When Liability is Not Enough: Regulating Bonus Payments in Markets With Advice

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Abstract

We introduce a model of advice in which firms steer advisors through nonlinear incentive schemes. In addition to developing an isomorphism to pricing with mixed bundling, we obtain three main insights. First, firms optimally use nonlinear bonuses to economize on the rent paid to advisors. Second, equilibrium bonus payments induce advisors to make biased recommendations that are artificially contingent on each other, resulting in an inefficient allocation. Third, if advisor liability is stepped up, firms respond by increasing the size of the bonus, leaving advisor bias unchanged. These results support direct regulatory interference on the way advisors are compensated.

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1 Introduction

Compensation of financial intermediaries and advisors has come under close scrutiny since the 2008 financial crisis when policymakers pointed to distorted incentives as a major culprit.[1] In many jurisdictions and industries, inducements paid by providers of retail investment services are now heavily regulated, including a full ban of commissions for advisors.[2] When inducements are still possible, however, their shape seems to frequently escape regulatory scrutiny. In the prominent case of mortgage brokers, product providers can still pay inducements also to non-tied brokers both in Europe and the US. In fact, it is common industry practice to make bonus payments contingent on total sales volume. Actually, Section 1403 of the US Dodd-Frank Wall Street Reform and Consumer Protection Act explicitly exempts mortgage origination from regulation that restricts incentive payments based on volume:

> "(4) RULES OF CONSTRUCTION.—No provision of this subsection shall be construed as— . . .
> (D) prohibiting incentive payments to a mortgage originator based on the number of residential mortgage loans originated within a specified period of time."[3]

In other areas, such as retail investment services or consumer (non-mortgage) loan origination, US regulation has targeted bonuses. For instance, a 2010 amendment of Regulation Z (Loan Originator Compensation and Steering 12 CFR 226) prohibits various compensation practices.

Should policy makers regulate nonlinear incentives in markets with advice? Should policy makers just step up liability or should they regulate more directly the shape of the compensation structure of advisors? The paper addresses these questions in a model of advice that endogenizes the introduction of nonlinear incentives by competing product

[1] In the wake of the financial crisis, the insurance business was in the limelight with a high-profile lawsuit against Marsh McLennan, the largest US insurance brokerage firm. The case centered on how clients were steered toward insurers with which Marsh McLennan had lucrative undisclosed contingent commission (or "placement service") agreements and was settled with a record $850 million compensation to clients. See Cummins and Doherty (2006) for a detailed discussion and analysis of brokerage intermediation in the US insurance market.

[2] See Financial Stability Board (2011). The key legislative measures in Europe are the Markets in Financial Instruments Directives, MiFID and MiFID II (banning various commissons as of January 2018).

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providers. Regardless of whether advice is given simultaneously or sequentially to different customers, bonus contracts distort advice by generating an artificial link between otherwise independent transactions. In the baseline model without regulation, we show that bonus incentives always arise; a linear component is also present, in contrast notably to extant models where intermediaries must exert costly effort. Importantly, stepping up advisor liability does not reduce the bias because product providers, while reducing the overall value of inducements, optimally restructure incentives to fully compensate for the increase in liability. Predicting the implications of such a policy change while ignoring how incentives schemes adapt to the policy would fall foul of the Lucas critique. Nonlinear incentives and their adjustment thwart stepping up advisor liability, have detrimental effects on consumers, and thus should be carefully scrutinized by regulators.

Even though consumer protection for retail financial products has been strengthened over the last decade, bonus-driven remuneration practices remain widespread for mortgage brokers and other advisors and have escaped close scrutiny. Few jurisdictions, such as the UK and Australia, seem to recognize potential consumer detriment from bonus payments where inducements are still allowed. The UK's Financial Conduct Authority (FCA) explicitly reserves the right to intervene directly if individual incentive schemes are found to not be in the interest of consumers. To this end, the FCA practises a wider monitoring regime including on-site assessments of incentive arrangements, focusing in particular on bonuses and how these are earned.[4] Australia has recently announced a similar regime with respect to mortgage origination incentives.[5] Such monitoring and micro-management of individual compensation schemes may seem as second best compared to regulating participants' incentives, principally by imposing greater liability on advisors. Our results, however, justify such interference. As we show, product providers' adjustment of compensation undoes the desired effects of increasing advisor liability. While greater advisor liability dampens the overall use of inducements, it even makes bonus payments relatively more attractive compared to linear commissions.

While we conduct our comparative statics in terms of changes in liability, the agent's preferences for suitable advice may originate from various sources beyond liability. Following the 2008 financial crisis and evidence of misconduct to the detriment of purchasers of lending or investment products, observers lamented the lax ethical standards that prevail

[4]See the FCA's guidance on remuneration, published in 2018, https://www.fca.org.uk/publication/finalised-guidance/fg18-02.pdf.

[5]See the guidance to mortgage brokers published in 2020 by the Australian Securities and Investment Commission, https://download.asic.gov.au/media/5641325/rg273-published-24-june-2020.pdf.

in the finance profession.[6] There have been calls to improve standards of ethical conduct by introducing a Hippocratic oath for financial professionals.[7] In fact, ethics training has now been mandated in a number of leading business education programs. In our setting, however, following an increased concern of advisors for suitability, product providers would optimally provide steeper nonlinear incentives, thus completely undoing the effect of the change in preferences resulting from ethics training.[8]

Beyond the retail financial industry and insurance, expert recommendations in healthcare and other markets for other credence goods are also characterized by pervasive conflict of interest. For example, pharmaceutical firms have a strong incentive to bias the prescriptions physicians issue to patients. Regulatory interference in these markets, however, often remains soft and focuses mainly on disclosure.[9] More broadly, our analysis is also relevant for online commerce, where biased recommendations and self-preferencing are currently attracting regulatory scrutiny, again focusing mostly on disclosure.[10] When incentives have a non-linear component, recommendations become biased, even in a fully symmetric scenario, as advisors' individual recommendations become inefficiently contingent on each other.

Our model builds on previous research on how firms steer advice, which however has focused on linear incentives or, equivalently, on the case with a single customer. As shown by Inderst and Ottaviani (2009, 2012a), in this case competition for recommendations is largely isomorphic to competition for final consumers in a Hotelling model, with the

[6] Individuals who are particularly self selving might be attracted to lucrative jobs in finance. See Gil et al. (2015) for an experimental study and references therein.

[7] See Weitzel and Kirchler (2020) for an audit study of the introduction of a bankers' oath in the Netherlands.

[8] Increased liability would instead have an immediate impact if it was imposed on product providers, through secondary (also known as vicarious) liability; see Pitchford (1995) and Che and Spier (2008). In practice, however, there are serious legal obstacles to overcome when the advisor acts as an independent business, being neither an employee of a particular product provider nor in an exclusive relationship. In fact, we are not aware of cases where the sole presence of bonus payments was sufficient to impose vicarious liability for financial losses associated with retail lending or investment products. Typically, liability was instead established only when consumer loss (misselling) could be traced back to the way the products were designed, packaged, or advertised by the provider.

[9] A notable example is the Physician Payment Sunshine Act of 2010. For a discussion see the articles in the May 2017 issue of the Journal of the American Medical Association and for a recent empirical analysis see Guo, Sriram, and Manchanda (2021).

[10] For instance, the UK's Competition Markets Authority (CMA) has formulated such a policy for the hotel booking sector, where platforms receive payments for preferred listings (CMA, "Consumer Protection Law Compliance: Principles for Businesses Offering Online Accomodation Booking Services", 2019). Regulatory interference with platforms has also targeted the practice of self-preferencing by intermediary recommendations; see the Digital Markets Act in the Eurpean Commission and the proposed US Senate Bill "American Innovation and Choice Online Act".

responsiveness of demand depending on the advisor's liability—rather than on consumer preference heterogeneity, as in Hotelling's classic framework. In a version of the model with nonlinear advisor incentives, we uncover a similar isomorphism with the workhorse model of pricing under mixed bundling.

To derive these results, our key departure from the growing literature on steering advice—Inderst and Ottaviani (2009, 2012a), Armstrong and Zhou (2011), Hagiu and Jullien (2011), de Corniére and Taylor (2019), and Teh and Wright (2020)—is the explicit consideration of nonlinear incentives.[11] We initially consider the case in which the advisor make sequential recommendations to customers, and later extend the analysis to the case with simultaneous recommendations. The case with simultaneous arrival of customers is isomorphic to models of mixed bundling. There, multi-product firms face a consumer who may want to purchase one or more of a firms' products and firms charge individual prices as well as prices for bundles. The discount that is offered with a bundle in McAfee et al. (1988) and Armstrong and Vickers (2010) is akin to the bonus that the advisor earns in our model when giving the same advice to different consumers who arrive simultaneously.

We note another important connection to antitrust policy. In many cases antitrust agencies have scrutinized loyalty rebates involving intermediaries, such as travel agencies or garages, rather than final consumers. Casual evidence suggests that these intermediaries also engage in advice. While traditionally antitrust focuses on possible foreclosure of smaller (or single-product) competitors, our analysis points to biased advice as another important source of consumer detriment from nonlinear incentives.[12] As we show, nonlinear incentives bias advice and thereby lead to consumer harm, even when they are provided by firms that are equally well positioned to capture market share, thereby obviating the risk of foreclosure. Our analysis thus strengthens the case of antitrust authorities against nonlinear incentives.

From a normative perspective, our focus on the benefits of regulating bonus payments ties into the growing literature on regulating firms' compensation practices more generally. Here, the literature has dealt with increased transparency of incentives for consumers (Inderst and Ottaviani (2009)), the overall size of compensation (Thanassoulis (2012)), or minimum deferral periods (Hoffman et al. (2021)). Much of the literature on the regulation

[11] That said, we do not incorporate other features that this literature has introduced, such as advisor competition or a downward sloping demand function (arising, for instance, from consumer private information); cf. our concluding remarks.

[12] As noticed by Innes (1990), nonlinear incentives for intermediaries may efficiently reduce the cost of eliciting effort. In our model, nonlinear incentives arise as a cost-effective way for product providers to steer recommendations.

of compensation focuses on excessive risk taking or managerial short-sightedness as an outcome of governance problems (cf. also Bénabou and Tirole (2016), Albuquerque et al. (2016), or Bebchuck and Fried (2010)).

Our work relates also to various strands of empirical literature. Sharing with our contribution the focus on nonlinear incentives, a small empirical literature mostly in marketing shows how bonuses affect the behavior of a firm's sales force; see Misra and Nair (2011) for sellers of contact lenses and Larkin (2014) for sellers of enterprise software. Using data from the pharmaceutical industry, Kishore et al. (2013) show that an early fulfillment of a sales target dampens further sales. Tzioumis and Gee (2013) have recently studied the remuneration of mortgage officers, showing a spike in sales and a notable reduction in processing time at the end of the month. In the case of sequential recommendations, in our model distortions arise because nonlinear incentives make incentives for subsequent recommendations dependent on earlier sales. Steenburgh (2008) and Chung et al. (2013) provide a simulated quantification exercise in a model where incentives induce costly effort in the spirit of Innes (1990) and Holmström and Milgrom (1987).

We share the focus on welfare and regulation with a growing literature that uses structural approaches to assess the welfare impact of regulatory intervention affecting the compensation of advisors. Robles-Garcia (2020) undertakes a number of counterfactual policy analyses for the UK mortgage market, including a full ban on payments. In healthcare, recent papers by Carey et al. (2020) and Grennan et al. (2020) study the implications of payments made by pharmaceutical firms to physicians. While this recent work has taken up the challenges of estimating the welfare implications of incentive pay for intermediaries, to our knowledge the models and counterfactual analyses have not yet dealt with the role of nonlinear incentives.[13]

The rest of this paper is organized as follows. Section 2 introduces the baseline model with sequentially arriving consumers. Without fully solving for the equilibrium, Section 3 shows that compensation always involves bonus payments and that advice must therefore be biased. Section 4 derives the equilibrium compensation and shows how compensation reacts to increased liability and thereby counteracts completely the direct effect on advice. In Section 5 we close the model, including equilibrium product prices, both with and without regulation. Section 6 extends results to the case where consumers arrive simultaneously. In our concluding remarks in Section 7 we discuss both avenues for future

[13] Various empirical studies have documented the presence of biased advice in the areas of retail finance, insurance, and healthcare, often without a full welfare analysis and also without detailed information on commissions. For instance, for healthcare evidence from Japan see Iizuka (2007, 2012).

research and why the imposition of excessive liability, which would drive out all incentive payments, should often be inefficient. The appendix collects the proofs, while the online appendix contains various additional results mentioned in the main text.

2 The Model

The model features firms that compete by steering advisors. Departing from the existing literature, we analyze the role and possible regulation of nonlinear incentives and focus on how incentives affect the suitability of advice.

Consider two firms or product providers, $n = A, B$, that sell their products through a single advisor. While we stipulate that firms sell only indirectly through the advisor, we also provide conditions under which non-advised sales do not arise in equilibrium. Firms' per-unit production costs are denoted by c_n and product prices by p_n. Given our interest in situations in which private contracting through warranties fails, we rule out payments from or to customers that are contingent on the realized utility.[14] Normalizing the utility of customers from not purchasing to zero, the utility of a given customer j from purchasing either product depends on a binary state variable $\theta_j \in \{A, B\}$, which captures product suitability as follows: the customer derives utility $v_h > 0$ if the product matches the state and utility $v_l < v_h$ otherwise, where v_l may be negative.

