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TAXATION AND THE OPTIMIZATION OF OIL EXPLORATION AND PRODUCTION: THE UK CONTINENTAL SHELF*

By CARLO A. FAVERO

1. Introduction

In a recent paper Pesaran (1990) has developed an econometric model for the analysis of the exploration and extraction policies of 'price taking' suppliers of oil and has applied it to the UK continental shelf (UKCS). The model takes explicit account of the process of oil discovery and of the intertemporal nature of the exploration and production decisions. Estimation of the model over the period 1978(1)–1986(4) produces an important trade-off between statistical fit and the plausibility of the estimates. The use of rational expectations delivers statistically significant results with estimates of the structural parameters that have the theoretically expected signs, but average marginal extraction costs over the sample take an implausibly high value of over $100 and the 'shadow price' of oil in the ground is not always positive. Sensitivity analysis reveals that one important reason for the implausibly high average estimate of the marginal extraction cost is the low estimate obtained for the intertemporal discount rate: the most plausible estimates for the marginal extraction costs are obtained by setting the discount rate to infinity, ie. by assuming that the future is irrelevant to the exploration and production decisions of the firm. The aim of this paper is to evaluate the sensitivity of this result to the inclusion of taxation in an intertemporal model of exploration and production of North Sea oil. The necessary condition to omit taxation from an economic and econometric model is that the tax system is *neutral.* A system can be considered neutral if it does not affect the decision of economic agents. A non-neutral tax system has to be explicitly modelled because, by definition, it affects firms' decisions with respect to exploration, development and production activities. When a non-neutral tax system is omitted, its *instability* might cause the break-down of the econometric model every time the tax system is modified.[1] The first section of the paper is devoted to a description of the North Sea oil tax-system over the period 1978–86 in order to assess its neutrality and stability. In the second section tax-dependent supply and exploration equations are

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[1] Instability certainly causes break down of a backward looking econometric model in which taxation is omitted and the true structure is forward looking (Lucas (1976)), but it does not affect the stability of the coefficients in a forward looking model, when the changes in the tax-system are not predictable by the optimizing agents.

derived from an intertemporal optimization framework. The third section is devoted to the estimation of a two-equation econometric model.

2. The UKCS oil fiscal regime

The United Kingdom Continental Shelf (UKCS) oil fiscal regime was introduced by the Oil Taxation Act in 1975 and operates essentially in three stages (Mabro *et al.* (1986)):

1. The first stage is the payment of a royalty based on the field gross revenues. The royalty can be paid in cash or in kind and it has been fixed at 12.5% of the revenue over the 1978–1986 period.[2]
2. The second stage is the Petroleum Revenue Tax (PRT).

The PRT is assessed on a field by field basis: around each field a notional 'ring fence' prevents external influences affecting the PRT bill paid. In practice a company has as many PRT assessments as it has shares in different fields and company losses in one field cannot be set against profit in other fields.

PRT is assessed on total revenues from a field, royalties and allowable expenditure plus an uplift are deductible. In addition there are an oil allowance and a safeguard. Allowable expenditure includes all the expenditure incurred in discovering and developing a field. Costs incurred in exploration and development are increased by the amount of an uplift which was originally set at 75% and was later reduced to 35% in 1979; from the start of 1981 a cut-off date is calculated as being the end of the period in which the field reaches payback (ie. when the cumulative revenues of a field exceed the cumulative outgoings). Before 1979 the oil allowance was a deduction from profits of 500,000 tonnes of oil for each six-month period up to a cumulative maximum of 10 million tonnes, from 1979 onwards 250,000 tonnes were allowed for each period with a maximum cumulative allowance of five million tonnes.

The safeguard is a limitation on PRT that is restricted to be no more than the 80% of the amount by which the adjusted profits (assessable profits plus oil allowances and capital allowances) exceed 15% of the accumulated capital expenditure at the end of the period. The PRT rate has varied considerably over the sample period, in ranges from 45–75% (Fig. 1).

With the Finance Act of 1983 important changes in PRT have been introduced, which apply only to new oil fields. The rationale behind the changes to PRT has been to keep intact the revenues from existing fields, simultaneously offering incentives to the development of new fields. Three major changes were introduced in the taxation of new fields: the oil allowance was doubled, an exemption from the payment of royalty was fully granted, and the ring fence was partially lifted for exploration and appraisal costs incurred after March 1983. Subsequently, in March 1987 the Cross Field Allowance for new fields extended the facilitations

[2] Some modifications in the payment of the royalty occurred from 1983 onward but they affected only new fields.

FIG. 1. PRT = petroleum revenue tax.

to development and the development costs of a new oil field were allowed to be offset against the PRT liability on other existing fields.

In addition to PRT, a Supplementary Petroleum Duty (SPD) was also introduced in the 1981 budget. The new tax was charged at a rate of 20% of gross production revenue from each field less an allowance of one million tonnes per annum. It was calculated on a field by field basis and it was deductible in calculating profits liable to both PRT and Corporation Taxes.

At the end of 1982 SPD was succeeded by the Advance Petroleum Revenue Tax (APRT) which is essentially a method of collecting PRT liability in advance. APRT was originally calculated the same way as SPD but it was not deductible before PRT and corporation taxes were calculated, although the extent to which APRT was paid had the effect of reducing subsequent PRT bills accordingly. Also the rate of APRT varied within the sample (Fig. 2) and the tax was phased out from 1983 onwards.

3. The third stage in the oil taxation system is the corporation tax (CT).

CT is levied on the operating company and not on individual fields, both the royalties and the PRT are deductible from the CT. The changes in the CT rate over the sample are shown in Fig. 4.

On the basis of the above brief description of the UKCS tax-system we can now discuss its neutrality and its instability. As already mentioned in the introduction non-neutrality and instability of a tax system play a different role in a forward looking model of intertemporal optimization. Non-neutrality is crucial from the theoretical point of view because the optimal decision rule is affected by the existence of taxation. Stability would only affect the econometric

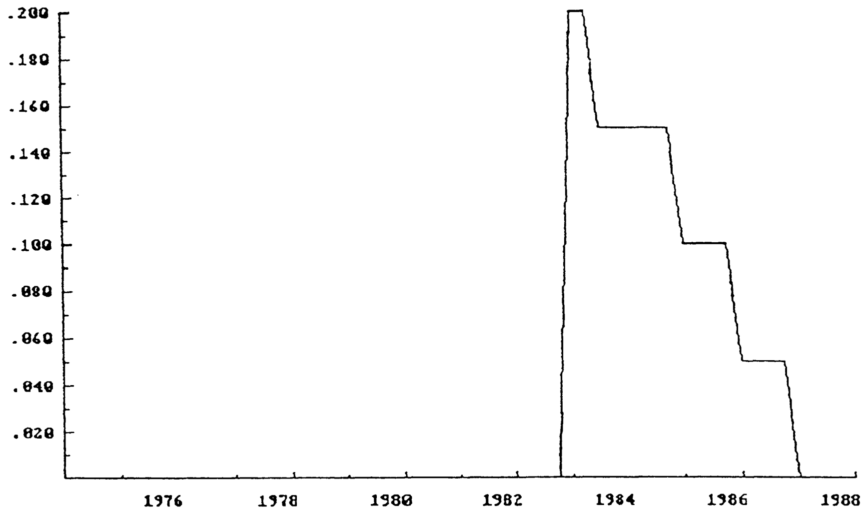

FIG. 2. APRT = advanced petroleum revenue tax.

estimates and their interpretation. When taxation is omitted, the estimated parameter becomes convolution of deep parameters and taxation parameters with two consequences: it is never possible to interpret the coefficients of the model without taxes as 'deep' coefficients, and the parameters of the model undergo a change every time there is an *expected* modification in the tax rates. In a forward looking model of intertemporal optimization *unexpected* modification of the tax rates does not affect the properties of estimators, being by definition orthogonal to the variables included into the model. This aspect is very important to our analysis. In fact, as it will be clear from the following discussion, the modifications of the tax system on the UKCS can be considered as unexpected when the exploration and production decisions are taken.

