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Price Competition for an Informed Buyer[1]

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This paper investigates price competition with private information on the demand side. Two sellers each offer a different variety of a good to a buyer endowed with a private binary signal on their relative quality. The model provides an informational foundation to differentiation in Hotelling's price competition game. Equilibrium comparative statics is performed with respect to the prior belief and the precision of the private information. Competition is fierce when the prior strongly favors one seller and private signals are relatively uninformative. Sellers' equilibrium profits may fall with the revelation of public information and are nonmonotonic in the prior belief. *Journal of Economic Literature* Classification Numbers: C72, D43, D82, L15. © 2001 Elsevier Science

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1. INTRODUCTION

Martha is determined to hire the best decorator to revamp her vast town house in central London with stucco curved ceilings, stone-finished walls,

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and mosaic floors. The classical design of Sanderson—a decorator with a long-standing tradition in London—has clearly an edge on the funkier design of Conran. Each decorator privately gives her a detailed description and demonstration of the services to be provided and quotes an inclusive price. Martha then decides which decorator to hire, if either, on the basis of this information. What is the outcome of price competition in this situation? Do the sellers benefit from revealing public information on the relative quality of their products? When do they prefer to sell to a better informed buyer?

In a number of real-world markets for consumer services and production inputs, the price-taking party knows better than the competing price-setters the match of her preferences (or technology) with the intrinsic characteristics of the good or service exchanged. For example, a subcontractor often knows better the relative cost of the services required by competing contractors. A small business has information on its specific needs for a local network. A customer is privately informed on the relative desirability of insurance deals offered by competing insurers. Consultants compete for jobs while having less information on their relative competence in providing the service. Local jurisdictions bid to attract foreign direct investment by privately informed firms.

This paper extends Bertrand's [3] classic model of simultaneous price competition to asymmetric settings with private information on the demand side. First, sellers may differ *ex ante*, when it is commonly known in the market that one variety of the good (Sanderson) is more likely to be superior to the other (Conran) before the customer receives any private information. Second, sellers appeal *ex post* in different ways to the buyer, depending on the realization of a signal (description and demonstration of the services to be provided) privately observed by the buyer before the purchase decision is made. The simplest possible model of static competition for an informed buyer is constructed: *Two* sellers make simultaneous price offers to a single buyer who observes a private *binary* signal on the *relative* quality of their goods. Ex-post differentiation between the sellers then depends on two parameters, the first measuring prior differentiation and the second the quality of the buyer's private information.

The buyer's uncertainty lends itself to two formally equivalent interpretations: the buyer is unsure about either her taste for the goods or their objective quality. According to the taste interpretation, the buyer has an imperfect signal about her idiosyncratic preference for the goods. According to the quality interpretation, the buyer has some private knowledge of the goods' intrinsic value. The model applies equally well to common values situations with multiple buyers, either when different buyers have perfectly correlated signals or when a continuum of buyers have conditionally independent realizations of the signals. In both cases, the probability of each

signal is equal to the fraction of the population expected by the sellers to receive that signal realization. In the presence of many buyers, the assumption that they take the price as given is rather compelling. More generally, we assume price-taking behavior for convenience.

The first contribution of the paper is a complete characterization of the equilibrium prices and profits of the sellers as functions of the parameters. In two extreme regions of parameters the equilibrium is in pure strategies, while the sellers play a mixed strategy in the remaining intermediate region. When the prior belief on the relative quality of the two goods is sufficiently balanced and the private signal of the customer sufficiently precise, there is little competitive pressure on prices. In the resulting *separating equilibrium* the sellers become local monopolists, thereby enjoying high profits and leaving no rents to the buyer. Each seller targets only the buyer with a favorable private signal, giving up the other to the competitor.

When instead the prior is biased enough in favor of one seller and the precision of the private signal is low, the weak power of private information cannot reverse the strong ex-ante inclination of the customer for one of the two goods. Competition results then in a *pooling equilibrium*, whereby the seller favored by the prior belief covers the entire market by posting a limit price that excludes the competitor. This price is necessarily low, relative to the level of ex-ante quality, because it must induce the customer to buy even after receiving an unfavorable signal. Competition has therefore a tendency to become fiercer and to lead to lower prices as uncertainty on the relative quality decreases. In this region of parameters both types of customer enjoy rents.

In this model the payoff functions of the sellers are discontinuous. For certain prices, by slightly reducing the price a seller can obtain a discrete gain in quantity demanded and therefore in expected profits. In cases of mild final differentiation, the equilibrium is then in *mixed strategies*. The technical problems encountered are similar to those solved by Osborne and Pitchik [23] in the classic Hotelling [16] model of price competition between two sellers located at the interior of the product line and uniform distribution of consumers with linear transportation cost. While they can only provide a partial characterization in the continuous case, the two-point distribution of buyers allows us to obtain a complete characterization of the equilibrium in the pricing game.[2]

The second contribution of the paper is the analysis of how the equilibrium changes with the parameters determining the distribution of the

[2] A simple extension of our methods can be used to characterize the equilibrium in the Hotelling model with binary (rather than uniform) distribution. Such an extension can be useful for investigating price competition among differentiated sellers when consumers are asymmetrically distributed.

buyer's valuations. Uniqueness of equilibrium allows unambiguous comparative statics predictions on equilibrium strategies and payoffs. This exercise allows us to gain insights on the incentives for information policies. Furthermore, this is a preliminary step to the analysis of strategic pricing in learning models with privately informed buyers. In dynamic extensions of this model, the level of ex-ante differentiation is the natural state variable evolving over time with the accumulation of information on relative quality.[3]

Consider changes in the prior belief, achieved by revelation of public information to both buyer and sellers. Abstracting from distributional issues by keeping prices fixed, this information is socially beneficial. Public information is also beneficial to the sellers when competing for a buyer who does not have private information on relative quality. With a privately informed buyer, the logic of Milgrom and Weber's [21] linkage principle implies that a single seller with monopoly power achieves higher expected profits by publicly revealing additional affiliated information. We show that the linkage principle can break down when the competing sellers' equilibrium reactions to the realization of the public signal are accounted for. The reason is that competition is fierce for beliefs biased in favor of one seller (where monopolization results). Surprisingly, the strategic effect is so strong that the equilibrium payoff of the ex-ante superior seller is not even monotonic in the prior belief, in contrast to the monopoly case.

Next, consider changes in the precision of the buyer's private information. In contrast to the prediction in a fixed-price environment, the payoff of the buyer is nonmonotonic in the precision of her own private signal. The degree of ex-post differentiation increases with the quality of the consumers' private information, and more differentiation relaxes price competition, potentially hurting the buyer. We show that the buyer is strictly worse off by overtly acquiring costless private information beyond a certain level. Similarly, the payoff of a seller is nonmonotonic in the precision of the buyer's private information, even though more precise signals raise the total surplus to be shared among the society of sellers and buyer. This extends the findings of Lewis and Sappington [17] for a monopoly setting to this duopoly situation.

The relation of this paper with the literature on competition in the presence of imperfect information is discussed throughout the paper. Harrington [15] extends the model of learning of the market demand from the monopoly to an oligopoly setting with product differentiation. While Harrington looks at the case where the sellers are learning the degree of

[3] Recent literature has looked at the dynamics of price competition as public information spreads in the absence of such private information (see e.g. Bergemann and Välimäki [1], [2]).

substitutability between their products, in our model it is not known which of the two products is superior but it is common knowledge that one is. In this sense, in our model the goods sold are perfect substitutes.[4] In the industrial organization literature, Stole [31] studies price discrimination with non-linear pricing in differentiated oligopoly when consumers have private information on their preferences for brand and quality. In our model with one unit there is instead no scope for action on the quantity dimension, even when sellers are allowed to offer menus of lotteries. In the auction literature, McAfee [20] constructs a dynamic model of price formation where sellers compete in designing mechanisms to sell identical objects to buyers. In our setting instead sellers offer differentiated goods and the buyer has private information on their relative quality.

The paper proceeds as follows: Section 2 introduces our basic duopoly model of competition for an informed buyer and draws the analogy with Hotelling's model. Section 3 characterizes the equilibrium for all parameter values. Section 4 reports on the value of information for the buyer and the sellers. Section 5 comments on the robustness of our results to changes in some of the assumptions. Section 6 concludes the paper.

2. MODEL

Setup

On the supply side of the market there are two sellers who simultaneously post prices. Each duopolist, denoted by $j \in \{0, 1\}$, posts price P_j for her variety of the good and commits to sell at that price if the buyer agrees. The marginal cost of both suppliers is set equal to zero for convenience of notation. Each firm is risk neutral and maximizes expected profits.

On the demand side there is a single risk-neutral buyer with a unit demand for an indivisible good. The payoff in the case of no purchase (action $a_\varnothing$) is 0. There are two states of nature, ω_0 and ω_1, the subscript indicating the superior good. Since good 1 is better than good 1 in state ω_1, while the opposite is true in state ω_0, we assume for convenience that the (gross of price) payoff of purchasing good i (action a_i) in state ω_j is 1 if $i = j$ and 0 if $i \neq j$. Relative quality indicates the match of the preferences of the buyer with the intrinsic characteristics of the good. The buyer maximizes the expected valuation net of the price paid.

Sellers and buyer share the same *prior belief* $q = \Pr(\omega_1)$ on the state of nature. Prior to purchase, the buyer observes the prices posted by both

[4] Schlee [28] considers the value of public information in a model where buyer and seller share the same belief on quality, but in his setting the buyer does not possess any private information while demanding multiple units.

sellers as well as the realization of a private signal on the relative quality of the two varieties. In particular, we consider the case where the signal $\sigma \in \{\sigma_0, \sigma_1\}$ is binary with conditional probability distributions,

$$\Pr(\sigma_i \mid \omega_j) = \begin{cases} \alpha & \text{for} \quad j = i \\ 1-\alpha & \text{for} \quad j \neq i, \end{cases} \tag{2.1}$$

with $i, j \in \{0, 1\}$. Note that for simplicity we are considering the symmetric case where $\Pr(\sigma_i \mid \omega_i) = \alpha$ for $i = 0, 1$. Without loss of generality we restrict our attention to $\alpha \in [1/2, 1]$, since $\alpha \in [0, 1/2]$ would be equivalent to relabeling the signals. The binary signal distribution allows a simple parametrization of the *quality of information*: A higher α corresponds to a more informative experiment in the sense of Blackwell.[5]

The firms' common probability assessment that signal σ_i is received by the customer is

$$\Pr(\sigma_i \mid q, \alpha) \equiv q \Pr(\sigma_i \mid \omega_1) + (1-q) \Pr(\sigma_i \mid \omega_0). \tag{2.2}$$

Let $f_i(q, \alpha) \equiv \Pr(\omega_1 \mid \sigma_i)$ be the buyer's posterior belief that the state is ω_1 after observing the signal realization σ_i. Bayes' rule yields

$$f_0(q, \alpha) = \frac{(1-\alpha)\, q}{\alpha(1-q) + (1-\alpha)\, q}, \qquad f_1(q, \alpha) = \frac{\alpha q}{\alpha q + (1-\alpha)(1-q)}. \tag{2.3}$$

The updated belief represents, given the zero/one payoffs, the customer's expected valuation for good 1 in monetary terms. The ex-ante valuations are q for good 1 and $1-q$ for good 0, and the ex-post valuations of the buyer with signal σ_i are f_i for good 1 and $1-f_i$ for good 0.

As illustrated in Fig. 1, the firms simultaneously quote prices without knowing the signal drawn by nature according to (2.1). Upon observation of the realization σ_i, the *type-i* customer updates the belief from q to f_i and decides whether to buy and from which firm to buy. The payoff to a firm when selling is equal to the price charged and the payoff of the customer is equal to the valuation for the good bought minus the price paid for it. The game being symmetric with respect to $q = 1/2$, attention is restricted to $q \in [1/2, 1]$.

[5] The binary signal formulation is widely adopted in information economics (see e.g. Broecker's [6] study of competition with adverse selection). We refer the reader to Section 5 for a discussion of the robustness of our results to alternative specifications of the signal structure.