A key feature of our model is that the advisor has private information about product suitability for a given customer. As in Inderst and Ottaviani (2012a, 2012b), we directly work with the advisor's posterior beliefs. Based on his private information, the advisor's posterior belief is given by $q_j = \Pr(\theta_j = A) \in [0, 1]$, which captures the likelihood that product A is more suitable. A priori, q_j is i.i.d. with cumulative distribution function $G(q_j)$ and density $g(q_j) > 0$ on $[0, 1]$.[15]

The advisor is motivated both by the suitability of the choice made by customers and by the incentive payments received from product providers, as specified below. The advisor derives utility w_h if the purchased product is suitable and $w_l < w_h$ otherwise. When a customer does not purchase any of the two products, we denote the advisor's utility gross of any payments by w_0. Assuming that $w_0 < w_l$, the advisor always recommends one of the two products, so that firms are always in competition through compensation,

[14] We also do not allow consumers to pay separately for (unbiased) advice, given that this is rarely the case without regulation. The trade-off between inducements and payments made only by consumers is analyzed in Gravelle (1994), Stoughton et al. (2011), and Inderst and Ottaviani (2012b).

[15] The prior beliefs of all parties are thus $\Pr(\theta_j = A) = \int_0^1 qg(q)dq$ and $\Pr(\theta_j = B) = \int_0^1 (1-q)g(q)dq$.

corresponding to the full-coverage assumption in Hotelling competition.[16] The difference $w = w_h - w_l$ captures the advisor's concern for suitability. We recall however from the introduction that a change w may also originate from other sources that affect the advisor's preferences, such as a change in her own standards of professional ethics and conduct.

Firms can make their incentive schemes contingent on the number of sales s_n, with non-negative payment $F_n(s_n) \geq 0$ because of limited liability.[17] We assume that the advisor has an outside option of value zero. It follows that firms have no incentives to pay a base salary, so that $F_n(0) = 0$. To study the implications and regulation of nonlinear incentives, we restrict attention to the case with at most two customers, so that we can conveniently express compensation as follows. Define f_n as a per-unit commission and b_n as an additional bonus, so that $F_n(1) = f_n$ and $F_n(2) = 2f_n + b_n$. In the online appendix we show how nonlinear incentives and biased advice also result when the number of arriving customers is arbitrary.

To close the model, we next specify the timing. At $t = 1$, firms simultaneously offer incentives (f_n, b_n), which the advisor can accept or reject. At $t = 2$, firms simultaneously set product prices p_n. At $t = 3$, after observing q_1, the advisor provides a recommendation to the first customer, who then decides whether or not to purchase and, if so, which product. Next, after the second customer arrives, the advisor makes a recommendation based on q_2 and the second purchase decision is made. The order of arrival is randomly assigned and denoted by $i = 1, 2$; customers do not know whether they are first or second in line. The advisor privately observes product suitability q_i for the i-th arriving customer and advice is provided by sending a message $m_i \in \{A, B\}$. Section 6 extends the analysis to the case with simultaneous arrival of consumers, when the advisor simultaneously observes q_1 and q_2. Figure 1 illustrates the structure of the model.

Note that we do not allow firms to condition incentives, as well as prices p_n, on the timing and order of advised customers.[18] Likewise, we do not allow firms to renegotiate contracts after the first recommendation and sale have been concluded. Rather than applying a fully-fledged mechanism design approach, our aim is to start from what we regard as empirically reasonable contractual instruments, which are furthermore identical

[16] Without the full-coverage assumption, the analyzed problem boils down to either two separate monopoly problems or the same problem we analyze here. In the online appendix we show how our key result about nonlinear incentives and distorted advice also carries over to the monopoly context.

[17] Note that $F_n(s_n) \geq 0$ implies that firm n cannot force the advisor to hand over any compensation received from the other firm.

[18] This also rules out menus so that the advisor could pick contracts depending on s_1 and (s_1, s_2), respectively.

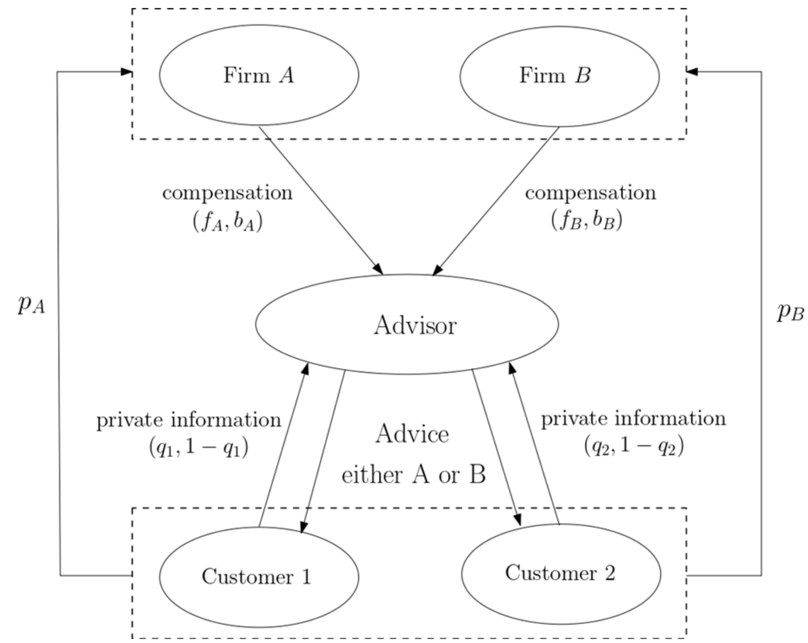

Figure 1: The Model

in both the models with simultaneous and sequential arrival.

In what follows, we focus on the case with symmetric product providers, $c_n = c$, and further assume that G is symmetric around the common prior belief $q = 1/2$, $G(q) = 1 - G(1-q)$. Note that in models with a single customer this symmetry setting ensures that the advice is unbiased, regardless of the size of the incentives, which in equilibrium are symmetric. By focusing on the symmetric case, we thus make transparent the novel implications that follow when product providers can offer nonlinear incentives in the presence of several customers. The online appendix derives additional results for the case with cost asymmetry. To ensure uniqueness we stipulate that the hazard rate, $g(q)/[1-G(q)]$, is increasing in $q \in [0,1]$. Together with symmetry of G, this implies that the reverse hazard rate, $g(q)/G(q)$, is decreasing. Denote now $v_A(q) = qv_h + (1-q)v_l$ and $v_B(q) = (1-q)v_h + qv_l$. Requiring

$$\int_0^1 v_A(q)g(q)dq = \int_0^1 v_B(q)g(q)dq = \frac{v_l + v_h}{2} < c \tag{1}$$

ensures that advice is essential for selling either product.

3 The Impossibility of Unbiased Advice Without Regulation

Before solving for the optimal compensation of firms and the resulting market equilibrium in the absence of regulation, we show first that advice will always be biased at equilibrium. To this end, we first solve stage $t = 3$, in which the advisor makes recommendations to customers, and show that advice is always biased when firms use nonlinear incentives. We then show—still without a full characterization—that nonlinear incentives must necessarily arise in equilibrium when firms optimize.

Pattern of (Biased) Advice. In $t = 3$, we focus on pure strategy equilibria in which advice is informative. Ignoring the payoff-equivalent outcome in which messages are swapped, we show subsequently that in equilibrium the i-th arriving customer follows the advisor's respective recommendation through message $m_i = A$ or B. For now we thus postulate that customers follow the advisor's recommendations.

We first consider the pattern of advice for the customer who arrives second. When product A is sold to the first customer, the advisor anticipates to receive an expected payoff equal to $f_A + b_A + q_2 w + w_l$ from recommending product A (through message $m_2 = A$) to the second customer when product suitability is $q_2 \in [0, 1]$, and $f_B + (1 - q_2)w + w_l$ from recommending product B (through message $m_2 = B$). Comparing payoffs yields the threshold

$$\bar{q}_2^A = \frac{1}{2} - \frac{f_A - f_B + b_A}{2w}, \tag{2}$$

so that the advisor prefers $m_2 = A$ if $q_2 \geq \bar{q}_2^A$ and $m_2 = B$ otherwise.[19] The subscript 2 in $\bar{q}_2^A$ denotes the second customer, and the superscript A indicates that the advisor has sold product A to the first customer. Analogously, for the case where the first customer purchased product B we obtain

$$\bar{q}_2^B = \frac{1}{2} - \frac{f_A - f_B - b_B}{2w}. \tag{3}$$

Intuitively, the responsiveness of the advisor's cutoff to monetary incentives depends negatively on the liability w. This is thus akin to how the cutoff in a Hotelling model of competition depends on consumers' horizontal preferences. This analogy was already pointed out by Inderst and Ottaviani (2009) for the case with a single customer, even though there compensation involved only a single instrument.

[19] To deal with corner solutions, we set $\bar{q}_2^A = 0$ if $w \leq f_A - f_B + b_A$ and $\bar{q}_2^A = 1$ if $w \leq -(f_A - f_B + b_A)$.

Turning to the first customer, the advisor's recommendation now depends on the current compensation and the anticipated compensation to be obtained on the second customer. To capture the latter, we denote the expected continuation payoff after advising the first consumer to purchase A by

$$Z^A = \int_0^{\bar{q}_2^A} (f_B + (1-q)w + w_l)g(q)dq + \int_{\bar{q}_2^A}^1 (f_A + b_A + qw + w_l)g(q)dq,$$

where the first (respectively second) term is the advisor payoff when recommending B (respectively A) to the second customer. Denoting by Z^B the expected continuation payoff after advising the first consumer to purchase B, when observing product suitability q_1 for the first customer the advisor realizes $f_A + q_1 w + w_l + Z^A$ by recommending product A and $f_B + (1-q_1)w + w_l + Z^B$ by recommending product B. Comparing payoffs, the indifference threshold the first consumer is

$$\bar{q}_1 = \frac{1}{2} - \frac{f_A - f_B + Z^A - Z^B}{2w}. \tag{4}$$

Lemma 1 characterizes the resulting pattern of advice, where it is clearly inconsequential how the (zero-probability) indifference is resolved.

Lemma 1 *When customers follow the recommendation, the advisor's optimal recommendation is characterized by*

$$(m_1, m_2) = \begin{cases} (A, A) & \textit{if } q_1 \in [\bar{q}_1, 1] \textit{ and } q_2 \in [\bar{q}_2^A, 1], \\ (A, B) & \textit{if } q_1 \in [\bar{q}_1, 1] \textit{ and } q_2 \in [0, \bar{q}_2^A], \\ (B, A) & \textit{if } q_1 \in [0, \bar{q}_1] \textit{ and } q_2 \in [\bar{q}_2^B, 1], \\ (B, B) & \textit{if } q_1 \in [0, \bar{q}_1] \textit{ and } q_2 \in [0, \bar{q}_2^B]. \end{cases}$$

To illustrate Lemma 1, consider a symmetric compensation scheme (f, b). In the case of no bonus ($b = 0$), as then $\bar{q}_2^A = \bar{q}_2^B = 1/2$ holds and so also $\bar{q}_1 = 1/2$, the advisor recommends product A to the i-th arriving customer (sends message $m_i = A$) if $q_i \geq 1/2$ and B otherwise, irrespective of the order of their arrival. This recommendation rule, which would also arise without monetary incentives, is depicted in the left panel of Figure 2. Precisely, there the cases where the same product is recommended to both customers are found in the lower left and the higher right square, with $(m_1 = B, m_2 = B)$ and $(m_1 = A, m_2 = A)$, respectively. And the cases where different products are recommended are found the upper left and the lower right square, with $(m_1 = B, m_2 = A)$ and $(m_1 = A, m_2 = B)$, respectively.

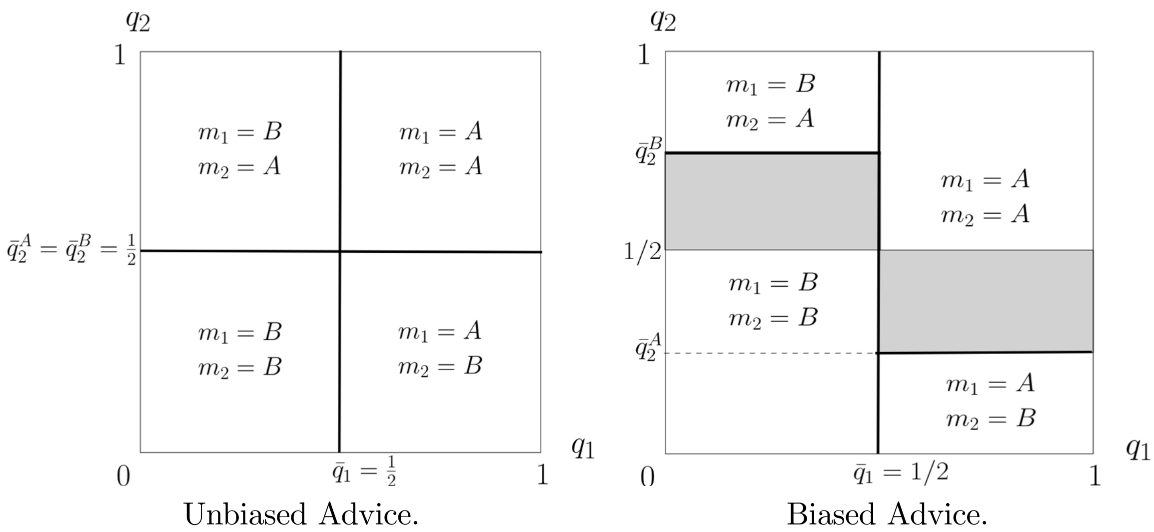

Figure 2: Pattern of Biased Advice.

Now, with $b_n = b > 0$ advice cutoffs are given by $\bar{q}_2^A = 1/2 - b/(2w) = 1 - \bar{q}_2^B \in (0, 1/2)$, while still $\bar{q}_1 = 1/2$ given the assumed symmetry of contracts. This pattern of advice with a strictly positive bonus is illustrated in the right panel of Figure 2. There, the gray-colored regions represent the effect of the bonus on advice, compared to the case with $b = 0$: The lower left square with $(m_1 = B, m_2 = B)$ expands upwards to $\bar{q}_2^B > 1/2$ and the upper right square with $(m_1 = A, m_2 = A)$ expands downwards to $\bar{q}_2^A < 1/2$. When the bonus is strictly positive, the advisor's recommendation to the second customer depends thus on the recommendation made to the first customer. The advisor is then more likely to recommend the product that was already recommended to the first customer. A strictly positive bonus therefore generates a link between recommendations to different customers that is not justified by efficiency considerations and that leads to biased advice. To formalize the concept of biased advice, we compute the resulting welfare.

