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Demographics and US Stock Market Fluctuations*

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Abstract

This article illustrates how the information component determining long-horizon US stock market returns can be related to a demographic variable, MY the ratio of middle-aged to young adults. In fact, MY can be seen as the major determinants of a slowly evolving time-varying mean of the dividend/price ratio. A forecasting model for stock market returns over a century of US annual data that uses as predictors the dividend/price ratio and MY overcomes all the statistical difficulties related to the high persistence of the dividend/price ratio and performs very well in forecasting long-horizon stock market returns. Moreover, the use of demographic variables as a predictor for long-run stock market returns delivers a steeply downward sloping term structure of stock market risk. (JEL codes: G17, C53, E44)

Keywords: dynamic dividend growth model, long run returns predictability, stock market risk, demographics, direct regressions, multi-period iterated forecasts.

1 Introduction: noise and information in US stock market returns

This article is motivated by the view that stock market returns are determined by a permanent 'information' component and by a temporary 'noise' component.[1] The noise component dominates the data at high frequency, while the information component emerges when high-frequency observations are aggregated over time to construct long-horizon returns. Figure 1 makes the point by showing one century data on 1- and 20-year annualized US stock market returns (S&P 500 index).

This view is not new (see, for example, Cochrane 1994). In fact, the Dynamic Dividend Growth (DDG) model proposed by Campbell and Shiller (1988) can be interpreted in the light of the 'noise'–'information' decomposition as an identification strategy based on the use of dividends as the variable that captures information. Our main contribution lies in the relation of the information component to a demographic variable, MY, the ratio of middle-aged to young population. Intuitive reasoning and formal modeling (see, for example, Geneakopoulos et al. 2004) hints

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1 The use of the term 'noise' and 'information' has been inspired by our reading of Chapter 3 of Taleb (2001).

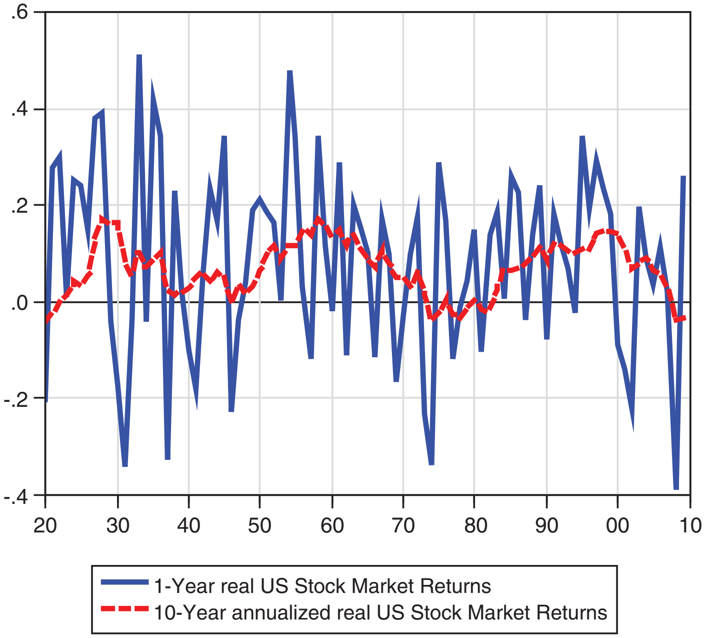

Figure 1 Noise and information in US stock market returns.

at demography as an important variable to determine the long-run behavior of the stock market, while it is difficult to imagine a relationship between high-frequency fluctuations in stock market prices and a slow-moving trend determined by demographic factors. We show how MY can be used in the DDG to complement the information component of dividends for long-run stock price fluctuations.

Moreover, the fact that a slow moving variable determined by demographics has very little impact on predictability of stock market returns at high frequency but a sizeable and strongly significant impact at low frequency has some obvious consequences on the slope of stock market risk, defined as the conditional variance and covariance per period of asset returns. When demographic trends are used to model the slow-moving fluctuations in the dividend/price ratio the decomposition of this variable into an high volatility 'noise' component and a low-volatility 'information' component naturally emerges. The dominance of the 'noise' component at high frequency and of the 'information' component at low frequency should lead naturally to a positive relation between predictability of returns and forecasting horizon and to a negatively sloped term structure of risk.

The importance of a demographic variable for capturing information-related long-horizon stock market fluctuations has a number of wide-ranging implications not limited to strategic asset allocation (e.g. Campbell and Viceira 2002). In fact, the decomposition of stock market returns into 'noise' and 'information'-related fluctuations makes clear that the choice of the frequency at which returns are defined becomes crucial for constructing a model aimed at integrating macroeconomic fluctuations and asset price fluctuations. It would be very hard to find any relation between macroeconomic variable and asset price fluctuations whenever asset price fluctuations are considered at frequencies dominated by the 'noise' component. Whereas the analysis of comovements between asset prices and macroeconomic fundamentals at frequencies where the 'information' component dominates would clearly increase the capability of macro-finance model to fit the data.

