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Naive audience and communication bias

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Abstract We introduce the possibility that the receiver naively believes the sender's message in a game of information transmission with partially aligned objectives. We characterize an equilibrium in which the communication language is inflated, the action taken is biased, and the information transmitted is more precise than in the benchmark fully-strategic model. We provide comparative statics results with respect to the amount of asymmetric information, the proportion of naive receivers, and the size of the sender's bias. As the state space grows unbounded, the equilibrium converges to the fully-revealing equilibrium that results in the limit case with unbounded state space.

Keywords Strategic information transmission · Naive audience · Bounded support

JEL Classification C72, D72, D83

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1 Introduction

The study of cheap talk games is one of the main themes in theoretical political economy.[1] Following Crawford and Sobel (1982), one of the fundamental tenets of the standard analysis is that players are fully strategic. As a result, deception is impossible in equilibrium. Whenever information is transmitted, the receiver is able to undo any bias in the sender's message, and to take equilibrium actions that are unbiased conditional on the information transmitted. The only effect of the conflict of interest between the players is to reduce the amount of information credibly transmitted in equilibrium. For the same reasons, the communication language cannot be determined in equilibrium and there is no reason for an inflated language to be adopted.

In reality, however, language inflation and deception seem to be widespread in the political sphere. For example, electoral campaigns broadcast biased adverts to swing the opinions of voters. Media outlets provide partisan news accounts, plausibly to accommodate the possibly biased agenda of stakeholders (e.g. Groseclose and Milyo 2003). Experts sponsored by lobbies and interest groups attempt to manipulate the legislators with openly biased reports. Politicians often try to impose their partisan agenda on the Parliament. In addition, there is empirical (Malmendier and Shanthikumar 2005) and experimental (Dickhaut et al. 1995; Forsythe et al. 1999; Cai and Wang 2006) evidence that some of the receivers of these biased reports are deceived.

As we argue in this paper and in Kartik et al. (2006), language inflation and deception are natural outcomes of communication when the audience has heterogeneous strategic sophistication. While the companion paper focuses on the case in which the state space is unbounded, here we consider the case of bounded state space, which is closer to the basic structure of Crawford and Sobel (1982). For the case with bounded state space, we derive here a number of comparative statics predictions and normative implications. We argue that our construction can account for the experimental findings by Dickhaut et al. (1995) and Cai and Wang (2006).

Our point of departure is Crawford and Sobel (1982) model of information transmission. A sender, privately informed of a unidimensional, continuous state of the world, reports to a decision-maker (receiver). The players' preferences are represented by quadratic loss functions. The sender is biased upwards relative to the receiver. Our aim is to characterize the outcomes of communication when the receiver is allowed to be naively unaware of the sender's incentives and to erroneously believe that the sender always reports truthfully.

The presence of non-strategic behavior drastically alters the outcomes of communication. We show that there is an equilibrium in which (1) the language used is pinned down in equilibrium and inflated and (2) the action taken by the

[1] Cheap talk games have been applied to legislative debate (Austen-Smith 1990), political bargaining between the President and the Congress (Matthews 1989), committee referral within Congress (Gilligan and Krehbiel 1989; Battaglini 2002), lobbying (Grossman and Helpman 2001, Chap. 4), and macroeconomic policy announcements (Stein 1989).

receiver (i.e., outcome) is ex-ante biased, so that naive receivers are deceived. Furthermore, we find that the amount of information transmitted in equilibrium is greater than predicted by the standard fully-strategic model.

If the state space is unbounded, Kartik et al. (2006) show, there is a fully-revealing equilibrium. The equilibrium communication function is globally invertible. If sophisticated, the receiver correctly de-biases the sender's message, and determines the actual state of the world. Hence, the sender has an incentive to add more bias to her already inflated report. As long as the equilibrium message is already above the sender's bliss point, however, this further bias will damage the sender if the receiver is naive and blindly follows the sender's recommendation. The equilibrium strategy equalizes the marginal benefit induced by the strategic type of receiver with the marginal cost induced by the naive type.

This result stands in stark contrast to the characterization in the fully-strategic communication model, where the equilibrium is partitional, but crucially hinges on the assumption that the state space is unbounded. In order to penalize a sender's deviation from an equilibrium invertible message function, it is necessary that the equilibrium message be more biased than the sender's bliss point. Typically, this would require the sender to send messages that lie outside the state space, when the state of the world is close enough to the upper bound of the state space.

For the case with bounded state space, this paper constructs an equilibrium that is fully revealing in a low range of the state space and partitional in the top range. When the state of the world is small, the sender may fully reveal it by inflating the message; but full revelation is impossible when the state of the world is large, because the required inflated message would lie outside the state space. In this context, we study how the information transmitted depends on the four parameters of the model: the fraction of naive receivers in the population; the size of the state space; the sender's informational advantage; and the divergence between the preferences of sender and receiver. We show that the relative size of the fully-revealing range increases in the fraction of naive receivers and in the informational advantage of the sender, whereas it decreases in the bias level.

These comparative statics results yield the following normative implications. A possible strategy for protecting naive agents from biased communication is to make them aware of the possibility of biased advice. But in our equilibrium, a decrease in the fraction of naive agents results in a shrinking of the full-revelation range. Educating naive agents may reduce the amount of information transmitted in equilibrium. "Hard" policies that protect naive agents and attempt to regulate communication are immune to this perverse effect. In our model, interventions aimed at reducing the sender's bias result in an increase of the fully-revealing segment.

Furthermore, we find that the relative size of the equilibrium partitional interval vanishes as the size of the state space goes to infinity, holding the bias level fixed. This result confirms the robustness of our findings for the unbounded state space case. As the state space grows unbounded, fully-revealing communication takes place on the entire state space except for a set whose relative size vanishes. We conclude by discussing our results in relation to experimental findings in Crawford and Sobel's games with a finite number of states. We show that our results are in broad agreement with the experimental findings, which consistently report that language is inflated, that communication is more informative than is predicted by the fully-rational model, and that messages bunched at the top of the message set are less informative than low messages.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 summarizes some results obtained by Kartik et al. (2006) for the case with unbounded state space. In Sect. 4 we characterize a partially-revealing equilibrium for the model with bounded state space. Section 5 analyzes the robustness of partitional equilibria. Section 6 concludes.

2 Related literature

Our work is mostly related to the following papers. Morgan and Stocken (2003) study a bounded-state space communication model where players are fully strategic and the sender is either biased or unbiased.[2] They find a semi-responsive equilibrium that is fully-revealing for low states and partitional for high states. Intuitively, the biased sender has no interest in mimicking the unbiased sender, when the latter reveals that the state is low. In their model, as in other cheap talk models with fully rational receivers, there is no deception—the expected action of the receiver is unbiased. In our model, instead, the naive receiver is deceived and ends up taking an action that is more biased than the receiver's bliss point.

While our paper focuses on the effect of naiveté on communication of information, Crawford (2003) introduces naive players in the study of communication of actions planned (intentions) in a subsequent asymmetric matching-pennies game. In a related vein, Forges and Koessler (2003) study the set of communication equilibria that can be achieved by adding general communication systems to Bayesian games in which players' types are partially verifiable.