As far as neutrality is concerned we notice that one of the main features of the tax-system is the lack of progressivity. The tax rates are constant, and there are very few elements of progressivity in the tax-base, the most noticeable being the oil allowance. Leaving aside equity considerations, the absence of a progressive tax system has three major implications (Clunies Ross (1982), Hann and Rowland (1986)):

(i) For a given tax revenue target a progressive tax system is less likely to affect projects in such a way that development decisions are jeopardized: the impact of a progressive tax-system on companies earning marginal profits is negligible.
(ii) A progressive tax system will not magnify the impact of other imperfections in the tax-system. The non-progressivity of the tax system

will amplify the impact of any reduction of the marginal field profitability caused by a mistaken inclusion of part of the normal profits in the tax-base.

(iii) A non-progressive tax-system causes higher variability of post-tax profits than a progressive one. Given the variability of oil prices, the risk associated with the development of marginal fields is higher when the tax-system is not progressive.

We can conclude that the lack of progressivity of the tax-system is a strong reason to consider it as non-neutral.

Another reason for non-neutrality is the relation between the tax-system and risk in the exploratory and development activity. Exploration and development in the North Sea are risky activities because of the possibility of drilling dry holes, the difficulty of delineating deposits, the problem of estimating reserves, and the unknown geology of the areas investigated. The ring-fence provision, which precludes setting losses in any fields against profit obtained by the same company from other fields, does not allow companies to spread their risk. The fact that the risk is entirely born by a risk-averse taxpayer increases the non-neutrality of the taxes.[3]

Moving to stability, we notice immediately that the absence of progressivity is in itself a potential cause of instability of the tax-system: in fact only in a progressive tax system price rises would automatically generate an increase in tax revenue without any need for modifications of the system's rates.

The empirical evidence shows that the time span between the date of proposal and the date of implementation of modifications in the tax system ranges from a minimum of −2 months (PRT was proposed in the March 1975 budget, but put into effect from January 1975) to a maximum of two years (see Hann and Rowland (1986, pp. 13–14)). Since the development lag (the time-span between the discovery and the production start-up dates) ranges from three years and ten months to twelve years (see Mabro *et al.* (1986, Table S3, p. 320)) it follows that all the changes in the tax system must be considered as unknown at the time exploration and output decisions are taken. From Figs 1–4 we can see that there has been a greal deal of variation in the tax parameters, and the relevant tax parameters are not only *unknown* to the firm but also *unpredictable*, as stated earlier, to both the UK Offshore Operators Association and BRINDEX (the association of about forty smaller oil companies) (Clunies Ross (1982)). The reason for the unpredictability is ascribed by some economists (Hann (1985)) to the fact that the reform in oil taxation is driven more by political and bureaucratic pressures than by economic factors predictable by firms.

We can therefore conclude that, in view of the non-neutrality of the UKCS tax system, the inclusion of taxation in a model of exploration and production of North Sea Oil in the UKCS is a task of some relevance.

[3] This consideration, however, is not relevant to the 'representative' firm model adopted in Pesaran (1990).

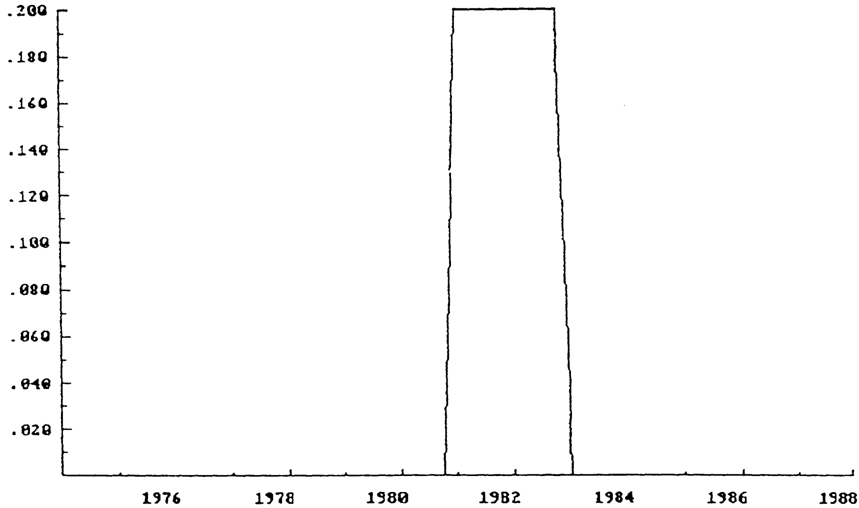

FIG. 3. SPD = supplementary petroleum duty.

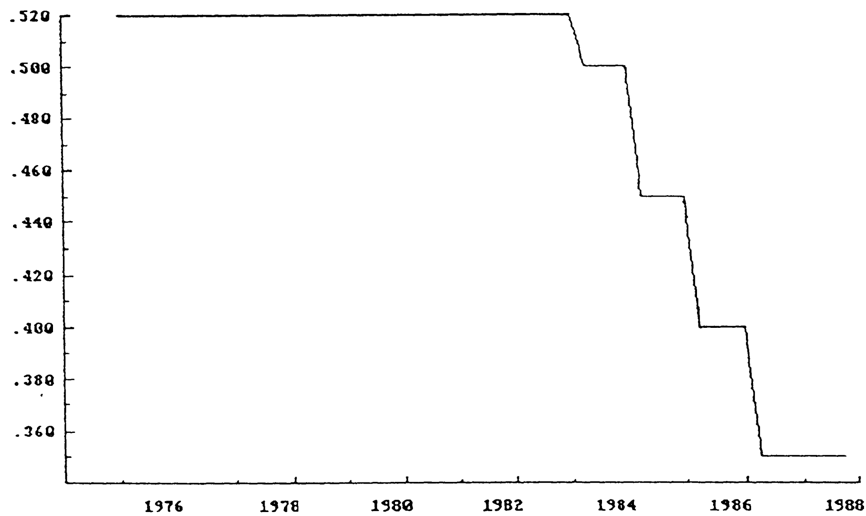

FIG. 4. CT = corporation tax.

