\})$ by verifying that $(\hat{\nu}, \hat{\tau})$ satisfies the constraints (19) of the monopolist problem. There are two cases: Either $M = \hat{\nu}(w)$ for given t and w, in which case the analogous equilibrium condition (16) with the beliefs $\beta^*(z|M^*(w), t) = \Pr(z|w, t)$ implies that $\hat{\tau}$ satisfies (19). Or $M \neq \hat{\nu}(w)$, in which case $\hat{\tau}$ satisfies (19) because τ° satisfies it by definition. We conclude that $\pi_e(Z, T) = \hat{\pi} \leq \pi(W_e, \{T, W_e\})$. $\square$

While no affiliation assumption is made for this result, affiliation is instead used to show that the monopolist should commit to revealing Z rather than its garbling W_e, so that:

Theorem 4 *If S, T, Z are affiliated random variables, the monopolist achieves higher expected profits by committing to reveal her private information,* $\pi(Z, \{T, Z\}) \geq \pi(Z, T)$.

Proof. For any PBE e of $G(Z,T)$, $\pi_e(Z,T) \leq \pi(W_e, \{T, W_e\})$ by Theorem 3. Theorem 2 then implies $\pi(W_e, \{T, W_e\}) \leq \pi(Z, \{T, Z\})$, as W_e is a garbling of Z. Hence, $\sup_e \pi_e(Z,T) \leq \pi(Z, \{T, Z\})$. $\square$

Clearly, the results hold a fortiori for refinements of PBE. To see the importance of affiliation for this final result, consider the following example where affiliation fails and revealing private information hurts the monopolist. As in the examples of the previous section, $\mathcal{Q} = \{0, 1\}$, $v(q, s) \equiv 0$, $u(q, s) = qs$. Each random variable has two unconditionally equiprobable realizations, with $\mathcal{S} = \{8, 10\}$, $\mathcal{T} = \{-1, 1\}$, and $\mathcal{Z} = \{-1, 1\}$. T and Z are uninformative if observed alone, but their joint observation perfectly reveals the state: S, T, and Z are pairwise independent and linked by the deterministic relationship $s = 9 + tz$. In $G(Z,T)$, the monopolist achieves highest expected profits in a (robust to refinements) pooling equilibrium by selling at $p = 9$ independently of z. Instead, in $G(Z, \{T, Z\})$ the buyer knows s perfectly (but the monopolist is still ignorant), so that the optimal policy is to sell always at $p = 8$ independently of z. Here, $\pi(Z,T) > \pi(Z, \{T, Z\})$.

4.1 Relation to the Informed Principal and Rent Extraction Literatures

Theorem 4 is valid when the monopolist moves first by offering a menu of contracts and the buyer then chooses a contract within this menu. While the restriction to this mechanism is natural in our monopoly problem, it is not imposed in the literature on mechanism design by an informed principal (Myerson (1983) and Maskin and Tirole (1990, 1992)). In particular, we do not allow for more complex mechanisms in which the seller proposes a menu of quantity-price pairs contingent on messages to be sent by both buyer and seller simultaneously.

The following simple example shows that, when such simultaneous mechanisms are allowed, the seller may prefer to keep an affiliated signal private. Let $\mathcal{S} = \{s_1 = 1, s_2 = 2, s_3 = 3\}$, $\mathcal{T} = \{t_1, t_2\}$, $\mathcal{Z} = \{z_1, z_2\}$, $\mathcal{Q} = \{0, 1, 2\}$, $v(q, s) = 0$ for $q = 0, 1$ and $v(2, s) = -100$, $u(q, s) = qs$. The states are a priori equally likely and the signals Z and T are affiliated, with $\Pr(t_1|s_1) = \Pr(t_2|s_2) = \Pr(t_2|s_3) = 1$ and $\Pr(z_1|s_1) = \Pr(z_1|s_2) = \Pr(z_2|s_3) = 1$. First, profits are $\pi(Z, \{T, Z\}) = (2/3)1 + (1/3)3 = 5/3$ when Z is publicly revealed. Second, consider the simultaneous mechanism $\{q(t_1, z_1) = 1, p(t_1, z_1) = 1, q(t_1, z_2) = 2, p(t_1, z_2) = 50, q(t_2, z_1) = 1, p(t_2, z_1) = 2, q(t_2, z_2) = 1, p(t_2, z_2) = 3\}$, where $q(b, m)$ is the allocation and $p(b, m)$ the transfer following reports of messages b and m by the buyer and the seller, and the buyer is also allowed to opt out with $q_0 = p_0 = 0$. This mechanism is incentive compatible for both buyer and seller and allows the seller to achieve profits $2 > 5/3$ by fully extracting the rent of the buyer.