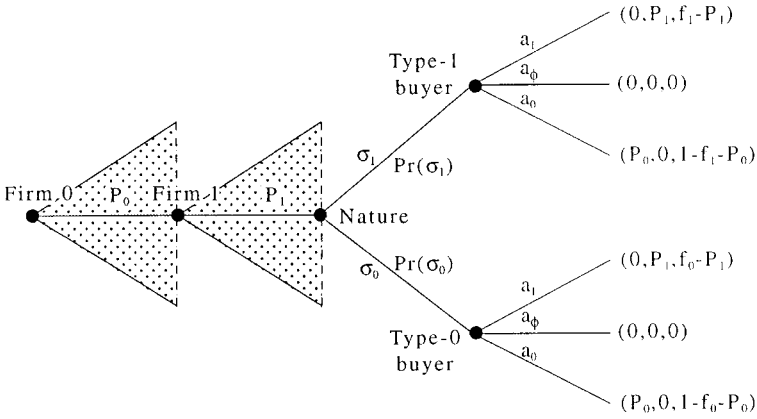

FIG. 1. The extensive form of the game is shown. The payoffs are given at the terminal nodes in this order: seller 0, seller 1, and the buyer.

Hotelling Analogy

This model provides a simple informational foundation to the pricing stage of Hotelling's model of competition for given locations of the sellers. Figure 2 represents the model in the Hotelling line. Each seller is located at the endpoints of a segment of unit length, seller 0 at the origin and seller 1 at the other end. The consumer is located in the interior of the interval, depending on the realization of the private information on the relative desirability of the products. With probability $\Pr(\sigma_1 \mid q, \alpha)$ the consumer is at $f_1(q, \alpha)$ and with complementary probability at $f_0(q, \alpha)$. The distance of consumer i from seller j is denoted by $D(i, j)$. The consumer of type i enjoys net utility $1 - D(i, j) - P_j$ when purchasing from seller j at price P_j and 0 when not buying any good. The unconditional probability of a buyer of either type plays the same role in our model as the mass of consumers located to the side of each seller in the standard Hotelling model.[6]

The outcome of competition between sellers depends on the distribution of the buyer. A prior more biased in favor of one seller shifts the distribution closer to that seller, and more accurate information of the buyer corresponds to a mean-preserving spread in the distribution. Bayesian updating imposes restrictions on the comparative statics exercise: changes

[6] In a similar vein, Gabszewicz and Grilo [14] study price competition conditional on quality in a duopoly market where firms sell vertically differentiated products and consumers have heterogeneous beliefs on quality. In order to tackle the problem of information acquisition, we instead perform unconditional analysis in a market for a single buyer with beliefs derived from a common prior.

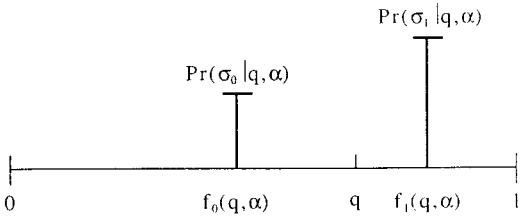

FIG. 2. The Hotelling lin is shown. The buyer is located at $f_i(q, \alpha)$ with probability $\mathrm{PR}(\sigma_i \mid q, \alpha)$. Seller j is located at j.

in q and α affect *simultaneously* the location and the distribution of demand according to Bayes' rule.

The ex-post valuations f_0 and f_1 for good 1 of both buyers' types increase in the prior belief: $\partial f_0/\partial q > 0$ and $\partial f_1/\partial q > 0$. Given the symmetry of payoffs, any change in the ex-post valuation for one good is associated with the same change in the ex-post valuation for the other good. The valuation spread $\Delta f \equiv f_1 - f_0 > 0$ is the difference between the type i and the type $1-i$ buyer in the valuation for good i. Note that the valuation spread is decreasing in q for $q > 1/2$, because Δf is a concave function of q (by $\partial^2 f_1/\partial q^2 < 0 < \partial^2 f_0/\partial q^2$) maximized at $q = 1/2$. The reduction in the spread is a by-product of Bayesian updating with two states of nature. Regarding the distribution of demand at the two locations, q raises $\Pr(\sigma_1)$ but reduces $\Pr(\sigma_0)$.

An increase in the quality of the buyer's information spreads the buyer's posterior valuations further apart by increasing f_1 and reducing f_0: $\partial f_0/\partial \alpha < 0 < \partial f_1/\partial \alpha$. The effect on the distribution at those locations is similar: $\partial \Pr(\sigma_0)/\partial \alpha < 0 < \partial \Pr(\sigma_1)/\partial \alpha$. Overall, an increase in α results in a mean-preserving spread of the distribution of posterior valuation.

Ex-ante differentiation is defined as $|q - 1/2|$ and measures the degree of vertical differentiation between the two goods. High ex-ante differentiation obtains in asymmetric situations, where the quality of one seller is expected to be higher than that of the competitor. Given our restriction to $q \geqslant 1/2$, the prior valuation for good 1 (equal to the prior belief q) parametrizes ex-ante differentiation. By the martingale property of beliefs, the expectation of the posterior valuation for good 1 is equal to the prior valuation q.

Ex-post differentiation is defined as the variance of the posterior valuation $\Pr(\sigma_1)(f_1)^2 + \Pr(\sigma_0)(f_0)^2 - q^2$ and measures the degree of horizontal differentiation generated by the private information of the buyer on the relative quality of the goods. Ex-post differentiation is easily verified to increase in the quality of private information α and decrease in the level of ex-ante differentiation q. Furthermore, it is flat as a function of q around the two extreme values $q = 1/2$ and $q = 1$.

3. EQUILIBRIUM

Each duopolist wishes to extract the maximum rent from the buyer for given expected quantity demanded and at the same time is tempted to steal demand from the competitor. The *Bayes–Nash equilibrium* of the game strikes a balance between these two forces, depending on the parameters of the model, i.e., the quality of private information $\alpha \in [1/2, 1]$ and the prior belief $q \in [1/2, 1]$.

Buyer's behavior. The customer chooses the good yielding the highest expected payoff net of the price, provided that such a net payoff is non-negative because of the outside option of not purchasing. Good 1 is preferred to good 0 by the type-i buyer when

$$f_i - P_1 > 1 - f_i - P_0, \qquad (IC_{i,1})$$

where $IC_{i,j}$ stands for the *incentive compatibility* for the buyer of type i to buy good j. By reversing the inequality one obtains the $IC_{i,0}$ constraint. The type-i customer is exactly indifferent for prices when IC_i binds

$$P_1 = 2f_i - 1 + P_0. \qquad (IC_i)$$

It is convenient to denote the price of firm j corresponding to P_{1-j} through the IC_i constraint by $IC_i(P_{1-j})$; i.e., $IC_i(P_0) = 2f_i - 1 + P_0$ and $IC_i(P_1) = 1 - 2f_i + P_1$. The maximum willingness to pay for good j by the type-i customer is determined by the *individual rationality* constraint $IR_{i,j}$

$$f_i - P_1 \geqslant 0, \qquad (IR_{i,1})$$

$$1 - f_i - P_0 \geqslant 0. \qquad (IR_{i,0})$$

The type-i customer buys good 1 if $P_1 < \min\langle 2f_i - 1 + P_0, f_i \rangle$ on good 0 if $P_0 < \min\langle 1 - 2f_i + P_1, 1 - f_i \rangle$. Figure 3 represents the IC's and IR's constraints in the P_0, P_1 space. The $IR_{i,1}$ lines are horizontal and the $IR_{i,0}$ lines vertical. The IC_i lines both have unit slope. As illustrated in the figure, $IC_{1,1}$ is binding for $P_0 < 1 - f_1$ and $IR_{1,1}$ is binding otherwise. $IC_{1,0}$, rather than $IR_{1,0}$, is instead always binding in the relevant range of prices.

Sellers' Best Replies. Given a price P_0 posted by firm 0, only three strategies are not patently dominated for firm 1: not selling at all, selling only to type-1 customer at the *separating* price

$$P_1^S(P_0) \equiv \begin{cases} 2f_1 - 1 + P_0 & \text{for} \quad P_0 \leqslant 1 - f_1 \\ f_1 & \text{for} \quad P_0 \geqslant 1 - f_1 \end{cases} \qquad (3.1)$$

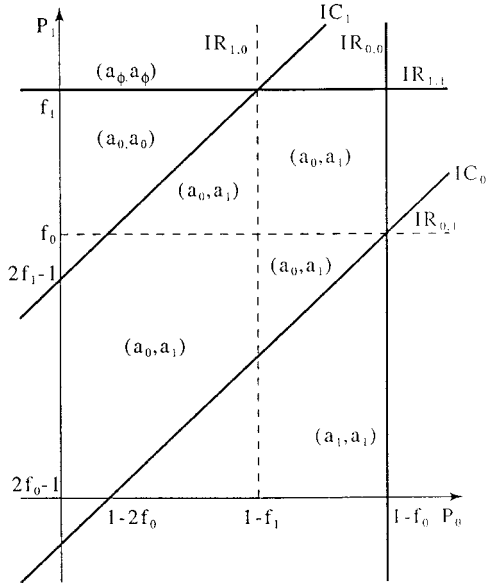

FIG. 3. The constraints in the price space. In the three regions of price combinations where purchase takes place, the buyer's decision is represented by two numbers: (a_i, a_j) means that the type-0 buyer purchases good i and the type-1 buyer purchases good j.

or to both customer types at the *pooling* price

$$P_1^P(P_0) \equiv 2f_0 - 1 + P_0. \tag{3.2}$$

The best reply is then the strategy which achieves $\max\langle 0, \Pr(\sigma_1)\, P_1^S(P_0), P_1^P(P_0)\rangle$. At the (unique) *switching price* $\hat{P}_j$ the best reply function of firm $1-j$ jumps down from the separating price $P_{1-j}^S(\cdot)$ to the pooling price $P_{1-j}^P(\cdot)$, so that $\Pr(\sigma_{1-j})\, P_{1-j}^S(\hat{P}_j) \equiv P_{1-j}^P(\hat{P}_j)$.

Nontrivial issues of equilibrium existence arise in this class of games with discontinuous payoffs (see e.g. Dasgupta and Maskin [12]), resolved here by constructing the equilibrium for all parameter configurations. Depending on the parameters, the equilibrium is either in pure strategies (separating or pooling) or in mixed strategies.

3.1. *Separating Equilibrium*

The unique equilibrium is *separating* in the region of parameters indicated by S in Fig. 4: The customer of type i buys from seller i. For these parameter configurations the switching prices exceed the maximum valuations for the two goods, so that the best reply correspondences of the two firms cross at the corner point $P_0 = 1 - f_0$, $P_1 = f_1$ (see Fig. 5a). The no-deviation condition for

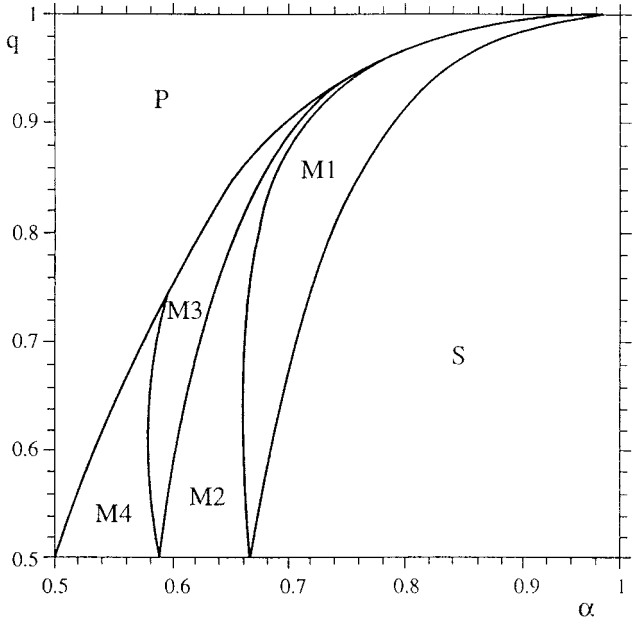

FIG. 4. The parameter space with the six equilibrium regimes. The separating region is marked by S, the pooling region by P, and the mixing regions by M1, M2, M3, and M4.

firm 1 requires that pooling both types of buyer is less profitable than selling only to the ex-post favorable customer at the separating price, $2f_0 - 1 + P_0 = f_0 \leqslant \Pr(\sigma_1) f_1$, or equivalently $q < q^S(\alpha) \equiv (\alpha^2 + \alpha - 1)/[\alpha(2\alpha - 1)]$ with $dq^S/d\alpha > 0$. The no-deviation condition for seller 0, equivalent to $q \geqslant 1 - q^S$, is implied by $q \geqslant q^S$ for $q \geqslant 1/2$. For this equilibrium to exist, it is necessary that the quality of private information be large enough ($\alpha \geqslant 2/3$).