3. An intertemporal model of exploration and extraction with taxation

To model exploration and extraction in the UKCS we assume that producers are risk neutral and decide on the rates of extraction $q_t, q_{t+1}, \ldots$ and the rates of exploratory efforts, $x_t, x_{t+1} \ldots$, by maximizing the discounted future streams of profits conditional on the information set Ω_{t-1}, which includes the taxation system in force at time $t - 1$. Therefore the desired values for extraction and production are found by solving the intertemporal optimization problem

$$\underset{\substack{q_t, q_{t+1}\ldots \\ x_t, x_{t+1}\ldots}}{\text{Max}} \; E\left\{ \sum_{\tau=0}^{\infty} \beta^{\tau}\Pi_{t+\tau} \middle| \Omega_{t-1} \right\} \tag{1}$$

where $0 \leqslant \beta < 1$ is the discount factor and Π_t is the producer profit net of taxes defined as follows

$$\begin{aligned} \Pi_t = p_t q_t - C(q_t, R_{t-1}) - w_t x_t - \{&(\tau_{1t} + \tau_{4t}) p_t q_t \\ &+ \tau_{2t}[p_t q_t - C(q_t, R_{t-1}) - up_t w_t x_t - (\tau_{1t} + \tau_{4t}) p_t q_t] \\ &+ \tau_{ct}[p_t q_t - (\tau_{1t} + \tau_{4t}) p_t q_t] \\ &- \tau_{2t}[p_t q_t - C(q_t, R_{t-1}) - up_t w_t x_t - (\tau_{1t} + \tau_{4t}) p_t q_t]\} \end{aligned} \tag{2}$$

which can be rewritten as

$$\Pi_t = \alpha_{1t} p_t q_t - \alpha_{2t} C(q_t, R_{t-1}) - \alpha_{3t} w_t x_t$$

where

$$\begin{aligned} \alpha_{1t} &= (1 - \tau_{2t})(1 - \tau_{1t} - \tau_{4t})(1 - \tau_{ct}) \\ \alpha_{2t} &= [1 - \tau_{2t} - \tau_{ct}\tau_{2t}] \\ \alpha_{3t} &= [1 - \tau_{2t} up_t - \tau_{ct}\tau_{2t} up_t] \end{aligned}$$

q_t :rate of extraction

R_t :level of proven reserves

p_t :well-head price

τ_{2t} Petroleum Revenue Tax

τ_{4t}:Supplementary Petroleum Duty

x_t :rate of exploratory effort

w_t :unit cost of exploratory effort

τ_{1t} :royalty

up_t :1 + uplift on exploration costs

τ_{ct} :Corporation Tax

Several comments on the profit function are in order. The proposed parameterization of the function is a general specification which embeds all the various features of the tax system in place over the sample period. Switches from one regime to the other are captured by the tax parameters, which can be considered as dummies set to zero when the tax is not applied and set to the appropriate value when the tax is imposed. In the net profit function (2) $[p_t q_t - C(q_t, R_{t-1}) - w_t x_t]$ is the profit before taxes, and $C(q_t, R_{t-1})$ is a convex function capturing the cost of development and extraction. $C(q_t, R_{t-1})$ is assumed to vary positively with the rate of extraction and negatively with the level of remaining proven reserves.[4]

$(\tau_{1t} + \tau_{4t}) p_t q_t$ captures the effect of the royalty payment and of the SPD (when the SPD is on), which are both levied on the gross field revenues and represents the first stage of the Oil Taxation System (OTS) previously discussed. The second stage of the OTS is captured by the following expression $\tau_{2t}[p_t q_t - C(q_t, R_{t-1}) - up_t w_t x_t - (\tau_{1t} + \tau_{4t}) p_t q_t]$ which represents the PRT bill: royalties and SPD are deduced from gross revenues together with

[4] The inclusion of R_{t-1} is justified by Pesaran (1990) on the basis of the engineering information on the determinants of the pressure dynamics of the petroleum reserves.

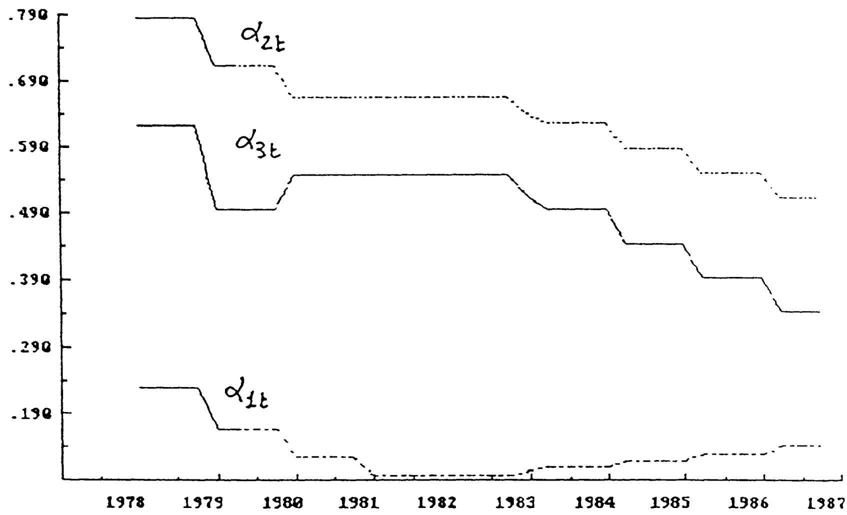

FIG. 5. The tax variables.

production costs and exploration costs augmented by the uplift. Finally, in the third stage of the OTS, the Corporation Tax is $\tau_{ct}[p_tq_t - (\tau_{1t} + \tau_{4t})p_tq_t - \tau_{2t}[p_tq_t - C(q_t, R_{t-1}) - up_tw_tx_t - (\tau_{1t} + \tau_{4t})p_tq_t]$, in which payments for SPD, royalties and PRT are deduced from taxable profits to determine the corporation tax bill. We notice that, since APRT it is a way of collecting PRT in advance, its effect is captured by the PRT parameter. Lastly, it has to be said that the aggregate nature of the data does not allow the consideration of field allowances, whose effect can be modelled properly only when a field by field disaggregated analysis is considered.

To summarize we can state that the impact of the tax system in our model is captured by the three variables $\alpha_{1t}, \alpha_{2t}, \alpha_{3t}$, where α_{1t} is a measure of the reduction in the marginal revenue due to taxation (the smaller α_{1t} the larger the difference between pre-tax and post-tax revenue), while α_{2t} and α_{3t} are respectively measures of the reduction in the marginal extraction cost and the marginal exploration cost due to allowances in the tax system (the smaller α_{2t} and α_{3t} the higher the allowance). By plotting $\alpha_{1t}, \alpha_{2t}, \alpha_{3t}$ over the sample period (Fig. 5) we notice immediately a turning point in 1983, in fact from 1983 onward the wedge between pre-tax profits and post-tax profits has been decreasing uniformly.

In optimizing the firm faces the following constraints:

$$R_{t+\tau} - R_{t+\tau-1} = d_{t+\tau} + e_{t+\tau} - q_{t+\tau} \qquad \tau = 0, 1, 2, \ldots \tag{3}$$

$$X_t = X_{t-1} + x_t \tag{4}$$

where d_t denotes the addition to proven reserves during period $t - 1$ to t from new discoveries, and e_t the revisions/extensions to previously discovered reserves, X_t represents the level of cumulative exploratory effort at time t. To determine d_t we follow, as in Pesaran (1990), a version of the Kaufman (1975) model and specify that

$$d_t = F(x_t, X_{t-1})\varepsilon_t \tag{5}$$

where ε_t is assumed to satify the orthogonality condition

$$E(\varepsilon_t | \Omega_{t-1}) = 1 \tag{6}$$

The information set at time $t-1$ is assumed to contain observations on at least the current and past values of q, x and past values of R, w, p and all the tax parameters at time t (the modifications of the tax-system are usually announced at the time of the budget and implemented successively).

The following conditions are satisfied:

$$\partial F_t / \partial x_t > 0, \quad \frac{\partial^2 F_t}{\partial x_t^2} < 0, \quad \partial F_t / \partial x_{t-1} < 0 \text{ for } t > m, \quad \lim_{X_{t-1} \to \infty} F(x_t, X_{t-1}) = 0 \tag{7}$$

Therefore the marginal product of exploratory effort is positive but diminishing, and there is a discovery decline phenomenon; so that as exploration proceeds the effects of reserves exhaustion dominates the influence of the accumulation of geological knowledge.