Kartik (2005) studies a version of Crawford and Sobel (1982) bounded-state model in which the sender may send both costly reports and cheap-talk messages. He characterizes equilibria subject to a forward-induction refinement. Lying costs play the same role as our naive receivers in turning the fully-strategic cheap talk game into a game with costly signaling and a continuum of types.[3] As in our analysis, he finds that equilibrium is fully revealing in a low range of the state space and partitional in the top range. Unlike our analysis, his characterization hinges on the possibility that the sender sends messages about which the receiver is fully sophisticated. When costs become small, he

[2] See Sobel (1985), Benabou and Laroque (1992) and Morris (2001) for dynamic models of communication by advisers who may be biased.

[3] Because the action of the naive receiver follows the sender's recommendation, the sender's payoff is directly influenced by her message regardless of the equilibrium strategies, as is the case in games with costly signaling and unlike in fully-strategic cheap talk games.

shows that only the most informative Crawford and Sobel (1982) equilibrium survives.[4]

Chen (2005) studies a bounded-support quadratic-loss preferences communication model with small fractions of honest senders who are always truthful and naive receivers who always blindly believe the sender. As we noted, the addition of these behavioral types turns the cheap talk game in a continuous types costly signaling game. Following Manelli (1996), she proves the existence and uniqueness of monotonic equilibrium in the cheap-talk extension of the game, where the players also communicate with an additional message about which they are fully strategic. She shows that as the fraction of naive players become small, the equilibrium converges to the most informative Crawford and Sobel (1982) equilibrium. By holding the fraction of naive receivers constant, while taking the size of state space to infinity, we consider the opposite polar case here. We show that the resulting equilibrium mimics our unbounded state-space fully-revealing equilibrium, on all of the state space except for a set whose relative measure vanishes.

A few papers have considered some aspects of non-strategic information transmission in the context of Downsian elections. Baron (1994) studies a Downsian model where uninformed voters naively believe campaign adverts. The probability that they vote for a candidate depends on the candidate's expenditure in election campaigns. Because campaign funding is provided by interest lobbies, electoral candidates face a trade-off between choosing policies to attract informed voters, and raising funds to attract naive voters. Convergence to the median policy occurs when policies influence all interest groups, regardless of whether or not they contribute. Callander and Wilkie (2005) and Kartik et al. (2006) analyze models of Downsian competition in which some candidates may be honest and truthfully report their policy preferences. As in our model, the strategic sophistication of candidates is private information.

3 Benchmark with unbounded support

Our communication model closely follows Crawford and Sobel (1982) but accounts for the possibility that the receiver is naive and blindly believes the sender's recommendation. After being privately informed of the state of the world, $x \in \mathbb{R} \equiv X$, with cumulative distribution function $F \in \mathcal{C}^2$ and density f, a sender (S) sends a message $m \in \mathbb{R} \equiv M$ to a receiver (R). Upon receiving the message, R takes a payoff-relevant action $y \in \mathbb{R} \equiv Y$. The von Neuman–Morgenstern utilities of the players are $U^S(y,x,b) \in \mathcal{C}^2$, and $U^R(y,x) \in \mathcal{C}^2$, where the bias $b \in \mathbb{R}$ is common knowledge among the players. For the sake of concreteness we focus on the quadratic-loss case. Hence we assume that $U^S(y,x,b) = -(y-(x+b))^2$ and $U^R(y,x) = -(y-x)^2$.

[4] Blume et al. (1993) develop evolutionary based refinements for finite-state signaling games, where the sender can communicate at a cost smaller than the smallest payoff difference. For games of partial common interest (such as a finite-state version of the Crawford and Sobel 1982, model), they find that the babbling equilibrium cannot be evolutionary stable.

A message strategy is a family $(\nu(\cdot|x))_{x\in\mathbb{R}}$ where for each x, $\nu(\cdot|x)$ is a c.d.f. on the message space. Given the message strategy, the beliefs are a family $(\beta(\cdot|m))_{m\in\mathbb{R}}$ where for each m, $\beta(\cdot|m)$ is a c.d.f. on the state space. In equilibrium, beliefs are derived by Bayes' rule whenever possible. The receiver does not ever play a mixed strategy, because $U^R_{11} < 0$. Hence, the receiver's action strategy is a function $s : m \mapsto y$. When $\nu(\cdot|x)$ is degenerate for all x, we represent $(\nu(\cdot|x))_{x\in\mathbb{R}}$ by means of a function $\mu : x \mapsto m$. With a common notational violation, we say that $\beta(m) = x$ when $\beta(\cdot|m)$ is degenerate on a state x. Given the players' equilibrium strategies, the equilibrium outcome is represented by the family $(\xi(\cdot|x))_{x\in\mathbb{R}}$, where for any x, $\xi(\cdot|x)$ is a measure on the action space. When $\xi(\cdot|x)$ is degenerate for all x, we represent $(\xi(\cdot|x))_{x\in\mathbb{R}_+}$ by means of a function $\zeta : x \mapsto y$.

We modify the fully-strategic communication model to account for the possibility that the receiver is naive and matches her action y^N with the sender's message m. In an expanded communication game Γ_α, the receiver is strategic with probability $1-\alpha$ and naive with probability α. In equilibrium, the strategic receiver knows the sender's message strategy ν, and best responds to it. The sender does not know if her opponent is naive or strategic.

For the case in which the support of the cumulative distribution F is unbounded, Kartik et al. (2006) show that there always exists an equilibrium in which the sender fully reveals the state to strategic receivers.[5] For any $\alpha > 0$, the game Γ_α has an equilibrium where the message strategy is invertible. Remarkably, this occurs independent of the prior signal distribution. In these fully-revealing equilibria, the sender exaggerates the state of the world even beyond her own bias. Communication is inflated, yet detailed. Instead of reporting the general result, we introduce the main features of the fully revealing equilibrium in the quadratic loss environment.

Suppose that in equilibrium, the sender adopts an invertible function μ as her communication strategy. When a message m is sent, the strategic receiver correctly infers the state $\mu^{-1}(m)$ and the naive receiver plays the action m regardless of strategic considerations. Hence the sender will not deviate from the strategy μ only if for any x,

$$\mu(x) \in \arg\max_m -(1-\alpha)\left(\mu^{-1}(m)-(x+b)\right)^2 - \alpha\,(m-(x+b))^2 .$$

The first order condition, is

$$-2(1-\alpha)\left(\mu^{-1}(m)-x-b\right)\left(\mu^{-1}(m)\right)' - 2\alpha\,(m-x-b) = 0.$$

By substituting $\mu^{-1}(m)$ with x and m with $\mu(x)$ we obtain the differential equation

5 More generally, Kartik et al. (2006) show that there exists a fully revealing equilibrium when the sender's preferences are shape invariant (see their Theorem 3).

$$-2(1-\alpha)(x-x-b)-2(\mu(x)-x-b)\mu'(x)\alpha=0,$$

with the linear strictly-increasing solution

$$\mu(x)=x+\frac{b}{\alpha}.$$

This strategy may be interpreted as revealing the actual state of the world, but inflating the communication by an amount b/α. The factor by which communication is inflated is inversely proportional to the fraction of naive receivers in the population. Verification of the sufficient second-order condition is immediate. This concludes our equilibrium construction.