In this example, the designer can exploit the correlation between Z and T to extract the rent of the buyer by means of stochastic contracts, similarly to what happens in the literature on rent extraction (e.g. Crémer and McLean (1985)). The difference is that in our setting the correlated information is possessed by the designer rather than by other agents. Full rent extraction is then generally not possible, as can be seen when the example given above is modified to have $\mathcal{Q} = \{0, 1\}$.

5. APPLICATIONS

The linkage principle can be applied literally to public certification. An independent agency which publicly certifies the quality of the product of a monopolist offers a valuable service to the monopolist, provided the reports are affiliated to the quality of the good. Similarly, a monopolist profits by committing to reveal the level of satisfaction of other consumers.

Our result has also powerful implications for the dynamics of monopoly pricing with social learning (Ottaviani (1996)). The monopolist selects a dynamic pricing policy to sell to a sequence of privately informed buyers deciding one after the other. Buyers learn from the observable purchase behavior of previous buyers, as in the models of Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). Consider a two-period example, where the menu offered to the first buyer as well as the contract chosen by the first buyer are publicly observed by the monopolist and the second buyer. By the choice of the menu offered in the first period, a patient monopolist trades off static profit maximization with the value of future public information revelation. Theorem 2 implies immediately that a patient monopolist will deviate from the myopically optimal menu by offering a menu which does not result in the revelation of less public information.

The set of prices supporting trade in Akerlof's (1970) lemons market can be affected in a non-monotonic fashion as the information asymmetry decreases because of the public revelation of affiliated information. This can be easily seen in simple examples, analogous to those constructed by Levin (1998) to compare the possibility of trade as the private information of the informed party improves. Even if the set of prices supporting trade is non-monotonic in the amount of public information, our general result guarantees that the price-setter is necessarily better off when affiliated information is revealed publicly.

6. CONCLUSION

Since its discovery by Milgrom and Weber (1982), the linkage principle is acquiring a central role in models of pricing. We have shown here that its logic extends to the classic environment of a price-discriminating monopolist selling multiple units (or single units of heterogenous quality) of a good to a single buyer. In the monopoly problem we have studied here the buyer's type has one dimension only. Our proofs rely on the structure of the monopoly solution for the unidimensional case and do not readily extend to the characterizations provided by the recent literature on multidimensional monopoly (Armstrong (1996) and Rochet and Choné (1998)). It is an open question whether our results extend to the multidimensional case.

While the linkage principle generalizes in some interesting directions, two negative results have been recently provided. Perry and Reny (1999) have recently shown that the principle does not generalize to multi-unit Vickrey auctions with more than one buyer, each demanding more than one unit.[2] Moscarini and Ottaviani (1998) show that the linkage principle does not hold when competing principals sell to a buyer with private information on the relative value of goods.

In this paper we do not discuss the value for the monopolist of the *private* information of the buyer. By selecting trial and return policies, the seller can often controls the amount of private information available to the buyer when purchasing the product. Lewis and Sappington (1995) offer a series of interesting examples to illustrate how the seller's profits change as the buyer becomes better informed about the quality of the product. In contrast to the case of public information, no general principle has yet emerged on the value of private information in monopoly.

[2] In a Vickrey auction equilibrium, losing bids are based on underestimates of the signals of the competing bidders, while winning bids on overestimates. Revelation of affiliated public information not only results on average in an increase of losing bids but also in a decrease of winning bids. When bidders demands multiple units, the second effect becomes relevant for the seller's revenue and may dominate the first one.

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Supplementary Material

The Value of Public Information in Monopoly

(Marco Ottaviani and Andrea Prat)

A. Proof of Proposition 1

We provide the details of a revisitation in our setting of some standard results on the reduction of the self selection constraints (see Section 3 of Maskin and Riley's (1984)).