2 Noise, information and the DDG Model

The existence of an 'information'-related components that progressively emerges in stock markets returns can be framed within the DDG model originally proposed by Campbell and Shiller (1988). This model uses a log-linear approximation to the definition of returns on the stock market.

Under the assumption of stationarity of the log of price/dividend ratio, total stock market returns, r^s_{t+1}, can be approximated by the relation derived by linearizing $\ln\left(1+e^{pd_{t+1}}\right)$ around its steady-state value P/D:

$$r^s_{t+1} = \Delta d_{t+1} + k - \rho\left[dp_{t+1} - \overline{dp}\right] + \left[dp_t - \overline{dp}\right] \tag{1}$$

where $\rho = \frac{P/D}{1+P/D}$, $\overline{dp} = \ln(D/P)$. By iterating (1) forward for m periods, and taking expectations, we have:

$$E_t \sum_{j=1}^{m} \rho^{j-1}\left(r^s_{t+j}\right) = dp_t - \overline{dp} + E_t \sum_{j=1}^{m} \rho^{\,j-1}\left(\Delta d_{t+j}\right) + \rho^m E_t\left[dp_{t+m+1} - \overline{dp}\right] \tag{2}$$

Equation (2) states that, for m sufficiently large such that $\rho^m E_t(dp_{t+m+1}) \simeq 0$, long-run stock market expected returns $\left(E_t \sum_{j=1}^{m} \rho^{j-1}(r^s_{t+j})\right)$ depend on the deviation current dividend-price from a multiple of its steady-state value and on future expected dividend growth $\left(E_t \sum_{j=1}^{m} \rho^{j-1}\left(\Delta d_{t+j}\right)\right)$.

The DDG model delivers a number of predictions.

(i) Under the maintained hypothesis that stock market returns and dividend-growth are covariance stationary, Equation (2) restates that the log of the price/dividend ratio is stationary (the log of price and the log of dividend are cointegrated with a (−1,1) cointegrating vector), and that deviations of (log) prices from the common trend in (log) dividends summarize expectations of either stock market returns, or dividend growth or some combination of the two. In other words the permanent, 'information'-related, component in stock market prices should be well captured by dividends.

(ii) The forecasting performance of the dividend yield for stock market returns crucially depends on its forecasting performance for dividend growth. Note that when the dividend yield predicts expected dividend growth perfectly, then returns become not predictable within the DDG model. Interestingly, this is the only case in which the DDG model implies no-predictability of stock market returns.

(iii) However, the model implies predictability of 'long-run' returns. Equation (2) clearly states that the dividend/price ratio should have predictive power for m-period ahead stock market returns (and/or dividend growth), for sufficiently large m so that the transversality condition can be applied and the last term, capturing 'noise', becomes negligible. The model also implies that the degree of predictability should increase with the forecasting horizon. In other words, noise dominates short-run stock market fluctuations but as the forecasting horizon gets longer, a role for information progressively emerges.

The empirical investigation of the DDG model has established a few empirical results:

(i) dp_t is a very persistent time-series and forecasts stock market returns and excess returns over horizons of many years (Campbell and Shiller 1988; Fama and French 1988; Cochrane 2001, Ch. 20; Cochrane 2008). The dividend/price ratio has a very long-memory for 'noise'.

(ii) dp_t does not have important long-horizon forecasting power for future discounted dividend growth (Campbell 1991; Campbell et al. 1997; Cochrane 2001, 2008).

(iii) The very high persistence of dp_t has led some researchers to question the evidence of its forecasting power for returns, especially at short horizon. Careful statistical analysis that takes full account of the persistence in dp_t provides little evidence in favor of predictability of stock market returns and excess returns based on the log dividend/price ratio (Nelson and Kim 1993; Stambaugh 1999; Goyal and Welch 2003; Valkanov 2003; Ang and Bekaert 2007; Boudoukh et al. 2008; Goyal and Welch 2008). Structural breaks have also been found in the relation between dp_t and future returns (Neely and Weller 2000; Paye and Timmermann 2006; Rapach and Wohar 2006).

(iv) A recent strand of the empirical literature has related the contradictory evidence on the DDG model to the potential weakness of its fundamental hypothesis that log dividend/price ratio is a stationary process (Lettau and Van Nieuwerburgh, 2008, LVN henceforth). LVN use a century of US data to show evidence on the breaks in the constant mean $\overline{dp}$.