3.1. *The Euler equations*

Given price cost and tax parameter expectations $p^e_{t+k} = E(p_{t+k} | \Omega_{t-1})$, $w^e_{t+k} = E(w_{t+k} | \Omega_{t-1})$, $\tau^e_{i,t+k} = E(\tau_{i,t+k} | \Omega_{t-1})$, and an initial level of proven reserves the above relations completely define the decision environment of the firm.

The First Order Conditions (FOC) for optimality can be obtained from unconstrained optimization of the following Lagrangean function

$$L = E\left(\sum_{k=0}^{\infty} \beta^k G_{t+k} | \Omega_{t-1} \right)$$

with respect to $q_{t+k}, x_{t+k}, R_{t+k}, X_{t+k}$, for $k = 0, 1, 2, \ldots$ where

$$G_t = \Pi_t + \lambda_t (d_t + e_t - q_t - R_t + R_{t-1}) + \mu_t (X_t - X_{t-1} - x_t)$$

the auxiliary variables λ_t and μ_t are the Lagrange multipliers.

The FOC for maximization can be written as follows[5]

$$E_{t-1}\left(\alpha_{1t+\tau} p_{t+\tau} - \alpha_{2t+\tau} \frac{\partial C_{t+\tau}}{\partial q_{t+\tau}} - \lambda_{t+\tau} \right) = 0 \tag{8a}$$

$$E_{t-1}\left(\beta \lambda_{t+\tau+1} - \lambda_{t+\tau} - \alpha_{2t+\tau+1} \beta \frac{\partial C_{t+\tau+1}}{\partial R_{t+\tau}} \right) = 0 \tag{8b}$$

[5] The transversality conditions relevant to this optimization are assumed to be satisfied.

$$E_{t-1}\left(\mu_{t+\tau} + \alpha_{3t+\tau}w_{t+\tau} - \lambda_{t+\tau}\frac{\partial d_{t+\tau}}{\partial x_{t+\tau}}\right) = 0 \tag{8c}$$

$$E_{t-1}\left(\mu_{t+\tau} - \beta\mu_{t+\tau+1} + \beta\lambda_{t+\tau+1}\frac{\partial d_{t+\tau+1}}{\partial X_{t+\tau}}\right) = 0 \tag{8d}$$

$$E_{t-1}(R_{t+\tau} - R_{t+r-1} - d_{t+\tau} - e_{t+\tau} + q_{t+\tau}) = 0 \tag{8e}$$

$$E_{t-1}(X_{t+\tau} - X_{t+\tau+1} - x_{t+\tau}) = 0 \tag{8f}$$

From this set of equations we can interpret economically the two Lagrange multipliers λ_t and μ_t:λ_t represents the net of tax shadow price of reserves in the ground, while μ_t can be interpreted as the net value of the marginal product of reserves discovery (see Pesaran (1990)).

Conditions (8a)–(8f) are non-linear but can be transformed to derive exploration and extraction rules that can be consistently estimated. If we focus on current decision variables q_t and x_t and rewrite relations (8a)–(8f) for $\tau = 0$ we have

$$E_{t-1}(\lambda_t) = E_{t-1}\left(\alpha_{1t}p_t - \alpha_{2t}\frac{\partial C_t}{\partial q_t}\right) \tag{9a}$$

$$E_{t-1}(\lambda_t) = \beta E_{t-1}\left(\lambda_{t+1} - \alpha_{2t+1}\frac{\partial C_{t+1}}{\partial R_t}\right) \tag{9b}$$

$$E_{t-1}(\mu_t) = E_{t-1}\left(\lambda_t\frac{\partial d_t}{\partial x_t} - \alpha_{3t}w_t\right) \tag{9c}$$

$$E_{t-1}(\mu_t) = \beta E_{t-1}\left(\mu_{t+1} - \lambda_{t+1}\frac{\partial d_{t+1}}{\partial X_t}\right) \tag{9d}$$

Equation (9a) gives an expression for the expected shadow price of oil in the ground; it is noticeable that both the expected well-head price and the expected marginal extraction cost are effected by the tax system. Equation (9b) gives the intertemporal condition for the extraction of oil over time. Equations (9c) and (9d) give the necessary conditions for the determination of the optimum level of exploration. Again it is noticeable that the expected return from exploration, defined by the right hand side of (9c) is affected by the tax system.

3.1.a. *The production equation*

We can derive an output equation from condition (9a)–(9d) by eliminating all the unobservables.

From 9a we have

$$E_{t-1}(\lambda_{t+1}) = E_{t-1}\left(E_t\alpha_{1t+1}p_{t+1} - E_t\alpha_{2t+1}\frac{\partial C_{t+1}}{\partial q_{t+1}}\right) \tag{10a}$$

and, assuming expectations are formed consistently yields

$$E_{t-1}(\lambda_{t+1}) = E_{t-1}\left(\alpha_{1t+1}p_{t+1} - \alpha_{2t+1}\frac{\partial C_{t+1}}{\partial q_{t+1}}\right) \tag{10a'}$$

Substituting this result in (9b) and using (9a) to eliminate $E_{t-1}(\lambda_t)$, we obtain

$$\alpha_{2t}E_{t-1}\left(\frac{\partial C_t}{\partial q_t}\right) = [\alpha_{1t}E_{t-1}p_t - \beta E_{t-1}\alpha_{1t+1}p_{t+1}]$$

$$+ \beta E_{t-1}\left(\alpha_{2t+1}\left(\frac{\partial C_{t+1}}{\partial R_t} + \frac{\partial C_{t+1}}{\partial q_{t+1}}\right)\right) \tag{11}$$

This equation does not depend on the Lagrnage multipliers and, for a given specification of the extraction cost function, can be consistently estimated. The following non-linear form (Pesaran (1990)) is adopted

$$C(q_t, R_{t-1}) = \delta_0 + \delta_1 q_t + \frac{1}{2}\left(\delta_2 + \frac{\delta_3}{R_{t-1}}\right)q_t^2 + u_t q_t \tag{12}$$

where u_t represents unobserved random shocks to marginal extraction cost, which are assumed to be orthogonal to the information set Ω_{t-1}. All the parameters in the cost function are expected to be positive except for δ_1 which could be negative so long as $E_{t-1}(\partial C_t/\partial q_t) > 0$. Using (12) we arrive at the following specification for the optimal level of extraction q_t^*,

$$\delta_1 + \frac{\delta_2 q_t^*}{z_{t-1}} = E_{t-1}\left[(\alpha_{1t}/\alpha_{2t})p_t - \beta\left(\frac{\alpha_{1t+1}}{\alpha_{2t}}p_{t+1}\right)\right.$$

$$\left. + \beta\frac{\alpha_{2t+1}}{\alpha_{2t}}\left(\delta_1 + q_{t+1}\frac{\delta_2}{z_t} - \frac{1}{2}\delta_3\frac{q_{t+1}^2}{R_t^2}\right)\right] \tag{13}$$

This result generalizes equation (13) in Pesaran (1990) to give the desired level of output as a function of reserves, price expectations and firm's planned or expected future output and reserves, and the expected level and the change in future tax parameters. We can write (13) as

$$q_t^* = (\delta_1/\delta_2)E_{t-1}\left[\frac{\beta\alpha_{2t+1}}{\alpha_{2t}}z_{t-1}\right] - (\delta_1/\delta_2)z_{t-1} + \delta_2^{-1}z_{t-1}E_{t-1}[(\alpha_{1t}/\alpha_{2t})p_t]$$

$$- \delta_2^{-1}z_{t-1}E_{t-1}\left[\frac{\beta\alpha_{1t+1}}{\alpha_{2t}}p_{t+1}\right]$$

$$+ \beta z_{t-1}E_{t-1}\left(\frac{\alpha_{2t+1}}{\alpha_{2t}}q_{t+1}\right) + \beta\gamma z_{t-1}E_{t-1}\left(\frac{\alpha_{2t+1}}{\alpha_{2t}}h_{t+1}\right) \tag{14}$$