Proposition 1 Let $U(q,t)$ be strictly supermodular in q and t. Then: (i) q is implementable if and only if it is monotonic, $q_0 \le \cdots \le q_n$; (ii) Given an implementable q, at a profit maximizing price vector p the local downward incentive compatibility constraints are binding,

$$p_i = p_{i-1} + U(q_i,t_i) - U(q_{i-1},t_i) \qquad \forall i,$$

with $p_0 \equiv 0$.

This is proved by the following four results.

Lemma 3 *q is implementable only if $q_0 \le \cdots \le q_n$.*

Proof. Suppose not, i.e. $q_i < q_k$ for an $i > k$. Supermodularity of U (with $t_i \ge t_k$ and $q_i < q_k$) implies

$$U(q_i,t_i) + U(q_k,t_k) < U(q_k,t_i) + U(q_i,t_k). \tag{20}$$

Implementability of q implies

$$U(q_i,t_i) + U(q_k,t_k) \ge U(q_k,t_i) + U(q_i,t_k),$$

obtained by summing $(IC_{i,k})$ and $(IC_{k,i})$, in contradiction with (20). □

Lemma 4 *Suppose $q_0 \le \cdots \le q_n$. Consider $\langle q,p\rangle$. If all the adjacent downward incentive compatibility constraints $IC_{i,i-1}$ hold as equalities, then all other IC's are satisfied.*

Proof. The statement is proven in two steps: First, all upward constraints are satisfied. Second, all downward constraints are satisfied.

Step 1: $(IC_{i,i-1}), \forall i \Rightarrow (IC_{k,i}), \forall k < i$. The claim follows immediately from:

$$\begin{aligned}
& p_i - p_k \\
= \; & (p_i - p_{i-1}) + (p_{i-1} - p_{i-2}) + \ldots + (p_{k+1} - p_k) \\
= \; & (U(q_i,t_i) - U(q_{i-1},t_i)) + (U(q_{i-1},t_{i-1}) - U(q_{i-2},t_{i-1})) + \ldots + (U(q_{k+1},t_{k+1}) - U(q_k,t_{k+1})) \\
\ge \; & (U(q_i,t_k) - U(q_{i-1},t_k)) + (U(q_{i-1},t_k) - U(q_{i-2},t_k)) + \ldots + (U(q_{k+1},t_k) - U(q_k,t_k)) \\
= \; & U(q_i,t_k) - U(q_k,t_k),
\end{aligned}$$

where the second equality is $(IC_{i,i-1})$, the inequality comes from supermodularity (and the assumption that q is nondecreasing), and the last equality is an immediate simplification.

Step 2: $(IC_{i,i-1}), \forall i \Rightarrow (IC_{i,k}), \forall k < i$. As above,

$$\begin{aligned}
& p_i - p_k \\
= \; & (U(q_i,t_i) - U(q_{i-1},t_i)) + (U(q_{i-1},t_{i-1}) - U(q_{i-2},t_{i-1})) + \ldots + (U(q_{k+1},t_{k+1}) - U(q_k,t_{k+1})) \\
\leq \; & (U(q_i,t_i) - U(q_{i-1},t_i)) + (U(q_{i-1},t_i) - U(q_{i-2},t_i)) + \ldots + (U(q_{k+1},t_i) - U(q_k,t_i)) \\
= \; & U(q_i,t_i) - U(q_k,t_i),
\end{aligned}$$

where the argument is analogous to Step 1. □

Corollary 1 *If $q_0 \leq \cdots \leq q_n$, then q is implementable.*

Proof. For any q, it is always possible to construct a p such that all $(IC_{i,i-1})$ hold as equalities. If $q_1 \leq \cdots \leq q_n$, Lemma 4 guarantees that $\langle q, p \rangle$ also satisfy the other IC's. □

Corollary 2 *For any implementable q, the monopolist maximizes profits by making $(IC_{i,i-1})$ binding.*

Proof. Immediate from Lemmas 3 and 4. The monopolist can always increase profits by eliminating slack from $(IC_{i,i-1})$. □