The essence of the debate on the predictability of long-horizon stock market returns using the dividend/price ratio as the 'information' variable is illustrated by Figure 2 that reports the US dividend/price ratio along with the 10-year annualized stock market returns.

The figure shows the presence of some comovement between the two variables which is somewhat limited by the fact that the dividend–price shows a very high degree of persistence that does match the mean reversion of the returns. This high degree of persistence contradicts one of the crucial hypothesis of the DDG model, and it is at the root of the debate on the robustness of the statistical evidence on the predictability of stock market returns.

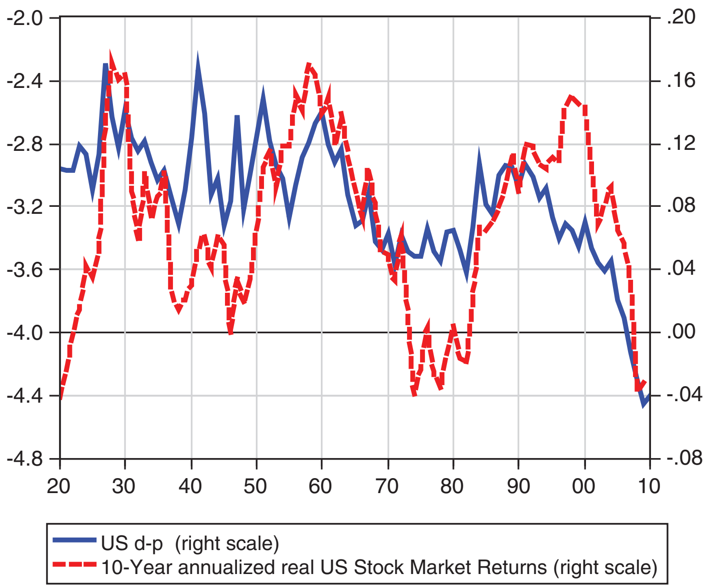

Figure 2 Information and the dividend/price ratio.

3 Is there a role for demographics?

Intuitive reasoning hints at demographics as a natural candidate to explain the long-run behavior of the stock market, while it is difficult to imagine a relationship between high-frequency fluctuations in stock market prices and a slow-moving trend determined by demographic factors.

In fact, a theoretical model by Geanakopoulos et al. (2004, henceforth, GMQ) predicts that a specific demographic variable, MY, the ratio of middle-aged to young population, explains fluctuations in the dividend yield. GMQ consider an overlapping generation model in which the demographic structure mimics the pattern of live births in the US that have featured alternating 20-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior, which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired, plays an important role in determining equilibrium asset prices. Consumption smoothing by the agents, given the assumed demographic structure requires that when the MY ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. As the dividend/price ratio is negatively related to fluctuations in prices, the model predicts a negative relation between this variable and MY. Favero et al. (2010), take the GMQ model to the data via the conjecture that fluctuations in MY could capture a slowly evolving mean in the dividend/price ratio within the DDG model, to find strong evidence in favor of using this variable together with the dividend/price ratio in long-run forecasting regressions for stock market returns. Interestingly, MY is shown to dominate alternative approaches proposed in the literature to capture an evolving mean in the dividend/price ratio, such as, $dp_{\mathrm{t}}^{\mathrm{LVN}}$, the (log) dividend/price corrected for breaks in LVN and $dp_{\mathrm{t}}^{\mathrm{BMRR}}$, the cash flow-based net payout yield (dividends plus repurchases minus issuances) proposed by Boudoukh et al. (2007). Also the performance in predicting long-run stock market returns is shown to be superior to that of all the traditionally adopted financial ratios, such as the detrended short-term interest rate (Campbell 1991; Hodrick 1992), the log dividend earnings ratio and the log price earning ratio (Lamont 1998), the spread of long-term bond yield (10Y) over 3M Treasury bill, and the spread between the BAA and the AAA corporate bond rates. In fact, the best forecasting model for real stock market returns found by Favero et al. (2010), is the one combining MY_t with cdy_t, a variable constructed by Lettau and Ludvigson (2005) to capture excess consumption with respect to its long-run equilibrium value. This evidence is taken as strongly supportive of the DDG model with an evolving mean, determined by MY_t. In fact, the model predicts that long-horizon returns should depend on the deviations of the dividend/price ratio from its mean and on long-run dividend growth. MY_t models the mean of the dividend/price ratio while cdy_t is a predictor of long-horizon dividend growth, confirming the evidence in Lettau and Ludvigson (2005). Finally, fluctuations in MY are also capable of explaining the breaks in the dividend/price ratio found via statistical analysis by LVN. We report in Figure 3 the dividend/price ratio and MY to show how, in line with the predictions of the GMQ, a negative relation between MY and the dividend price is present in the data.