For these parameters the sellers are weakly ex-ante differentiated, but strongly ex-post differentiated. The customer is much more keen to buy from one of the two sellers depending on the realization of the private signal received. In turn, each supplier targets only the customer with favorable information for its own good, since it would be too costly to steal from the competitor the customer with unfavorable information. In sharp contrast with the standard Bertrand paradox, in this equilibrium there is no competitive pressure on prices. The sellers become local monopolists and make high profits by fully extracting the customer's surplus.[7] In summary, we have

[7] Full rent extraction only arises because the number of sellers (goods) is equal to that of buyer types (signal realizations). With nonbinary signals, some buyer types would enjoy rents even in a separating equilibrium.

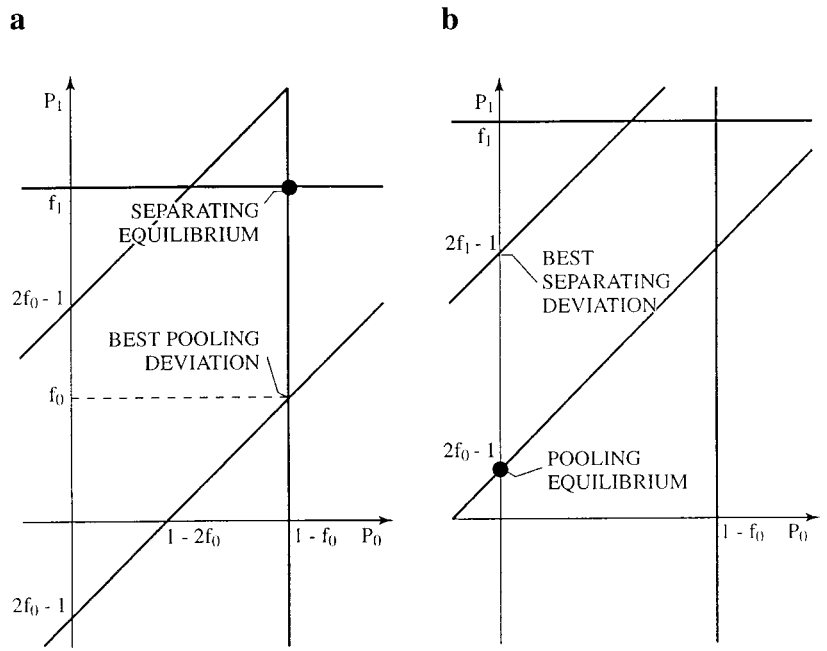

FIG. 5. (a) The separating equilibrium in the price space. (b) The pooling equilibrium in the price space.

PROPOSITION 1 (Seperating Equilibrium). *For* $q < q^S(\alpha)$ *the unique equilibrium is separating*: *prices are* $P_0(q, \alpha) = 1 - f_0(q, \alpha)$, $P_1(q, \alpha) = f_1(q, \alpha)$, *and the buyer who receives signal* σ_i *purchases from seller i. Expected profits for the sellers are* $V_0(q, \alpha) = \alpha(1-q)$, $V_1(q, \alpha) = \alpha q$ *and the buyer's expected rent is* $V_B(q, \alpha) = 0$.

3.2. *Pooling Equilibrium*

When the switching price of one firm is below the marginal cost (e.g. $\hat{P}_0 \leqslant 0$), the only equilibrium is *pooling* on the good sold by the other firm, in this case good 1: The customer buys from seller 1 regardless of the realization of the signal. Consider the prices $P_0 = 0$, $P_1 = IC_0(0) = 2f_0 - 1 > 0$, at which the indifferent type-0 buyer breaks the tie in favor of the high price seller 1 who sells with probability one. See Fig. 5b for an illustration in the price space. Clearly, seller 0 has no profitable deviation since any nonnegative price would not sell. Seller 1 must prefer not to deviate to the separating price $IC_1(0) = 2f_1 - 1$ along the IC_1 constraint where the $IR_{1,1}$ constraint is satisfied with strict inequality. Since the separating price $IC_1(0)$ sells with probability $\Pr(\sigma_1)$, it is needed that $IC_0(0) \geqslant \Pr(\sigma_1)\, IC_1(0)$. In the limit as q tends to 1, f_1 and f_0 both converge

to 1, so that the separating price converges to the pooling one, while the probability of selling at the separating price tends to α. For q large enough it is then optimal for seller 1 to charge the pooling price, thereby selling with probability 1. A pooling equilibrium of this sort exists if and only if $q \geqslant q^P(\alpha)$, where $q^P(\alpha)$ is the largest root of

$$2f_0(q, \alpha) - 1 = q - (1 - \alpha), \tag{3.3}$$

with $q^P(\alpha) \in (\alpha, 1)$ and $dq^P/d\alpha > 0$. The pooling equilibrium region is marked by P in Fig. 4.

Note that the nonselling firm in a pooling equilibrium could reduce the profit of the selling firm to any nonnegative level by posting a negative price (a weakly dominated strategy). For $q > q^P(\alpha)$ there is a continuum of pure strategy pooling equilibria, where the nonselling firm posts $P_0 \in [1 - 2f_0, 0]$ and firm 1 sells at price $P_1 = 2f_0 - 1 + P_0 \in [0, 2f_0 - 1]$, thereby achieving profits $V_1 = P_1$. In order to exclude these undesirable equilibria we follow Bergemann and Välimäki [1] by requiring equilibria to be *cautious*: The nonselling firm must be indifferent between selling and not at the posted price.[8]

In the pooling region the prior belief favors one seller and the private signal precision α is low. The final result of strong ex-ante heterogeneity and of mild ex-post spread is strongly biased for the seller who is favored by the prior belief. Therefore, in equilibrium this seller becomes a global monopolist and covers the entire market by posting a limit price which excludes the competitor. With private information of bounded precision, the ex-ante superior seller finds it profitable to post a relatively low price, so that the buyer buys also when receiving an unfavorable private signal. The competitive pressure by the nonselling firm keeps the price low and leaves rents to both types of buyer. Although the pooling price is low relative to the prior willingness to pay of the buyer, it is high in absolute terms. In summary, we have

PROPOSITION 2 (Pooling Equilibrium). *For $q > q^P(\alpha)$ the unique cautious equilibrium is pooling on good* 1: *prices are $P_0(q, \alpha) = 0$, $P_1(q, \alpha) = 2f_0(q, \alpha) - 1$, and both types of buyer purchase from seller* 1. *Expected profits for the sellers are $V_0(q, \alpha) = 0$, $V_1(q, \alpha) = 2f_0(q, \alpha) - 1$ and the buyer's expected rent is $V_B(q, \alpha) = 1 + q - 2f_0(q, \alpha)$.*

3.3. *Mixed Strategy Equilibrium*

In the classic Hotelling [16] pricing game with uniform distribution of consumers, a purestrategy equilibrium fails to exist when the sellers are

[8] Uniqueness of equilibrium can be obtained also by eliminating strategies which are dominated according to the definition given by Börgers [5, p. 168].

located relatively close to each other (see e.g. d'Aspremont, *et al.* [11]). Similarly, in our model the equilibrium is in mixed strategies for weak ex-post differentiation, corresponding to intermediate levels of horizontal differentiation, $\max\langle 1/2, q^S(\alpha)\rangle < q < q^P(\alpha)$. The characterization of the mixed-strategy equilibrium is similar to the one obtained by Osborne and Pitchik [23] in the original Hotelling model.[9] A sequence of preliminary steps (Lemmata 1–6 in the Appendix) gives joint restrictions on the equilibrium randomizations that simplify the construction of the equilibrium and allows us to prove uniqueness. In the spirit of Simon and Zame [30], an endogenous tie-breaking rule is used. However, the tie-breaking rule is immaterial because in any mixed strategy equilibrium ties occur with probability zero.

As illustrated in Fig. 4, the mixed strategy region in the parameter space can be partitioned into four subregions (M1, M2, M3, and M4), each corresponding to a different specification of the equilibrium strategies. While the details on the construction of the equilibrium are left to the Appendix and our working paper [22], we provide here a brief summary. Seller $j=0, 1$ randomizes on a set $\mathscr{P}_j \subseteq [\underline{P}_j, \bar{P}_j]$, with *upper and lower bounds* taking values in the following intervals: $\underline{P}_0 \in [0, \bar{P}_0)$, $\bar{P}_0 \in (1-f_1, 1-f_0]$, $\underline{P}_1 \in [2f_0-1, \bar{P}_1]$, and $\bar{P}_1 \in [2f_1-1, f_1]$. Let $\tilde{P}_j \in \mathscr{P}_j \subset [\underline{P}_j, \bar{P}_j]$ be the *fully separating* price at which seller j sells with probability $\Pr(\sigma_j)$ to its own customer, given the strategy of the competitor. The following nonlinear map links this price to the upper and lower bounds of the support:

$$p_j(\underline{P}_j, \bar{P}_j) \equiv \frac{\Pr(\sigma_j)\cdot \underline{P}_j \cdot \bar{P}_j}{\Pr(\sigma_j)\cdot \bar{P}_j - \Pr(\sigma_{1-j})\cdot \underline{P}_j}.$$

For any $\alpha \in [0.5, 1)$ and for any $q \in [q^S(\alpha), q^P(\alpha)]$ there is a mixed strategy equilibrium where, for each seller, upper bounds and lower bounds of the support and of the fully separating price, $\{\bar{P}_j, \underline{P}_j, \tilde{P}_j\}_{j=0,1}$, are the unique solution to the nonlinear system of equations

$$\begin{aligned} \tilde{P}_0 &= \min\langle 1-f_0, p_0(\underline{P}_0, \bar{P}_0)\rangle & \tilde{P}_1 &= \min\langle f_1, p_1(\underline{P}_1, \bar{P}_1)\rangle \\ \bar{P}_0 &= \min\langle 1-f_0, 1-2f_0+\tilde{P}_1\rangle & \bar{P}_1 &= \min\langle f_1, 2f_1-1+\tilde{P}_0\rangle \\ \underline{P}_0 &= 1-2f_1+\tilde{P}_1 & \underline{P}_1 &= 2f_0-1+\tilde{P}_0 \end{aligned} \tag{3.4}$$

[9] Mixed strategy equilibria have been constructed also in rather different models of price competition. Rosenthal and Weiss [27] solved the Rothschild–Stiglitz nonexistence puzzle by characterizing the mixed-strategy equilibrium in a model of competition in the presence of signaling by an agent with two types. The features of their equilibrium are completely different from ours, because in their model the competitors are identical and therefore make no profits in equilibrium. Similarly, in Varian's [32] model identical sellers engage in sales behavior in an attempt to price discriminate between informed and uninformed consumers in a Butters-like [7] world.

which also satisfies

$$\underline{P}_0 < \min\langle \alpha(1-q), 1-f_1 \rangle, \qquad \underline{P}_0 > \max\langle 0, 1-2f_1+f_0 \rangle. \tag{3.5}$$

The support of seller *j's* strategy $\mathscr{P}_j$ is the *entire* interval $[\underline{P}_j, \bar{P}_j]$ except (i) if the solution to (3.4) and (3.5) entails $\tilde{P}_0 > 1-f_1$, as in the M1 and M2 regions (resp., $\tilde{P}_1 > f_0$, as in M1, M2, and M3), then $\mathscr{P}_0$ (resp. $\mathscr{P}_1$) does not include the interval $(1-f_1, \tilde{P}_0)$ (resp. $(f_0, \tilde{P}_1)$) and correspondingly seller 1's (resp. seller 0's) strategy has an atom on the maximum price f_1 (resp. $1-f_0$); (ii) if $\tilde{P}_1 = f_1$, as in M1, then $\mathscr{P}_0 = [\tilde{P}_0, 1-f_0]$. The equilibrium payoff of seller j is $V_j = \Pr(\sigma_j)\,\tilde{P}_j$.

As ex-post differentiation declines, the price required from firm 1 to attract a hostile customer increases, making more tempting the option of selling to the entire market. Firm 0 responds by pricing low. Moving from the separating to the pooling region the equilibrium randomizations put more weight on low prices, as the seller favored ex ante has more incentive to undercut the competitor. The equilibrium randomizations are illustrated in the four panels of Fig. 6. As the equilibrium regime changes, strategies change in a discontinuous fashion but the payoffs to the three parties change continuously. Finally, we have

PROPOSITION 3 (Uniqueness of Equilibrium). *The cautious equilibrium is unique for almost all parameters* $\alpha \in [1/2, 1]$ *and* $q \in [1/2, 1]$.