Welfare. When the advisor applies for a given customer the cutoff $\bar{q}$, define the expected valuation

$$\mathbb{E}[v \mid \bar{q}] = \int_0^{\bar{q}} v_B(q) g(q) dq + \int_{\bar{q}}^1 v_A(q) g(q) dq.$$

Abstracting from prices and taking account of the random order of customer arrivals, a customer's expected gross utility from following advice is thus

$$U = (1/2) \left\{ \mathbb{E}[v \mid \bar{q}_1] + G(\bar{q}_1) \mathbb{E}[v \mid \bar{q}_2^B] + [1 - G(\bar{q}_1)] \mathbb{E}[v \mid \bar{q}_2^A] \right\}, \tag{5}$$

which will be useful also later when determining the equilibrium price. When w arises from penalties imposed on the sale of an unsuitable product, such monetary transfers do not enter social welfare. As the expected product cost per customer is always c, expected welfare per customer is thus $W = U - c$. This is clearly maximized when the advisor sets the same threshold $q^{FB} = 1/2$ with each customer, recommending A if and only if $q_i \geq q^{FB}$. Put differently, welfare is maximized when the advisor treats all customers equally with $\bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B = q^{FB}$.

Firm Profits. To define firm profits we introduce some auxiliary shorthand notation. We denote the ex-ante probability that the advisor makes different recommendations to the two customers by

$$\Pr(1) = G(\bar{q}_1)\left[1 - G(\bar{q}_2^B)\right] + \left[1 - G(\bar{q}_1)\right] G(\bar{q}_2^A)$$

and the ex-ante probability of recommending twice the same product by

$$\Pr_A(2) = \left[1 - G(\bar{q}_1)\right]\left[1 - G(\bar{q}_2^A)\right] \text{ and } \Pr_B(2) = G(\bar{q}_1)G(\bar{q}_2^B).$$

Consequently, expected sales for firm n are

$$\mathrm{S}_n = \Pr(1) + 2\Pr_n(2)$$

and firm profits

$$\pi_n = \mathrm{S}_n(p_n - c_n - f_n) - \Pr_n(2)b_n. \tag{6}$$

The last term in π_n accounts for compensation costs that arise from the payment of a bonus b_n, which is made with probability $\Pr_n(2)$.

We now analyze how compensation affects firm profits. There are two first-order effects: a profit gain by the increase in sales and a profit loss by the increase in expected compensation. Differentiating firm n's profit (6) with respect to f_n and b_n yields the marginal profits

$$\frac{\partial \pi_n}{\partial f_n} = \mathrm{S}_n^f(p_n - c_n - f_n) - \Pr_n^f(2)b_n - \mathrm{S}_n \tag{7}$$

and

$$\frac{\partial \pi_n}{\partial b_n} = \mathrm{S}_n^b(p_n - c_n - f_n) - \Pr_n^b(2)b_n - \Pr_n(2), \tag{8}$$

where we have used shorthand notation for the partial derivatives, with $\Pr_n^f(2)$ denoting the partial derivative with respect to f_n.

The Impossibility of Unbiased Advice. So far we have shown that a strictly positive bonus invariably leads to biased advice, because recommendations to different customers are then linked even though costumer decisions should be kept separate to achieve efficiency. We now show that a strictly positive bonus will always arise in an unregulated equilibrium. Taken together this proves that advice will be biased in an unregulated equilibrium. We already note at this point that typically optimal compensation will involve also a strictly positive base commission.[20]

Our argument at this stage proceeds by contradiction. Supposing thus that $b_n = 0$, we can benefit from the simplification that then all cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ will be equal. Denoting this common cutoff by $\bar{q} \in (0,1)$, note that it lies in the open interval $(0,1)$, as otherwise the advisor would always recommend a particular firm's product to customers in contradiction to assumption (1). We also remark that the argument that follows does not rely on firm symmetry and thus holds irrespective of the symmetry of $G(q)$ and of $c_n = c$.

Starting thus from $b_n = 0$, we show that any of the two firms would strictly benefit from paying a strictly positive bonus, while possibly reducing its commission. For this we consider for a given firm n a marginal increase in its bonus and, at the same time, a marginal decrease in its commission so that total expected sales remain unchanged. We then show that along this gradient the firm's profit strictly increases, contradicting the optimality of $b_n = 0$.

Consider thus marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ such that indeed total expected sales remain unchanged, that is,

$$\mathrm{S}_n^f df_n + \mathrm{S}_n^b db_n = 0. \tag{9}$$

Next, note that, starting from the partial derivatives (7) and (8), the total derivative of firm profits simplifies to

$$d\pi_n = -\mathrm{S}_n df_n - \mathrm{Pr}_n(2) db_n \tag{10}$$

as we can use that $b_n = 0$ and that $dS_n = 0$ by construction in (9). To simplify expressions, we conduct our argument for a single firm, choosing $n = A$. Since from $b_n = 0$ all advice cutoffs are equal to $\bar{q}$, we have $\mathrm{Pr}_A(2) = [1 - G(\bar{q})]^2$ and, with $\mathrm{Pr}(1) = 2G(\bar{q})\,[1 - G(\bar{q})]$, $\mathrm{S}_A = 2\,[1 - G(\bar{q})]$, so that expression (10) becomes

$$d\pi_A = -2\,[1 - G(\bar{q})]\, df_A - [1 - G(\bar{q})]^2\, db_A. \tag{11}$$

[20] As discussed in the Introduction, our setting is notably different from agency problems in which the agent has to spend costly effort (on multiple projects or tasks).

A key step is now to show that, still starting from $b_n = 0$, total sales remain constant, so that (9) is satisfied, if

$$df_A = -\left[1 - G(\bar{q})\right] db_A. \tag{12}$$

This expression (derived formally in the proof) is a consequence of the fact that, starting from $b_n = 0$, the marginal impact of increasing the bonus is exactly equal to the marginal impact of an increase in the commission multiplied by the likelihood with which the advisor recommends the respective product "a second time", here product A, i.e., $1 - G(\bar{q})$. Substituting (12) into (11), the total derivative of profits becomes stepwise

$$\begin{aligned} d\pi_A &= -2\left[1 - G(\bar{q})\right] df_A - \left[1 - G(\bar{q})\right]^2 db_A \\ &= 2\left[1 - G(\bar{q})\right]^2 db_A - \left[1 - G(\bar{q})\right]^2 db_A \\ &= \left[1 - G(\bar{q})\right]^2 db_A, \end{aligned}$$

which concludes the argument. Starting from $b_n = 0$, it is strictly profitable to increase b_A, while adjusting f_A to keep total expected sales constant, as given by (9).

Proposition 1 *Nonlinear incentives are part of any unregulated equilibrium when compensation is positive, i.e., there is no equilibrium in which compensation is positive but* $b_n = b = 0$.

In essence, starting from $b_n = 0$, the benefit-to-cost ratio of increasing the bonus strictly exceeds that of increasing the commission.[21] The bonus is more effective as, compared to an increase in the commission, it tilts the advisor's first recommendation also when the advisor would otherwise be just indifferent with respect to only the first customer. We note however as well that when already $b_n > 0$, the then strictly positive bonus needs to be paid with a higher likelihood also when the first recommendation threshold shifts. This is why the argument for why $b_n > 0$ does not generally imply that *only* $b_n > 0$ while $f_n = 0$, which we show below.

We find it instructive to further support the intuition for why a positive bonus is paid by considering the following auxiliary model. Suppose that a firm could make commissions contingent on when a sale took place, by paying f_A^1 for a sale to the first and f_A^2 for a

[21] In what basically amounts only to a rewriting of the preceding steps, we note again that the impact on sales of a marginally higher bonus equals that of a marginally higher commission multiplied by $1 - G(\bar{q})$. As the higher commission is paid with likelihood $2\left[1 - G(\bar{q})\right] G(\bar{q}) + 2\left[1 - G(\bar{q})\right]^2 = 2\left[1 - G(\bar{q})\right]$ and the higher bonus with likelihood $\left[1 - G(\bar{q})\right]^2$, the corresponding ratio for costs is $\left[1 - G(\bar{q})\right]/2$, which is indeed strictly smaller than $1 - G(\bar{q})$.

sale to the second customer.[22] When both commissions are chosen optimally, the choice of f_A^2 satisfies the corresponding first-order condition: the costs of marginally increasing the commission by one currency unit whenever the advisor recommends A to customer 2 just balances the incremental benefits from increasing the likelihood of selling to customer 2. Instead of marginally raising f_A^2, consider now the introduction of a marginal bonus b_A of one currency unit as well and its impact on recommendations made by the advisor who had already recommended A to the first customer. With respect to the advisor's recommendation to customer 2, the effects of a marginal bonus trade off in exactly the same way as those of a marginal increase in f_A^2. From this perspective alone, introducing a bonus would not dominate an increase in f_A^2. However, this analysis so far fully neglects the impact that a bonus, but not a change in f_A^2, has on the recommendation to customer 1. Through the bonus, the advisor's recommendation is tilted toward product A when the advisor was previously (just) indifferent, because now the advisor expects to earn b_A from the second customer with some probability.

4 The Limitations of Liability

We now derive the optimal compensation for a symmetric equilibrium, with $f > 0$ and $b > 0$. As we show below, such an equilibrium is unique and exists whenever w is not too large. We also show that for larger w the equilibrium compensation features $f = 0$. Note that in this section we take the prices $p_n = p$ as given; we derive the full equilibrium with endogenous prices in Section 5.

Our characterization borrows from the proof by contradiction for why $b_n = 0$ cannot be optimal (Proposition 1). That is, we characterize the optimal compensation by invoking the first-order condition along the gradient of the profit function where (9) is satisfied, so that a firm's total expected sales remain constant. Hence, making use of the partial derivatives (7) and (8), again we start out with the total derivative

$$d\pi_n = -df_n \left[\mathrm{Pr}_n^f(2)b_n + 1\right] - db_n \left[\mathrm{Pr}_n^b(2)b_n + \mathrm{Pr}_n(2)\right], \tag{13}$$

where we have already used that, in a symmetric equilibrium, $\mathrm{S}_n = \mathrm{Pr}_n(1) + 2\mathrm{Pr}_n(2) = 1$ and that this stays constant at the considered gradient. In the appendix we proceed from (13) by substituting for the corresponding terms. While this does not allow to derive the equilibrium compensation explicitly, we obtain the following first characterization result:

[22] This discussion thus also highlights that our results do not depend on our restriction to commissions f_n that can not be made contingent on the identity of the customer.

Lemma 2 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, the optimal choice of the bonus must satisfy*

$$b_n = b = 4\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)}w. \tag{14}$$

We first note that there exists as well a symmetric characterization of b in terms of the cutoff $\bar{q}_2^B = 1 - \bar{q}_2^A$. In (14) the cutoff $\bar{q}_2^A$ is endogenous and depends also on b. Still, we can use (14) to obtain one of our key results. For this we note that with symmetry and thus $b_n = b$ and $f_n = f$, from (2) the cutoff simplifies to

$$\bar{q}_2^A = \frac{1}{2} - \frac{b}{2w},$$

which we can substitute into the right-hand side of (14).

Lemma 3 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, the advice cutoff for the second customer, conditional on that A was recommended to the first customer, is uniquely determined by*

$$\bar{q}_2^A = \frac{1}{2} - 2\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)}. \tag{15}$$

The respective advice cutoff when B was recommended to the first customer, is symmetric and given by $\bar{q}_2^B = 1 - \bar{q}_2^A$.

Proof. See the Appendix.

Equation (15) has two main implications. First, we have that $\bar{q}_2^A < 1/2$ and $\bar{q}_2^B > 1/2$, so that advice is always biased in an unregulated equilibrium, which confirms our previous result (proved without using symmetry). Second, the two cutoffs are independent of the agent's liability w.

The result that the pattern of advice and thereby also the size of the bias are independent of the advisor's liability may at first seem counterintuitive, given that this implies a strictly higher expected liability payment by the agent as w increases. In fact, if we were to hold the compensation fixed, the bias would be reduced as w increases: $\bar{q}_2^A$ and $\bar{q}_2^B$ would be closer to 1/2. This would also be the naive prediction that would only focus on the agent's behavior and incentives, thereby ignoring firms' incentives to change compensation structure. In this case where both $b_n > 0$ and $f_n > 0$ (see below for the remaining case where $b_n > 0$ but $f_n = 0$), firms react to the increased liability by stepping up the bonus. This follows already from the characterization in (14), using that $\bar{q}_2^A$ does not change with w. As we show next, firms reduce commissions at the same time, reacting to the fact that the advisor becomes less responsive to monetary incentives. Focusing thus alone on the advisor's change of preferences would thus be subject to the Lucas critique.

From the first-order condition of firm profits with respect to commission f_n, using the derivative (7), we next obtain:

Lemma 4 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, given $\bar{q}_2^A$ from (15), the optimal commission satisfies*

$$f = p - c - 2wH(\bar{q}_2^A), \tag{16}$$

where $H(\bar{q}_2^A)$ is defined by

$$H(\bar{q}_2^A) = \frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)} + \frac{1}{4g\,(1/2)\,[1 - G(\bar{q}_2^A)]^2 + g(\bar{q}_2^A)}. \tag{17}$$

Proof. See the Appendix.