The demographic variable captures the slowly evolving information component in the dividend/price ratio and helps a better identification of the information component from the noise component. To better illustrate this point consider the following small structural model that extends the DDG framework to allow for an explicit role of MY.

$$r^s_{t+1} = \Delta d_{t+1} - \rho\left[dp_{t+1} - \overline{dp}_{t+1}\right] + \left[dp_t - \overline{dp}_t\right] \tag{3}$$

$$\Delta d_{t+1} = \varepsilon_{1t+1} \tag{4}$$

$$dp_{t+1} = \varphi_{22} dp_t + \varphi_{23} \mathrm{MY}_{t+1} + \varepsilon_{2t+1} \tag{5}$$

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{matrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{matrix} \right]$$

Equation (3) defines real returns using the log-linearized approximation proposed by Campbell and Shiller (1988), and has no error

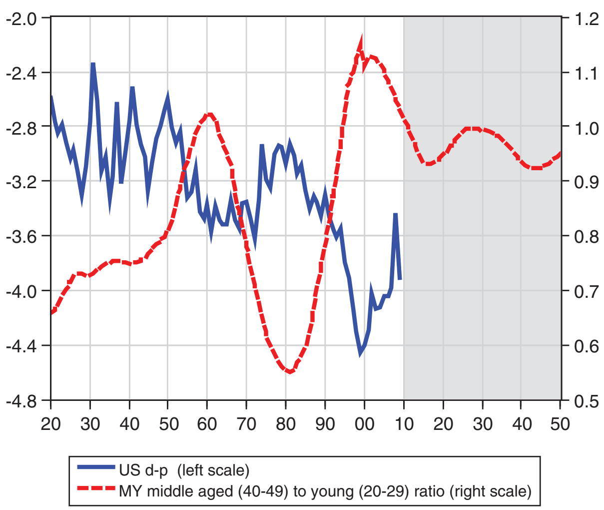

Figure 3 Dividend/price ratio and Demographics (MY).

attached to it.[2] The specification of this equation differs form the standard linearization only in that the equilibrium long-run mean around which the dividend/price ratio is linearized is not constant. In fact, the dividend/price ratio itself depends on the age structure of the population, MY, a slowly evolving highly predictable variable (the Bureau of Census makes available through its web page projections of this variable up to 2050). Such a modification is justified by the theoretical model of Geneakopoulos et al. (2004), and by the empirical evidence provided in Favero et al. (2010). MY constitutes the information component of the dividend/price ratio and there is no uncertainty attached to it: we take it as an exogenous variable whose path for the relevant future is known. However, the dividend/price is also affected by some short-term idiosyncratic noise ε_{2t+1}. In the process of generating dividends we take $|\varphi_{22}| < 1$: dividend/price are mean reverting toward their long-run trend determined by the information variable and the effect of the noise shock on the process is only temporary. In fact, our empirical results show that the speed of mean reversion of the dividend/price ratio toward its long-run mean determined by demographic trends is much higher than that of the dividend/price process itself and there is little doubt on the stationarity of dividend/price ratio around a demographic trend. Importantly, stability analysis conducted via the Quandt–Andrews test (Andrews 1993) for unknown breakpoints confirms the evidence of instability discussed in LVN for the parameters of a simple autoregressive process for dp_{t+1}, while the null of no-break cannot be rejected when the autoregressive model is augmented with MY_{t+1}.[3] There is a second shock in the model specification, the innovation to real dividend growth ε_{1t+1}. This is a simple parameterization for the dividend growth process that is fully consistent with the evidence of very little predictability of dividend growth.

By solving (3) forward we obtain:

$$\sum_{j=1}^{m} \rho^{j-1}\left(r^s_{t+j}\right) = \left[dp_t - \overline{dp}_t\right] + \sum_{j=1}^{m} \rho^{j-1}\left(\Delta d_{t+j}\right) - \rho^m\left[dp_{t+m+1} - \overline{dp}_{t+m+1}\right] \tag{6}$$

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[2]
$$r^s_{t+1} = \ln(1 + H^s_{t+1}) = \ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right),\ dp_t = \ln\left(\frac{D_t}{P_t}\right)$$

[3] The Quandt–Andrews test for unknown breakpoints (with a trimming of 10% of the observations) takes a maximum Wald statistic of 20.06 in 1954 with a tail probability of 0.001 for the parameters in the autoregeressive process for dp. When the same test is applied to Equation (5) the Maximum Wald statistic takes a value 11.68 with a tail probability of 0.076 and the null of parameters stability cannot be rejected.