Notice that equation (14) depends on the ratios α_{1t}/α_{2t}, $\alpha_{1t+1}/\alpha_{2t}$ and $\alpha_{2t+1}/\alpha_{2t}$, suggesting that the omission of the tax system is going to affect the econometric performance of the model unless the ratios are constant over time. These three

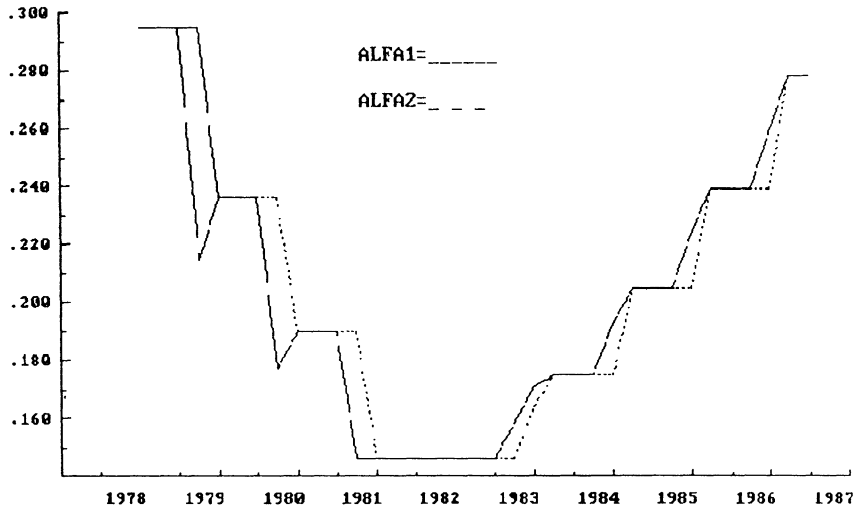

FIG. 6.1. The tax variables relevant to production.

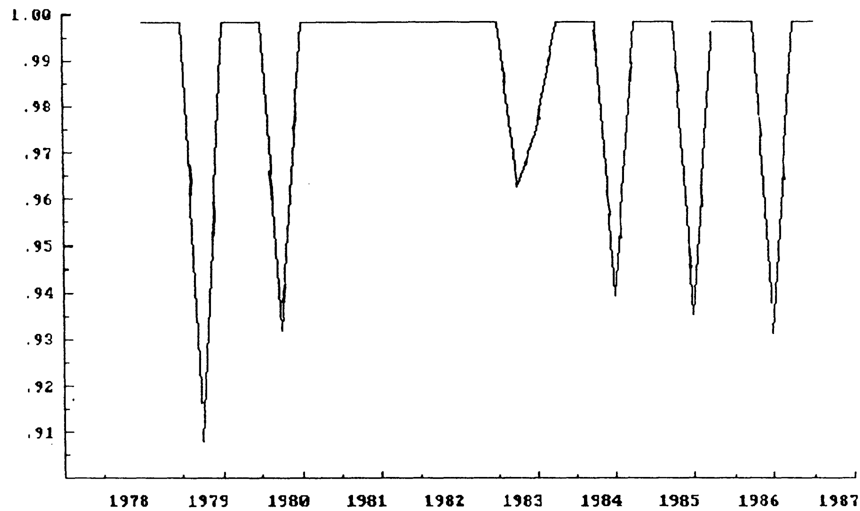

FIG. 6.2. The tax variables relevant to production: ALFA3.

ratios capture entirely the impact of the tax system and from now on we will denote them respectively as $\theta_{1t}, \theta_{2t}, \theta_{3t}$. Figure 6 reports the variation over the sample 1978:1–1986:4 of $\theta_{1t}, \theta_{2t}, \theta_{3t}$.

By assuming that the relationship between the actual rate of extraction and the firm's desired rate of extraction can be characterized by the following simple partial adjustment model

$$q_t - q_{t-1} = \phi(q_t^* - q_{t-1}) \tag{15}$$

we have

$$q_t = (1 - \phi)q_{t-1} + b_0 z_{t-1} E_{t-1}\boldsymbol{\theta}_{3t} + b_1 z_{t-1} + b_2 z_{t-1} E_{t-1}[\boldsymbol{\theta}_{1t} p_t - \beta \boldsymbol{\theta}_{2t} p_{t+1}]$$
$$+ b_3 z_{t-1} E_{t-1}(\boldsymbol{\theta}_{3t} q_{t+1}) + b_4 z_{t-1} E_{t-1}(\boldsymbol{\theta}_{3t} h_{t+1})$$

where

$$b_0 = \phi\beta\delta_1/\delta_2; \qquad b_1 = -\phi\delta_1/\delta_2 \leqslant 0$$
$$b_2 = \phi\delta_2^{-1} \geqslant 0; \qquad b_3 = \phi\beta \geqslant 0 \; b_4 = \phi\beta\gamma \geqslant)$$

To model oil prices expectations and tax parameters' expectations we consider two alternative hypotheses. Under the first hypothesis expectations on both tax parameters and oil price are assumed to be rational and we have

$$E_{t-1}(\boldsymbol{\theta}_{1t} p_t) = \boldsymbol{\theta}_{1t} p_t + \xi_{1t}$$
$$E_{t-1}[\boldsymbol{\theta}_{2t} p_{t+1}] = \boldsymbol{\theta}_{2t} p_{t+1} + \xi_{1t+1}$$

where ξ_{it+k} satisfies the orthogonality property $E(\xi_{it+k}|\Omega_{t-1}) = 0$

We notice that, although the changes in the tax parameters are usually announced in the budget, we still maintain the possibility of forecasting errors in predicting future tax parameters, generated by the length of the development lag.

Our second alternative for expectations formation is constituted by an adaptive expectations scheme for price combined with a rational expectations scheme for the tax parameters. We have

$$E_{t-1} p_t = E_{t-1} p_{t+1} = (1 - v) \sum_{i=1}^{\infty} \theta^{i-1} p_{t-i} = \tilde{p}(v) \text{ where } 0 \leqslant v < 1$$

$$E_{t-1}\boldsymbol{\theta}_{2t} = \boldsymbol{\theta}_{2t} + \xi_{2t}$$

under this alternative we consider the possibility of a backward looking behaviour by agents in the formation of price expectations, possibly justified by the high volatility of oil prices within the sample period. However, we do not remove the hypothesis of rational expectations on the tax parameters because the relaxations of this hypothesis would have the implication that announcements made at time $t - 1$ of future modifications in the tax parameters are ignored by the representative firm. In order to apply this alternative we have also to assume that p_{t+1} is distributed independently from $\boldsymbol{\theta}_{1t}$ and $\boldsymbol{\theta}_{2t}$.

Summarising, we will consider the two following empirical alternatives for the supply equation

(i) Rational Expectations model

$$q_t = (1 - \phi)q_{t-1} + b_0 z_{t-1}\boldsymbol{\theta}_{3t} + b_1 z_{t-1} + b_2 z_{t-1}[\boldsymbol{\theta}_{1t} p_t - \beta\boldsymbol{\theta}_{2t} p_{t+1}]$$
$$+ b_3 z_{t-1}(\boldsymbol{\theta}_{3t} q_{t+1}) + b_4 z_{t-1}(\boldsymbol{\theta}_{3t} h_{t+1}) + \varepsilon_{1t} \tag{16}$$

where

$$\begin{aligned}\varepsilon_{1t} = {} & b_0 z_{t-1}[E_{t-1}\theta_{3t} - \theta_{3t}] \\ & + b_2 z_{t-1}\{E_{t-1}[\theta_{1t}p_t - \beta\theta_{2t}p_{t+1}] - \theta_{1t}p_t + \beta\theta_{2t}p_{t+1}\} \\ & + b_3 z_{t-1}\{E_{t-1}(\theta_{3t}q_{t+1}) - \theta_{3t}q_{t+1}\} \\ & + b_4 z_{t-1}(E_{t-1}\theta_{3t}h_{t+1} - \theta_{3t}h_{t+1})\end{aligned}$$

satisfies the orthogonality conditions $E(\varepsilon_{1t}|\Omega_{t-1}) = 0$.