Note again that in this case (which arises provided the liability w is not too high, as shown below) firms pay both a bonus and a commission. In particular, it is thus not generally optimal for firms to only pay a bonus. Evidently, other than in moral hazard models with costly effort, such a flattening of incentives does not hinge on risk aversion. When a firm pays only a bonus, it risks losing sales precisely when the two realizations q_1 and q_2 are particularly diverse as products A and B are particularly suitable for only one of the two customers respectively.

Inspection of expression (16) immediately confirms that in response to higher liability firms decrease their commissions, while, as we already know, increasing their bonus. The equilibrium commission is a function also of firms' margin (gross of compensation cost), $p - c$, which we determine below. It is, however, useful to note already now that the product price will remain unaffected, so that the present comparative analysis still holds.

We can also see immediately that with an increase in w the steepness of the compensation strictly increases, which holds irrespective of whether we consider the bonus-to-commission ratio b/f or the ratio of the expected bonus payment, $b/2$, to the expected total compensation, $f + b/2$.[23] When we consider expected total compensation, we can make, in addition, the following observation. Substituting the expressions for the compensation components, we obtain the expected compensation paid by a given firm

$$f + b/2 = p - c + 2w\frac{1}{4g\,(1/2)\,[1 - G(\bar{q}_2^A)]^2 + g(\bar{q}_2^A)}.$$

[23] Also the latter is obvious as the ratio $(b/2)/(f + b/2)$ is montonic in b/f.

As $\bar{q}_2^A$ remains unchanged, expected compensation thus strictly increases with the advisor's liability. We note that, next to the invariance of the advice cutoffs to changes in w, also this implication is orthogonal to that with only linear incentives (as in the extant literature reviewed in the introduction). There, as the imposition of greater liability makes the single advice cutoff less responsive to commissions, product providers respond by reducing commissions, resulting also in lower aggregate compensation for advisors. Now, with nonlinear incentives, commissions still decrease, but the increase in the bonus more than compensates for this, increasing overall compensation. From the perspective of a regulator, observed increases in the bonus, the steepness of compensation, and also total compensation may thus be signs that adjustments in the market, here effected by product providers, thwart the intended effects of stricter liability.

We now turn to the condition that both compensation instruments are indeed strictly positive, $b_n > 0$ and $f_n > 0$. The optimal commission in (16) is indeed positive as long as w is below a certain level, otherwise it equals zero. We denote by w_f^* the resulting threshold, given by

$$w_f^* = \frac{p - c}{2H(\bar{q}_2^A)}, \tag{18}$$

where $\bar{q}_2^A$ is uniquely determined by (15). We can now collect our characterization of the equilibrium nonlinear incentive scheme when $f_n > 0$ and $b_n > 0$.

Proposition 2 *Suppose that the advisor's liability w is below the threshold w_f^* defined by (18). Then, there is a unique symmetric equilibrium compensation scheme, giving rise to unique advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, which are all independent of w. The commission f decreases with the size of w while the bonus b increases. Precisely, $\bar{q}_1 = 1/2$, $\bar{q}_2^A = 1 - \bar{q}_2^B$ is determined by (15), and (f, b) solve (16) and (14), respectively.*

Once w reaches the threshold $w_f^* > 0$, setting a positive commission is no longer profitable and firms will react to a change in w by reducing their still positive bonus, which then has an immediate implication for the advice cutoffs. More explicitly, simplifying the derivative (8) using $f_n = 0$ and symmetry, the first-order-condition with respect to the symmetric bonus is then $\mathrm{S}_n^b(p-c) - \mathrm{Pr}_n^b(2)b_n = \mathrm{Pr}_n(2)$. From this we can derive an implicit expression for $b_n = b$, reported in the proof of the next result. The size of w at which also the bonus becomes zero as a function of the price p is

$$w_b^* = 2g(1/2)(p - c). \tag{19}$$

Note that the symmetric price p used to define w_f^* and w_b^* is not the same. We turn to this in the next section where we fully characterize the equilibrium.

Proposition 3 *Let w_f^* and w_b^* be the thresholds of w defined by (18) and (19), respectively. If $w \in (w_f^*, w_b^*)$, the optimal commission is zero and the optimal bonus is given by (38). For $w \geq w_b^*$ total compensation is zero.*

Proof. See the Appendix.

5 Equilibrium Analysis

So far we have taken prices as given when determining the optimal incentive scheme. In this section, we derive the full equilibrium of the game. To this aim, we first provide a definition of the equilibrium. We then prove existence of a unique equilibrium, and finally turn to the comparative statics.

Recall that we suppose that customers cannot observe the incentive schemes, so that they have to form rational expectations. For this we assume that they hold passive beliefs out of equilibrium. To derive customers' conditional valuations, we define their beliefs about the (non-observed) compensation by $(\hat{f}_n, \hat{b}_n)$ and the corresponding rationally anticipated advice cutoffs by $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2^A, \hat{q}_2^B)$. For some arbitrary expected advice cutoff $\hat{q}$, let

$$\mathbb{E}[v_A \mid \hat{q}] \equiv \frac{1}{1 - G(\hat{q})} \int_{\hat{q}}^{1} v_A(q) g(q) dq \quad \text{and} \quad \mathbb{E}[v_B \mid \hat{q}] \equiv \frac{1}{G(\hat{q})} \int_{0}^{\hat{q}} v_B(q) g(q) dq$$

be the customer's conditional expected valuation for products A and B, respectively. Taking account of sequential arrivals in a random order, each customer anticipates that the expected valuation conditional on being recommended product $n(= A, B)$ would be[24]

$$\mathbb{E}[v_n \mid \hat{\mathbf{q}}] \equiv \frac{1}{2} \left\{ \mathbb{E}[v_n \mid \hat{q}_1] + G(\hat{q}_1) \mathbb{E}[v_n \mid \hat{q}_2^B] + [1 - G(\hat{q}_1)] \, \mathbb{E}[v_n \mid \hat{q}_2^A] \right\}. \tag{20}$$

Given passive beliefs, in equilibrium each firm optimally extracts the full conditional valuation, so that $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}]$. With this we are now in a position to define an equilibrium.

An equilibrium is characterized by a tuple of firm strategies (f_n, b_n, p_n), the advisor's cutoff strategy $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, and customer beliefs about compensation $(\hat{f}_n, \hat{b}_n)$, which give rise to the expected cutoffs $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2^A, \hat{q}_2^B)$, so that the following conditions hold. Incentive schemes (f_n, b_n) must be optimal, given prices p_n. Prices in turn must be optimal, which is the case if $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}]$. The advisor makes optimal recommendations, implying that cutoffs $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ are given by Lemma 1. Finally, customers' beliefs are rational as $(\hat{f}_n, \hat{b}_n) = (f_n, b_n)$ and thereby also $\hat{\mathbf{q}} = \mathbf{q}$.

[24] This mirrors expression (5).

We now turn to the determination of the prices that are used to define the thresholds w_f^* and w_b^*. Starting with w_b^*, where for $w \geq w_b^*$ compensation is zero, p is obtained from unbiased advice, $p = p_n = \mathbb{E}[v_n \mid (1/2, 1/2, 1/2)]$. The threshold w_f^*, where for $w \geq w_f^*$ commission is zero, instead is obtained from biased advice, $p = p_n = \mathbb{E}[v_n \mid (1/2, \bar{q}_2^A, \bar{q}_2^B)]$ with $\bar{q}_2^A$ given by (15) and $\bar{q}_2^B = 1 - \bar{q}_2^A$.

Proposition 4 *When $w < w_f^*$, there exists a unique unregulated equilibrium where firms pay both positive commissions and bonuses $f_n = f > 0$ and $b_n = b > 0$, as characterized in Proposition 2. Advice is biased and the bias is also not mitigated when liability w marginally increases. When liability is instead high with $w \geq w_b^*$, firms do not provide incentives to the advisor. In the intermediate case, where $w_f^* \leq w < w_b^*$, we have $f_n = f = 0$ and $b_n = b > 0$, as in Lemma 3. Then, as w increases, advice becomes less biased, as $\bar{q}_1 = 1/2$, while $1/2 - \bar{q}_2^A > 0$ and $\bar{q}_2^B - 1/2 > 0$ strictly decrease.*

Proof. See the Appendix.

As long as $w < w_f^*$, an increase in liability w affects neither the suitability of advice nor the maximum price that firms can charge. Liability only affects the outcome when it is sufficiently high, in particular when it fully crowds out any compensation. In our concluding remarks we argue why setting an arbitrarily high liability may not be a realistic solution, because of the risk of various unwanted consequences.

Example: Uniform Distribution. Suppose that G is the uniform distribution on $[0, 1]$. By Proposition 2, we can derive a unique advice cutoff $\bar{q}_2^A = 1 - \bar{q}_2^B \in (0, 1/2)$ from (15), which yields $\bar{q}_2^A = 1/6$ and $\bar{q}_2^B = 1 - \bar{q}_2^A = 5/6$. Applying $\bar{q}_2^A = 1/6$ to (14) leads to the optimal bonus given by $b = \frac{2}{3}w$. To fully pin down the optimal commission, we derive the function $H(\bar{q}_2^A)$ defined by (17). With $\bar{q}_2^A = 1/6$ we obtain $H(\bar{q}_2^A) = 12/51$, so that from (16) $f = p - c - 44/51w$. Next, as $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) = (1/2, 1/6, 5/6)$, we have for the optimal price

$$p_n = p = \mathbb{E}[v_A \mid \mathbf{q}] = \frac{1}{2}\left[2\left(\frac{3v_h + v_l}{8}\right) + \frac{1}{2}\left(\frac{3v_h + v_l}{2}\right)\right] = \frac{3v_h + v_l}{4}.$$

All compensation instruments are indeed positive if w is below the threshold defined by (18), which now becomes

$$w_f^* = \frac{p - c}{2H(\bar{q}_2^A)} = \frac{51}{44}(p - c) = \frac{51}{176}(3v_h + v_l - 4c).$$

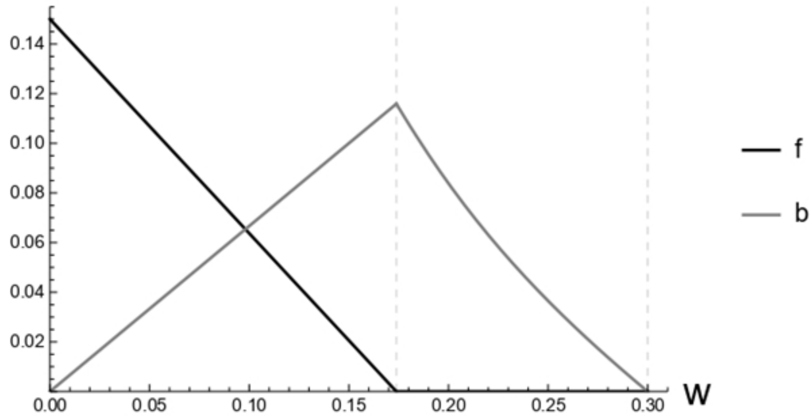

Figure 2: Uniform distribution.

Next, we consider the case where w is both above w_f^* and below w_b^*, with

$$w_b^* = 2(p-c) = \frac{1}{2}(3v_h + v_l - 4c).$$

In this case, the optimal bonus is given by (38), which simplifies to $b = w(1-2\bar{q}_2^A)$.[25]

Figure 2 illustrates the characterization of incentives (f, b) as a function of w for parameters $(v_h, v_l, c) = (1, 0, 0.6)$. The commission f decreases with the size of w while the bonus b increases if w is below the threshold $w_f^*(\approx 0.17)$. When w is higher, the bonus b decreases as w further increases, up to the threshold $w_b^*(= 0.3)$, above which all compensation is zero.

Comparative Statics in the Precision of the Advisor's Information. As a second specification for the advisor's posterior beliefs, consider a truncated normal distribution with mean 1/2, variance $\sigma > 0$, and truncation $[0, 1]$. An increase in σ results in a mean-preserving spread of the posterior distribution, so that the advisor becomes better informed about products' suitability. Ceteris paribus, as the advisor becomes better informed, this increases the suitability of advice and welfare. But such a consideration neglects the change in compensation—just as a ceteris paribus consideration of an increase in liability would wrongly suggest a reduction in the advisor's bias. Now, as the advisor becomes better informed, we find that this makes it again profitable for firms to step up their bonus and thereby increase the bias of advice.

[25] Here, $\bar{q}_2^A$ solves the equation $w\left(2(1-\bar{q}_2^A)^2+1\right)(1-2\bar{q}_2^A) = \left(4(1-\bar{q}_2^A)^2+1\right)(p-c) - 2w(1-\bar{q}_2^A)$.

Proposition 5 *Suppose $G(q)$ is a truncated normal distribution with mean $1/2$. As the advisor becomes better informed, as reflected in a higher variance of the distribution of posterior beliefs, product providers increase their bonus and the advice cutoff for the second customer becomes more biased away from the first-best, that is $1/2 - q_2^A$ and $q_2^B - 1/2$ increase.*

Proof. See the Appendix.

Equilibrium when Bonus Payments to Advisors are Prohibited. As discussed in the Introduction, some regulatory authorities intervene directly in firms' choice of compensation. We now consider the outcome when such a regulator bans bonus payments, $b_n = 0$. As is immediate from our preceding observations, then the recommendations given to different customers become independent, as dictated by efficiency. For each customer the advisor applies the threshold

$$\bar{q} = \frac{1}{2} - \frac{f_A - f_B}{2w}, \tag{21}$$

which depends only on the remaining commissions and which leads to unbiased advice, $\bar{q} = 1/2$, when commissions are symmetric, $f_n = f$. For completeness we derive in the appendix the following characterization for the case where bonus payments are prohibited.

Proposition 6 *There exists a unique regulated equilibrium in which the optimal commission $f_n = f^R$ is given by*

$$f^R = p - c - \frac{w}{g(1/2)} \tag{22}$$

if $w \leq g(1/2)(p - c)$, and it is zero otherwise, where $p = 2\int_{1/2}^{1} \upsilon_A(q)g(q)dq$. Advice is always unbiased.