Equation (6) clearly shows that the model implies the predictability of 'long-run' returns. In fact, the equation states that the deviations of the dividend/price ratio from its equilibrium value should have predictive power for m-period ahead stock market returns (and/or dividend growth), for sufficiently large m so that the transversality condition can be applied and the last term in the equation becomes of negligible importance. To bring the model to the data, we assume that the relevant linearization value for computing returns from time t to time $t+m$ is the conditional expectation of the dividend yield for time $t+m$, given the information available at time t. We then have

$$\sum_{j=1}^{m} \rho^{j-1}\left(r^{s}_{t+j}\right) = dp_t - \left[\varphi_{22}^{m} dp_t + \sum_{j=1}^{m} \varphi_{22}^{j-1}\varphi_{23}\mathrm{MY}_{t+m+1-j}\right] + u_{t+m}$$

$$= \left(1-\varphi_{22}^{m}\right)dp_t - \sum_{j=1}^{m}\varphi_{22}^{j-1}\varphi_{23}\mathrm{MY}_{t+m+1-j} + u_{t+m} \qquad (7)$$

$$u_{t+m} = \sum_{j=1}^{m}\rho^{j-1}\left(\varepsilon_{1t+j}\right) - \rho^{m}\sum_{j=1}^{m}\varphi_{22}^{j-1}\varepsilon_{2t+m+1-j}$$

Note that the relevance of the noisy component in the distribution of m-period returns natural decreases with the horizon: as the horizon gets longer the mean reversion of the dividend-yield process around the information variable makes the informative content of this variable dominant. The speed at which the noise disappears depends on the speed of mean reversion of the dividend process and on the discount parameter ρ. However, even for values of ρ close to unity, the mean reversion in dividend/prices is sufficient to cause a cancellation of the noise ε_{2t}. The second component of the noise in m-period returns is the uncertainty in the dividend process that dies out much more slowly than the effect of the noise ε_{2t+j} and it becomes persistent when ρ approaches the unit value. Equation (7) implies that the fit of direct predictive regressions projecting returns at different horizon on the information available at time t should improve with the horizon. It also predicts that the residuals such as predictive regressions have a moving-average component that should be taken care of in estimation. This is a well-known result (see for example, Valkanov 2003). Interestingly, the model also predicts that the coefficient on the dividend yield in the projections of long-horizon returns on this variable should be increasing with the horizon. Finally, the direct regression of returns at different horizons on the relevant predictors should deliver the following term structure of stock market risk at horizon m:

$$\begin{aligned}
\sigma_r^2(m) &= \psi_1(m)\sigma_1^2 + \psi_2(m)\sigma_2^2 \\
\psi_1(m) &= \frac{1}{m}\sum_{j=1}^{m}\rho^{2(j-1)} \\
\psi_2(m) &= \frac{\rho^{2m}}{m}\sum_{j=1}^{m}\varphi_{22}^{2(j-1)},
\end{aligned} \tag{8}$$

which is downward sloping as the effect of the noisy component of the dividend/price dies out as the horizon m increases.

4 The empirical evidence

We measure the term structure of stock market risk by estimating, via Generalized Method of Moments (GMM) the following 'structural' system of 11 equations:[4]

$$\begin{aligned}
\frac{1}{\sqrt{m}}\sum_{j=1}^{m}\left(r_{t+j}^{s}\right) &= \delta_0 + \frac{1}{\sqrt{m}}\left(1-\varphi_{22}^{m}\right)dp_t - \frac{\varphi_{23}}{\sqrt{m}}\left(\sum_{j=1}^{m}\varphi_{22}^{j-1}\mathrm{MY}_{t+m+1-j}\right) + u_{t+m} \\
m &= 1,\ldots,10 \\
dp_{t+1} &= \varphi_{20} + \varphi_{22}dp_t + \varphi_{23}\mathrm{MY}_{t+1} + \varepsilon_{2t+1}
\end{aligned} \tag{9}$$

The specification of (9) slightly differs from the model in that we use as a dependent variable the unweighted annualized period returns ($\rho = 1$). This is because the objective of our exercise is to compare the term structure of stock market risk obtained by direct regression and by iterative multi-step iterated Vector Auto Regressive (VAR)-based forecasts. To assess the potential cost of the approximation introduced by using $\frac{1}{\sqrt{m}}\sum_{j=1}^{m}(r_{t+j}^{s})$ instead of $\sum_{j=1}^{m}\rho^{j-1}(r_{t+j}^{s})$ an unrestricted version of (9) is also estimated to perform a test of the validity of the relevant restrictions:

$$\begin{aligned}
\frac{1}{\sqrt{m}}\sum_{j=1}^{m}\left(r_{t+j}^{s}\right) &= \delta_0 + \frac{\delta_{1m}}{\sqrt{m}}dp_t + \frac{\delta_{2m}}{\sqrt{m}}\left(\sum_{j=1}^{m}\varphi_{22}^{j-1}\mathrm{MY}_{t+m+1-j}\right) + u_{t+m} \\
m &= 1,\ldots,10 \\
dp_{t+1} &= \varphi_{20} + \varphi_{22}dp_t + \varphi_{23}\mathrm{MY}_{t+1} + \varepsilon_{2t+1}
\end{aligned} \tag{10}$$

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4 Our 'structural' estimation is similar to that by Van Binsbergen and Koijen (2010) with two main differences: equations at all relevant horizons are simultaneously estimated and all variables included in the model are observable.