(ii) Mixed Adaptive and Rational Expectations model

$$\begin{aligned}q_t = {} & (1-\phi)q_{t-1} + b_0 z_{t-1}\theta_{3t} + b_1 z_{t-1} + b_2 z_{t-1}[\theta_{1t}\tilde{p}(v) - \beta\theta_{2t}\tilde{p}(v)] \\ & + b_3 z_{t-1}(\theta_{3t}q_{t+1}) + b_4 z_{t-1}(\theta_{3t}h_{t+1}) + \varepsilon_{2t} \qquad (17)\end{aligned}$$

where

$$\begin{aligned}\varepsilon_{2t} = {} & b_0 z_{t-1}[E_{t-1}\theta_{3t} - \theta_{3t}] + b_2 z_{t-1}\{E_{t-1}[\theta_{1t}\tilde{p}(v) - \beta\theta_{2t}\tilde{p}(v)] - \theta_{1t}\tilde{p}(v) \\ & + \beta\theta_{2t}\tilde{p}(v)\} + b_3 z_{t-1}\{E_{t-1}\theta_{3t}q_{t+1}) - \theta_{3t}q_{t+1}\} \\ & + b_4 z_{t-1}\{E_{t-1}\theta_{3t}h_{t+1}) - \theta_{3t}h_{t+1}\}\end{aligned}$$

$$E(\varepsilon_{2t}|\Omega_{t-1}) = 0$$

As far as estimation is concerned we notice that both ε_{1t} and ε_{2t} are orthogonal to the information set Ω_{t-1}, equations (16) and (17) can be therefore consistently estimated by implementing the 'Error in variables' method (McCallum (1976)) using instruments dated $t-2$ or earlier, assuming, of course, that the parameters are in fact identified.

3.1.b. *The exploration equation*

The first step to generate an estimable exploration equation is to combine (9c) and (9d) in the following condition

$$E_{t-1}\left(\lambda_t \frac{\partial d_t}{\partial x_t}\right) = E_{t-1}(\alpha_{3t}w_t) - \beta E_{t-1}\left(\alpha_{3t+1}w_{t+1} - \lambda_{t+1}\left(\frac{\partial d_{t+1}}{\partial x_{t+1}} - \frac{\partial d_{t+1}}{\partial X_t}\right)\right) \qquad (18)$$

Then we assume that the discovery function takes the form advocated by Uhler (1976, p. 79) in his empirical analysis of oil and gas discovery in the province of Alberta

$$F(x_t, X_{t-1}) = Ax_t^\rho \exp(b_1 X_{t-1} - b_2 X_{t-1}^2) \qquad (19)$$

for positive values of A, b_1 and b_2 and for $0 < \rho < 1$ the function satisfies all the required properties, in particular the threshold value for the cumulative exploratory effort, after which we have the discovery decline phenomenon, is given by $X_m = b_1/2b_2$.

Even a discovery function like (18) produces a highly non-linear functional form in the general case. However if we focus on the simple case $\beta = 0$ we have the following relation for the desired rate of exploration

$$\log x_t^* = d_0 + d_1 X_{t-1} + d_2 X_{t-1}^2 + d_3 \log[E_{t-1}\lambda_t / E_{t-1}(w_t \alpha_{3t})] \quad (20)$$

where $d_0 = (1-\rho)^{-1}\log(A\rho) \qquad d_1 = (1-\rho)^{-1} b_1 \geqslant 0$

$$d_2 = (1-\rho)^{-1} b_2 < 0 \qquad d_3 = (1-\rho)^{-1} > 0$$

using again a partial adjustment mechanism, this time on the logs, we have

$$\log x_t = (1-\phi)\log x_{t-1} + \phi d_0 + \phi d_1 X_{t-1} + \phi d_2 X_{t-1}^2 + \phi d_3 \log(E_{t-1}\lambda_t / E_{t-1} w_t \alpha_{3t}) \quad (21)$$

and $E_{t-1}\lambda_t$ can be eliminated by proxying it with $\hat{\lambda}_t$

$$\hat{\lambda}_t = (\alpha_{1t} p_t) - \alpha_{2t}\hat{\delta}_1 - \hat{\delta}_2(\alpha_{2t} q_t / z_{t-1})$$

which represents the shadow price of oil in the ground and whose value is determined by using the parameter estimates from the production equation.

The estimable equation will then be

$$\log x_t = (1-\phi)\log x_{t-1} + \phi d_0 + \phi d_1 X_{t-1} + \phi d_2 X_{t-1}^2 + \phi a_3 \log(\hat{\lambda}_t / w_t \alpha_{3t}) + u_t \quad (22)$$

where, under the REH, the disturbances u_t satisfy the orthogonality condition

$$E(u_t | \Omega_{t-1}) = 0.$$

4. The empirical results

Our empirical analysis will focus on the sample period 1978(1)–1986(4) to allow direct comparability with the results obtained by Pesaran (1990).

4.1. *Supply equation*

In order to estimate the Rational Expectations version of the supply equation we have to take in account the possible correlation between the composite disturbance in equation (16) and the regressors. Under the rational expectations hypothesis ε_{1t} is orthogonal to the information set available at time $t-1$, therefore the Error in Variables procedure (McCallum (1976)) can be implemented by using instruments dated $t-1$ or earlier.[6] The Non-linear Two Stage Least squares method (Gallant (1987, ch. 6)) allows us to model the non-linearity in parameters of equation (17).[7]

Possible heteroscedasticity of residuals can be taken into account by basing

[6] In practice however we found first order serial correlation in the residuals and we have used instruments dated $t-2$ and earlier.

[7] Notice however that the high degree of non-linearity does not allow to check the conditions for identification of parameters and the orthogonality between instruments and residuals is only a necessary condition for consistency of the resultant estimators (see Pesaran (1987, ch. 7)).

TABLE 1
*Estimates of the Oil supply function Under Alternative Price Expectations Formation Models**

Parameters	1978:1–1986:4 Equation 16 REH	Equation 17 AEH
ϕ	0.368	0.377
	(0.129)	(0.128)
$\hat{\delta}_2$	0.116	0.32
	(0.171)	(3.61)
$\hat{\delta}_3$	99.57	417.0
	(339.1)	(4577)
$\hat{\beta}$	1.19	1.11
	(0.40)	(0.21)
$\hat{\theta}$	—	0.96
σ	9.01	9.24
μ	186.78	186.78
$X^2_{SC}(4)$	17.97	17.86
$X^2_{FF}(1)$	0.62	0.87
$X^2_N(2)$	0.37	0.59
$X^2_H(1)$	11.09	12.53

The figures in ( ) are the White's (1982) heteroscedasticity-consistent estimates adjusted for the degrees of freedom, $\hat{\sigma}$ is the estimated standard error of the regression, μ is the mean of the dependent variable $X^2_{SC}(4)$, $X^2_{FF}(1)$, $X^2_N(2)$, $X^2_H(1)$ are diagnostic statistics distributed approximately as chi-squared variables (with the degrees of freedom reported in parentheses) for tests for residual serial correlation (up to the fourth order), functional mis-specification form, non-normal errors and heteroscedasticity respectively. See Pesaran and Pesaran (1989).