Proof. See the Appendix.

6 Simultaneous Advice and Connection to Mixed Bundling

So far, we have supposed that customers arrive in a sequential order. We now consider the case of simultaneous arrival. The key difference is that the advisor then observes product suitability for both customers at the same time and makes recommendations simultaneously. Given the similarity with the baseline model with sequential arrival, we keep the analysis of the unregulated outcome short.

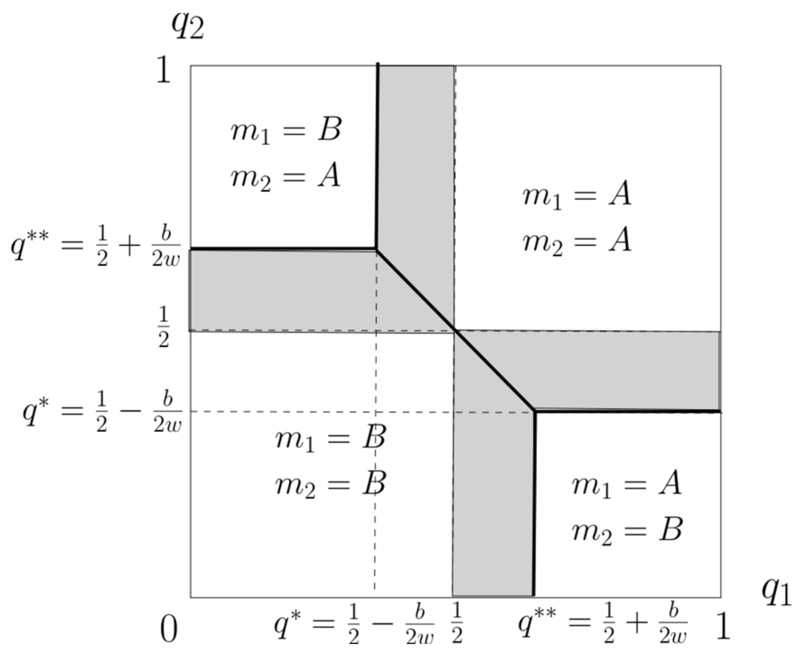

Figure 3: Biased Advice with Simultaneous Arrival.

The Pattern of (Biased) Advice with Simultaneous Arrival. We give a brief description of how biased advice results when both firms choose symmetric compensation schemes, $f_n = f$ and $b_n = b$; see the appendix for a complete analysis. The pattern of advice is depicted in Figure 3, where again the gray areas show the deviations from the benchmark of unbiased advice when $b > 0$. A positive bonus leads to an inefficient expansion of the areas in which the same product is recommended to both consumers. To see this, consider the case when the advisor recommends different products. The advisor recommends (B, A), as depicted in the left upper square, only when

$$q_1 \leq q^* = \frac{1}{2} - \frac{b}{2w} \text{ and } q_2 \geq q^{**} = \frac{1}{2} + \frac{b}{2w}, \tag{23}$$

and recommends (A, B), as depicted in the right lower square, only when

$$q_1 \geq q^{**} = \frac{1}{2} + \frac{b}{2w} \text{ and } q_2 \leq q^* = \frac{1}{2} - \frac{b}{2w}. \tag{24}$$

This shows immediately how the gray areas and thus the bias increase in b.

Equilibrium Compensation. In the main text we confine ourselves to a characterization of the equilibrium compensation when liability w is not too high so that still both compensation instruments are used. Defining now for any given $q^* \in (0, 1/2)$

$$H(q^*) = \frac{G(q^*)}{g(q^*)} + \frac{1}{2(G(q^*)g(q^*) + \int_{q^*}^{1-q^*} g^2(q)dq)}, \tag{25}$$

we have the following result:

Proposition 7 *Consider the case with simultaneously arriving customers. Generally, $b_n = b = 0$ is again not an equilibrium when there is positive compensation. If there exists an equilibrium where both the commission and the bonus are strictly positive, $f > 0$ and $b > 0$, then it is again unique and characterized as follows. The advisor recommends different products to the two customers only when conditions (23) and (24) are satisfied, where the cutoff $q^* = 1 - q^{**} \in (0, 1/2)$ is uniquely determined by*

$$q^* = \frac{1}{2} - \frac{G(q^*)}{g(q^*)}, \tag{26}$$

so that the strictly positive bias is independent of liability w. The optimal bonus

$$b = 2\frac{G(q^*)}{g(q^*)}w$$

strictly increases with liability.

Proof. See the Appendix B.

This extends the two main results from the unregulated equilibrium with sequential arrival: (i) the optimality of nonlinear incentives and the resulting bias of advice (Proposition 1) and (ii) the characterization of the optimal nonlinear incentive scheme (Proposition 2).

Relation to Mixed Bundling. To industrial organization researchers, the pattern of biased advice in Figure 4 should look familiar, as it resembles the segmentation of consumer purchases when firms offer products both separately and in a discounted bundle. We refer notably to Armstrong and Vickers (2010) who analyze symmetric duopolistic competition (cf. their Figure 3). With simultaneous arrival, one can construct indeed an isomorphism between our problem and models of competition in so-called mixed bundles; see the online appendix for details. This observation extends the work of Inderst and Ottaviani (2012a) who have shown such an isomorphism to the Hotelling model of single-product competition.

Relation to Antitrust. Mixed bundling and the use of loyalty rebates across different products are practices frequently scrutinized by antitrust agencies. A well known example is the European Commission's case against the bonus incentive system for travel agents used by British Airways to incentive scheme for travel agents to sell British Airways sell tickets.[26] While the literature on mixed bundling mainly models competition for final consumers, many of these cases involve intermediary industries; consider the well-known European case about rebate scheme the tire manufacturer Michelin offered to their dealers. Against the claim of antitrust authorities that such incentives have the objective, or at least the effect, of foreclosing the market to notably smaller rivals or newcomers, economists have pointed to possible efficiencies, linked to improved incentives to intermediaries as in Innes (1990). Interestingly, these cases involve intermediaries, such as travel agencies and garages, from which consumers typically receive advice. Our analysis may thus strengthen the case of authorities against such nonlinear incentives. Even when these nonlinear incentives are provided by firms that are equally well positioned to capture market share, they risk biasing advice and thereby lead to consumer harm.

7 Concluding Remarks

To conclude, we stress two insights from our analysis, which apply both with sequential and simultaneous arrival of customers. First, when firms want to steer advisors' recommendations, they optimally use volume-contingent bonuses, thus inducing biased advice. Because of these non-linear incentives, recommendations become implicitly contingent on other recommendations, resulting in a welfare loss. Second, at least when the agent's liability is not too high, imposing stricter liability does not affect this bias. Product providers fully counteract the agent's higher liability by sufficiently stepping up the bonus. The reason for this stark result is that the pattern of advice and thus the bias stem alone from product providers' trade-off between the base commission and the bonus, which is not affected by the agent's liability. Directly interfering with firms' incentives, by requiring that these are proportional to sales rather than contingent on certain sales targets, leads to unbiased advice, while stepping up liability alone may be ineffective—this is a key conclusion of our analysis.

A first caveat to this conclusion may be that imposing sufficiently high liability would also lead to unbiased advice, e.g. when this fully drives out product providers' compensation. When the agent becomes sufficiently unresponsive to incentives, these no longer arise in equilibrium. While in our setting there is indeed no reason for why imposing such strict liability may be counterproductive, in practice this should be different. For instance, a higher liability may lead to the exit of advisors, increasing the remaining advisors' market

[26] This case was decided in 1999 and the decision was finally upheld in 2007 by the European Court of Justice.

power. Advisors may also face obstacles in generating revenues directly from customers. In fact, there may be reasons why in many markets customers do not directly pay for advice; we leave these extensions to future work.

Imposing linear incentives may also be counterproductive in some circumstances, and policymakers should at least be aware of this possibility. As we show in the online appendix, when products differ in costs, so that efficiency requires asymmetric market shares, prescribing linear incentives that do not allow for a bonus may backfire by inefficiently reducing the market share of the more efficient product. Also, the imposition of linear incentives can reduce welfare when it lowers the advisor's overall (per-customer) compensation and when this inefficiently reduces incentives to acquire customers. This may be harmful particularly for products such as pensions and savings plans for which customers typically exhibit considerable inertia. In this direction, the analysis could be extended by adding effort provision to our model of advice, as in Innes (1990). Stifling effort provision would have a first-order effect because even without regulation effort provision would not be first best, given that the provision of incentives is a public good under common agency. We leave this extension to future research.

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8 Appendix A: Omitted Proofs for the Case With Sequential Arrival

For the subsequent derivations it is helpful to express $\bar{q}_1$ as a function of the subsequently applied thresholds $\bar{q}_2^A$ and $\bar{q}_2^B$. For this we substitute for $Z(\bar{q}_2^A)-Z(\bar{q}_2^B)$ in (4) the expression $b_A - 2w\int_{\bar{q}_2^A}^{\bar{q}_2^B} G(q)dq$, which yields the following:

Lemma 5 *If* $(\bar{q}_2^A, \bar{q}_2^B) \in (0,1)^2$,

$$\bar{q}_1 = \bar{q}_2^A + \int_{\bar{q}_2^A}^{\bar{q}_2^B} G(q)dq. \tag{27}$$

Proof of Proposition 1. We first examine the effects of the marginal increases in f_n and b_n on the advice cutoffs at $\bar{q} = \bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B \in (0,1)$, which holds when $b_n = 0$ for both firms. We have

$$\frac{\partial \bar{q}_2^A}{\partial f_n} = \frac{\partial \bar{q}_2^B}{\partial f_n} = \frac{\partial \bar{q}_1}{\partial f_n} = \begin{cases} -\frac{1}{2w}, & \text{if } n = A, \\ \frac{1}{2w}, & \text{if } n = B, \end{cases} \tag{28}$$

where, focussing on f_A, we have used

$$\frac{\partial \bar{q}_1}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_A} + \left[1 - G(\bar{q}_2^A)\right]\frac{\partial \bar{q}_2^A}{\partial f_A} = \frac{\partial \bar{q}_2^B}{\partial f_A} = \frac{\partial \bar{q}_2^A}{\partial f_A} = -\frac{1}{2w},$$

with $\bar{q}_1$ described by (27) due to Lemma 5 and with $G(\bar{q}_2^A) = G(\bar{q}_2^B)$ at $\bar{q}_2^A = \bar{q}_2^B = \bar{q}$. We show next that

$$\left(\frac{\partial \bar{q}_2^A}{\partial b_n}, \frac{\partial \bar{q}_2^B}{\partial b_n}, \frac{\partial \bar{q}_1}{\partial b_n}\right) = \begin{cases} \left(-\frac{1}{2w}, 0, -\frac{(1-G(\bar{q}))}{2w}\right), & \text{if } n = A, \\ \left(0, \frac{1}{2w}, \frac{G(\bar{q})}{2w}\right), & \text{if } n = B. \end{cases} \tag{29}$$

Focussing again on firm A, we have used there that

$$\frac{\partial \bar{q}_1}{\partial b_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial b_A} + \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial b_A} = \left(1 - G(\bar{q})\right)\frac{\partial \bar{q}_2^A}{\partial f_A},$$

where the last equality follows from $\partial \bar{q}_2^B/\partial b_A = 0$, $\partial \bar{q}_2^A/\partial b_A = -1/(2w)$, and $\bar{q}_2^A = \bar{q}$. Focussing still on firm A, recall that

$$\begin{aligned} \mathrm{S}_A &= \Pr(1) + 2\Pr_A(2) \\ &= G(\bar{q}_1)\left[1 - G(\bar{q}_2^B)\right] + \left[1 - G(\bar{q}_1)\right]G(\bar{q}_2^A) + 2\left[1 - G(\bar{q}_1)\right]\left[1 - G(\bar{q}_2^A)\right]. \end{aligned} \tag{30}$$

Differentiating (30) with respect to $x \in \{f_A, b_A\}$ yields $\mathrm{S}_A^x = \mathrm{Pr}^x(1) + 2\mathrm{Pr}_A^x(2)$, which can be written as

$$\begin{aligned}
\mathrm{S}_A^x &= -\left[G(\bar{q}_2^B) + 1 - G(\bar{q}_2^A)\right] g(\bar{q}_1)\frac{\partial \bar{q}_1}{\partial x} - G(\bar{q}_1)g(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial x} - \left[1 - G(\bar{q}_1)\right] g(\bar{q}_2^A)\frac{\partial \bar{q}_2^A}{\partial x} \\
&= -g(\bar{q})\left(\frac{\partial \bar{q}_1}{\partial x} + G(\bar{q})\frac{\partial \bar{q}_2^B}{\partial x} + \left[1 - G(\bar{q})\right]\frac{\partial \bar{q}_2^A}{\partial x}\right) \\
&= \begin{cases} -2g(\bar{q})\frac{\partial \bar{q}_2^A}{\partial x}, & \text{if } x = f_A, \\ -2\left[1 - G(\bar{q})\right] g(\bar{q})\frac{\partial \bar{q}_2^A}{\partial x}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $\bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B = \bar{q}$ and the second from (28) and (29). We note that this leads to $S_A^b = \left[1 - G(\bar{q})\right] S_A^f$, while we can show symmetrically that $S_B^b = G(\bar{q})\mathrm{S}_B^f$, so that we obtain along the gradient where expected sales remain constant

$$df_n = \begin{cases} -\left[1 - G(\bar{q})\right] db_A, & \text{if } n = A, \\ -G(\bar{q})db_B, & \text{if } n = B. \end{cases} \tag{31}$$