Note that (9) and (10) are both specified with $\frac{1}{\sqrt{m}}\sum_{j=1}^{m}(r_{t+j}^{s})$ as the dependent variable to obtain directly the conditional annualized standard error of returns from the standard error of the regression.

Following Favero and Tamoni (2010), we estimate the model on a data set of annual observations for the period 1910–2008. The data are from Goyal and Welch (2008),[5] who provide detailed descriptions of the data and their sources. Stock returns are measured as continuously compounded returns on the S&P 500 index, including dividends. To compute real returns, we calculate inflation rate from the Consumer Price Index (all urban consumers). The predictor for the equity premium is the dividend/price ratio, computed as the difference between the log of dividends paid on the S&P 500 index and log of stock prices (S&P 500 index), where dividends are measured using a 1-year moving sum.

The results of the estimation are reported in Table 1.

The estimation of the restricted model shows an highly significant effect of MY both in the equation for dp_t and in all 10 predictive regressions. The performance of the restricted model, which estimates only two parameters in addition to 11 constants, is very similar in term of adjusted R^2 and standard error of the equations to that of the unrestricted model that estimates 20 more parameters and the restrictions are not rejected by the relevant chi-square test. The estimates of the parameters φ_{22} and φ_{23} show that demographics are clearly significant in explaining the dividend/price ratio and that the dividend/price ratio is clearly mean reverting around a mean determined by MY. The term structure of stock market risk described by the estimation of the structural system of direct predictive regressions[6] is steeply downward sloping as it can be read directly off the standard errors of regressions reported in Table 1.

Importantly, the profile of stock market risk is much steeper than that obtained by Campbell and Viceira (2002), using iterated forecasts from a VAR that does not include any demographic variable.

The importance of MY for capturing the relevant information to predict long-horizon returns is visually illustrated in Figure 4 in which we report the 10-year stock market returns and the deviation of the dividend/price ratio from its slowly time-varying mean determined by MY.

5 The data are available at www.bus.emory.edu/AGoyal/Research.html.

6 Two basic alternative strategies are available in the literature for generating multi-period forecasts. The first one is the estimation of a dynamic model for data observed at the highest available frequency, and then use the chain rule to generate a forecast at the desired horizon, h. This is defined as the 'iterated' or 'indirect' approach. The second approach is the estimation a model for the variable measured h-periods ahead as a function of current information. This leads to so-called 'direct' forecasts (see, for example, Marcellino et al. 2006).

Table 1 System estimation (1910–2009)

$$dp_{t+1} = \varphi_{20} + \varphi_{22} dp_t + \varphi_{23} \mathrm{MY}_{t+1} + \varepsilon_{2t+1}$$

UM: $\frac{1}{\sqrt{m}} \sum_{j=1}^{m} (r^s_{t+j}) = \delta_{0m} + \frac{\delta_{1m}}{\sqrt{m}} dp_t + \frac{\delta_{2m}}{\sqrt{m}} (\sum_{j=1}^{m} \varphi_{22}^{j-1} \mathrm{MY}_{t+j}) + u_{t+m} \quad m = 1, \ldots, 10$

RM: $\frac{1}{\sqrt{m}} \sum_{j=1}^{m} (r^s_{t+j}) = \delta_{0m} + \frac{1}{\sqrt{m}} (1 - \varphi_{22}^{m}) dp_t - \frac{\varphi_{23}}{\sqrt{m}} \left(\sum_{j=1}^{m} \varphi_{22}^{j-1} \mathrm{MY}_{t+j} \right) + u_{t+m}$