* Estimation by Non linear Two-stage Least Squares computed using seasonal dummies, $q_{t-2}, p_{t-2}, h_{t-2}, R_{t-2}, R^2_{t-2}, R_{t-2}q_{t-2}, \alpha_{1t-2}, \alpha_{2t-2}, \Delta\alpha_{1t-2}, \Delta\alpha_{2t-2}$ as instruments.

The results are obtained by assuming $\gamma = \delta_3/\delta_2 = 2000$, which results from a grid search on the parameters.

The value of 0.96 reported for the adaptive expectations coefficient v is the maximum likelihood estimate of v computed by grid search method over the range $0 \leqslant v \leqslant 1$.

inference on White's (1982) heteroscedasticity consistent estimator of the covariance matrix of the instrumental variables (IV) estimators.

Following the empirical evidence provided by Mabro *et al.* (1986) we have included seasonal dummies in the estimated equation and, in order to ensure that the seasonal effect add up to zero over a given year, we used $s_{it} - s_{4t}$ $i = 1, 2, 3$ as seasonal variable, where $s_{it} = 1$ in the ith quarter and zero elsewhere. The instrumental variables used are seasonal dummies, $q_{t-2}, p_{t-2}, h_{t-2}, R_{t-2}, R^2_{t-2}, R_{t-2}q_{t-2}, \alpha_{1t-2}, \alpha_{2t-2}, \Delta\alpha_{1t-2}, \Delta\alpha_{2t-2}$, which all satisfy the orthogonality condition. The estimates of the structural parameters are reported in Table 1 together with the heteroscedasticity corrected standard errors. All the estimates

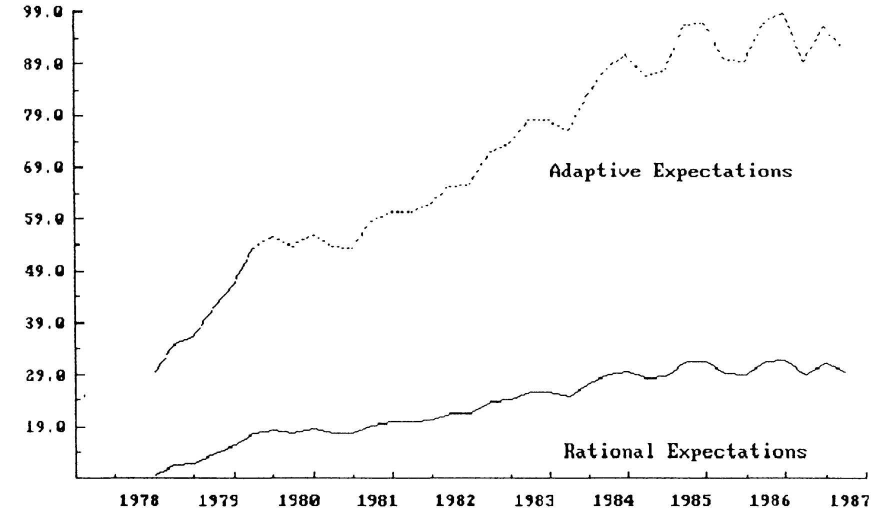

FIG. 7. The marginal extraction costs with Beta different from 0.

have the expected signs although $\delta_1, \delta_2, \delta_3$ are poorly determined and δ_1 is very far from being significant. In the reported specification δ_1 has been set to zero but the uncertainty on δ_2 and δ_3 remains high. The point estimate of the discount factor β is outside the admissible range, although the hypothesis that $\beta = .9$ cannot be rejected statistically.

The adaptive expectations equation fits worse than the REH version and the t-statistics on all the parameters estimates are lower. As in Pesaran (1990) we find heteroscedasticity and autocorrelation in the residuals, more importantly the marginal extraction costs implied by the estimated parameters in equation (16) and (17), plotted in Fig. 7, although smaller than those obtained in the model without taxation, are still too high to be taken seriously.

As in Pesaran (1990), we find empirically a negative relation between the discount factor and plausibility of the marginal extraction cost. We therefore re-estimated the two alternative models under the hypothesis that $\beta = 0$. The results are reported in Table 2. We notice that both the serial correlation and the heteroscedasticity problems disappear. The REH equation is preferable to the AEH equation according to all the reported criteria. The marginal extraction costs derived from these parameters estimates (Fig. 8) under the RE hypothesis ($\beta = 0, \delta_2 = .0108, \delta_3 = 21.64$) range from \$1.04 to \$3.66 in a perfectly acceptable region. In the basis of our results we can draw the conclusion that the most satisfactory model of oil supply in the UKCS supports the hypothesis of rational expectations with a discount factor of zero. The explicit inclusion of taxation in the intertemporal model does not modify the result obtained by Pesaran (1990) that the future is not relevant to today's oil supply decision. The main modification regards the marginal costs of extraction, which range between

TABLE 2

*Estimates of the Oil supply function with zero discount factor ($\beta = 0$) Under Alternative Price Expectations Formation Models**

Regressors	1978:1–1986:4 Equation 16* REH	Equation 17** AEH	Combined model
$s_{1t} - s_{4t}$	0.107	0.286	0.308
	(2.97)	(3.21)	(3.06)
$s_{2t} - s_{4t}$	−6.15	−6.19	−6.19
	(2.96)	(3.19)	(3.04)
$s_{3t} - s_{4t}$	1.35	1.09	1.24
	(2.96)	(3.21)	(3.04)
q_{t-1}	0.95	0.96	0.92
	(0.027)	(0.04)	(0.04)
$z_{t-1}\alpha_3 p_t$	4.62	—	4.98
	(1.80)		(1.94)
$z_{t-1}\alpha_3\tilde{p}(0.96)$	—	4.28	1.67
		(4.03)	(3.73)
σ	10.28	11.05	10.52
μ	186.78	186.78	186.78
$X^2_{SC}(4)$	1.01	2.28	0.88
$X^2_{FF}(1)$	1.73	7.21	1.06
$X^2_N(2)$	0.64	1.02	0.53
$X^2_H(1)$	0.15	1.25	0.21
Sargan X^2	7.48(8)	10.05(4)	6.05(8)

The figures in () are the White's (1982) heteroscedasticity-consistent estimates adjusted for the degrees of freedom, $\hat{\sigma}$ is the estimated standard error of the regression, μ is the mean of the dependent variable $X^2_{SC}(4)$, $X^2_{FF}(1)$, $X^2_N(2)$, $X^2_H(1)$ are diagnostic statistics distributed approximately as chi-squared variables (with the degrees of freedom reported in parentheses) for tests for residual serial correlation (up to the fourth order), functional mis-specification form, non-normal errors and heteroscedasticity respectively. Sargan X^2 is a test for the validity of instruments (degrees of freedom within brackets). See Pesaran and Pesaran (1990).

* Estimation by Instrumental Variables computed using seasonal dummies, q_{t-1}, q_{t-2}, p_{t-2}, h_{t-2}, R_{t-2}, R^2_{t-2}, $R_{t-2}q_{t-2}$, α_{1t-2}, α_{2t-2}, $\Delta\alpha_{1t-2}$, $\Delta\alpha_{2t-2}$ as instruments.

** Estimation by Instrumental Variables computed using seasonal dummies, q_{t-1}, α_{1t-2}, α_{2t-2}, $\Delta\alpha_{1t-2}$, $\Delta\alpha_{2t-2}$ as instruments.