To gain some intuition on the equilibrium construction above, we note that truthful communication is established in equilibrium by postulating that for any state x, the sender reports the message m that reveals the state x according to an invertible message function μ, and by checking that she has no incentive to deviate. Since the sophisticated receiver correctly de-biases the sender's message, and determines $x=\mu^{-1}(m)$, the sender has an incentive to add more bias to the report $\mu(x)$. But if she does so, the naive receiver will believe this and end up damaging her, as long as $\mu(x)$ is already above the sender's bliss point $x+b$. For any $\alpha>0$, since the sender's quadratic utility is strictly concave in the final outcome, it is possible to find $\mu(x)$ large enough so that the rate at which the sender's utility drops because of the naive receiver's response is fast enough to make up for the gain achieved through the response of the sophisticated sender. Because the sender's utility is quadratic, the marginal benefit to inflate communication beyond μ is exactly offset by the marginal cost when $\mu(x)=x+b/\alpha$.

4 Partially revealing equilibrium with bounded support

In the previous section the construction of our fully-revealing inflated-communication equilibrium hinges crucially on the hypothesis of unbounded state space. Because the sophisticated receiver correctly inverts any equilibrium communication strategy, it is optimal for the sender to use a fully-revealing strategy μ only if she is penalized by the naive receiver when adding more bias to the equilibrium message $\mu(x)$. This requires that $\mu(x)$ is above the sender's bliss point $y^S(x,b)=x+b$. This is always possible if the state space is unbounded above. But when the state space is bounded, sending messages $\mu(x)$ above the bliss point $x+b$ is impossible, as such messages lie outside the state space X.[6]

[6] When the state space is bounded, our construction embeds the naive receivers with some degree of strategic sophistication. Say that the state space is $[0,U]$. In principle, the sender may send a message m larger than U. We implicitly assume that the receiver maintains her prior belief that the state x belongs to $[0,U]$. If a naive receiver were to believe that the state x coincided with the message m even when m is larger than U, then a fully revealing equilibrium would exist even though the state space is bounded.

Consider the equilibrium identified in the previous section. When the state space is bounded above by U, the sender cannot achieve the outcome $\mu(x) = x+b/\alpha$ from the naive receiver when $\mu(x) > U$. This means that the equilibrium is necessarily partitional for a high interval in the state space. Nevertheless, we will show that in a simple environment with bounded state space, there exists an equilibrium that maintains the most interesting properties of our equilibrium of the unbounded state space model, albeit for only a range of states that are not too close to the upper bound of the state space.

To facilitate comparison with the analysis of the previous section, we assume that the state is uniformly distributed, and let the state space (as well as the message and action spaces) be $X = M = Y = [0, U]$: the limit case with $U \to \infty$ is an unbounded support environment. Formally, we let $F(x) = 0$ if $x < 0$, $F(x) = x/U$ if $0 \leq x \leq U$, and $F(x) = 1$ if $x > U$, and assume that the naive receiver takes the action $s = 0$ upon receiving a message $m < 0$ and the action $s = U$ upon being sent a message $m > U$.

4.1 Equilibrium characterization

We proceed by constructing our semi-partitional equilibrium in several steps. After specifying our equilibrium strategies and beliefs, we calculate the expressions for the break points between the fully-revealing range and the segments in the partitional range. We then derive conditions such that these break-points are admissible and such that the sender is unwilling to deviate from equilibrium by sending messages off path.

4.1.1 Equilibrium specification

We construct an equilibrium that is fully-revealing in a low range of the state space and partitional in the top range. In the range of states $\mathcal{X}^P \equiv [a_0, U]$, our equilibrium is partitional: it is described by the collection of intervals $A = \{[a_{i-1}, a_i] : i = 1, \ldots, N\}$, where $a_N = U$. For any i, the sender sends a single message $\mu(x) = m_i$ for any $x \in [a_{i-1}, a_i]$, and $m_i \neq m_j$ for any $i \neq j$. In the range of states $x \in \mathcal{X}^R \equiv [0, a_0]$, the sender reports the fully revealing message $\mu(x) = x + b/\alpha$. Of course, our "semi-partitional" equilibrium cannot exist for all parameter configurations (b, α, U). At a minimum, it is required that $\mu(0) = 0 + b/\alpha$ be within the message space $M = [0, U]$; that is, $b/U < \alpha$. We establish below that this equilibrium exists when the bias, b, is small relative to the size of the state space, U, and the fraction of naive receivers, α.

The action of the naive receiver coincides with the received message m, while the equilibrium action of the sophisticated receivers in response to messages on the equilibrium path is $s(m) = m - b/\alpha$ for $m \in \left[b/\alpha, a_0 + b/\alpha\right]$ and $s(m) = \left[a_{i-1} + a_i\right]/2$ when $m = m_i$, for $i = 1, \ldots, N$. We conclude the equilibrium description by specifying the beliefs (and associated actions) of the strategic receiver following messages off the equilibrium path: $\mathcal{M}^B \equiv [0, \frac{b}{\alpha}]$ and $\mathcal{M}^T \equiv [a_0 + \frac{b}{\alpha}, U] \backslash \{m_i\}_{i=1,\ldots,N}$. While Bayes' rule imposes no restrictions on such beliefs, we must verify that the receiver's beliefs and consequent actions make the sender unwilling to deviate and send off-path messages. In order to give our candidate equilibrium the best fighting chance, we consider beliefs that the state is the highest possible when receiving off-path low messages, and the lowest possible in response to high messages: $\beta\left(\mathcal{M}^B\right) = U$ and $\beta\left(\mathcal{M}^T\right) = 0$. Hence $s(m) = U$ for all $m \in \mathcal{M}^B$ and $s(m) = 0$ for all $m \in \mathcal{M}^T$.

4.1.2 Indifference conditions between messages on path

The break points $a_0, \ldots, a_N$ are determined as follows. In the partitional range $\mathcal{X}^P$, the sender must be indifferent between sending message m_i and m_{i+1} at each break point a_i for $i = 1, \ldots, N-1$; this imposes that

$$-\alpha\left(m_i - b\right)^2 - (1-\alpha)\left(\frac{a_{i-1}+a_i}{2} - a_i - b\right)^2 = -\alpha\left(m_{i+1} - b\right)^2 - (1-\alpha) \times \left(\frac{a_i + a_{i+1}}{2} - a_i - b\right)^2.$$

Calculation of these break-points is greatly simplified in equilibria where the messages m_i, while all different from each other, are all approximately equal to the upper bound of the state space U. The indifference conditions approximate those derived by Crawford and Sobel (1982), and we therefore obtain their differential equation

$$a_{i+1} - a_i = a_i - a_{i-1} + 4b$$

for $i = 1, \ldots, N-1$, with final boundary condition $a_N = U$ and free initial condition a_0. The solution is

$$a_i = a_0 + (U - a_0)\frac{i}{N} - 2i(N-i)\,b.$$

When $x = a_0$, the sender must be indifferent between sending the fully revealing message $x + b/\alpha$ and the bunching message m_1, because the break-point a_0 is the upper bound of the revealing segment $\mathcal{X}^R$ as well as the lowest break point of the partitional range $\mathcal{X}^P$. This requires that

$$-\alpha\left(a_0 + \frac{b}{\alpha} - a_0 - b\right)^2 - (1-\alpha)\left(a_0 - a_0 - b\right)^2 = -\alpha\left(U - a_0 - b\right)^2 - (1-\alpha) \times \left(\frac{a_0 + a_1}{2} - a_0 - b\right)^2.$$