horizon m in years

		1	2	3	4	5	6	7	8	9	10
UM											
δ_{1m} (t–$stat$)		0.18 (3.87)	0.38 (5.57)	0.48 (5.61)	0.61 (6.96)	0.70 (7.99)	0.73 (7.17)	0.78 (7.25)	0.86 (8.73)	0.89 (7.63)	0.89 (6.15)
δ_{2m} (t–$stat$)		0.41 (3.23)	0.52 (3.69)	0.56 (3.85)	0.64 (4.28)	0.69 (4.55)	0.70 (4.69)	0.74 (4.78)	0.78 (4.93)	0.81 (4.94)	0.83 (5.01)
φ_{22} (t–$stat$)	0.61 (9.21)										
φ_{23} (t–$stat$)	−0.83 (−3.79)										
RM											
φ_{22} (t–$stat$)	0.76 (19.31)										
φ_{23} (t–$stat$)	−0.53 (4.41)										
χ^2_{12}	13.45 (0.34)	χ^2_{20}	17.19 (0.64)								
σ_{DepVar}		0.195	0.198	0.187	0.185	0.181	0.174	0.172	0.173	0.171	0.168
$\sigma_{u_{t+m}}$	UM	0.188	0.179	0.164	0.152	0.140	0.131	0.125	0.118	0.112	0.109
$\sigma_{u_{t+m}}$	RM	0.189	0.179	0.164	0.152	0.141	0.133	0.127	0.120	0.115	0.112
$adj\ R^2$	UM	–	0.18	0.24	0.33	0.41	0.44	0.48	0.54	0.57	0.58
$adj\ R^2$	RM	0.06	0.18	0.23	0.32	0.40	0.42	0.46	0.52	0.54	0.55

This table compares the univariate Ordinary Least Squares (OLS) long-horizon regression coefficients to the GMM estimates that impose the present-value restriction. The estimation is by GMM, where the moments are the OLS normal conditions. Standard errors are by Newey–West with optimal bandwidth selection. The first-stage weighting matrix is the identity matrix. σ_{DepVar} is the annualized unconditional standard deviation. $\sigma_{u_{t+m}}$ is the annualized conditional standard deviation of the compounded (over m periods) returns, i.e. our measure of stock market risk. The effective sample period is 1910–2009.

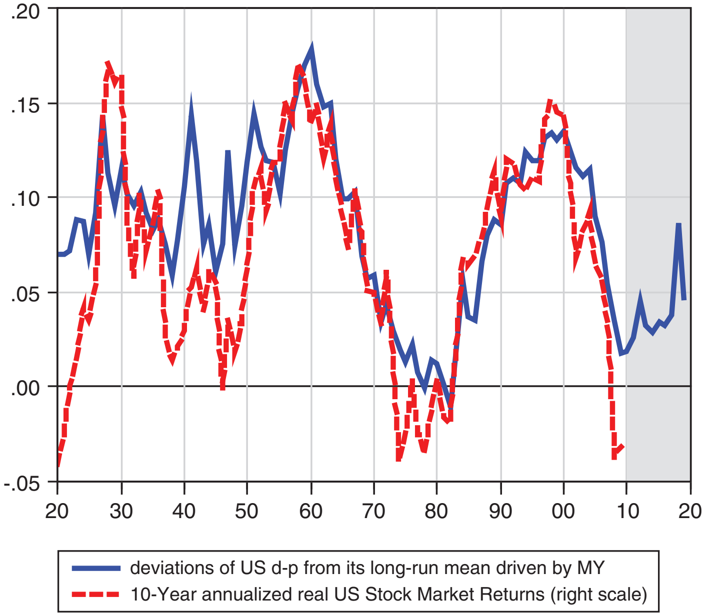

Figure 4 A role for demographics.

Figure 4 also illustrates an additional interesting feature of MY: long-run forecasts for this (exogenous) variable are readily available. In fact, the Bureau of Census (BoC) provides projections up to 2050 for MY. In fact, Favero et al. (2010), exploit the exogeneity and the predictability of MY to project the equity risk premia up to 2050. The simulations point to an average equity risk premium of about 5% for the next 40 years. Importantly, the exercise also shows that the good within-sample performance of the model is confirmed by out-of-sample analysis of the forecasting performance.

5 Demographics and the term structure of stock market risk

The empirical results illustrated in the previous section show that the emergence of relative importance of 'information' versus 'noise' in the determination of stock market returns at different horizons generates a downward sloping term structure of stock market risk. There are two main ingredients to our downward sloping term structure of stock market risk. First, we use a demographic variable to capture the slow-moving information component in the dividend/price ratio and in stock market returns.

Second, we derive the term structure of stock market risk via direct regressions based on the structural estimation of a forward-looking specification consistent with the DDG model.

Interestingly, the slope of the term structure of stock market risk is a topic heavily debated in the recent literature. Campbell and Viceira (2002, 2005) estimated a VAR model for returns and the dividend yield and used VAR-based multi-period iterated forecasts to find that the conditional variance of stock return does not grow in proportion with the investment horizon but it grows more slowly. As a consequence the term structure of stock market risk is downward sloping. However, this result has been recently questioned by Pastor and Stambaugh (2008, 2009), who show that, allowing for coefficient uncertainty and imperfect predictors[7] in a Campbell-Viceira type of VAR, the conditional variance of stock returns does increase with the horizon and it can even exceed the unconditional variance and the current variance.