B. Example of Ambiguous Effect on Social Welfare

Here is a simple example where revelation of an affiliated public signal results in a reduction in social welfare. Assume that $\mathcal{Q} = \{0,1\}$, $v(q,s) \equiv 0$, $u(q,s) = qs$. There are three states: $s_1 = 2$, $s_2 = 3$, and $s_3 = 10$, with prior probabilities $\Pr(s_1) = 9/10$ and $\Pr(s_2) = \Pr(s_3) = 1/20$. The signal T is perfectly informative, $\Pr(t_1|s_1) = \Pr(t_2|s_2) = \Pr(t_3|s_3) = 1$, while Z is a binary partition, $\Pr(z_1|s_1) = \Pr(z_2|s_2) = \Pr(z_2|s_3) = 1$. Without public information the monopolist sells to all three buyer's types, thereby implementing the efficient allocation. With public information, the monopolist optimally excludes type t_2 when z_2 is observed. The resulting allocation is socially inefficient, so that the introduction of public information results in a reduction of the expected social welfare.

C. When Affiliation Fails: Two More Counter-examples

In Section 3.2. we claim that Theorem 1 breaks down when any of the pairwise affiliations is relaxed. In the text we provide only one example. Here are the other two. In all these examples the monopolist can sell zero or one unit at no cost and the buyer's utility has a simple multiplicative structure: $\mathcal{Q} = \{0,1\}$, $v(q,s) \equiv 0$, $u(q,s) = qs$.

Relaxing affiliation of T and S. The monopolist is worse off by committing to reveal the public signal Z' affiliated to the valuation S, when the private signal T' of the buyer is not affiliated to S. Take $Z' = T$ and $T' = Z$ of the previous example. The only assumption not satisfied is affiliation of T and S conditional on some z. In this case, profit without public information are $E[S|t_1'] = E[S|t_2'] = 11$. With public information, expected profits are $\Pr(z_1')\,E[S|t_1', z_1'] + \Pr(z_2')\,E[S|t_1', z_2'] = 20/3 + 12/3 < 11$.

Relaxing affiliation of Z and T In this example Z and S are affiliated conditional on t, T and S are affiliated conditional on z, but Z and T are not affiliated. Unlike the previous examples, we need Z and T not independent conditional on S. Consider two equally likely states, $\{s_1 = 10, s_2 = 11\}$. The private signal T alone is uninformative about S: there are two possible realizations, with $\Pr(t|s_1) = \Pr(t|s_2) = 1/2$ for $t = 0, 1$. In the absence of public information, maximal profits are equal to $21/2$. Consider the effect of the public signal $Z = S - T$. By observing both Z and T, the buyer can infer the state perfectly. With revelation of Z, the expected profit for the monopolist is $(3/4)\,10 + (1/4)\,11 < 21/2$.

D. Non Necessity of Affiliation for Value of Public Information

Affiliation is not necessary for public information to be valuable, even when the monopolist is constrained not to change the menu as a result of the revelation of public information. Take $\mathcal{Q} = \{0, 1\}$, $v(q, s) \equiv 0$, $u(q, s) = qs$. Consider three equally likely states, $\{s_1 = 10, s_2 = 11, s_3 = 12\}$. Consider a case where the private signal T is affiliated to the buyer's valuation S, but profits are increased by publicly revealing a signal Z *not* affiliated to S, even when the monopolist does not change the targeted type of buyer. The probability distribution of T is $\Pr(t_1|s_1) = 1$ and $\Pr(t_2|s_2) = \Pr(t_2|s_3) = 1$. The public signal has two possible realizations $\mathcal{Z} = \{z_1, z_2\}$ with $\Pr(z_1|s_1) = \Pr(z_1|s_3) = 1$ and $\Pr(z_2|s_2) = 1$. Without public information profits are $E[S|t_1] = 10$. With public information, $\Pr(z_1)\,E[S|t_1, z_1] + \Pr(z_2)\,E[S|z_2] = 20/3 + 11/3 > 10$. This shows that affiliation is a sufficient but not a necessary condition for the value of public information. For brevity, this observation is not included in the paper.