With this at hands, for $n = A$ we can already refer to the main text for the final steps to show that the total derivative of profits (10) then becomes $d\pi_n = \left[1 - G(\bar{q})\right]^2 db_A$, thus concluding the proof for $n = A$. To prove that $b_n = 0$ can not be an equilibrium for both firms, this would be sufficient. For completeness, we briefly also consider the case of $n = B$. There, we now use that $\mathrm{Pr}_B(2) = G(\bar{q})^2$ and, again with $\mathrm{Pr}(1) = 2G(\bar{q})\left[1 - G(\bar{q})\right]$, $\mathrm{S}_B = G(\bar{q})$, so that $d\pi_B = -2G(\bar{q})df_B - G(\bar{q})^2 db_B$, which from (31) yields $d\pi_n = G(\bar{q})^2 db_B$. **Q.E.D.**

Proof of Lemma 2. Under symmetric compensation (f, b), advice cutoffs $\bar{q}_2^A$ and $\bar{q}_2^B$ simplify to $1/2 - b/(2w)$ and $1/2 + b/(2w)$, while $\bar{q}_1 = 1/2$. Focusing first on firm A, note next that $\mathrm{Pr}_A(1) = \left[1 - G(\bar{q}_1)\right] G(\bar{q}_2^A) + G(\bar{q}_1)\left[1 - G(\bar{q}_2^B)\right] = G(\bar{q}_2^A)$ and $\mathrm{Pr}_A(2) = (1/2)\left[1 - G(\bar{q}_2^A)\right] = (1/2)G(\bar{q}_2^B)$. The expressions for B are symmetric. In what follows, we will occasionally suppress the superscript (for firms) to highlight that the respective expressions applies to both firms under symmetry. We now show the following:

Claim: *For any given symmetric compensation $(f_n, b_n) = (f, b)$ with $0 < b < w$, $dS_n = 0$ holds iff*

$$df_n = -\frac{1}{2}db_n. \tag{32}$$

We show first that

$$\frac{\partial \bar{q}_1}{\partial f_n} = 2\frac{\partial \bar{q}_1}{\partial b_n} = 2G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_n} = 2\left[1 - G(\bar{q}_2^A)\right]\frac{\partial \bar{q}_2^A}{\partial f_n}.$$

We know from the proof of Proposition 1 that $(\partial\bar{q}_2^B/\partial x, \partial\bar{q}_2^A/\partial x)$ for $x \in \{f_n, b_n\}$ are given by (28) and (29), independent of $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) \in (0,1)^3$. We focus on firm A and consider a marginal shift of the advice cutoff $\bar{q}_1$. Differentiating $\bar{q}_1$, defined by (27), with respect to f_A and evaluating it at $\bar{q}_2^A = 1 - \bar{q}_2^B$ yields

$$\frac{\partial\bar{q}_1}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial\bar{q}_2^B}{\partial f_A} + \left[1 - G(\bar{q}_2^A)\right]\frac{\partial\bar{q}_2^A}{\partial f_A} = 2G(\bar{q}_2^B)\frac{\partial\bar{q}_2^B}{\partial f_A} = 2\left[1 - G(\bar{q}_2^A)\right]\frac{\partial\bar{q}_2^A}{\partial f_A},$$

where the second and third equalities follow from both $G(\bar{q}_2^A) = 1 - G(\bar{q}_2^B)$ by symmetry of G and $\partial\bar{q}_2^A/\partial f_A = \partial\bar{q}_2^B/\partial f_A$ by (28). Similarly,

$$\frac{\partial\bar{q}_1}{\partial b_A} = G(\bar{q}_2^B)\frac{\partial\bar{q}_2^B}{\partial b_A} + \left[1 - G(\bar{q}_2^A)\right]\frac{\partial\bar{q}_2^A}{\partial b_A} = \left[1 - G(\bar{q}_2^A)\right]\frac{\partial\bar{q}_2^A}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial\bar{q}_2^B}{\partial f_A},$$

where the second equality follows from both $\partial\bar{q}_2^B/\partial b_A = 0$ and $\partial\bar{q}_2^A/\partial b_A = \partial\bar{q}_2^A/\partial f_A$ by (29) and the third from both $G(\bar{q}_2^B) = 1 - G(\bar{q}_2^A)$ and $\partial\bar{q}_2^A/\partial f_A = \partial\bar{q}_2^B/\partial f_A$ by (28). Taken together, at $\bar{q}_2^A = 1 - \bar{q}_2^B$ we have $\partial\bar{q}_1/\partial f_A = 2(\partial\bar{q}_1/\partial b_A) = 2G(\bar{q}_2^B)\left(\partial\bar{q}_2^B/\partial f_A\right) = 2\left[1 - G(\bar{q}_2^A)\right](\partial\bar{q}_2^A/\partial f_A)$, which completes the argument for $n = A$. The same argument applies to $n = B$.

Next, we obtain the derivatives of sales. Considering again first firm A, $\mathrm{S}_A^x = \mathrm{Pr}^x(1) + 2\mathrm{Pr}_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned}
\mathrm{S}_A^x &= -\left[G(\bar{q}_2^B) + 1 - G(\bar{q}_2^A)\right] g(\bar{q}_1)\frac{\partial\bar{q}_1}{\partial x} - G(\bar{q}_1)g(\bar{q}_2^B)\frac{\partial\bar{q}_2^B}{\partial x} - \left[1 - G(\bar{q}_1)\right] g(\bar{q}_2^A)\frac{\partial\bar{q}_2^A}{\partial x} \\
&= -2\left[1 - G(\bar{q}_2^A)\right] g(1/2)\frac{\partial\bar{q}_1}{\partial x} - \frac{1}{2}g(\bar{q}_2^A)\left(\frac{\partial\bar{q}_2^B}{\partial x} + \frac{\partial\bar{q}_2^A}{\partial x}\right) \\
&= \begin{cases} -\left(4\left[1 - G(\bar{q}_2^A)\right]^2 g(1/2) + g(\bar{q}_2^A)\right)\frac{\partial\bar{q}_2^A}{\partial f_A}, & \text{if } x = f_A, \\ -\frac{1}{2}\left(4\left[1 - G(\bar{q}_2^A)\right]^2 g(1/2) + g(\bar{q}_2^A)\right)\frac{\partial\bar{q}_2^A}{\partial f_A}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $\bar{q}_1 = 1/2$, $\bar{q}_2^A = 1 - \bar{q}_2^B$, and symmetry of G around $1/2$ with $g(q) = g(1-q)$ for any given $q \in [0,1]$ and the second from (28) and (29), by which

$$\frac{\partial\bar{q}_2^B}{\partial f_A} = \frac{\partial\bar{q}_2^A}{\partial f_A} = \frac{\partial\bar{q}_2^A}{\partial b_A}, \frac{\partial\bar{q}_2^B}{\partial b_A} = 0, \text{ and } \frac{\partial\bar{q}_1}{\partial f_A} = 2\frac{\partial\bar{q}_1}{\partial b_A} = 2\left[1 - G(\bar{q}_2^A)\right]\frac{\partial\bar{q}_2^A}{\partial f_A}. \tag{33}$$

This leads to $S_A^f = 2S_A^b$. Similarly, we can consider firm B and derive $S_B^f = 2S_B^b$. Taken together, we have thus shown that, using symmetry,

$$\mathrm{S}^f = 2\mathrm{S}^b. \tag{34}$$

The final step is now to consider again marginal adjustments (df_n, db_n) satisfying (9), so that total sales S_n remain unchanged, i.e., with symmetry $\mathrm{S}^f df + \mathrm{S}^b db = 0$. The assertion in (32) follows then immediately from substitution. **Q.E.D. (Claim)**

We now recall the total derivative (13), which we rewrite as[27]

$$d\pi_n = \frac{1}{2}\left[\left(\mathrm{Pr}_n^f(2) - 2\mathrm{Pr}_n^b(2)\right) b_n + \mathrm{Pr}_n(1)\right] db_n.$$

As $d\pi_n = 0$ at an optimum, rearranging yields the requirement

$$\left(2\mathrm{Pr}_n^b(2) - \mathrm{Pr}_n^f(2)\right) b_n = \mathrm{Pr}_n(1). \tag{35}$$

We note that in (35) we apply the subscript n, though it must hold for both firms (and the respective condition is indeed identical as we start from symmetry). What remains is just a substitution of the respective expressions. In the subsequent claim we show that

$$2\mathrm{Pr}_n^b(2) - \mathrm{Pr}_n^f(2) = \left(\frac{1}{2w}\right) g(\bar{q}_2^A)\left[1 - G(\bar{q}_1)\right] = \frac{g(\bar{q}_2^A)}{4w}, \tag{36}$$

where the last step uses, in a symmetric equilibrium, that $1 - G(\bar{q}_1) = 1/2$. Noting that $\mathrm{Pr}_n(1) = G(\bar{q}_2^A)$, we obtain from (35) the characterization of the equilibrium bonus in (36) of Lemma 2.

Proof of the transformation in (36). For this we continue to focus first on firm A. Using (33) at $\bar{q}_1 = 1/2$ and $\bar{q}_2^A = 1 - \bar{q}_2^B$, we can derive $\mathrm{Pr}^x(1)$ and $\mathrm{Pr}_A^x(2) = \mathrm{Pr}^x(2)$ for $x \in \{f_A, b_A\}$ and $s = 1, 2$ as follows:

$$\begin{aligned}
\mathrm{Pr}^x(1) &= \frac{\partial \bar{q}_1}{\partial x} g(\bar{q}_1)\left[1 - G(\bar{q}_2^B) - G(\bar{q}_2^A)\right] - \frac{\partial \bar{q}_2^B}{\partial x} g(\bar{q}_2^B) G(\bar{q}_1) + \frac{\partial \bar{q}_2^A}{\partial x} g(\bar{q}_2^A)\left[1 - G(\bar{q}_1)\right] \\
&= \begin{cases} 0, & \text{if } x = f_A, \\ -\frac{g(\bar{q}_2^A)}{4w}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from (i) $1 - G(\bar{q}_2^B) = G(\bar{q}_2^A)$ and $g(\bar{q}_2^B) = g(\bar{q}_2^A)$ as $\bar{q}_2^A = 1 - \bar{q}_2^B$ and G is symmetric around $1/2$ and (ii) $\partial \bar{q}_2^A / \partial f_A = \partial \bar{q}_2^B / \partial f_A = \partial \bar{q}_2^A / \partial b_A$ and $\partial \bar{q}_2^B / \partial b_A = 0$ by (28) and (29) with $G(\bar{q}_1) = G(1/2) = 1/2$;

$$\begin{aligned}
\mathrm{Pr}_A^x(2) &= -\frac{\partial \bar{q}_1}{\partial x} g(\bar{q}_1)\left[1 - G(\bar{q}_2^A)\right] - \frac{\partial \bar{q}_2^A}{\partial x} g(\bar{q}_2^A)\left[1 - G(\bar{q}_1)\right] \\
&= \begin{cases} \frac{1}{4w}\left(4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)\right), & \text{if } x = f_A, \\ \frac{1}{4w}\left(2g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

[27] This expression has a simple intuition that derives again from the considered gradient. As total sales are constant and thereby also total paid commissions, what remains is thus the impact on the size of the bonus payment (as b_n changes and f_n must adjust accordingly). This is captured in the rectangular brackets. Note here that using that $df_n = -\frac{1}{2}db_n$, $2\mathrm{Pr}_n^b(2) - \mathrm{Pr}_n^f(2)$ is the change in the likelihood with which the bonus is paid.

where the second equality follows from (i) $\partial \bar{q}_2^A/\partial f_A = \partial \bar{q}_2^A/\partial b_A = -1/(2w)$ and $\partial \bar{q}_1/\partial f_A = 2(\partial \bar{q}_1/\partial b_A) = 2\left[1 - G(\bar{q}_2^A)\right](\partial \bar{q}_2^A/\partial f_A)$ by (28) and (29) and (ii) $G(\bar{q}_1) = G(1/2) = 1/2$.

Similarly, we can derive the derivatives for firm B with respect to f_B and b_B, and then show that $\Pr_A^x(s) = \Pr_B^x(s) = \Pr^x(s)$ holds for $x \in \{f, b\}$ and $s = 1, 2$. Thus, we have

$$\Pr^f(1) = 0 \text{ and } \Pr^b(1) = -\frac{g(\bar{q}_2^A)}{4w} = -\frac{g(\bar{q}_2^B)}{4w},$$

so that

$$\begin{aligned}
\Pr^f(1) + 2\Pr^f(2) &= 2(\Pr^b(1) + 2\Pr^b(2)) \\
&= -\frac{1}{2w}\left(4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)\right) \\
&= -\frac{1}{2w}\left(4g(1/2)G(\bar{q}_2^B)^2 + g(\bar{q}_2^B)\right),
\end{aligned}$$

by which we can obtain $\Pr^f(2) - 2\Pr^b(2) = \Pr^b(1)$, leading to (36). **Q.E.D.**

Proof of Lemma 3. We show the uniqueness of the advice cutoff $\bar{q}_2^A = 1/2 - b/(2w) \in (0, 1/2)$ determined by (15). The left-hand-side of the equation (15) is a bijective (or one-to-one) function of $\bar{q}_2^A \in (0, 1/2)$ and converges to $1/2$ in the limit as $\bar{q}_2^A$ approaches $1/2$ from below, while the right-hand-side of (15) is decreasing in $\bar{q}_2^A$ due to the hazard rate condition and converges to $1/2$ in the limit as $\bar{q}_2^A$ goes to zero from above. Taken together, there must be a fixed point $\bar{q}_2^A \in (0, 1/2)$ such that the left-hand- and right-hand sides intersect only once, thus equation (15) holds for a unique value $\bar{q}_2^A \in (0, 1/2)$. **Q.E.D.**

Proof of Lemma 4. We derive the optimal commission given by (16). Recall that, as shown in the proof of Lemma 2, $\Pr^f(1) = 0$ and

$$\Pr^f(2) = \frac{1}{4w}\left(4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)\right). \tag{37}$$

With $\Pr(1) + 2\Pr(2) = 1$, the first-order condition with respect to f_A, evaluated at a symmetric equilibrium, can now be transformed stepwise as follows:

$$\begin{aligned}
f &= p - c - \frac{1}{2}\left(b + \frac{1}{\Pr^f(2)}\right) \\
&= p - c - 2w\left(\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)} + \frac{1}{4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)}\right) \\
&= p - c - 2wH(\bar{q}_2^A).
\end{aligned}$$

Q.E.D.