3.4. *Illustration*

By way of introduction to the comparative statics exercise performed in the next section, it is useful to summarize the structure of the equilibrium in four special cases:

First, when there is no private information on the demand side ($\alpha = 1/2$), we are back to the Bertrand case with heterogeneous suppliers. In the unique cautious Nash equilibrium firm 0 posts $P_0 = 0$ without being able to sell and makes zero profits $V_0(q, 1/2) = 0$ and firm 1 sells to the unique type of the buyer at price $P_1 = 2q - 1$ and makes profits $V_1(q, 1/2) = 2q - 1$, while the indifferent buyer enjoys the net payoff $V_B(q, 1/2) = 1 - q$.

Second, with a perfectly informed ex post customer ($\alpha = 1$) in equilibrium $P_0(q, 1) = P_1(q, 1) = 1$, $V_0(q, 1) = 1 - q$, $V_1(q, 1) = q$, and $V_B(q, 1) = 0$. Compared to the previous polar case of no private information ($\alpha = 1/2$), it can already be seen that the buyer is worse off when (known to be) perfectly informed ($0 < 1 - q$), even though the sum of the payoffs of buyer and sellers is highest at $\alpha = 1$.

Third, when the buyer is perfectly informed ex ante ($q = 1$) the superior seller 1 monopolizes the market and extracts the entire surplus of the buyer: $P_0(1, \alpha) = V_0(1, \alpha) = 0$, $P_1(1, \alpha) = V_1(1, \alpha) = 1$, and $V_B(1, \alpha) = 0$.

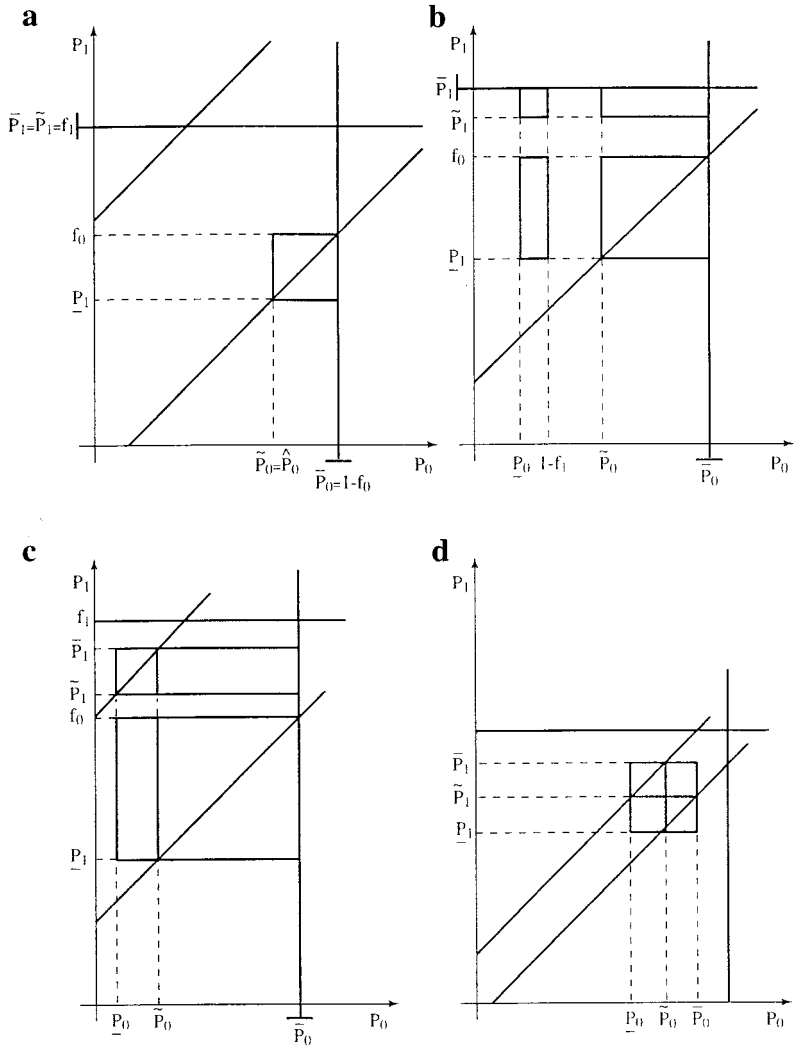

FIG. 6. The four mixed strategy equilibrium regimes in the price space are shown: (a) M1, (b) M2, (c) M3, and (d) M4.

Fourth, in the symmetric game with ex-ante identical sellers ($q = 1/2$) there is an interval of α corresponding to each of the three types of equilibria which are not asymmetric in nature (M4, M2, and Separating). For $\alpha \in [1/2, 2-\sqrt{2}]$ the unique equilibrium is of type M4, defined by $\underline{P}_0 = \underline{P}_1 = \sqrt{2-8\alpha+8\alpha^2}$) and $\tilde{P}_0 = \tilde{P}_1 = (1+\sqrt{2})(2\alpha-1)$ and with profit $V_0(1/2, \alpha) = V_1(1/2, \alpha) = (1+\sqrt{2})(2\alpha-1)/2$ for both sellers. For $\alpha \in [2-\sqrt{2}, 2/3]$ the unique equilibrium

is of type M2, defined by $\underline{P} = 1 - 3\alpha - 2\alpha^2/(1 - 4\alpha + \sqrt{1 - 8\alpha + 12\alpha^2})$ and $\tilde{P} = (2\alpha - 1 + \sqrt{1 - 8\alpha + 12\alpha^2})/2$ and with profit $V(1/2, \alpha) = (2\alpha - 1 + \sqrt{1 - 8\alpha + 12\alpha^2})/4$. Finally, for $\alpha \in [2/3, 1]$ the separating equilibrium is $P = \alpha$, and $V(1/2, \alpha) = \alpha/2$ for both sellers.

4. COMPARATIVE STATICS

This section illustrates the effects of changes in the prior belief q and the quality of the buyer's private information α on the players' payoffs in the unique equilibrium of the duopoly model. In order to uncover the role of strategic interaction for our results, we first describe the efficient allocation and the solution of the problem of the monopolist.

4.1. *Efficient Allocation*

With zero production costs the total surplus to be divided among the three players is equal to the valuation of the buyer. In the efficient allocation the consumer buys from the ex-post superior seller: the consumer of type i chooses good 1 if $f_i \geqslant 1 - f_i$. In general, the social optimum can be easily implemented by giving the bargaining power to the informed buyer.[10] For $q \in [1/2, \alpha]$ the signal is relevant for the optimal decision, and the social surplus is $W(q, \alpha) = \Pr(\sigma_1)\ f_1 + \Pr(\sigma_0)(1 - f_0) = \alpha$. For $q \in [\alpha, 1]$ it is efficient to buy good 1 regardless of the signal, so that the expected social surplus is equal to the buyer's ex ante valuation for that good, $W(q, \alpha) = \Pr(\sigma_1)\ f_1 + \Pr(\sigma_0)\ f_0 = q$. Overall, social surplus is a continuous, weakly increasing, and convex function of q for given α and a concave function of α for given q. A more precise signal (higher α) leads to an increase in the total surplus only if it is strong enough to potentially reverse the prior; otherwise information is socially worthless. Revelation of additional information introduces a spread in the belief which can only increase social welfare by allowing for better decisions.

The equilibrium outcome is not necessarily efficient. When the sellers compete ex ante, the buyer does not necessarily purchase the ex-post superior good, as would be required to achieve allocative efficiency. In particular, for mild final differentiation (in the mixed strategy region and in part of the separating region) the equilibrium is inefficient because the ex-ante superior seller has an incentive to keep the price high in order to extract more rent from the consumer.

[10] Note that in this binary signal model, the efficient outcome would equally result when at least two (equally efficient) sellers of each type simultaneously compete in prices.

4.2. *Monopoly Benchmark*

Consider the simple optimization problem of a monopolist competing against a good sold at fixed price (set to zero for convenience). It is immediate to verify that the optimum profit (or value) function of the monopolist is

$$V_M(q,\alpha)=\begin{cases}0 & \text{for} \quad q\leqslant 1-\alpha\\ q-(1-\alpha) & \text{for} \quad 1-\alpha\leqslant q\leqslant q^P(\alpha)\\ 2\,f_0(q,\alpha)-1 & \text{for} \quad q\geqslant q^P(\alpha),\end{cases} \tag{4.1}$$

where $q^P(\alpha)$ is again the largest root of the quadratic equation (3.3). For a low enough prior belief ($q<1-\alpha$) the monopolist prefers not to sell since in this region even the separating price $2f_1-1$ is negative. For intermediate beliefs the separating price gives a higher expected payoff than the pooling price $2f_0-1$. For a high enough prior ($q\geqslant q^P$) pooling becomes optimal for a reason similar to that discussed in Section 3.2.

First, the optimum profit function is (strictly) increasing (when positive) and globally convex in q, being the maximum of convex functions. It can be shown that convexity of the monopolist's value function in the prior distribution always holds for a general number of signals in the monopoly pricing model with binary state.[11] This is a manifestation of the linkage principle of Milgrom and Weber (1982): Revealing public affiliated information is always beneficial to the seller in auctions with affiliated values.[12] Therefore, a monopolist always benefits from revealing as much public affiliated information as possible by committing to public testing, credible certifiers of qualities, and revelation of the satisfaction of other consumers. This fact has also important dynamic consequences, as a patient monopolist would be willing to spend resources in the short run to foster revelation of public information on the quality of the good.

Second, monopoly profits are nonmonotonic in the precision α of the buyer's signal, as first stressed by Lewis and Sappington [17].[13] As it is seen immediately from (4.1), the monopolist's profit function is decreasing

[11] It is required that the conditional signal distributions satisfy the monotone likelihood ratio property, which is without loss of generality with two states of nature. More generally, in an affiliated environment revelation of public information raises the expected profits of a price discriminating monopolist selling to a privately informed buyer (Ottaviani and Prat [24]).

[12] Note that monopoly pricing with a single seller can be seen as a second-price auction with a single bidder, where the monopoly price plays the role of the reserve price. The bid of the buyer is either above the reserve price, in which case the reserve price is the second-price paid by the buyer, or below it when the buyer decides not to buy.

[13] See Courty [9] for an application of these findings to the dynamics of price discimination in ticket markets.

in α for $\alpha \leqslant (q^P)^{-1}(q)$ and increasing for $\alpha \geqslant (q^P)^{-1}(q)$. The monopolist benefits from a stronger signal of the buyer when posting the separating price. In the pooling region the monopolist is instead forced to reduce further the price to sell to the buyer with a more precise unfavorable signal.

4.3. *Value of Public Information in Equilibrium*

We are now ready to discuss whether the properties of the profit function of the monopolist extend to a strategic setting. The equilibrium payoff of each duopolist is continuous in both parameters, as can easily be verified algebraically at the borders across the parameter regions. The *form* of the equilibrium randomizations need not be continuous when crossing such borders between regions. In such cases it is verified that there are two equilibria, so that the equilibrium correspondence is upper-hemicontinuous in the parameters.

An increase in the common prior belief q that good 1 is superior then has two contrasting effects on profits. The first is the direct effect generated by the increase in the relative valuation for good 1 of all types of buyer (ex-ante differentiation). According to this effect, firm 1 enjoys a stronger advantage on the competitor and should be able to make higher profits. There is a second effect, acting indirectly through the reduction in the variance of the posterior valuation (ex-post differentiation). It is then less costly for seller 1 to attract the hostile type-0 customer. By strategic complementarity of Bertrand competition, this triggers an aggressive response by the competitor, who quotes low prices to avoid being cornered out of the market. In equilibrium firm 1 is then forced to reduce its price. Seller 1's profits rise in q less than proportionally, as the firm goes from fully extracting the buyer's rent in the separating region to a limit price which leaves some rents to both types of buyer in the pooling region.