E. Additional Details for the Proof of Theorem 3

Claim 1: $\beta^*(z|M^*(w), t) = \Pr(z|w, t) \quad \forall t, \forall w, \forall z.$

Check: By (17),

$$\begin{aligned}
\beta^*(z|M^*(w), t) &= \frac{\mu^*(M^*(w)|z)\Pr(z|t)}{\sum_{\tilde{z}\in\mathcal{Z}}\mu^*(M^*(w)|\tilde{z})\Pr(\tilde{z}|t)} = \frac{\Pr(w|z)\Pr(z|t)}{\sum_{\tilde{z}\in\mathcal{Z}}\Pr(w|\tilde{z})\Pr(\tilde{z}|t)} \\
&= \frac{\Pr(w|t,z)\Pr(z|t)}{\sum_{\tilde{z}\in\mathcal{Z}}\Pr(w|t,\tilde{z})\Pr(\tilde{z}|t)} = \frac{\Pr(w,z|t)}{\sum_{\tilde{z}\in\mathcal{Z}}\Pr(w,\tilde{z}|t)} = \frac{\Pr(w,z|t)}{\Pr(w|t)} = \Pr(z|w,t),
\end{aligned}$$

where the third equality follows from sufficiency of Z for W.

Claim 2: $\sum_{z\in\mathcal{Z}} \Pr(z|w,t)[U(q,t,z)-p] = U(q,t,w)-p \quad \forall t, \forall w, \forall z.$
Check: By the definition of U,

$$\begin{aligned}\sum_{z\in\mathcal{Z}} \Pr(z|w,t)[U(q,t,z)-p] &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(z|t,w)\Pr(s|t,z)[u(q,s)-p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(z|t,w)\Pr(s|t,w,z)[u(q,s)-p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(s,z|t,w)[u(q,s)-p] \\ &= \sum_{s\in\mathcal{S}} \Pr(s|t,w)[u(q,s)-p] = U(q,t,w)-p\end{aligned}$$

Claim 3: $\sum_{z\in\mathcal{Z}} \Pr(w|t,z)\Pr(t,z)[V(q,t,z)+p] = \Pr(t,w)[V(q,t,w)+p] \quad \forall t, \forall w.$
Check: By the definition of V,

$$\begin{aligned}\sum_{z\in\mathcal{Z}} \Pr(w|t,z)\Pr(t,z)[V(q,t,z)+p] &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(t,w,z)\Pr(s|t,z)[v(q,s)-p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(t,w,z)\Pr(s|t,w,z)[v(q,s)-p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{s\in\mathcal{S}} \Pr(s,t,w,z)[v(q,s)-p] \\ &= \sum_{s\in\mathcal{S}} \Pr(s,t,w)[v(q,s)-p] \\ &= \sum_{s\in\mathcal{S}} \Pr(t,w)\Pr(s|t,w)[v(q,s)-p] \\ &= \Pr(t,w)[V(q,t,w)+p],\end{aligned}$$

where sufficiency of Z for W is used for the second equality.

Claim 4: $\hat{\pi} = \pi_e(Z,T)$.
Check: By the definition of perfect Bayesian equilibrium e,

$$\begin{aligned}\pi_e(Z,T) &= \sum_{z\in\mathcal{Z}}\sum_{M\in\mathcal{M}^*(z)}\sum_{t\in\mathcal{T}}\sum_{(q,p)\in M} \mu^*(M|z)\Pr(t,z)\sigma^*((q,p)|M,t)[V(q,t,z)+p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{w\in\mathcal{W}_e}\sum_{t\in\mathcal{T}}\sum_{(q,p)\in M^*(w)} \Pr(w|z)\Pr(t,z)\sigma^*((q,p)|M^*(w),t)[V(q,t,z)+p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{w\in\mathcal{W}_e}\sum_{t\in\mathcal{T}}\sum_{(q,p)\in M^*(w)} \Pr(w|t,z)\Pr(t,z)\sigma^*((q,p)|M^*(w),t)[V(q,t,z)+p] \\ &= \sum_{z\in\mathcal{Z}}\sum_{w\in\mathcal{W}_e}\sum_{t\in\mathcal{T}}\sum_{(q,p)\in\hat{\nu}(w)} \Pr(w|t,z)\Pr(t,z)\hat{\tau}((q,p)|\hat{\nu}(w),t,w)[V(q,t,z)+p] \\ &= \sum_{w\in\mathcal{W}_e}\sum_{t\in\mathcal{T}}\sum_{(q,p)\in M} \Pr(t,w)\hat{\tau}((q,p)|\hat{\nu}(w),t,w)[V(q,t,w)+p] = \hat{\pi}\end{aligned}$$

where the second equality is due to the definition of w, the third is due to the fact that W_e is less informative than Z, the fourth follows from the definition of $(\hat{\nu}, \hat{\tau})$, and the fifth is due to Claim 3.