Our empirical results show that such a controversy is a by-product of the use of the VAR framework that is not capable, by its nature, to eliminate the effect of the noise component at lower frequencies. The intuitive explanation for this result is as follows. VAR are specified for high-frequency, one-period, returns and conditional expectations for multi-period returns are then obtained via the aggregation of multi-step ahead one-period iterated forecasts, then conditional variances are derived from the backward solution of the reduced form model. We have shown that, when predictors for stock market return can be decomposed in a slow-moving information component and a noise component, the forward solution of the DDG model (Campbell and Shiller 1988) would naturally progressively eliminate the noise component as the horizon increases. Favero and Tamoni (2010) show that direct regressions of returns at different horizons on the relevant predictors capture this feature of the model, while VAR-based multi-period iterated forecasts do not. Our derived term structure of stock market risk is downward sloping and it is steeper than that estimated by Campbell–Viceira for two reasons. First, the information component in the dividend/price ratio is explicitly modeled by making it function of a slow-moving highly predictable demographic variable. As a consequence, the speed of mean reversion of deviation of the dividend/price from its demographic trend is much faster than that of the dividend/price itself and the elimination of the effect of the 'noise' component occurs more quickly as the forecasting

[7] The imperfect predictors problem occurs when the relevant 'true' predictors are unobservable and they are function of the observable predictors and some measurement error. The variance of the measurement error adds to the uncertainty when multi-step iterated prediction are constructed.

horizon increases. Second, the use of direct regression does naturally progressively eliminate the relevance of the noise component from the distribution of returns at lower frequencies.

Importantly, the measure of the term structure of risk based on the direct regression is very little affected by the 'imperfect predictors' problem pointed out by Pastor–Stambaugh. In fact, the existence of imperfect predictors would change the interpretation but not the shape of the term structure based on direct regression. Only parameter uncertainty could be an issue, but this is an issue of a certainly limited relevance as the term structure of risk derived from our structural system is based on the estimation of very few parameters, all them very well determined.

6 An agenda for further research

Our empirical results on the predictability of long-horizon returns and on the slope of stock market risk clearly identify demographics as a natural input into the optimal asset allocation decision of a long-horizon investor. However, the importance of demographics in modeling the information component of stock prices opens a number of avenues for further research that goes beyond the asset allocation implications. First, the importance of MY in determining the slow-moving component of the dividend/price ratio depends on the saving decisions of different generations. Therefore, the success of demographics in modeling the permanent component of stock prices should imply its effectiveness also in modeling the permanent component of GNP, that Cochrane (1994) has related to consumption and saving decisions. The adoption of a common demographics-related factor for modeling the permanent and transitory component of GNP and stock prices should also provide the basis for finding the correct frequency for the construction of a macro-finance model. Our proposed decomposition of stock market fluctuations in a temporary, 'noise'-related component and in a permanent 'information'-related component naturally raises the question of the correct frequency at which one should consider the data for constructing a model to capture the interaction between stock market returns and macroeconomic variables. The simultaneous modeling of the transitory and permanent component of GNP and stock prices should help in finding the right empirical answer to this question.

Second, in the GMQ model bond and stock are perfect substitutes, therefore the evaluation of the performance of MY_t in forecasting yields to maturity of long-term bonds seems a natural extension of our empirical investigation. In fact, the debate on the so-called Federal Reserve (FED) model (Lander et al. 1997) of the stock market, based on a long-run relation between the price-earning ratio and the long-term bond yield, brings some interesting evidence on this issue. The FED model is based on the equalization, up to a constant, between long-run stock and bond market returns. This feature is shared by the GMQ framework, and it requires a constant relation between the risk premium on long-term bonds and the risk premium on stocks. It has been shown that, although the FED model performs well in period where the stock and bond market risk premia are strongly correlated, some measure of the fluctuations in their relative premium is necessary to model periods in which volatilities in the two markets have been different (see, for example, Asness 2003). As a consequence, to put MY_t at work to explain the bond yields, some modeling of the relative bond/stock risk premia is also in order. We consider this as an interesting extension that might also be able to relate to demographics, the persistent component in bond yields whose empirical relevance has been recently highlighted by Fama (2006).

Third, what is the international evidence on the relation between demographics and asset prices? Our empirical results are so far limited to the US case only, but it is important to assess the effect of extending the model to other countries. The importance of such step goes beyond a natural robustness analysis. In fact, with the progressive integration of world financial markets, there is no doubt that the relevant age structure of population to determine asset price equilibria cannot be anymore than that of a single country.

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