Substituting $a_1 = a_0 + (U - a_0)\frac{1}{N} - 2(N-1)b$ in this indifference condition, simplifying, dividing through by b^2 and defining $K = (U - a_0)/b$, we obtain the quadratic equation

$$\left(\alpha + \frac{1}{4N^2}(1-\alpha)\right)K^2 - (\alpha+1)K + \alpha + \left(N^2 - \frac{1}{\alpha}\right)(1-\alpha) = 0.$$

Because $\alpha + \frac{1}{4N^2}(1-\alpha) > 0$, the left-hand side is negative when K lies within the two roots of this quadratic equation. As it must be optimal for the sender to send the message $x + b/\alpha$ for all $x < a_0$, and the message m_1 for all $a_0 < x < a_1$, the unique admissible root of the quadratic equation is

$$K(\alpha, N) = \frac{(1+\alpha) + \sqrt{(1+\alpha)^2 - 4\left(\alpha + \frac{1}{4N^2}(1-\alpha)\right)\left(\alpha + \left(N^2 - \frac{1}{\alpha}\right)(1-\alpha)\right)}}{2\left(\alpha + \frac{1}{4N^2}(1-\alpha)\right)}.$$

We then have $a_0 = U - bK(\alpha, N)$. Because of the monotonic structure of the quadratic loss – uniform prior specification, we conclude that it is optimal for the sender to send message $\mu(x) = x + b/\alpha$ when $x \in [0, a_0]$ and $\mu(x) = m_i$ when $x \in [a_{i-1}, a_i]$ for all $i = 1, \ldots,, N-1$, where the break-points a_i, for $i = 0, 1, \ldots, N$ are approximated by the above calculations.

4.1.3 Admissibility conditions for the break point a_0

We now determine under which conditions the break-point a_0 is admissible, i.e. within the state space $X = [0, U]$, and the messages $\mu(x) = x + b/\alpha$ are within the message space $M = [0, U]$ for all $x \in [0, a_0]$. The latter constraint is satisfied if $a_0 + b/\alpha < U$, or equivalently if $K(\alpha, N) > 1/\alpha$. Because $\alpha + \frac{1}{4N^2}(1-\alpha) > 0$, after simplification this condition is equivalent to:

$$\left(2\alpha N^2 - 2\alpha N - 1\right)\left(2\alpha N^2 + 2\alpha N - 1\right) < 0, \text{ i.e. } \alpha \in \left(\frac{1}{2(N+1)N}, \frac{1}{2N(N-1)}\right).$$

Surprisingly, *the possible number of intervals in the top range of the state space* $[a_0, U]$ *in this "semi-partitional" equilibrium is unique* and determined by α; e.g. there is an equilibrium where $[a_0, U]$ is a single pooling interval if and only if $\alpha > 1/4$, a 2-interval semi-partitional equilibrium exists if and only if $\alpha \in (1/12, 1/4)$, and so on. Specifically, the number of intervals partitioning $[a_0, U]$ in equilibrium for any given α is

$$N(\alpha) \equiv \max\left\{N \in \mathbb{N} : \alpha < \frac{1}{2(N-1)N}\right\},$$

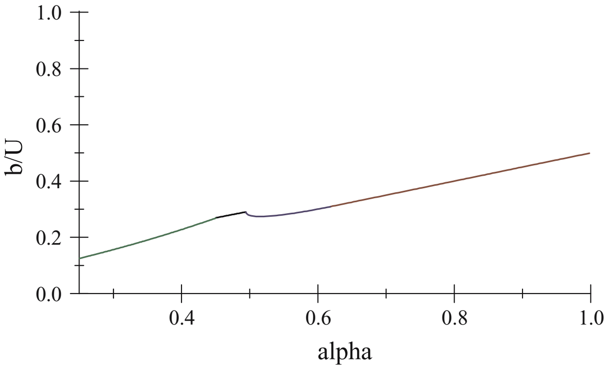

Fig. 1 Equilibrium ExistenceRegion for $N = 1$ $(\alpha > 1/4)$

where $\mathbb{N} = \{1, 2, \ldots,\}$ is the set of natural numbers, and we introduce the function $\mathcal{K}(\alpha) = K(\alpha, N(\alpha))$. We show in the Appendix that $\mathcal{K}(\alpha)$ is continuous and decreasing in α.

The constraint that a_0 is within the bounds of the state space $X = [0, U]$ is satisfied if $a_0 = U - b\mathcal{K}(\alpha) > 0$. This condition is satisfied if

$$\frac{b}{U} < \frac{1}{\mathcal{K}(\alpha)}, \tag{1}$$

where $1/\mathcal{K}(\alpha)$ is a continuous threshold function increasing in α.

4.1.4 Conditions ruling out deviations off path

We now determine the set of parameters (b, α, U) for which the sender is unwilling to deviate and send an off-path message $m \in \mathcal{M}^B \cup \mathcal{M}^T$. For the case $N = 1$, i.e. for $\alpha \geq 1/4$, the resulting equilibrium existence region (detailed calculations are available upon request) is

$$\frac{b}{U} < \bar{b}(\alpha) = \begin{cases} \dfrac{1 - \alpha - \sqrt{(1-\alpha)^2 - \alpha\left(1 - 2\alpha - (1-\alpha)\left(\frac{1}{2}\mathcal{K}(\alpha) + 1\right)^2 + \alpha\left(\frac{1}{\alpha} - 1\right)^2\right)}}{1 - 2\alpha - (1-\alpha)\left(\frac{1}{2}\mathcal{K}(\alpha) + 1\right)^2 + \alpha\left(\frac{1}{\alpha} - 1\right)^2} & \alpha \in [1/4, \bar{\alpha}_0] \\ \alpha / \left[1 + \sqrt{\alpha}\right] & \alpha \in [\bar{\alpha}_0, \bar{\alpha}_1] \\ \dfrac{1 - \alpha(1 - \mathcal{K}(\alpha)) - \sqrt{(\alpha(1 - \mathcal{K}(\alpha)) - 1)^2 - \alpha\mathcal{K}(\alpha)^2}}{\mathcal{K}(\alpha)^2} & \alpha \in \left[\bar{\alpha}_1, \left(\sqrt{5} - 1\right)/2\right] \\ \alpha/2 & \alpha \in \left[\left(\sqrt{5} - 1\right)/2, 1\right] \end{cases}$$

where $\bar{\alpha}_0 = 0.45295$ and $\bar{\alpha}_1 = 0.49433$ approximately, and corresponds to the area below the curve in Fig. 1. It is unfeasible to draw the precise threshold functions for every N, but we nevertheless verify in the Appendix that the sender has no incentive to deviate when the bias to information ratio b/U is small with respect to the fraction α of naive receivers. Hence, the equilibrium analysis is summarized in the following result.