In this model, the second effect can be so strong as to dominate the first. Figure 7 plots equilibrium profits for the two firms as functions of the prior q for private information of quality $\alpha = 0.69$. By using $V_1(q, \alpha) = V_0(1-q, \alpha)$, it is immediate to construct the value function for each seller for all $q \in [0, 1]$. As verified in the Appendix, we have

PROPOSITION 4 (Nonmonotonicity of Profits in Prior Belief). *For high enough quality of private information $\alpha \in (2/3, 1)$, firm 1's equilibrium profits $V_1(\alpha, \cdot)$ are strictly decreasing in the prior q, for q belonging to a nonempty interval $I(\alpha) \subset [q^S(\alpha), q^P(\alpha)]$.*

Combining the fact that profits are decreasing for $q \in [q^S(\alpha), q^P(\alpha)]$ and increasing after q^P with the fact that profits are increasing over some lower range one obtains:

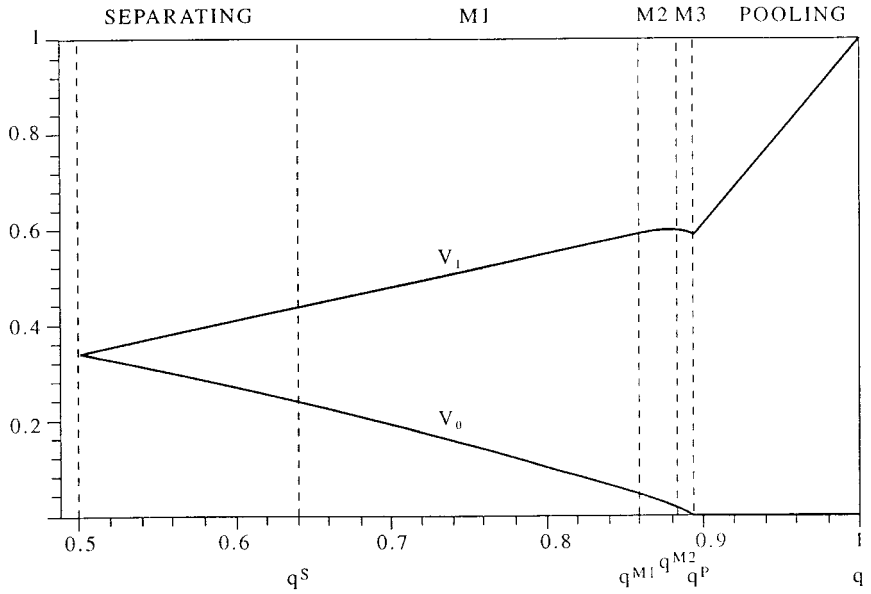

FIG. 7. The equilibrium profits of the two sellers, V_0 and V_1, as functions of the prior q for fixed qulity of information $\alpha = 0.69$.

PROPOSITION 5 (Nonconvexity of Profits in Prior Belief). *For high enough quality of private information* $\alpha \in (2/3, 1)$, *firm* 1*'s equilibrium profits* $V_1(\alpha, \cdot)$ *are not convex in the prior* q.[14]

More directly, the payoff cannot be convex because $\alpha q^P(\alpha) \geqslant q^P(\alpha) - (1-\alpha)$; i.e., the linear projection of the separating equilibrium payoff αq to the prior $q = q^P(\alpha)$ on the boundary of the pooling region is strictly higher than the pooling payoff achieved at that belief. To see that nonconvexity in the prior belief requires the combination of strategic interaction with private information of the buyer, note: (i) the convexity of the social payoff in the efficient allocation (Section 4.1); (ii) the convexity of the profit function of a monopolist selling one of the two goods against a competitive sector offering the other (Section 4.2); (iii) the convexity of the profit function of a multiproduct monopolist carrying both goods, equal to $\max\langle 1-f_1, \alpha, f_0\rangle$; and (iv) the convexity of equilibrium payoffs of competing sellers when the buyer has no private information (see e.g. the model of strategic pricing in the presence of public learning by Bergemann and Välimäki [2]).

An important implication of nonconvexity of the sellers' profits in the belief is that they might dislike revelation of (socially valuable) information

[14] This result does *not* follow from discontinuity of the equilibrium value function, which is instead continuous in the parameters.

publicly observed by all parties. For intermediate values of the prior belief and when the quality of the contemporaneous private information of the buyer is large enough, seller 1 strictly prefers to avoid diffusion of *some* public signal.[15] This implies that competing sellers do not necessarily wish to commit to transparency rules, in contrast to the monopoly case. This result also points to the fact that Milgrom and Weber's [21] revenue ranking of auctions does not extend to settings where mechanism designers compete for buyers with private information on the relative values of the goods.[16]

The violation of the linkage principle is clearly a robust feature of competition for a buyer who is privately informed about the relative quality of the goods sold. Revelation of public affiliated information has two effects on profits. The first positive effect is due to the (average) increase in the price that each type of buyer is willing to pay for the good of a single seller, for any fixed price of the competitor. When the competitor is allowed to change the price in response to the realization of the public signal, there is a second counteracting effect. Competition becomes more aggressive due to the reduction in the variance of the posterior valuation (ex-post differentiation) resulting from revelation of public information. This second effect is clearly not specific to our formulation. Our model allows us to show that this second effect can be strong enough to dominate the first.

It is useful to compare our findings with those of Harrington [15]. In his model, equilibrium profit is convex (concave) in the mean level of substitutability when goods are relatively substitutable (differentiated). Differently from Harrington, the asymmetry of our game drives our nonmonotonicity and nonconvexity results. In our setting, the profit of the ex-ante superior firm turns out to be locally convex in q when q and α are both close to $1/2$, so that the products are good substitutes, but it is nonconvex for high α and intermediate q when they are bad substitutes. The novelty of our approach and results is confirmed by the asymmetric effects of changes in q on the two firms' profits. For $q > 1/2$, firm 0 always loses from an increase in q, while firm 1 gains or loses depending on the value of q. Similarly, the convexities of the two profit functions do not agree (see Fig. 7 for an example). Therefore the two firms may have contrasting preferences regarding the diffusion of public information, depending on q, whereas in Harrington their interests in this respect always coincide. This also suggests that dynamic versions of our game would uncover strategic aspects so far hidden by symmetry assumptions.

[15] However, note that seller 1 always strictly prefers that *perfect information* is revealed, resulting in equilibrium expected profits equal to q, always larger than $V_1(\alpha, q)$.

[16] See Perry and Reny [25] for another counterexample to the linkage principle, but in a multi-unit auction.

The effect acting through ex-post differentiation is so strong that even the *sum* of the payoffs of the two sellers is nonmonotonic in the level of vertical differentiation $|q-1/2|$ for any level of $\alpha \in (1/2, 1)$. The result is proved easily by comparing the sum of the equilibrium profits at $q=1/2$ and $q=q^P(\alpha)$. Note that $\sum_{j=0}^{1} V_j(q^P(\alpha), \alpha) = q^P(\alpha) + \alpha - 1$, and $\sum_{j=0}^{1} V_j(1/2, \alpha) = 2V(1/2, \alpha)$ as reported in Section 3.4: by direct comparison the first quantity is strictly larger than the second for any α, so that the sum must be decreasing in q in part of the interval $[1/2, q^P(\alpha)]$. The sum of the profits is instead strictly increasing in q for $q \geqslant q^P(\alpha)$, being equal to the pooling profits of the ex-ante superior seller 1. In the absence of private information, as in the model of Shaked and Sutton [29], an increase in the level of ex-ante differentiation leads to less competition and more profits for the sellers. With private information, while this effect is still present, the induced lower ex-post differentiation increases the competitive pressure on prices and tends to reduce profits for the sellers. The latter effect dominates the former for intermediate prior beliefs (relative to private information), while the opposite is true for extreme priors.

The nonconvexity in public prior belief also extends to the buyer's expected payoff and to the sum of the payoffs of both sellers, as well as to the total surplus achieved in equilibrium by the society of buyer and sellers.[17] The increase in asymmetry resulting from additional (but still imperfect) information on relative quality leads to stronger competitive pressure on prices, thereby reducing total duopoly profits. We show how to verify only the nonconvexity of the total surplus. In the pooling region the equilibrium is efficient and the sum of players' payoffs is q. In the separating region, the firms extract the entire rent of the buyer and achieve a total payoff of α. The total surplus in equilibrium is continuous in q, but the separating equilibrium extends beyond the efficient boundary $q^S(\alpha) = \alpha$. Nonconvexity is verified by projecting the linear segment q to $q = q^S(\alpha) > \alpha$ (for $\alpha > \sqrt{2}/2$) where the total (separating) payoff is equal to α.

4.4. *Value of Private Information in Equilibrium*

What would happen if the consumer chose how much private information to acquire on the relative quality of the products prior to receiving price quotes? Typically, sellers can control the quality of the buyer's information by allowing them to try the product or to return it if unsatisfied. Consider the case of overt information acquisition, whereby the sellers know the quality (α) of the costless private information acquired by the buyer. While in the pooling region it is harder to attract a more informed hostile customer, more precise private signals raise ex-post differentiation of valuations and thereby sellers

[17] Contrast this with the convexity of the social payoff at the efficient allocation (Section 4.1).

can extract more rents in the separating region and in part of the mixed strategy region. Analogously with our finding for monopoly, we have

PROPOSITION 6 (Nonmonotonicity of Profits in Quality of Private Information). *Seller* 1*'s equilibrium profits* $V_1(\cdot, q)$ *decrease* with *in the quality of* the *buyer's information* α *for* $\alpha \leqslant (q^P)^{-1}(q)$ *and increase for* $\alpha \geqslant (q^S)^{-1}(q)$.

Similarly, the buyer benefits from an informative private signal of low precision, but too precise a signal results in a reduction in her equilibrium payoff. The buyer's payoff in the pooling equilibrium region (low α) is increasing in α. Instead, when the sellers are aware that the buyer has a strong informational advantage (high α), competition is relaxed and the buyer ends up worse off. For high enough α, the equilibrium is separating and the buyer has zero expected rent. While a poorly informed buyer likes better information because it triggers more competition, a well-informed one dislikes it. The optimal amount of costless information acquired by the buyer is interior to the mixed strategy region.[18]

5. ROBUSTNESS

Restrictions on the Strategy Space. The assumption that the offers made by each uninformed party cannot be contingent on the terms offered by the competitor is restrictive (see Epstein and Peters [13] for a general investigation of competition with such a dependence). The assumption that sellers are not allowed to screen different types of buyers is instead not restrictive in this environment. As shown in the working paper version of this article (Moscarini and Ottaviani [22]), simple price competition (with take-it-or-leave-it price offers) results when competing sellers optimally design the mechanism and are allowed to offer arbitrary menus of lotteries. This follows from the fact that here the payoff of the risk-neutral buyer is linear in the valuations for the indivisible good, in contrast with screening models.

Bargaining Power to the Buyer. The model can also be interpreted as a first-price procurement auction where the buyer decides which good to buy depending on the price bids and the realized private signal. Clearly, the optimal procurement mechanism for a buyer with all the bargaining power

[18] In the case of covert and costless information acquisition, the buyer would be clearly fully informed in any equilibrium. An interesting alternative would be to analyze a game of covert information acquisition by the buyer where the amount of costly information is determined in equilibrium, as was done for instance by Crémer *et al.* [10] in a single-principal setting. The extension of our model with competing principals to the case of contemporaneous costly information acquisition is left to future research.

in this environment would not be a first-price auction, but a credible offer of a price equal to zero to the seller with ex-post superior quality. This would also result in the efficient allocation.[19]

Communication from the Buyer. Suppose that the buyer were allowed to signal her type by publicly communicating with the sellers in a phase preliminary to competition. Clearly, the equilibrium constructed in this paper survives when communication is allowed, since babbling is always a perfect Bayesian equilibrium. In addition, new equilibria can arise. For example, truthful revelation by the buyer is credible if $q \leqslant \alpha$. Prices in the competition subgame are $P_0 = 0, P_1 = 2 f_1 - 1$ if the sellers believe the buyer's type to be 1, and $P_0 = 1 - 2 f_0 > 0, P_1 = 0$ if the buyer is believed to be of type 0. In the communication stage a type-1 buyer has rent $1 - f_1$ by telling the truth and rent $1 - f_1 - (1 - 2 f_0) < 1 - f_1$ by pretending to be of type 0. Similarly, a type-0 buyer has no incentive to lie, as $f_0 - (2 f_1 - 1) < f_0$.

Renegotiation. This model separates information from bargaining power by assuming that the uninformed parties (sellers) have all the bargaining power. The buyer is not allowed to make a counteroffer to the offers made under commitment by the sellers. It is natural to assume that single buyers have no bargaining power in environments with many buyers, as when the demand side is interpreted as resulting from a continuum of consumers with perfectly correlated signals on the relative quality of the goods. Otherwise, a seller might be tempted to privately negotiate with a buyer who rejected the initial offers. Nevertheless, sellers with a long-run horizon might be concerned about the loss of reputation from engaging in such private negotiations with the buyers.