Claim 5: If $M = \hat{\nu}(w)$ for given t and w, then $\hat{\tau}$ satisfies (19).

Check: Notice that (16) implies:

$$\begin{aligned} & \sum_{z \in \mathcal{Z}} \sum_{(q,p) \in M^*(w)} \beta^*(z|M^*(w), t)\sigma^*((q,p)|M^*(w), t)[U(q,t,z) - p] \\ \geq & \sum_{z \in \mathcal{Z}} \beta^*(z|M^*(w), t)[U(q,t,z) - p] \quad \forall (q,p) \in M^*(w) \end{aligned}$$

which, by Claim 1 and the definition of $(\hat{\nu}, \hat{\tau})$, rewrites as

$$\begin{aligned} & \sum_{z \in \mathcal{Z}} \sum_{(q,p) \in \hat{\nu}(w)} \Pr(z|w,t)\hat{\tau}((q,p)|\hat{\nu}(w), t, w)[U(q,t,z) - p] \\ \geq & \sum_{z \in \mathcal{Z}} \Pr(z|w,t)[U(q,t,z) - p] \quad \forall (q,p) \in \hat{\nu}(w). \end{aligned}$$

By Claim 2, the latter reduces to

$$\sum_{(q,p) \in \hat{\nu}(w)} \hat{\tau}((q,p)|\hat{\nu}(w), t, w)[U(q,t,w) - p] \geq U(q,t,w) - p \quad \forall (q,p) \in \hat{\nu}(w).$$

Equivalently, if $M = \hat{\nu}(w)$, then $\hat{\tau}$ satisfies (19).

F. Lemons Example

In the last paragraph of Section 5, it is mentioned that the set of prices supporting trade in Akerlof's (1970) lemons market can be affected in a non-monotonic fashion as the information asymmetry decreases because of the public revelation of affiliated information. In the following example $\mathcal{Q} = \{0, 1\}$. There are three equally likely states $\{L, M, H\}$ (quality levels) with payoffs $\{8, 31, 52\}$ to the buyer and $\{0, 20, 40\}$ to the seller. The buyer is initially uninformed, while the seller's information is $\{\{L\}, \{M, H\}\}$. The buyer is the price-setter (here we are considering monopsony pricing by the uninformed buyer) and the informed seller decides whether to accept that price. There is trade for prices $p \in [0, 8]$ (car of quality L only sold) and $p \in [30, 91/3]$ (car of all qualities sold). Now reveal public (affiliated) information $\{\{L, M\}, \{H\}\}$. If $\{L, M\}$ is revealed there can only be trade for prices $p \in [0, 8]$ (car of quality L only sold), but the car of quality M is never sold, as $39/2 < 20$. Here, more public information decreases trade opportunities. Nevertheless, our general result applies, so that the buyer's welfare is higher with public information than without. This can be easily verified in this example.

First, without public information, the buyer can either offer to buy at $p = 0$ (which is going to be accepted only if the seller has a lemon, i.e. with probability 1/3), or

$p = 30$ (which is going to be accepted always). The resulting payoffs of the buyer are: $(1/3)(8-0) = 8/3$ from $p = 0$ and $(1)(91/3 - 30) = 1/3$ from $p = 30$. Clearly, $p = 0$ is optimal, resulting in expected payoff 8/3. Second, what if the public (affiliated) information $\{\{L, M\}, \{H\}\}$ is revealed? Conditional on the realization $\{L, M\}$, $p = 0$ is optimal for the buyer, resulting in a payoff of $(1/2)(8-0)$ for the buyer. If instead $\{H\}$ is revealed, $p = 40$ is optimal, with buyer's payoff $52 - 40 = 12$. In expectation, the buyer now gets $(2/3)(1/2)(8) + (1/3)(12) = 20/3 > 8/3$. As expected, public information increases the expected payoff of the buyer.