Proposition 1 *There exists a threshold function* $\bar{b}(\cdot) > 0$, *such that for any* α, *and* $b/U < \bar{b}(\alpha)$,[7] *there is an equilibrium that is fully-revealing for* $x \in [0, a_0)$ *with communication strategy* $\mu(x) = x + b/\alpha$, *and that partitions the upper range* $[a_0, U]$ *in* $N(\alpha)$ *intervals with end-points* $\{a_0, \ldots, a_{N(\alpha)}\}$, *where*

$$N(\alpha) \equiv \max\left\{N \in \mathbb{N} : \alpha < \frac{1}{2(N-1)N}\right\}$$

and the thresholds are approximately

$$a_0 = U - b\left(\frac{(1+\alpha) + \sqrt{(1+\alpha)^2 - 4\left(\alpha + \frac{1}{4N(\alpha)^2}(1-\alpha)\right)\left(\alpha + \left(N(\alpha)^2 - \frac{1}{\alpha}\right)(1-\alpha)\right)}}{2\left(\alpha + \frac{1}{4N(\alpha)^2}(1-\alpha)\right)}\right),$$

$$a_i = a_0 + (U - a_0)\frac{i}{N(\alpha)} - 2i(N(\alpha) - i)\,b, \quad \text{for } i = 1, \ldots, N(\alpha).$$

To gain some intuition on the above characterization, we start by discussing the polar case in which the receiver is naive with probability $\alpha = 1$. It is easy to see that in this case the number $N(1)$ of intervals in the partitional part of the equilibrium equals 1. As a result, the lower threshold a_0 equals $U - b$. At the same time, the communication strategy $\mu(x)$ equals $x + b$ for all $x \in [0, a_0)$, and $\mu(x) = U$ for all $x \in [a_0, U]$. For any state of the world x, the sender convinces the naive receivers to play her best possible outcome $x + b$, as long as this recommendation is "credible" in the sense that it belongs to the state space $[0, U]$. When $x + b \geq U$, the sender resorts to the highest possible recommendation U. The equilibrium when the receiver is naive with probability 1 is equivalent to the receiver *delegating* the choice to the sender, with the constraint that the action must be in the feasible set $[0, U]$.

As the proportion of sophisticated receivers increases, i.e. as α decreases, the sender's message $\mu(x) = x + b/\alpha$ is inflated beyond the sender's bliss point $x + b$ for any state $x \leq a_0$. The equilibrium is necessarily partitional in an upper interval of the state space, because the message cannot be higher than the upper bound of the action set. The sender resorts to sending messages close to U, one message for each interval $[a_{i-1}, a_i]$ for $i = 1, \ldots, N(\alpha)$ of the partitional range of the state space. The requirements pinning down $N(\alpha)$, as well as the thresholds $a_0, \ldots, a_{N(\alpha)}$, are borne by the necessity that the sender does not have any incentive to deviate when the state x is in the partitional range of the equilibrium, and when taking into account the best response of the rational receiver to the equilibrium message strategies.

[7] The strict inequality is required because our previous calculations allow us only to approximate the equilibrium break points $a_0, \ldots, a_N$. But note that for $N = 1$ we can set $m_1 = U$ and hence the threshold a_0 is precisely pinned down in equilibrium.

4.2 Comparative statics

We now study how the communication properties of the equilibrium that we have approximated change when the model's parameters change. The quantity a_0/U measures the size of the range $\mathcal{X}^R = [0, a_0]$ where the communication is fully revealing, relative to the size of the state space $X = [0, U]$, the ex-ante uncertainty. We present here some comparative statics results for this measure of the amount of fully-revealing information transmission. The size of the support U measures the amount of asymmetric information in the problem. In the limit case with $U = 0$, in fact, there is no asymmetric information. As U grows, the importance of the sender's private information becomes larger for the receiver.

Result 1 *The relative size of the fully-revealing segment a_0/U is increasing in the fraction of naive receivers α, positively related to the amount of asymmetric information U, and negatively related to the relative bias b/U.*

Since $a_0 = U - b\mathcal{K}(\alpha)$, we have

$$\frac{a_0}{U} = 1 - \frac{b}{U}\mathcal{K}(\alpha),$$

and hence a_0/U decreases in b/U because $\mathcal{K}(\alpha) > 0$. Intuitively, communication becomes more difficult when the bias increases. The relative size of the fully-revealing segment, a_0/U, increases in α because $\mathcal{K}(\cdot)$ decreases in α. Intuitively, an increase in α results in a reduction in the penalty that the naive receiver must impose on the sender to sustain the equilibrium, thus improving the scope for communication.

Result 2 *The relative size of the fully-revealing segment a_0/U increases as the size of state space U increases, and converges to one in the limit case where $U \to \infty$, so that the state space $X = [0, U]$ becomes unbounded.*

Equilibrium communication is fully revealing in the entire state space X, and we approximate the equilibrium presented in Sect. 3 for the case with unbounded support. This result identifies key robustness properties of the main results of this paper: equilibrium communication may be inflated and fully-revealing in communication models with unbounded support, if the receiver is naive with positive probability.

Our results are compatible with experimental results obtained by Dickhaut et al. (1995) (DMM) and Cai and Wang (2006) (CW) in Crawford and Sobel's games with a finite number of states. They find strong evidence that communication becomes less informative as the bias increases and that the payoffs of both players are decreasing in the bias. These observations are compatible with the comparative statics of the most informative Crawford and Sobel equilibrium as well as of our non-fully strategic equilibrium. The addition of non-strategic behavior can explain the otherwise puzzling finding that communication is more informative than is predicted by the fully-rational model (see

DMM Table 3 and CW Sect. 4). DMM do not report the messages used by their subjects. CW's data display overwhelming language inflation (e.g., the highest message (9) is sent 57% of the time when the bias is large). The average action is monotonically increasing in the message and messages bunched at the top of the message set are less informative than low messages. These findings are broadly consistent with our inflated communication equilibrium.[8]

4.3 Welfare

We now turn to the welfare properties of our equilibrium. We begin with the welfare of the receiver. The rational receiver's utility is maximized at $U^R(y,x) = 0$ for $x < a_0$; whereas for any $i < N(\alpha)$, and $x \in (a_i, a_{i+1})$, the utility is $U^R(y,x) = -\left(\frac{a_i+a_{i+1}}{2} - x\right)^2$. For $x < a_0$, the naive receiver's utility equals $-(x + b/\alpha - x)^2 = -b^2/\alpha^2$, and it approximates $-(U-x)^2$ for $x > a_0$. Because $a_{N(\alpha)} = U$, it is easy to see that the rational receiver's utility is higher than the naive receiver's utility for any x. Intuitively, the rational receiver exploits the presence of the naive receiver to improve the inference on the state of the world, but the naive receiver is not able to fully exploit the information transmitted by the sender. The sender's utility is as follows: for $x < a_0$, $U^S(y,x) = -\frac{1-\alpha}{\alpha}b^2$ and for $x > a_0$, $U^S(y,x) = -\alpha(U - x - b)^2 - (1-\alpha)\left(\frac{a_{i-1}+a_i}{2} - x - b\right)^2$.

To investigate how the fraction of naive receivers in the population affects the receiver's utility, we first consider the key polar case in which the receiver is naive with probability $\alpha = 1$. As we have concluded before, the sender convinces the naive receiver to play the sender's optimal outcome $x + b$ whenever $x < U-b$, and when the state x is above $U-b$, she resorts to having the receiver play the highest possible outcome, U. The naive receiver's utility equals $-b^2$ for any state of the world $x < U - b$, and $-(U-x)^2$ when $x \geq U - b$.