Proof of Proposition 3. We can derive $\Pr^b(1) = -g(\bar{q}_2^A)/(4w)$ and

$$\Pr^b(2) = (1/(4w))(2g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)),$$

so that

$$\mathrm{S}^b = \Pr^b(1) + 2\Pr^b(2) = (1/(4w))(4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)).$$

Here, $\Pr(2) = (1/2)\left[1 - G(\bar{q}_2^A)\right]$. With this we can then substitute to obtain

$$b = \frac{\left(4g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)\right)(p-c) - 2w\left[1 - G(\bar{q}_2^A)\right]}{2g(1/2)\left[1 - G(\bar{q}_2^A)\right]^2 + g(\bar{q}_2^A)}, \tag{38}$$

where $\bar{q}_2^A = 1/2 - b/(2w) = 1 - \bar{q}_2^B \in (0, 1/2]$. Suppose now that $w \in (w_f^*, w_b^*)$ and that the optimal bonus satisfies (38). Firm A's marginal profit with respect to f_A, evaluated at $f_A = 0$, is written as

$$\begin{aligned}
& \mathrm{S}^f(p-c) - \Pr^f(2)b - \Pr(1) - 2\Pr(2) \\
= \;& 2\left(\mathrm{S}^b(p-c) - \Pr^b(2)b\right) - \Pr^b(1)b - \Pr(1) - 2\Pr(2) \\
= \;& 2\Pr(2) - \Pr^b(1)b - \Pr(1) - 2\Pr(2) = -\Pr^b(1)b - \Pr(1) < 0,
\end{aligned}$$

where the first equality follows from $\mathrm{S}^f = \Pr^f(1) + 2\Pr^f(2) = 2\mathrm{S}^b$ with $\Pr^f(1) = 0$, and the second from the first-order-condition with respect to b_A. Similarly, if $w \geq w_b^*$, we can show that the marginal profit with respect to f_A, evaluated at $(f_n, b_n) = (0, 0)$ for both $n = A, B$, is negative as

$$\begin{aligned}
& \mathrm{S}^f(p-c) - \Pr(1) - 2\Pr(2) \\
= \;& 2\mathrm{S}^b(p-c) - \Pr(1) - 2\Pr(2) = 2\left(\mathrm{S}^b(p-c) - \Pr(2)\right) - \Pr(1) \\
< \;& -\Pr(1) < 0,
\end{aligned}$$

where the first inequality follows from the fact that the marginal profit with respect to b_A is negative too. At both cutoffs w_f^* and w_b^* the assertion follows as we assume strict quasiconcavity of firm profits in the two instruments. **Q.E.D.**

Proof of Proposition 4. Take first the case with $w < w_f^*$. If there exists a symmetric equilibrium with $f > 0$ and $b > 0$, we know from Lemma 4 that the equilibrium must be unique: Advice cutoffs are uniquely determined and do not depend on p, the price is in turn uniquely determined by these cutoffs, and finally the level of commissions are determined by p. By construction of w_f^*, there is also no equilibrium where $f = 0$; cf. the proof of Lemma 3. From Lemma 3 follows also the unique characterization when $w \geq w_b^*$ and that in the intermediate range, we must have $f = 0$ and $b > 0$. As noted in the main text, what complicates the characterization in this case is that as b changes with w, so do the advice cutoffs and thus the price (that is, customers' conditional valuation). Still, that the bias must strictly decrease as w increases, follows from the following argument to a contradiction. For this we first rearrange the respective first-order condition for b to obtain, at a symmetric equilibrium,

$$\left(\Pr^b(1) + \Pr^b(2)\right)(p - c) + \Pr^b(2)(p - c - b) - \Pr(2) = 0. \tag{39}$$

We now evaluate this at a given (second-customer) cutoff $\bar{q}_2^A$, which, independently of w, also fixes $\bar{q}_2^B$ and p, as well as obviously $\Pr(2)$. Now suppose that (39) holds for given w and corresponding equilibrium b. We now consider a strictly lower w'. Evaluating this at the same $\bar{q}_2^A$, note that, first, this requires $b' < b$ and that, second, both $\Pr^b(1)+\Pr^b(2)$ and $\Pr^b(2)$ increase.[28] Invoking strict quasiconcavity of the profit function, this implies that at w', (39) is strictly positive when evaluated at b' so that the bias would remain unchanged. The claim follows then by invoking again strict quasiconcavity of the profit function (together with the first-order condition). **Q.E.D.**

Proof of Proposition 5. Consider a normal distribution with mean μ and variance $\sigma > 0$. Define the standard normal distribution and its density function by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$

and

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2},$$

respectively. With these functions, we write a truncated normal distribution function G with support $[0, 1]$ by

$$G(q) = \frac{\Phi(\frac{q-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}{\Phi(\frac{1-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}$$

and its density function g by

$$g(q) = \frac{\phi(\frac{q-\mu}{\sigma})}{\sigma\left(\Phi(\frac{1-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})\right)},$$

[28] Note that $\Pr(1)+\Pr(2)$ is the likelihood that a given firm sells at least one product. The respective derivative is evaluated at a symmetric choice $b_n = b$, but it holds constant the competitor's choice b (as otherwise it would remain unchanged).

where the mean of $G(q)$ is unchanged at $\mu = 1/2$ and and the variance is given by

$$\sigma^2\left[1-\frac{\mu\phi(\frac{-\mu}{\sigma})+(1-\mu)\phi(\frac{1-\mu}{\sigma})}{\sigma\left(\Phi(\frac{1-\mu}{\sigma})-\Phi(\frac{-\mu}{\sigma})\right)}-\left(\frac{\phi(\frac{-\mu}{\sigma})-\phi(\frac{1-\mu}{\sigma})}{\Phi(\frac{1-\mu}{\sigma})-\Phi(\frac{-\mu}{\sigma})}\right)^2\right].$$

We first show that the inverse of the reverse hazard rate $G(q)/g(q)$ is increasing in σ. Transforming the respective expression (using also a change of variable under integration from x to $y = -\sigma x$), we have

$$\frac{G(q)}{g(q)} = \int_{\frac{1}{2}-q}^{\frac{1}{2}} h(y \mid \sigma)dy \text{ with } h(y \mid \sigma) \equiv e^{-\frac{1}{2\sigma^2}\left(y-\frac{1}{2}+q\right)\left(y+\frac{1}{2}-q\right)}.$$

The assertion follows as h is increasing in σ for any given $y \in (\frac{1}{2}-q, 1/2]$ if $q \in (0,1/2)$ as $(y-1/2+q)(y+1/2-q) > 0$. Note also that $h(y \mid \sigma)$ converges to zero in the limit as σ goes to zero from above.

Using the fact that $G(q)/g(q)$ increases with the size of σ, we now show that in equilibrium as σ increases, the advice cutoff $\bar{q}_2^A = 1-\bar{q}_2^B$ decreases while the optimal bonus increases. Since $\bar{q}_2^A \in (0,1/2)$ is determined in equilibrium by equation (15), it decreases as $G(\bar{q}_2^A)/g(\bar{q}_2^A)$ increases proportional to σ. Since the optimal bonus can be written as $b/(2w) = 1/2 - \bar{q}_2^A$ with $(f_n, b_n) = (f, b)$ and thus decreases with $\bar{q}_2^A$, b is increasing in σ. **Q.E.D.**

Proof of Proposition 6. It remains to show uniqueness of the regulated equilibrium and its existence. Consider first the case where w is below the threshold of w^*. Note that $w_b^* = 2w^*$, where $p = \mathbb{E}[v_n \mid 1/2]$, and w^* is below w_b^*. In equilibrium, customers must hold rational beliefs with $\hat{f}_n = f^R$. Together with the pattern of advice defined by (21), this determines the equilibrium advice cutoffs $\hat{\mathbf{q}} = \mathbf{q} = (\bar{q}, \bar{q}, \bar{q})$ with $\bar{q} = 1/2$, irrespective of the size of w, as well as the price $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}] = \mathbb{E}[v_n \mid \mathbf{q}]$, which is common for both $n = A, B$ due to $\bar{q} = 1/2$ and symmetry of $v_A(q) = v_B(1-q)$ for any given $q \in [0,1]$, and which we denote by p. Firms set their optimal commissions $f_n = f^R$ by (22). These tuple of $(f^R, \mathbf{q}, p)$ with customers' rational beliefs of $\hat{f}_n = f^R$ and $\hat{\mathbf{q}} = \mathbf{q}$ constitutes a unique regulated equilibrium as long as $w < w^*$, otherwise $(0, \mathbf{q}, p)$ would be a unique equilibrium as a corner solution.

The existence of the equilibrium for any given $w > 0$ follows as the equilibrium price evaluated at $\mathbf{q} = (1/2, 1/2, 1/2)$ exceeds marginal cost c and therefore selling the product is profitable for the firm with equilibrium price $p = \mathbb{E}[v_n \mid 1/2]$:

$$\mathbb{E}[v_n \mid 1/2] = \int_{1/2}^{1} v_A(q)\frac{g(q)}{1-G(1/2)}dq = \int_0^{1/2} v_B(q)\frac{g(q)}{G(1/2)}dq > c.$$

Q.E.D.

9 Appendix B: Derivations for the Case With Simultaneous Arrival

In this part of the appendix we show how the results obtained with sequential arrival all extend (qualitatively) to the case with simultaneous arrival. This also proves Proposition 7.

We first complete the characterization of advice. When recommending different products through message $(m_1, m_2) = (A, B)$, the advisor's expected payoff equals

$$f_A + f_B + q_1 w + (1 - q_2) w + 2w_l,$$

and when recommending the same product to both customers through message $(m_1, m_2) = (A, A)$, the payoff equals

$$2f_A + b_A + q_1 w + q_2 w + 2w_l.$$

Comparing these two payoffs yields the threshold

$$q^* = \frac{1}{2} - \frac{f_A - f_B + b_A}{2w},$$

such that the advisor prefers (A, A) if $q_2 \geq q^*$ and (A, B) otherwise. Similarly, by considering the two payoffs

$$f_A + f_B + (1 - q_1) w + q_2 w + 2w_l$$

from sending $(m_1, m_2) = (B, A)$ and

$$2f_B + b_B + (1 - q_1) w + (1 - q_2) w + 2w_l$$

from $(m_1, m_2) = (B, B)$, we have

$$q^{**} = \frac{1}{2} - \frac{f_A - f_B - b_B}{2w},$$

such that the advisor prefers (B, A) if $q_2 \geq q^{**}$ and (B, B) otherwise. The advisor prefers (A, A) over (B, A) if $q_1 \geq q^*$ and (B, A) otherwise, and (A, B) over (B, B) if $q_1 \geq q^{**}$ and (B, B) otherwise. Considering finally the payoffs from sending $(m_1, m_2) = (A, A)$ and $(m_1, m_2) = (B, B)$, we have the threshold

$$\overline{q}_2(q_1) = 1 - q_1 - \frac{2(f_A - f_B) + b_A - b_B}{2w},$$

such that the advisor prefers message (A, A) if $q_2 \geq \overline{q}_2(q_1)$ and (B, B) otherwise. The thereby obtained thresholds $(q^*, q^{**}, \overline{q}_2(q_1))$ fully characterize the advisor's optimal recommendation for any incentive scheme (f_n, b_n).[29]

Lemma 6 *When customers follow his recommendation, the advisor's optimal recommendation with simultaneous arrivals is characterized as follows:*

$$(m_1, m_2) = \begin{cases} (A, A) & \text{if } (\forall i = 1, 2)\ q_i \in [q^*, 1] \text{ and } q_2 \geq \overline{q}_2(q_1), \\ (B, B) & \text{if } (\forall i = 1, 2)\ q_i \in [0, q^{**}] \text{ and } q_2 \leq \overline{q}_2(q_1), \\ (A, B) & \text{if } q_1 \in [q^{**}, 1],\ q_2 \in [0, q^*], \text{ and } q_1 \geq q_2, \\ (B, A) & \text{if } q_1 \in [0, q^*],\ q_2 \in [q^{**}, 1], \text{ and } q_1 \leq q_2. \end{cases}$$

Next, to describe firm profits, now $\Pr(1) = 2G(q^*)\,[1 - G(q^{**})]$ denotes the probability that the advisor makes recommendations for different products, which is common to both firms. Similarly, define by

$$\Pr_A(2) = [1 - G(q^*)]\,[1 - G(q^{**})] + \int_{q^*}^{q^{**}} [1 - G(\overline{q}_2(q_1))]\, g(q_1) dq_1$$

the probability that the advisor recommends product A to both customers, and by

$$\Pr_B(2) = G(q^*)G(q^{**}) + \int_{q^*}^{q^{**}} G(\overline{q}_2(q_1)) g(q_1) dq_1$$

the probability that the advisor recommends product B to both customers. For given compensation (f_n, b_n) and product price p_n, for $n = A, B$, expected profits are written in the same way as in the case of sequential advice, replacing probabilities $\Pr(1)$ and $\Pr_n(2)$ with the ones defined above. That is, $\pi_n = \mathrm{S}_n(p_n - c_n - f_n) - \Pr_n(2) b_n$.