While it is beyond the scope of this paper to derive foundations for the assumption of take-it-or-leave-it offers (as done by Riley and Zeckhauser [26] in the monopoly case), we briefly discuss renegotiation with a single seller in the following scenario. After the sellers make simultaneous initial offers to the buyer, the latter accepts either offer or makes a first counteroffer to a seller. This seller may then either accept the counteroffer and conclude the trade, or make a final counter-counteroffer to the buyer, which the buyer may finally accept or not.

The pure strategy equilibrium (pooling and separating) appears to survive to this form of renegotiation. For instance, consider the pooling equilibrium. Clearly, the buyer cannot obtain any better price for good 0 than the initial

[19] Manelli and Vincent [18] study optimal procurement mechanisms in environments where the valuation of the buyer depends on the sellers' private information. In their setting a procurement auction may be suboptimal from the point of view of the buyer, being dominated by credible take-it-or-leave-it offers. This is trivially the case in our setting, where the sellers do not have any private information.

offer $P_0 = 0$. The buyer would then counteroffer only to obtain with positive probability a price for good 1 lower than $P_1 = 2f_0 - 1$. But seller 1 would be willing to sell at such a lower price only to increase the chance of selling, which cannot be.

The robustness of the mixed strategy equilibrium depends instead on whether the sellers observe the realized price in the initial offer made by the competitor. To see that the mixed strategy equilibrium does not survive if the price drawn in the first period is public information, note that the mixed-strategy equilibrium always contains prices (P_0, P_1) such that $P_1 = 2f_0 - 1 + P_0 + \varepsilon$, for $\varepsilon > 0$ small. Following such a price realization, the seller would accept a pooling counteroffer whereby both buyer types propose to purchase good 1 at price $2f_0 - 1 + P_0$. A type-0 buyer is indifferent and a type-1 buyer is strictly better off. Given unchanged beliefs about types, the seller 1 is also strictly better off by accepting this offer for $\varepsilon > 0$ small enough, because a slight reduction in price results in an increase in the probability of selling by $\Pr(\sigma_1)$.

Distribution of Private Signal. The binary signal structure studied in the paper is admittedly restrictive and gives rise to nontrivial analytical problems, but allows us to investigate the dependence of the equilibrium on the prior belief and the quality of the private information. Natural alternative continuous formulations do not seem to simplify these tasks. In particular, we have considered a binary-state version of the model with continuous signals. If the likelihood ratio is bounded, a pooling equilibrium results for extreme prior beliefs. Once an equilibrium is constructed as a solution to a system of nonlinear equations in prices, comparative statics can be performed numerically.

6. CONCLUSION

We have investigated price competition in markets where quality is difficult to ascertain and the price-taking buyer has private information on the relative quality of the alternative competitors. When the prior belief is very biased toward one good and private signals are not too informative, sellers compete fiercely and leave rents to the buyer as in the classical Bertrand model. When instead the prior is balanced and signals are of bounded but strong precision, the sellers become local monopolists. Loosely speaking, the more spread the distribution of valuations is, due to a vaguer prior and stronger signals, the more rents the sellers can extract from the consumer in equilibrium. Our three main findings are: First, competing sellers may lose from the release of public information. Second, a moderately dominant seller loses in equilibrium from a slightly higher

ex-ante reputation. Third, the buyer does not want (to be known) to be too well informed about relative quality.

The insights gained on the value and incentives for information acquisition with strategic pricing could apply to a broad class of markets where sellers provide customized products to buyers at individualized prices. The stylized relationship between a buyer and two competing sellers could be enriched in order to consider problems arising in the labor, credit, and insurance markets. It is essential that the price-taking party has superior information on the relative desirability of the competing price-setters. As a labor market application with the role of buyer and seller reversed, consider the situation of a job applicant (seller) with private information on the net costs of working for different employers (buyers) who compete in wage offers. Finally, consider interjurisdictional competition for attracting business. This model can be applied to cases where the jurisdictions (or sellers) have all the bargaining power and a small firm (or buyer) has private information on the location-specific cost of new plants. Martin [19] considers instead how a firm should optimally solicit bids from several jurisdictions when costs are commonly known.

Our static model is a building block for dynamic models of strategic pricing with private learning. A natural dynamic extension of this model can allow for social learning about product quality with endogenous prices, as first investigated by Caminal and Vives [8]. The demand side of the market consists of a sequence of privately informed customers with the same preferences. Buyers are then able to partially infer the information possessed by other buyers by observing their purchase decisions as in the social learning model of Bikhchandani *et al.* [4]. On the supply side of the market, sellers engage in repeated price competition. In this context prices not only serve the usual allocative role, but also act as a screening device for the transmission of the private information held by previous buyers.

APPENDIX

A.1. *Properties of the Mixed Strategy Equilibrium*

This section illustrates general restrictions on the form of equilibrium mixed strategies for our game. All are valid for and only for the region of parameters where there is no pure strategy equilibrium, $q \in [\max \langle 1/2, q^S(\alpha) \rangle, q^P(\alpha)]$. The arguments can be followed easily with the help of the graphical representation of the constraints in the price space (Fig. 3). A *mixed strategy* by firm j, $j = 0, 1$, is a probability measure over Borel sets, with distribution G_j; the *support* $\mathscr{P}_j$ of the mixed strategy is the smallest Borel set of probability 1.

LEMMA 1 (Sellers' Payoffs). *Both firms make strictly positive profits in equilibrium.*

Proof. By way of contradiction, suppose that firm j makes zero profits in a mixed strategy equilibrium. If firm $1-j$ plays a positive mass of probability on prices above $IC_j(0)$, then firm j could post $P'_j=\varepsilon$ for some $\varepsilon>0$ and sell with positive probability, thereby making positive profits, in contradiction with the assumption. Otherwise, we are back to the pooling equilibrium on good $1-j$, which does not exist in this region of parameters. ∎

Recall that $\underline{P}_j$ and $\bar{P}_j$ denote the lower and upper bounds of the support of the equilibrium randomization of player j. Since, by Lemma 1, any price in the support—including the upper bound $\bar{P}_j$—must yield positive expected profits, we have

LEMMA 2 (Mass above $IC_j(\bar{P}_j)$). *Seller $1-j$ must play prices above $IC_j(\bar{P}_j)$ with positive probability.*

LEMMA 3 (Atoms and Gaps). (i) Atoms and gaps must correspond through IC constraints. *If in equilibrium seller j plays a price P_j with positive probability* (*an* atom), *then there is a corresponding* gap *in the support of the opponent's randomization*: $\exists \Gamma>0$ *such that* $(IC_i(P_j), IC_i(P_j)+\Gamma)\cap \mathscr{P}_{1-j}=\varnothing$ *for* $i=0, 1$. *The converse is also true, provided that seller* $1-j$ *plays prices weakly below the gap* (*some* $P_{1-j}\leqslant IC_i(P_j)$).

(ii) Atoms only at maximum prices. *Only the maximum prices that the buyer may accept,* $P_0=1-f_0$ *and* $P_1=f_1$, *can be played with positive probability by the sellers.*

Proof. (i) First, we show sufficiency of an atom for a corresponding gap. The price $IC_i(P_j)+\varepsilon$ for some $\varepsilon>0$ is strictly dominated for firm $1-j$ by $IC_i(P_j)-\delta$ for some $\delta>0$, because the latter steals a discrete mass of demand (the whole atom) from the competitor with loss of revenue $\delta+\varepsilon$, which can be made negligible for ε and δ small enough. Therefore $IC_i(P_j)+\varepsilon$ cannot possibly be in $\mathscr{P}_{1-j}$, for a set of $\varepsilon\in(0, \Gamma)$, with $\Gamma>0$ being the width of the gap.

Next, we show sufficiency of a gap $(IC_i(P_j), IC_i(P_j)+\Gamma)$ in the support $\mathscr{P}_{1-j}$, provided further that prices weakly below $IC_i(P_j)$ are also in $\mathscr{P}_{1-j}$, for a corresponding atom by firm j on P_j. First, given the gap by seller $1-j$, prices in $(P_j, P_j+\Gamma)$ cannot be in seller j's support $\mathscr{P}_j$, since they are dominated by $P_j+\Gamma$; so the two gaps correspond through IC_i. By contradiction, suppose there is no atom on P_j. Then firm $1-j$ would gain strictly from playing $IC_i(P_j)+\Gamma$ rather than $IC_i(P_j)-\varepsilon$ for $\varepsilon\geqslant 0$ small, because $(P_j, P_j+\Gamma)$ are not played by firm j; but this contradicts the

assumption that prices $IC_i(P_j)-\varepsilon$ for some $\varepsilon \geqslant 0$ small are played in equilibrium (are in $\mathscr{P}_{1-j}$).

(ii) Suppose that there is an atom at an interior price, e.g. at $P_1 < f_1$. Then, by (i) there is a corresponding gap in $\mathscr{P}_0$ containing either $IC_0(P_1)$ or $IC_1(P_1)$ (or both), and firm 0 does not play prices in $(IC_i(P_1), IC_i(P_1)+\varepsilon)$ for some $\varepsilon > 0$, for either i. But then firm 1 would gain over P_1 by deviating to a strictly higher price $P_1+\varepsilon$, which would sell with the same probability—positive by Lemma 1—as P_1. ∎

It follows immediately from claim (i) that, when there is an atom at $P_0 = 1-f_0$ (respectively, $P_1 = f_1$), there must be a gap in $\mathscr{P}_1$ containing $P_1 = f_0$ (respectively, in $\mathscr{P}_0$ containing $P_0 = 1-f_1$). From claim (ii), it follows that each seller's equilibrium randomization G_i is continuous and, being nondecreasing by definition, has a density $g_i = G_i'$ a.e. for prices smaller than the maximum ones acceptable by the buyer. Finally, since $1-f_0 \neq IC_i(f_1)$ for $i=0, 1$ in the mixed strategy equilibrium region,

COROLLARY 1. *Ties happen with probability zero in equilibrium.*

The previous results imply that the support of an equilibrium mixed strategy—a Borel set in $[0, 1]$, and thus a countable union of bounded intervals—is a collection of nondegenerate intervals, plus possibly the upper bound of the support. In fact, absent any atom on interior prices, we may exclude any isolated point other than the maximum feasible price by considering the smallest set of prices played by a firm with probability one. The next result is that the holes separating these intervals must be projections through one IC constraint of the holes in the opponent's support. Intuitively, if holes did not correspond, one of the two sellers would necessarily gain from realigning the holes, thereby raising the price at the lower bound of a hole without reducing the probability of selling.

LEMMA 4 (Corresponding Bounds). *The bounds of the disjoint intervals of prices that form the support of a player's equilibrium randomization must correspond through IC_i constraints to those of the other player.*

Proof. The proof is by contradiction. Let $\check{P}_j$ be a lower bound of one of these intervals, such that $\check{P}_{1-j} \neq IC_i(\check{P}_j)$ for both $i=0, 1$. Consider the case $\check{P}_{1-j} > IC_i(\check{P}_j)$. Then $P_j = IC_i(\check{P}_{1-j})$ dominates all prices in $(\check{P}_j, IC_i(\check{P}_{1-j}))$, in contradiction with the definition of equilibrium. A similar contradiction is reached if $\check{P}_{1-j} < IC_i(\check{P}_j)$. ∎

These results are closely connected to analogous restrictions derived by Osborne and Pitchik [23] in their analysis of the Hotelling pricing game

with linear demand. They first attempt to solve the equilibrium of the pricing game for all possible pairs of firms' locations. In their Appendix 1, they prove the claim of Lemma 1; their claim (b) is similar to our Corollary 1, their claim (j) to our Lemma 3 (ii), and their claim (m) to Lemma 3(i). As a consequence, their partial characterization of different equilibrium regimes (T1 and T2) resembles ours (M1–M4). However, they cannot obtain enough restrictions to pin down uniquely the equilibrium for all parameter values. In our discrete setting we are able to do this.

Consider a price P_0 in the support $\mathscr{P}_0$; by Lemma 4 there must be a price in $\mathscr{P}_1$ corresponding to P_0 through one of the two *IC* constraints. The next result shows that there is almost always only one such price: both the high price $IC_1(P_0)$ and the low price $IC_0(P_0)$ can be in $\mathscr{P}_1$ only for a countable set of prices P_0, and the case is similar for firm 0. Intuitively, the rate at which expected profits are lost by rasing a price, given the opponent's strategy, is different for the two *IC* constraints. This result greatly simplifies the search for mixed strategy equilibria over non countable action spaces.