When $\alpha = 1$, the game is equivalent to one in which the receiver *delegates* the final action to the sender, who is constrained to choose an action in the set $[0, U]$. Because $-(U-x)^2 \geq -b^2$ when $x \geq U - b$, the equilibrium improves the receiver's utility upon the *unconstrained delegation* outcome, where the sender plays $x + b$ for any states of the world x. Hence, the results by Dessein (2002) comparing unconstrained delegation and information transmission immediately imply that the (naive) receiver in the model with $\alpha = 1$ fares better in an ex-ante sense than the (rational) receiver in the Crawford and Sobel partitional equilibrium that results with $\alpha = 0$. But, while improving upon information transmission, our equilibrium fails to reach Holmström (1977) *optimal ex-ante solution* that implements action

$$a = \begin{cases} x + b & \text{for } x \in [0, U - 2b] \\ U - b & \text{for } x \in [U - 2b, U]. \end{cases}$$

[8] While Blume et al. (1998) find evidence of learning in simple communication games with 2 or 3 states, CW's findings are persistent over time.

This is the solution when the receiver can precommit to any outcome as a function of the sender's message, before receiving the message. Evidently, the sender's ex-ante welfare is also higher when $\alpha = 1$ than in Crawford and Sobel's partitional equilibrium (resulting with $\alpha = 0$).

Result 3 *If the receiver is naive with probability one, she fares better than if she is sophisticated with probability one.*

Now, consider the effect of an increase in the fraction of rational receivers on the welfare of naive receivers. As the fraction of naive receivers α decreases: (i) $-b^2/\alpha^2$ decreases, and hence the utility of the naive receiver is reduced for low states $x < a_0$; (ii) $-(U - x)^2$ remains constant; and (iii) a_0 decreases. Note that the utility $-(U - a_0)^2$ is smaller than $-b^2/\alpha^2$ because by construction $a_0 < a_0 + b/\alpha < U$. Hence, a reduction in α increases the loss in the revealing segment and increases the size of the partitional segment in which the naive receiver obtains a worse payoff than in the revealing segment. Overall, the naive receiver's utility decreases as the proportion of rational receivers increases.

Turning to rational receivers, their ex-ante welfare decreases as their proportion increases. This is because the utility of rational receivers is constant in α for $x < a_0$, and because a_0 decreases as α decreases: hence (i) the partitional segment $[a_0, U]$ increases in size and this reduces the (ex-ante) utility of the rational receiver; and (ii) the receiver's utility drops at the marginal state a_0 from zero to $-\left(\frac{a_1-a_0}{2}\right)^2$. Note that the size of the smallest message

$$a_1 - a_0 = (U - a_0)\,\frac{1}{N} - 2\,(N-1)\,b = b\left(\frac{K\,(\alpha, N)}{N} - 2\,(N-1)\right)$$

is approximately equal to zero when computed at $\alpha = \frac{1}{2N(N-1)}$ by (2). In conclusion:

Result 4 *The receiver's equilibrium welfare is monotonically increasing in the fraction of naive receivers, α.*

It is also interesting to see how the receivers' utility changes as U changes. Rational receivers achieve their bliss point when x belongs to the fully-revealing interval, whereas they achieve $-\left(\frac{a_i+a_{i+1}}{2} - x\right)^2 < 0$ for any $x \in [a_i, \frac{a_i+a_{i+1}}{2}) \cup (\frac{a_i+a_{i+1}}{2}, a_{i+1}]$. Because x is uniformly distributed, the probability that $x \in [0, a_0]$ is a_0/U. As we have concluded that a_0/U increases in U, it immediately follows that their ex-ante welfare increases in U. Naive receivers' utility when $x \in [0, a_0]$ is $-b^2/\alpha^2$. Whereas their average utility when $x \in [a_0, U]$ is:

$$\int_{a_0}^{U} \frac{-(U-x)^2\,\mathrm{d}x}{U-a_0} = \int_{U-bK(\alpha)}^{U} \frac{-(U-x)^2\,\mathrm{d}x}{b\mathcal{K}(\alpha)} = -\int_{bK(\alpha)}^{0} \frac{-(y)^2\,\mathrm{d}y}{b\mathcal{K}(\alpha)}$$

$$= -\int_{0}^{bK(\alpha)} \frac{y^2}{b\mathcal{K}(\alpha)}\mathrm{d}y$$

$$= -\frac{1}{3}\left[\frac{(b\mathcal{K}(\alpha))^3}{b\mathcal{K}(\alpha)}\right] = -\frac{1}{3}\mathcal{K}(\alpha)^2 b^2.$$

Direct calculations show that $-\frac{1}{3}\mathcal{K}(\alpha)^2 b^2 > -b^2/\alpha^2$, hence an increment in U reduces the ex-ante welfare of naive receivers. In conclusion:

Result 5 *The rational receiver's ex-ante welfare increases in the amount of private information U. The naive receiver's ex-ante welfare decreases in U.*

5 Partitional equilibria

While in the previous section we have focused on partially revealing equilibria, in this section we study whether Crawford and Sobel's partitional equilibria are robust with respect to the possibility of naive receivers. Our analysis does not depend on whether the state space is bounded or unbounded. We will show that, while not all partitional equilibria survive the introduction of naive receivers, all partitional equilibrium *outcomes* are robust.

A partitional equilibrium in a fully-strategic communication game generally can be expressed by a function ζ that partitions the state space into a collection $A = \{(a_{i-1}, a_i)\}_{i\in N}$ of intervals, for some finite (or possibly countably doubly infinite) index set N. For any i, any component (a_{i-1}, a_i), and any $x \in (a_{i-1}, a_i)$, it is the case that $\zeta(x) = \arg\max_{y\in\mathbb{R}} \int_{a_{i-1}}^{a_i} U^R(y,x)f(x)\mathrm{d}x$, and that $U^S(\zeta(a_{i-1}, a_i), a_i, b) = U^S(\zeta(a_i, a_{i+1}), a_i, b)$. The supports of $\nu(a_{i-1}, a_i)$ and $\nu(a_j, a_{j+1})$ are disjoint for any $j \neq i$.

In the fully-strategic model, the support of the message strategy ν is unspecified in any partitional equilibrium A. Instead, when some receivers are naive, the structure of the message strategy ν is severely limited. *For any interval $(a_{i-1}, a_i) \in A$, the sender can use only one message m_i on the equilibrium path; i.e., the support of $\nu(a_{i-1}, a_i)$ must be a singleton set.* Because $U^S_{12} > 0$, in fact, there cannot be any two distinct messages m and m' such that

$$\alpha U^S(m,x,b) + (1-\alpha)U^S(\zeta(a_{i-1},a_i),x,b) = \alpha U^S(m',x,b) + (1-\alpha)U^S(\zeta(a_{i-1},a_i),x,b),$$

for all $x \in (a_{i-1}, a_i)$. Hence the sender cannot be indifferent between sending m and m' if they induce the same equilibrium beliefs and action $\zeta(a_{i-1}, a_i)$ by the strategic receiver. Therefore, partitional equilibria with message strategies

ν, such that more than one message is sent on the equilibrium path for the same element of the partition, fail to exist in game Γ_α, for any $\alpha > 0$.