\$1.04 and \$3.66 in the model with taxes, and between \$2.19 and \$17.25 in Pesaran's original model over the sample period in a model without taxation.

From the economic point of view the result of a zero discount factor can be justified as an outcome of the high degree of uncertainty, generated both by price instability and the unpredictability of a non-stable and non-neutral tax-system: in a continuously changing environment the representative firm has a very short time-horizon in choosing the determinants of its productions because predictions more than one-period ahead are not reliable. In the context of the North Sea these economic considerations are strengthened by the technical cost of production: once a platform has been installed the high rate depreciation of capital and the prospective final cost of removal are important technical factors in reducing the sensitivity of production to future, highly unpredictable, economic factors. On the basis of this evidence the first item on the research

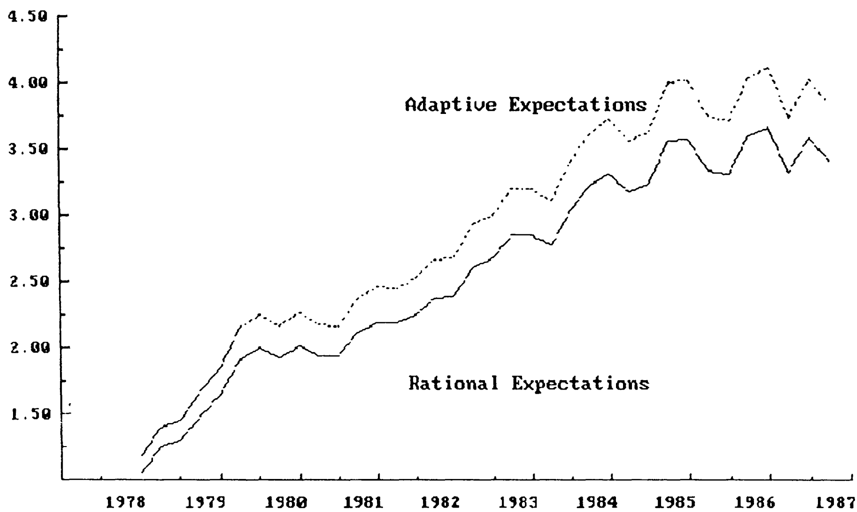

FIG. 8. The marginal extraction costs with beta = 0.

agenda is the incorporation of a 'development stage' in the optimization framework.

The separation of overall cost function $C(q_t, R_{t-1})$ in its two components, operating costs and development cost, and including the rate of development in the decision variables of the firms will allow the model to capture explicitly the dependence of the production stage on the development stage.

4.2. *The exploration equation*

In our theoretical model exploration is strictly linked to production through the shadow price of oil in the ground. The pre-tax and post-tax shadow prices of oil in the ground are plotted in Fig. 9. We notice that the pre-tax shadow price of oil in the ground is always positive while the post-tax price of oil in the ground becomes negative for the four quarters in 1986, which suggests that we are somewhat overestimating the impact of taxation towards the end of the sample. This is probably because most of the latest innovations in the tax system are oriented towards new fields, and there have been reductions in post-tax development cost, neither aspect is captured in our aggregate model. This evidence confirms the value of extending the model to include explicitly the development stage.

When the post-tax shadow price of oil in the ground is used to estimate the exploration equation over the sample 1978:1–1985:4 the following results are

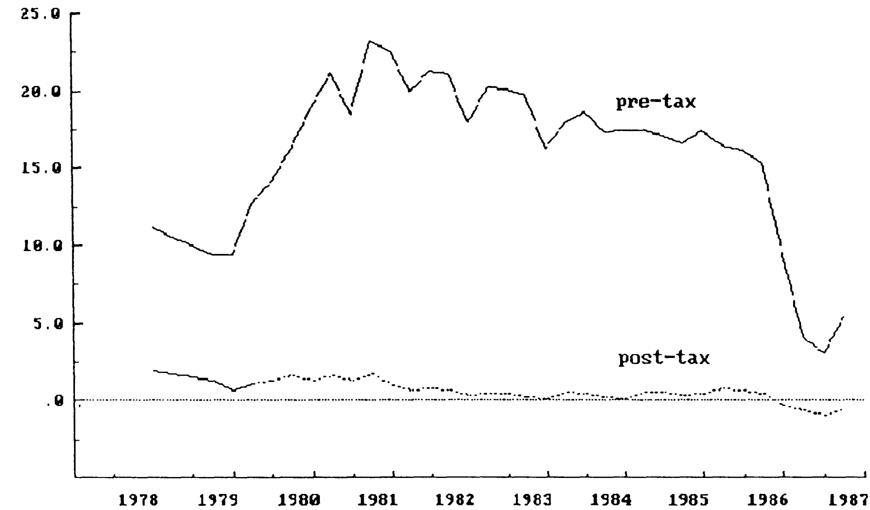

FIG. 9. The shadow prices of oil in the ground.

obtained

$$\log x_t = \underset{(.082)}{.46}\, \log x_{t-1} + \underset{(1.25)}{1.82} + \underset{(.0024)}{.007}\, X_{t-1} - \underset{(10^{-6}\times 1.5)}{10^{-6}\times 3.9}\, X_{t-1}^2$$

$$+ \underset{(.05)}{.10}\log(\hat{\lambda}_t) - \underset{(.08)}{.22}\log(w_t\alpha_{3t})$$

$$R^2 = .95 \qquad \sigma = .09 \qquad X_{SC}^2(4) = 4.13 \qquad X_{FF}^2(1) = .006$$

$$X_N^2(2) = 1.08 \qquad X_H^2(1) = 3.26$$

The model is estimated by Two Stage Least Squares ($\hat{\lambda}_t = \alpha_{1t}p_t - \alpha_{2t}.0108\, \hat{q}_t/z_{t-1}$, $\hat{q}_t$ are the estimated values from the supply equation) once again under the maintained hypothesis $\beta = 0$, standard errors are reported within brackets.

We notice that all the coefficients have the expected signs, although the restrictions that the coefficient on $\log \lambda_t$ and $\log(w_t\alpha_{3t})$ are equal with opposite sign is rejected by the data. The model passes all the diagnostics and the threshold value of cumulative level of exploration effort implied by the estimates is of 934 wells which in statistical terms does not support the strong implication, reached in the model without taxes, that a discovery decline phenomenon has already begun.[8]

[8] In the Pesaran (1990) model the point estimate of the threshold level is 894, while the cumulative number of exploration wells reached the figure of 943 wells in 1984.

5. Conclusions

The inclusion of taxation in an intertemporal econometric model of oil exploration and extraction in the UKCS reinforced the result obtained in Pesaran (1990) of a very short time horizon in the decision on the optimal supply, delivering much more plausible estimates of the marginal extraction costs. When the estimated marginal extraction costs are used to determine the shadow price of oil in the ground and to estimate the exploration equation, results consistent both with the theory and the recent empirical evidence in the UKCS are obtained. However the post-tax shadow price of oil in the ground becomes negative for the last four observations in our sample, suggesting that the model is overstating the impact of taxation on profit. This feature can be explained by the inability of the model to capture recent modifications in taxation aimed at helping development. The result also suggests that a further disaggregation of the investment and production decision into exploration development and extraction decisions may be worth considering, and this is on our agenda for future research.

Queen Mary College London

DATA APPENDIX

The data set used in this paper was kindly provided by Hashem Pesaran, for the definitions and sources of all the variables see the Data Appendix to Pesaran (1990). The fundamental source for the tax variables is Mabro *et al.* (1986), the update to the series was kindly provided by the Oxford Institute for Energy Studies.

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