LEMMA 5 (The Tie Principle). *For all values of the parameters* (q, α), *at equilibrium the set of prices in the support of seller j such that the two tying prices are both in the support of* $1-j$, $\{P_j \in \mathscr{P}_j : P'_{1-j} = IC_1(P_j) \in \mathscr{P}_{1-j}$ and $P''_{1-j} = IC_0(P_j) \in \mathscr{P}_{1-j}\}$, *has Lebesgue measure zero.*

Proof. Consider seller 0 and tying prices by firm 1, the other case being symmetric. Fix any price P_0 in the relevant range $[0, 1-f_1]$, where two feasible (IR) tying prices by firm 1 exist: i.e., $P'_1 = 2f_1 - 1 + P_0$ and $P''_1 = 2f_0 - 1 + P_0 < P'_1$. Let us first compute the expected payoffs associated with these two prices. P'_1 sells to both customers' types if seller 0's price —drawn according to the c.d.f. G_0—exceeds $P'_1 + 1 - 2f_0 = P_0 + 2\Delta f$, i.e., with chance $1 - G_0(P_0 + 2\Delta f)$ given the opponent's mixed strategy; and sells only to type 1 if seller 0 prices in $[P'_1 + 1 - 2f_1, P'_1 + 1 - 2f_0] = [P_0, P_0 + 2\Delta f]$, i.e., with chance $G_0(P_0) - G_0(P_0 + 2\Delta f)$; and sells to none otherwise. Thus the expected payoff to firm 1 from P'_1 is the weighted sum $\pi_1(P'_1) = \pi_{1,1}(P_0) = (2f_1 - 1 + P_0)[1 - \Pr(\sigma_0)\, G_0(P_0 + 2\Delta f) - \Pr(\sigma_1)\, G_0(P_0)]$. Similarly, the payoff from P''_1: $\pi_1(P''_1) = \pi_{1,0}(P_0) = (2f_0 - 1 + P_0)[1 - \Pr(\sigma_0)\, G_0(P_0) - G_0(P_0 - 2\Delta f)]$.

Contrary to the claim, suppose that there exists a nonzero Lebesgue measure set of prices played by seller 0, $\mathscr{X}_0 \subset \mathscr{P}_0$, such that $P'_1(P_0)$, $P''_1(P_0) \in \mathscr{P}_1$ for $P_0 \in \mathscr{X}_0$. Since $P_0 \leqslant 1 - f_1 < 1 - f_0$, G_0 has a density $g_0 = G'_0$ at almost all of the points we are considering. $P'_1 \in \mathscr{P}_1$ and $P''_1 \in \mathscr{P}_1$ imply that for all $P_0 \in \mathscr{X}_0$ the two expected payoffs to firm 1 are equal: i.e., $\pi_{1,0}(P_0) = \pi_{1,1}(P_0)$. Therefore their slopes must coincide on the same set, $\pi'_{1,0}(P_0) = \pi'_{1,1}(P_0)$ or:

$$
\begin{aligned}
0 = {} & g_0(P_0)[\Pr(\sigma_0)(2f_0 - 1 + P_0) - \Pr(\sigma_1)(2f_1 - 1 + P_0)] \\
& + \Pr(\sigma_0)[G_0(P_0) - G_0(P_0 + 2\Delta f)] + \Pr(\sigma_1)[G_0(P_0 - 2\Delta f) - G_0(P_0)] \\
& - (2f_1 - 1 + P_0)\Pr(\sigma_0)\, g_0(P_0 + 2\Delta f) \\
& + (2f_0 - 1 + P_0)\Pr(\sigma_1)\, g_0(P_0 - 2\Delta f) \qquad \text{(A.1)}
\end{aligned}
$$

All terms on the right-hand side are either nonpositive or strictly negative, except possibly the last one. Thus, for the equality to hold we require $g_0(P_0 - 2\Delta f) > 0$, namely, that the price $P_0 - 2\Delta f$ is in the support: $P_0 - 2\Delta f = IC_0(P_1'') \in \mathscr{P}_0$. Then one obtains that $P_1'' \in \mathscr{P}_1$, $IC_0(P_1'') \in \mathscr{P}_0$, and $IC_1(P_1'') = P_0 \in \mathscr{P}_0$, and the situation that we are trying to rule out for firm 0 at P_0 is replicated for firm 1 at P_1''. Symmetrically, this implies $g_1(P_1'' - 2\Delta f) > 0$. Then, recursively $P_0 - 2\Delta f$ must be in $\mathscr{P}_0$, and it ties with $P_1'' = IC_1(P_0 - 2\Delta f) \in \mathscr{P}_1$, as just seen, and $P_1'' - 2\Delta f = IC_0(P_0 - 2\Delta f) \in \mathscr{P}_1$ by $g_1(P_1'' - 2\Delta f) > 0$. Iterating to $P_0 - 2\Delta f$ this reasoning, initially applied to P_0, we require $g_0(P_0 - 4\Delta f) > 0$ and $g_0(P_0 - n \cdot 2\Delta f) > 0$ at any further step $n > 2$, for otherwise the whole argument would unravel. But clearly for $n = N$ large enough and for interior parameters such that $\Delta f = f_1 - f_0 > 0$, one must have $P_0 - 2N\Delta f < 0$ and thus $g_0(P_0 - 2N\Delta f) = 0$, giving the desired contradiction. ∎

For firm 1, which is favored by the prior belief, we can say even more: Given any randomization by firm 0, firm 1's profits are increasing faster on IC_0 than on IC_1: i.e.,

LEMMA 6 (From up to down). *For almost all $P_0 \leqslant 1 - f_1$ in the interior of $\mathscr{P}_0$ the profit of seller 1 increases less along IC_1 than along IC_0 in the price of the competitor*:

$$\frac{d\pi_1(IC_1(P_0))}{dP_0} < \frac{d\pi_1(IC_0(P_0))}{dP_0}.$$

Proof. To establish the claim it suffices to prove that $g_0(P_0 - 2\Delta f) = 0$ and to use Eq. (A.1). Suppose by contradiction that $g_0(P_0 - 2\Delta f) > 0$, hence $P_0 - 2\Delta f \in \mathscr{P}_0$. Let $P_1'' \equiv IC_1(P_0 - 2\Delta f) = IC_0(P_0)$ and $P_1' \equiv IC_0(P_0 - 2\Delta f) < P_1''$. By Lemma 5, ignoring zero Lebesgue measure sets, P_1' and P_1'' cannot both be in $\mathscr{P}_1$. However, as we know, at least (and therefore exactly) one of the two is, otherwise firm 0 would strictly gain by deviating from P_0 to some $P_0 + \varepsilon$. If $P_1'' \in \mathscr{P}_1$ Lemma 5 is contradicted, as $P_0 = IC_0(P_1'') \in \mathscr{P}_0$ by assumption and $P_0 - 2\Delta f = IC_1(P_1'') \in \mathscr{P}_0$, so both projections of P_1'' through the *IC* constraints are in the support of firm 0.

If instead $P'_1 \in \mathscr{P}_1$ and thus $P''_1 = IC_1(P_0 - 2\Delta f) \notin \mathscr{P}_1$, then $P'''_1 \equiv IC_1(P_0) \in \mathscr{P}_1$ since one of the two projections of $P_0 \in \mathscr{P}_0$ must be in the opponent's support, with $P'''_1 > P''_1$. So we have P'''_1, $P'_1 \in \mathscr{P}_1$ and $P''_1 \notin \mathscr{P}_1$, where $P''_1 \in (P'_1, P'''_1)$. Being concerned only with nonzero Lebesgue measure sets of such prices, this requires $(P''_1 - \delta, P''_1 + \delta) \cap \mathscr{P}_1 = \varnothing$ for some $\delta > 0$. No hole in $\mathscr{P}_0$ may correspond through IC_0 to the hole $(P''_1 - \delta, P''_1 + \delta)$ in $\mathscr{P}_1$, because the former would contain $(P_0 - \delta, P_0 + \delta)$ while $(P_0 - \varepsilon, P_0 + \varepsilon) \subset \mathscr{P}_0$ by assumption (P_0 is in the interior of $\mathscr{P}_0$); so by Lemma 4 a hole in $\mathscr{P}_0$ must project $(P''_1 - \delta, P''_1 + \delta)$ through the other constraint IC_1, and be of the form $(P_0 - \delta - 2\Delta f, P_0 + \delta + 2\Delta f)$. This hole contains $P_0 - 2\Delta f$, contradicting $g_0(P_0 - 2\Delta f) > 0$. ∎

A.2. *Description of the Mixed Strategy Equilibrium*

A preliminary discussion on the occurrence of ties is in order. In this game ties can happen with positive probability only if a seller is making zero profits in equilibrium. While in the pooling region the sharing rule selected (in favor of the high-price firm) is not only sufficient but also necessary for existence, in the mixed strategy equilibria this necessity disappears. By Corollary 1 ties happen with probability zero, so one can choose any sharing rule without affecting the equilibrium. To facilitate the construction of the equilibrium the following new sharing rule is selected, purely as an illustrative device.

Assumption 1 (Tie-Breaking Rule in Mixed Strategy Equilibrium). If seller j plays a price P_j with positive probability (an *atom*), the buyer breaks the tie(s) on either $IC_j(P_j)$ by buying from seller $1 - j$.

This tie-breaking rule requires the buyer to break the tie in favor of the seller without the atom and thus serves the important role of facilitating the check of unilateral profitable deviations. In particular, it guarantees that the supports of the randomizations of the sellers are a collection of closed intervals. With any other sharing rule, seller $1 - j$ would gain back the "missing" fraction of the atom of demand at price $P_{1-j} = IC_j(P_j)$ by insisting on an infinitesimally smaller price $IC_j(P_j) - \varepsilon$ with $\varepsilon > 0$ small enough. Therefore, the support would be a collection of disjoint but non-compact intervals, and the check for profitable deviations would be more cumbersome, although the equilibrium found clearly survives the specification of any tie-breaking rule.

The detailed construction of the mixed strategy equilibrium M1 is followed by a brief description of the other three types of equilibria.

M1. A small increase of the prior belief q from the separating level $q^S(\alpha)$, or similarly a reduction of the precision of the signal α, raises the posterior valuation for good 1 of the consumer with unfavorable signal σ_0,

giving firm 1 an incentive to be more aggressive. In particular, the best reply of firm 1 jumps from $IC_1(P_0)$ to $IC_0(P_0)$ at the interior switching price $\hat{P}_0 = 1 - 2f_0 + \alpha q < 1 - f_0$. In the region of parameters M1 (represented in Fig. 4), firm 1's equilibrium strategy puts some weight on low prices in order to attract the type-0 customer with some probability. Firm 0 responds by posting correspondingly low prices. Each seller still posts the highest feasible price, f_1 and $1 - f_0$, respectively, with positive probability (an atom) and spreads the remaining probability with an atomless distribution on an interval of prices. As illustrated in Fig. 6a, the probability mass by seller 1 on prices above f_0 consists only of an atom at $P_1 = f_1$. There is a gap in firm 1's support $\mathscr{P}_1$ between $f_0 = IC_0(1 - f_0)$ and f_1, in correspondence to the atom on $P_0 = 1 - f_0$ by firm 0.

Description. The equilibrium prices that define the support, payoffs and randomizations of M1 are $\{\tilde{P}_j, \bar{P}_j, \underline{P}_j\}$, $j = 0, 1$, which solve *uniquely* the system

$$\underline{P}_1 = \alpha q,\ \tilde{P}_1 = f_0,\ \bar{P}_1 = f_1, \qquad \underline{P}_0 = \tilde{P}_0 = 1 - 2f_1 + \underline{P}_1,\ \bar{P}_0 = 1 - f_0.$$

Seller 0 randomizes over an interval of prices $[\tilde{P}_0, 1 - f_0)$, where $\tilde{P}_0 = \underline{P}_0 > 1 - f_1$, with an atomless distribution G_0 of total mass $G_0(f_0) = 1 - \gamma_0$, and on the highest possible price $1 - f_0$ with an atom of probability mass

$$\gamma_0 = \frac{\alpha}{1 - \alpha} - \frac{\Pr(\sigma_1)}{\Pr(\sigma_0)}; \tag{A.2}$$

seller 1 randomizes on $[\underline{P}_1, f_0]$ with an atomless distribution G_1 of total mass $G_1(f_1) = 1 - \gamma_1$, and on f_1 with an atom of probability mass

$$\gamma_1 = \frac{1 - 2f_0 + \alpha q}{1 - f_0}. \tag{A.3}$$

According to the specified tie-breaking rule, at prices $\{1 - f_0, f_0\}$ the indifferent type-0 buyer chooses good 1 since firm 0 posts price f_0 with positive probability; at prices $\{1 - f_1, f_1\}$ the (indifferent) type-1 buyer goes to the low price seller 0. The equilibrium payoffs to the sellers are

$$V_0(q, \alpha) = \Pr(\sigma_0 \mid q, \alpha)[1 - 2f_0(q, \alpha) + \alpha q], \qquad V_1(q, \alpha) = \alpha q. \tag{A.4}$$

For these strategies to constitute an equilibrium, all prices in the support of the probability distribution of each seller must yield the same expected payoff and all other strategies must yield weakly lower payoff, given the strategy of the other seller.