While the equilibrium correspondence of the family of games Γ_α is not lower hemi-continuous in α, all the partitional equilibrium *outcomes* A of the fully-strategic model are robust with respect to the possibility of naive receivers. Informally, we construct an equilibrium by requiring that, for any interval $\left(a_{i-1}, a_i\right) \in A$ and state $x \in \left(a_{i-1}, a_i\right)$, the sender sends the message m_i that corresponds to the strategic receiver's optimal choice $\zeta(a_{i-1}, a_i)$. Following the equilibrium prescription, the sender obtains the outcome $\zeta\left(a_{i-1}, a_i\right)$ with probability 1. When sending an off-the-equilibrium-path message $m \in \left(a_{i-1}, a_i\right)$ smaller (or larger) than $\zeta\left(a_{i-1}, a_i\right)$, the beliefs that we assign to the sophisticated receiver lead to an action $s(m)$ that is larger (or smaller) than $\zeta\left(a_{i-1}, a_i\right)$. Sending an off-path message is thus equivalent to a bet that can be made arbitrarily unattractive by exploiting the concavity of the sender's utility in the final outcome. Deviation from equilibrium is deterred by setting $s(m)$ so that the expected outcome when sending message m is exactly equal to $\zeta\left(a_{i-1}, a_i\right)$.

Proposition 2 *For any partition A that identifies an equilibrium outcome in the game Γ_0, and any α small enough, the game Γ_α has an equilibrium that yields the partition A.*

Note that the equilibrium construction above requires somewhat implausible specific off-path beliefs by the sophisticated type of receiver. When the sender deviates from an equilibrium prescription $m_i = \zeta\left(a_{i-1}, a_i\right)$ by sending a message m smaller (or larger) than $\zeta\left(a_{i-1}, a_i\right)$, the equilibrium requires that the sophisticated receiver understands that she meant to communicate the opposite, i.e. that the optimal choice is larger (or smaller) than $\zeta\left(a_{i-1}, a_i\right)$. This problem is unavoidable, because for any interval $\left(a_i, a_{i+1}\right) \in A$, the support of $\nu\left(a_i, a_{i+1}\right)$ must be a singleton set. In the fully-revealing equilibrium reported in Sect. 3, in contrast, all messages are on path. Hence, the fully-revealing equilibrium of Sect. 3 cannot be criticized on the basis that it hinges on possibly implausible off-path equilibrium beliefs.

6 Conclusion

Departing from Crawford and Sobel (1982) basic communication model, we have introduced the possibility that receivers are naive and erroneously believe that the sender is truthful. Focusing on the case with quadratic preferences and a uniformly distributed state, we have shown that there is an equilibrium in which communication is fully-revealing on a low segment of the state space and partitional on a high segment. In this equilibrium, communication language is inflated, the equilibrium outcome is biased, and the information transmission is more precise than in the standard communication model. We have argued that these results are in broad agreement with empirical observations and experimental findings. We provide comparative statics results and highlight their normative implications.

Because the relative size of the fully-revealing segment increases in the proportion of naive receivers, educating receivers about the sender's strategic behavior bears the perverse effect of reducing the amount of communication conveyed in equilibrium. On the other hand, reducing the sender's bias has the effect of increasing the amount of information transmission. As the state space grows unbounded, we have shown that the equilibrium converges to the fully-revealing equilibrium that Kartik et al. (2006) derive for the limit case with unbounded state space.

In the context of media bias, our analysis gives a simple account for why political adverts are evidently biased. However, educating the decision maker to the possibly manipulative behavior of campaigners/partisan experts would only result in a reduction of the amount of information contained in their biased reports. It might be preferable instead to sever all links between experts and interest groups (such as sponsoring lobbies) through the institution of independent bodies of experts.

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Appendix

Lemma A.1 *The function $K(\alpha)$ is continuous and strictly decreasing in α.*

Proof Because

$$K\left(\frac{1}{2N\,(N-1)},N\right) = 2N\,(N-1) = K\left(\frac{1}{2N\,(N-1)},N-1\right), \tag{2}$$

the function $\mathcal{K}(\alpha)$ (and hence the break-point $a_0 = U - b\mathcal{K}(\alpha)$) change continuously at the critical value $\alpha = \left[2\,(N+1)\,N\right]^{-1}$, despite the fact that the number of intervals partitioning $[a_0, U]$ drops from N to $N-1$. It is immediate that $N(\cdot)$ is a decreasing function of α. We now show that the function $K\,(\alpha, N)$ decreases in α and increases in N for $\alpha \in \left(\left[2\,(N+1)\,N\right]^{-1}, \left[2\,(N-1)\,N\right]^{-1}\right)$.

The result that $K\,(\alpha, N)$ increases in N for $\alpha \in \left(\frac{1}{2(N+1)N}, \frac{1}{2N(N-1)}\right)$ immediately follows from the relation $K\,(\alpha, N) > 1/\alpha > K\,(\alpha, N-1)$ for $\alpha \in \left(\frac{1}{2(N+1)N}, \frac{1}{2N(N-1)}\right)$.

To show that $K\,(\alpha, N)$ decreases in α for $\alpha \in \left(\frac{1}{2(N+1)N}, \frac{1}{2N(N-1)}\right)$, we differentiate it and simplify, finding that

$$\frac{\partial K}{\partial \alpha} \propto \frac{-1 + 3\alpha - 12\alpha N^2 - 4\alpha^2 + 24\alpha^2 N^2 - 36\alpha^2 N^4 + 2\alpha^3 - 8\alpha^3 N^2 + 4\alpha^3 N^4 + 16\alpha^3 N^6}{4\alpha^2 N^4 \sqrt{(\alpha - 1)\,\dfrac{4\alpha^2 N^4 - 4\alpha^2 N^2 + \alpha^2 - 4\alpha N^2 + \alpha - 1}{\alpha N^2}}} - \frac{2N^2 - 1}{N^2}$$

where the second term is clearly negative and the denominator of the first term is positive. We only need to show that the term

$$-1+3\alpha-12\alpha N^2-4\alpha^2+24\alpha^2N^2-36\alpha^2N^4+2\alpha^3-8\alpha^3N^2+4\alpha^3N^4+16\alpha^3N^6$$

is negative for all α belonging to the interval $\left(\frac{1}{2N(N+1)},\frac{1}{2N(N-1)}\right)$. Replacing the positive α with its upper bound $\frac{1}{2N(N-1)}$ and the negative α with the lower bound $\frac{1}{2N(N+1)}$, we obtain

$$-\frac{1}{4}\left(\frac{-1-3N-21N^2+33N^3+38N^4-6N^5+44N^6-108N^7-120N^8+56N^9}{(N+1)^3\,N^3\,(N-1)^3}\right).$$

This term is negative for $N>2$. In addition, it is easy to verify directly that $\partial K(\alpha,N)/\partial\alpha<0$ for $N=1$ and $N=2$ in the relevant ranges $(1/4,1)$ and $(1/12,1/4)$. □

Proof of Proposition 1 There are four no-deviation conditions to be verified: the sender must be unwilling to send "bottom" off-path messages $\mathcal{M}^B$ when (i) the state x is in the "revealing" interval $\mathcal{X}^R$ and when (ii) x is in the "partitional" interval $\mathcal{X}^P$, and she must not send "top" off-path messages $\mathcal{M}^T$ when (iii) $x\in\mathcal{X}^R$ and when (iv) $x\in\mathcal{X}^P$. For any state x, we denote by $D(m,x)$ the sender's payoff when sending an off-path message m, and by $E(x)$ her equilibrium payoff. For each state x, we determine the highest possible deviation payoff $D^*(x,\mathcal{M})=\sup_{m\in\mathcal{M}}D(m,x)$ for $\mathcal{M}=\mathcal{M}^T,\mathcal{M}^B$. Then we determine for which parameter configurations there is no incentive to deviate in any state, i.e. $\sup_{x\in\mathcal{X}}D^*(x,\mathcal{M})-E(x)<0$ for $\mathcal{X}=\mathcal{X}^R,\mathcal{X}^P$.