We will provide a series of lemmas to extend both the optimality of nonlinear incentives and the characterization of the optimal nonlinear incentive scheme to the case with simultaneous advice. Suppose that firms set $b_A = b_B = 0$. In this case, advice cutoffs (q^*, q^{**}) should be equal, which we denote by $\bar{q} \in (0, 1)$.[30] Also, advice cutoff $\bar{q}_2(q_1)$ reduces to

$$\bar{q}_2(q_1) = 1 - q_1 - \frac{f_A - f_B}{w}.$$

Consider now firm n's marginal profits with respect to the commission and the bonus $x_n \in \{f_n, b_n\}$. We focus on firm A and first examine the effects of the marginal increases in f_A and b_A on the advice cutoffs $(q^*, q^{**}, \bar{q}_2(q_1)) \in (0, 1)^2$.

[29] To deal with corner solutions, define for ease of exposition the following: $q^* = 0$ if $w \leq f_A - f_B + b_A$ and $q^* = 1$ if $w \leq -(f_A - f_B + b_A)$; $q^{**} = 0$ if $w \leq f_A - f_B - b_B$ and $q^{**} = 1$ if $w \leq -(f_A - f_B - b_B)$; and $\overline{q}_2(q_1) = 0$ if $2w(1 - q_1) \leq 2(f_A - f_B) + b_A - b_B$ and $\overline{q}_2(q_1) = 1$ if $2wq_1 \leq -(2(f_A - f_B) + b_A - b_B)$.

[30] The advice cutoff $\bar{q}$ should lie in the open interval $(0, 1)$, as otherwise the advisor would always recommend a particular firm's product to customers, which contradicts assumption (1).

Lemma 7 *For any given* $(q^*, q^{**}, \bar{q}_2(q_1)) \in (0,1)^2$,

$$\frac{\partial q^*}{\partial f_n} = \frac{\partial q^{**}}{\partial f_n} = \frac{1}{2}\frac{\partial \bar{q}_2(q_1)}{\partial f_n} = \begin{cases} -\frac{1}{2w}, & \textit{if } n = A, \\ \frac{1}{2w}, & \textit{if } n = B, \end{cases} \tag{40}$$

and

$$\left(\frac{\partial q^*}{\partial b_n}, \frac{\partial q^{**}}{\partial b_n}, \frac{\partial \bar{q}_2(q_1)}{\partial b_n}\right) = \begin{cases} \left(-\frac{1}{2w}, 0, -\frac{1}{2w}\right), & \textit{if } n = A, \\ \left(0, \frac{1}{2w}, \frac{1}{2w}\right), & \textit{if } n = B. \end{cases} \tag{41}$$

With (40) and (41), consider now a marginal increase in sales, $\mathrm{S}_n^x = \Pr^x(1) + 2\Pr_n^x(2)$, with $x \in \{f_n, b_n\}$.

Lemma 8 *At* $\bar{q} = q^* = q^{**} \in (0,1)$*, we have*

$$S_n^b = \begin{cases} (1 - G(\bar{q}))S_A^f, & \textit{if } n = A, \\ G(\bar{q})S_B^f, & \textit{if } n = B. \end{cases} \tag{42}$$

Proof. We derive (42) using (40) and (41) in the case of $b_n = b = 0$. For now we restrict attention to firm A. By (40) and (41), $\mathrm{S}_A^x = \Pr^x(1) + 2\Pr_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned} \mathrm{S}_A^x &= 2\left[\begin{array}{c} -\left[1 - G(\bar{q}_2(q^*))\right] g(q^*)\frac{\partial q^*}{\partial x} - G(\bar{q}_2(q^{**}))g(q^{**})\frac{\partial q^{**}}{\partial x} \\ -\int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \end{array}\right] \\ &= -2g(\bar{q})\left(\left[1 - G(\bar{q}_2(\bar{q}))\right]\frac{\partial q^*}{\partial x} + G(\bar{q}_2(\bar{q}))\frac{\partial q^{**}}{\partial x}\right) \\ &= \begin{cases} -2g(\bar{q})\frac{\partial q^*}{\partial f_A}, & \text{if } x = f_A, \\ -2\left[1 - G(\bar{q})\right] g(\bar{q})\frac{\partial q^*}{\partial f_A}, & \text{if } x = b_A, \end{cases} \end{aligned}$$

where the first equality follows from $\bar{q} = q^* = q^{**}$ and the second from (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A$ due to (40), (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$ and $\partial q^{**}/\partial b_A = 0$ due to (41), and (iii) $\bar{q}_2(\bar{q}) = \bar{q}$. Thus, we obtain equation (42) in case of $n = A$. The same argument applies to $n = B$, leading to the remaining part of (42). **Q.E.D.**

Consider now marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ such that total sales remain unchanged, as in (9). Applying (42) to (9), we have:

Lemma 9 *For* $n = A, B$*, consider marginal adjustments* $(df_n, db_n) \in \mathbb{R}^2$ *as defined by (9). If* $b_n = b = 0$*,* (df_n, db_n) *must satisfy equation (31).*

We next examine the total derivative of π_n with respect to the marginal adjustments (df_n, db_n) such that total sales remain unchanged as (31) holds. Taking $b_n = b = 0$ as given, the total derivative can be written as (10), which gives rise to:

Proposition 8 *Nonlinear incentives are part of any equilibrium, i.e., there is no equilibrium in which $b_n = b = 0$.*

Suppose that firms set their compensation $(f_n, b_n) = (f, b)$ with $0 < b < w$. Under symmetric compensation (f, b), the advice cutoffs (q^*, q^{**}) satisfy $q^* = 1 - q^{**} \in (0, 1/2)$, and so $\Pr_A(2) = \Pr_B(2) = \Pr(2)$. Furthermore, using

$$\int_{q^*}^{q^{**}} G(q_1)g(q_1)dq_1 = \frac{G(q^{**})^2 - G(q^*)^2}{2},$$

we can rewrite

$$\Pr(2) = \frac{1 - 2G(q^*)^2}{2}.$$

Lemma 10 *At $q^* = 1 - q^{**} \in (0, 1/2)$, equation (34) holds true.*

Proof. We derive (34) using the derivatives of (q^*, q^{**}) given by (40) and (41) when $(f_n, b_n) = (f, b)$. For now we restrict attention to firm A. With $q^* = 1 - q^{**}$, $\mathrm{S}_A^x = \Pr^x(1) + 2\Pr_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned}
& 2\left[\begin{array}{c} -\left[1 - G(\bar{q}_2(q^*))\right] g(q^*)\frac{\partial q^*}{\partial x} - G(\bar{q}_2(q^{**}))g(q^{**})\frac{\partial q^{**}}{\partial x} \\ -\int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \end{array}\right] \\
= \ & -2\left(g(q^*)G(q^*)\left(\frac{\partial q^*}{\partial x} + \frac{\partial q^{**}}{\partial x}\right) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1 \frac{\partial \bar{q}_2(q_1)}{\partial x}\right) \\
= \ & \begin{cases} -4\frac{\partial q^*}{\partial f_A}\left(g(q^*)G(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right), & \text{if } x = f_A, \\ -2\frac{\partial q^*}{\partial f_A}\left(g(q^*)G(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $q^* = 1 - q^{**}$, $\bar{q}_2(q_1) = 1 - q_1$, and symmetry of G around 1/2 with $g(q) = g(1-q)$ for any given $q \in [0, 1]$ and the second from both (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = (1/2)(\partial \bar{q}_2(q_1)/\partial f_A)$ and (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$, $\partial q^{**}/\partial b_A = 0$, and $\partial \bar{q}_2(q_1)/\partial b_A = \partial q^*/\partial f_A$. This leads to $S_A^f = 2S_A^b$. By symmetry, $S_B^f = 2S_B^b$ holds true. Thus, we have derived (34). **Q.E.D.**

Consider now marginal adjustments (df_n, db_n) defined by (9) such that total sales remain unchanged. As we have symmetric incentive schemes (f, b) with $0 < b < w$, we omit subscript $n = A, B$. Again we have $df = -(1/2)db$. We use next that $\Pr(1) = 2G(q^*)\left[1 - G(q^{**})\right] = 2G(q^*)^2$, $\Pr(2) = (1/2)(1 - \Pr(1))$, and $\mathrm{S} = \Pr(1) + 2\Pr(2) = 1$ by symmetry, and

$$\Pr^f(2) - 2\Pr^b(2) = -G(q^*)\frac{g(q^*)}{w}.$$

Applying this, next to $df = -(1/2)db$, to the total derivative of π, we finally have that

$$d\pi = \frac{G(q^*)}{2}\left(-\frac{g(q^*)}{w}b + 2G(q^*)\right)db.$$

Lemma 11 *For any given $q^* \in (0, 1/2)$, the optimal bonus is uniquely determined by*

$$b \;=\; 2\frac{G(q^*)}{g(q^*)}w. \tag{43}$$

Proof. We focus on firm A. Using (40) and (41) with $q^* = 1 - q^{**}$, we derive $\Pr^x(1)$ and $\Pr_n^x(2) = \Pr^x(2)$ for $x \in \{f_A, b_A\}$ as follows:

$$\begin{aligned}
\Pr^x(1) &= 2\left(g(q^*)\left[1 - G(q^{**})\right]\frac{\partial q^*}{\partial x} - g(q^{**})G(q^*)\frac{\partial q^{**}}{\partial x}\right) \\
&= \begin{cases} 2g(q^*)G(q^*)\left(\frac{\partial q^*}{\partial f_A} - \frac{\partial q^{**}}{\partial f_A}\right) = 0, & \text{if } x = f_A, \\ 2g(q^*)G(q^*)\frac{\partial q^*}{\partial b_A} = -G(q^*)\frac{g(q^*)}{w}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from (i) $1 - G(q^{**}) = G(q^*)$, (ii) $g(q^*) = g(q^{**})$, and (iii) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = \partial q^*/\partial b_A = -1/(2w)$ and $\partial q^{**}/\partial b_A = 0$ due to (40) and (41);

$$\begin{aligned}
\Pr_A^x(2) &= -\left[1 - G(q^{**}) + 1 - G(\bar{q}_2(q^*))\right]g(q^*)\frac{\partial q^*}{\partial x} \\
&\quad - \left[1 - G(q^{**}) + 1 - G(\bar{q}_2(q^{**})\right]g(q^{**})\frac{\partial q^{**}}{\partial x} \\
&\quad - \int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \\
&= -\left(2G(q^*)g(q^*)\frac{\partial q^*}{\partial x} + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x}\right) \\
&= \begin{cases} -2\frac{\partial q^*}{\partial f_A}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right), & \text{if } x = f_A, \\ -\frac{\partial q^*}{\partial f_A}\left(2G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from $q^* = 1 - q^{**}$, $\bar{q}_2(q_1) = 1 - q_1$, and symmetry of G with $g(q) = g(1-q)$ for any given $q \in [0, 1]$ and the third from both (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = (1/2)(\partial \bar{q}_2(q_1)/\partial f_A)$ due to (40) and (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$, $\partial q^{**}/\partial b_A = 0$, and $\partial \bar{q}_2(q_1)/\partial b_A = \partial q^*/\partial f_A$ due to (41).

Similarly, we can derive the same derivatives for firm B with respect to f_B and b_B, and then $\Pr_A^x(s) = \Pr_B^x(s) = \Pr^x(s)$ for any given $x \in \{f, g\}$ and $s = 1, 2$. Thus, we have

$$\Pr^f(1) = 0 > \Pr^b(1) = -G(q^*)\frac{g(q^*)}{w}$$

and

$$\Pr^f(1) + 2\Pr^f(2) = 2(\Pr^b(1) + 2\Pr^b(2)) = \frac{2}{w}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right),$$

by which $\Pr^f(2) - 2\Pr^b(2) = \Pr^b(1)$, leading to (43). **Q.E.D.**

Combining condition (43) with $q^* = 1/2 - b/(2w)$ under symmetric compensation (f, b), as in the proof of Lemma 3, we can pin down q^* as follows:

Lemma 12 *The advice cutoff $q^* = 1 - q^{**} \in (0, 1/2)$ is uniquely determined by*

$$q^* = \frac{1}{2} - \frac{G(q^*)}{g(q^*)}.$$

Since $\Pr^f(1) = 0$ as shown in the proof of Lemma 11 and $\Pr(1) + 2\Pr(2) = 1$ at a symmetric equilibrium, we use also that b is given by (43) and that

$$\Pr^f(2) = \frac{1}{w}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1\right).$$

Define $H(q^*)$ by (25). Applying both (43) and $\Pr^f(2)$, together with $H(q^*)$, determines the optimal commission.

Lemma 13 *At a symmetric equilibrium with $q^* = 1 - q^{**} \in (0, 1/2)$ determined by $q^* = 1/2 - G(q^*)/g(q^*)$, the optimal commission is*

$$f = p - c - wH(q^*), \tag{44}$$

where $H(q^*)$ is defined by (25).

Proof. We derive the optimal commission given by (16). From the proof of Lemma 2, we know that $\Pr^f(2) = (1/w)(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1)$. Applying (43) and $\Pr^f(2)$, together with $H(q^*)$ defined by (25), yields

$$\begin{aligned} f &= p - c - \frac{1}{2}\left(2w\frac{G(q^*)}{g(q^*)} + \frac{w}{G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1}\right) \\ &= p - c - w\left(\frac{G(q^*)}{g(q^*)} + \frac{1}{2(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} g(q_1)^2 dq_1)}\right) \\ &= p - c - wH(q^*), \end{aligned}$$

which leads to (44). **Q.E.D.**

One can easily observe that the optimal commission (44) is positive as long as w is below a certain level, otherwise equals zero. Denote by w_f^* the corresponding threshold given by

$$w_f^* = \frac{p - c}{H(q^*)} \tag{45}$$

where q^* is uniquely determined by (26). Finally, we characterize the optimal nonlinear incentive scheme in an interior solution:

Proposition 9 *Suppose that prices are symmetric and the advisor's concern level w is below the threshold w_f^* defined by (45). Then, there are unique advice cutoffs (q^*, q^{**}) that are independent of w, where $q^* = 1 - q^{**}$ is determined by (26) and (f, b) solve (44) and (43), respectively. The commission f decreases with the size of w, while the bonus b increases.*