Payoffs. The expected payoffs associated with the benchmark prices in the support of firm 0 given the seller 1's strategy G_1 stated above are easily computed with the help of Fig. 6(a): $\pi_0(\tilde{P}_0) = \Pr(\sigma_0)\,\tilde{P}_0$, $\pi_0(1-f_0) = \gamma_1 \Pr(\sigma_0)(1-f_0)$. Similarly for firm 1: $\pi_1(\underline{P}_1) = \underline{P}_1$, $\pi_1(f_0) = [(1-\gamma_0)\Pr(\sigma_1) + \gamma_0]\, f_0$, $\pi_1(f_1) = \Pr(\sigma_1) f_1 = \alpha q$.

Solving for Strategies and Randomizations. All prices in the support must yield the same expected payoff V_1: from $\pi_1(\underline{P}_1) = \pi_1(f_1)$ we find $V_1 = \underline{P}_1 = \alpha q$ and from $\pi_1(f_0) = \pi_1(f_1)$ we obtain (A.2), the mass of the atom γ_0 played by seller 0 on the maximum price $1-f_0$. Note that $d\gamma_0/dq < 0$, $\gamma_0(q^S) = 1$, and $\gamma_0(1) = 0$. The prices $P_1 \in (\underline{P}_1, f_0)$ in the support are left to be considered. The randomization G_0 of seller 0 must be such that seller 1 is indifferent among all such prices in the support of G_1, so that $V_1 = [1 - G_0(1-2f_0+P_1) + G_0(1-2f_0+P_1)\Pr(\sigma_1)]P_1$. Equating this to $V_1 = \alpha q$ and substituting $P_0 = 1-2f_0+P_1$, we obtain $G_0(P_0) = (P_0 + 2f_0 - 1 - \alpha q)/\{\Pr(\sigma_0)[2f_0 - 1 + P_0]\}$. Note that $G_0(\underline{P}_0) = 0$ and that seller 0's density $g_0(P_0) = G'(P_0)$ is strictly decreasing in P_0.

Substituting back $\tilde{P}_0 = 1 - 2f_0 + \underline{P}_1 = 1 - 2f_0 + \alpha q$ in the two equations above, one obtains the equilibrium payoff for firm 0 given in (7.5) and the mass of the atom in (A.3). Note that $\gamma_1(q) < 1$ for $q \in [q^S, q^P]$. The c.d.f. G_1 played by seller 1 must make seller 0 indifferent among all the remaining prices in the stated support $P_0 \in (\underline{P}_0, 1-f_0)$, so that $V_0 = P_0 \Pr(\sigma_0)[1 - G_1(2f_0 - 1 + P_0)]$. Equating this with (A.4) obtained above and substituting $P_1 = 2f_0 - 1 + P_0$, one obtains $G_1(P_1) = (P_1 - \alpha q)/(1 - 2f_0 + P_1)$. Note that $G_1(\underline{P}_1) = 0$ and $G_1(f_0) = 1 - \gamma_1$, so that $G_1(f_1) = 1$. The density $g_1(P_1) = G_1'(P_1)$ is decreasing in P_1.

Deviations. Given seller 0's strategy, any price less than $\underline{P}_1$ is dominated by $\underline{P}_1$, and any $P_1 \in (f_0, f_1)$ by f_1, as immediately seen from Fig. 6(a). Given seller 1's strategy, seller 0's best deviation is $P_0' = 1 - f_1$: any price below P_0' would result in the same probability of selling as P_0' but at a lower price, and similarly for a price between P_0' and $\tilde{P}_0$ compared with $\tilde{P}_0$. This best deviation is not profitable provided that $\pi_0(P_0') = [\gamma_1 + (1-\gamma_1)\Pr(\sigma_0)](1-f_1) \leqslant V_0$. By (7.5), this is equivalent to $\psi(q, \alpha) \equiv (2\alpha - 1)(1 - 2f_0 + \alpha q) - \alpha(1-f_1) \geqslant 0$. First, note that $\psi''(q) = -2(2\alpha-1) f_0''(q) + \alpha f_1''(q) < 0$ because $f_1''(q) 0 > 0 > f_1''(q)$ for all q, so that ψ is strictly concave and quasi-concave in q: hence $\psi(q) \geqslant 0$ for q belonging to an interval Q_{M1}. Next, it can be verified that for $\alpha > 2/3$, i.e. whenever a separating equilibrium exists, $\psi(q) = 0$ has a unique root $q^{M1} > q^S$. For $q < q^S$ the atom on the separating price $P_1 = f_1$ in the M1 equilibrium would have a mass exceeding 1 (cf. 7.4), so that M1 may exist only in $Q_{M1} = [q^S, q^{M1}] \subset [q^S, q^P]$. Instead, for $\alpha < \frac{2}{3}$ there are two roots (q^{m1}, q^{M1}), with $q^{M1} < q^{M1}$ and $\psi'(q^{m1}) < 0 < \psi'(q^{M1})$, so that M1 may exist only in $Q_{M1} = [q^{m1}, q^{M1}] \subset [1/2, q^P]$.

M2. By increasing q (or reducing α) beyond the boundary between regions M1 and M2 in Fig. 4, the M1 equilibrium breaks down, since seller 0 would profit from deviating to price $1-f_1$, thereby gaining the demand of the type-1 consumer when the opponent posts f_1. Intuitively, with even lower ex-post differentiation in customers' valuations, the incentive for seller 1 to separate types is reduced and the incentive to insist on low (pooling) prices is enhanced. The equilibrium of type M2 (Fig. 6b) is like M1, with the addition of the interval $[\tilde{P}_1, f_1]$ to $\mathscr{P}_1$ and of the corresponding prices $[\underline{P}_0 = IC_1(\tilde{P}_1), 1-f_1]$ to $\mathscr{P}_0$. The probability mass by seller 1 on prices above f_0 consists not only of the atom on f_1 (as in M1) but also of the density on the interval $[\tilde{P}_1, f_1]$. There is a hole in the support of each player corresponding to the atom by the competitor on the highest price.

M3. By increasing further the prior belief (or decreasing further the signal precision), the effects illustrated in M2 are reinforced. Ex-post differentiation in the valuations becomes so low that the ex-ante superior seller 1 does not play the highest price f_1 at all. In the mixed strategy equilibrium of type M3 (Fig. 6c), seller 0's strategy has no holes and an atom on $1-f_0$; seller 1's strategy has a hole between $[\underline{P}_1, f_0]$ and $[\tilde{P}_1, \bar{P}_1]$, with $\tilde{P}_1 = 2f_1 - 1 + \underline{P}_0 > f_0$, and no atom.

M4. Finally, when the prior belief is rather balanced but the signal imprecise, players compete aggressively for customers who are mildly differentiated ex post. Ex-post differentiation is still strong enough to prevent a pooling equilibrium from arising on the ex ante superior good 1. In the M4 equilibrium (Fig. 6d) not even seller 0 plays the highest price $P_0 = 1-f_0$, and there are no atoms in the equilibrium randomizations nor holes in their supports, which are connected.

A.3. *Other Proofs*

Proof of Proposition 3 (Uniqueness of Equilibrium). In the pooling region there is a continuum of noncautious equilibria, while the cautious pooling equilibrium is always unique. The separating equilibrium is easily seen to be the unique equilibrium in the separating region. The various mixed-strategy equilibria constructed are mutually exclusive by construction, other than possibly at the boundaries between the different regions. To establish uniqueness we need to exclude mixed strategy equilibria that do not fall into one of the four classes M1–M4.

Claims (i) and (ii) of Lemma 3 imply that in an equilibrium there can be at most two atoms at the maximum prices of the relevant range and two corresponding gaps and the rest of the support is connected. Therefore, using Lemma 5 and Lemma 6, M4 is the only possible equilibrium if there are no atoms.

If there are two atoms the equilibrium is of type M1 or M2, since the other a priori possible forms of the equilibrium (similar to M1 with the indexes of the sellers interchanged) do not exist for $q \geqslant 1/2$, as we now show by contradiction. By symmetry with respect to M1 (see above), such an equilibrium would have $\underline{P}_0 = \alpha(1-q)$, $\underline{P}_1 = 2f_1 - 1 + \alpha(1-q)$, $V_0 = \underline{P}_0$, and $V_1 = \Pr(\sigma_1)\,\underline{P}_1$, with $\underline{P}_0 < 1 - f_1$ by construction. The atom on $\bar{P}_0 = 1 - f_0$ would have measure $\gamma_0 = \underline{P}_1 / f_1$ for seller 1 to be indifferent between the lower and upper bounds of the support ($\underline{P}_1$ and f_1) so that in order for 1 not to deviate to f_0 one needs $\pi_1(f_0) = f_0(\Pr(\sigma_1) + \gamma_0 \Pr(\sigma_0)) < V_1$, equivalent to $\underline{P}_1 = 2f_1 - 1 + \alpha(1-q) > f_0/(2\alpha - 1)$. But this inequality is incompatible with $\underline{P}_0 = \alpha(1-q) < 1 - f_1$ for $q \geqslant 1/2$.

Finally, from Lemma 5 again, M3 is the only possible form of the equilibrium which features only an atom by firm 0 on $P_0 = 1 - f_0$. So we are left to exclude the symmetric equilibrium with only an atom by firm 1 on $\bar{P}_1 = f_1$. This done by contradiction. By Lemma 2, in such an equilibrium firm 0 must play with positive probability prices above $1 - f_1$. By Lemma 4, to this atomless mass there must correspond a mass by seller 1 below f_0. If $2f_1 - 1 > f_0$, firm 1 randomizes below f_0 and on $\bar{P}_1 = f_1$, but not in the interval $[f_0, 2f_1 - 1]$ dominated by $2f_1 - 1$, so that there would be a hole in $\mathscr{P}_1$ without an atom by firm 0, contradicting Lemma 3. If instead $2f_1 - 1 \leqslant f_0$, absent atoms by firm 0, firm 1 must play below and above f_0, with no hole in $\mathscr{P}_1$. Firm 0 must play prices below $1 - f_1$, and then below the hole, for otherwise prices in (f_0, f_1) would be dominated by f_1 for firm 1, and there would be a hole in $\mathscr{P}_1$. Firm 0 must play all the way down to $1 - 2f_1 + f_0$ to avoid this hole, and not below it, otherwise Lemma 6. would apply. Therefore the following four prices are in the support of firm 0: $1 - 2f_1 + f_0$, $1 - f_1$, $\tilde{P}_0$, and $1 - f_0$. Equating the four payoffs yields a system of three equations in the three unknowns: $\tilde{P}_0$, the measure of firm 1's atom, and the probability mass played by firm 1 in $[f_0, f_1)$, which we have shown must be positive. Given the solution for $\tilde{P}_0$, consider the following prices in $\mathscr{P}_1$: $2f_0 - 1 + \tilde{P}_0$, f_0, and f_1. Equating the expressions for the payoffs of firm 1 at these prices yields two independent equations to determine one unknown only, the fraction of the probability played by firm 0 above the hole. Therefore the system is overdetermined and has no solution. ∎

Proof of Proposition 4 (Non-monotonicity of Profits in the Prior Belief). At belief $q = q^P(\alpha)$ on the boundary of the pooling equilibrium regime, firm 1's equilibrium profits are $V_1(\alpha, q) = 2f_0(q^P(\alpha), 1) = q^P(\alpha) + \alpha - 1$. At belief $q = q^{M1}(\alpha) \in (q^S(\alpha), q^P(\alpha))$ on the boundary between regions M1 and M2, $V_1(\alpha, q^{M1}(\alpha)) = \alpha q^{M1}(\alpha)$. By direct calculation $q^P(\alpha) + \alpha - 1 < \alpha q^{M1}(\alpha)$ for all α

in the stated range. By continuity, $V_1(\alpha, \cdot)$ must decrease in q for a nonempty set of prior beliefs contained in $[q^{M1}(\alpha), q^{P}(\alpha)]$. ∎

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