Fix α, which defines the corresponding number $N(\alpha)$ of segments in the partitional part of the equilibrium. We need only show that for sufficiently small b, there is no incentive for the sender to deviate and send off-path messages.

Let the sender's equilibrium payoff be

$$E^R(x)=-\alpha\left(x+\frac{b}{\alpha}-x-b\right)^2-(1-\alpha)(x-x-b)^2=-\frac{1-\alpha}{\alpha}b^2$$

for any x in the revealing segment $\mathcal{X}^R=[0,a_0]$ and

$$E^P(x)=-\alpha(U-x-b)^2-(1-\alpha)\left(\frac{a_{i-1}+a_i}{2}-x-b\right)^2$$

in the partitional segment $x\in[a_{i-1},a_i]$, with $i=1,\ldots,N(\alpha)$.

The largest payoff achieved by the sender by sending an off-path message in the bottom segment $\mathcal{M}^B=[0,b/\alpha]$ is

$$\bar{D}^B(x)=\max_{m\in\left[0,\frac{b}{\alpha}\right]}\left\{D^B(m,x)=-\alpha(m-x-b)^2-(1-\alpha)(U-x-b)^2\right\}.$$

Similarly, the largest payoff for the sender corresponding to an off-path message in the top segment $\mathcal{M}^T = \left[a_0 + \frac{b}{\alpha}, U\right]$ is

$$\bar{D}^T(x) = \max_{m \in \left[a_0 + \frac{b}{\alpha}, U\right]} \left\{ D^T(m,x) = -\alpha (m - x - b)^2 - (1-\alpha)(0 - x - b)^2 \right\}.$$

Consider the equations

$$a_0 = U - bK(\alpha, N(\alpha)),$$

$$a_i = a_0 + (U - a_0)\frac{i}{N(\alpha)} - 2i(N(\alpha) - i)\,b, \quad \text{for } i = 1, \ldots, N(\alpha),$$

approximating the equilibrium thresholds when the messages m_i associated with the intervals $[a_{i-1}, a_i]$, $i = 1, \ldots, N(\alpha)$ are all different, but arbitrarily close to U. It is apparent that each of the above threshold approximations converges to U as $b \to 0$. By taking each equilibrium message m_i closer and closer to U as b converges to 0, we obtain that the actual thresholds a_i, $i = 0, \ldots, N(\alpha)$ also converge to U as $b \to 0$.

Now the argument proceeds in two steps.

First, note that for any x, the limit of the equilibrium payoffs $E(x)$ converges to zero for any x as $b \to 0$. In the revealing segment we clearly have that $E^S(x) = -\frac{1-\alpha}{\alpha}b^2 \to 0^-$. In the partitional segment, for any $i = 1, \ldots, N(\alpha)$ and any $x \in \left[a_{i-1}, a_i\right]$, $\lim_{b\to 0} E^P(x) = 0$ because $\lim_{b\to 0} a_i = U$ for all $i = 0, \ldots, N(\alpha)$.

Second, for any x, by continuity of $D^B(m,x)$ and $D^T(m,x)$ in m, we have

$$\begin{aligned}\lim_{b\to 0} \bar{D}^B(x) &= \lim_{b\to 0} \max_{m\in\left[0,\frac{b}{\alpha}\right]} \Big\{ D^B(m,x) = -\alpha(m - x - b)^2 \\ &\qquad\qquad - (1-\alpha)(U - x - b)^2 \Big\} \\ &= -\alpha(0 - x - b)^2 (1-\alpha)(U - x - b)^2 \le -\alpha(1-\alpha)U^2\end{aligned}$$

and

$$\begin{aligned}\lim_{b\to 0} \bar{D}^T(x) &= \lim_{b\to 0} \max_{m\in\left[a_0+\frac{b}{\alpha},U\right]} \Big\{ D^T(m,x) = -\alpha(m - x - b)^2 \\ &\qquad\qquad - (1-\alpha)(0 - x - b)^2 \Big\} \\ &= -\alpha(U - x - b)^2 - (1-\alpha)(0 - x - b)^2 \le -\alpha(1-\alpha)U^2,\end{aligned}$$

because $\lim_{b\to 0}\left(a_0 + \frac{b}{\alpha}\right) = U$ when taking each equilibrium message m_i closer and closer to U as b converges to 0.

The sender's limit gains from deviating off path are negative and bounded away from zero as b vanishes:

$$\lim_{b\to 0} (\bar{D}^B(x) - E^B(x)) < 0 \quad \text{and} \quad \lim_{b\to 0} (\bar{D}^T(x) - E^T(x)) < 0.$$

By continuity of the payoff functions in b, we conclude that it is not profitable for the sender to deviate from equilibrium and send off-path messages provided that $b < \overline{b}(\alpha)$, for some threshold function $\overline{b}$ that is always strictly positive. □

Proof of Proposition 2 For any equilibrium partition A of game Γ^0, any $(a_{i-1}, a_i) \in A$ and any α, construct the strategy μ such that $\mu(x) = m_i = \zeta(a_{i-1}, a_i)$ for any $x \in (a_{i-1}, a_i)$. In game Γ^α, for any α, the naive receiver responds to m_i by playing $y = m_i$ and the sophisticated receiver optimally plays $s(m_i) = \zeta(a_{i-1}, a_i)$; and hence the sender of type a_i is indifferent between sending the message m_i and m_{i+1}, because by construction $U^S(\zeta(a_i, a_{i+1}), a_i, b) = U^S(\zeta(a_{i-1}, a_i), a_i, b)$.

Assign the off-path beliefs of sophisticated senders as follows. For any $m \in (a_{i-1}, a_i)$, $m \neq m_i$, the sophisticated receiver believes that the state x coincides with $y^{R^{-1}}\left(\frac{m_i - \alpha m}{1-\alpha}\right)$ with certainty. As a result, this type of receiver plays $s(m) : (1-\alpha)s(m) + \alpha m = m_i = \zeta(a_i, a_{i+1})$. Clearly if $m < \zeta(a_i, a_{i+1})$, then $s(m) > \zeta(a_i, a_{i+1})$ and vice versa.

Considering the sender's problem, for any $m \in (a_{i-1}, a_i)$, $m \neq m_i$, we obtain

$$\alpha U^S(m, x, b) + (1-\alpha) U^S(s(m), x, b) < \alpha U^S(m_i, x, b) + (1-\alpha) U^S(m_i, x, b),$$

where the inequality follows from $U_{11} < 0$, concavity of U in y, and Jensen inequality.

Also, note that for $m \in \{a_{i-1}, a_i\}$ we can repeat the same construction arbitrarily with i, with $i-1$ or with $i+1$. Since i is arbitrary, this concludes the construction of the equilibrium. □

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