Implications of Return Predictability for Consumption Dynamics and Asset Pricing*

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Abstract

Two broad classes of consumption dynamics - long-run risks and rare disasters - have proven successful in explaining the equity premium puzzle when used in conjunction with recursive preferences. We show that bounds a-là Gallant, Hansen and Tauchen (1990) that restrict the volatility of the Stochastic Discount Factor by conditioning on a set of return predictors constitute a useful tool to discriminate between these alternative dynamics. In particular we document that models that rely on rare disasters meet comfortably the bounds independently of the forecasting horizon and the asset classes used to construct the bounds. However, the specific nature of disasters is a relevant characteristic at the 1-year horizon: disasters that unfold over multiple years are more successful in meeting the predictors-based bounds than one-period disasters. Instead, over a longer, 5-year horizon, the sole presence of disasters - even if one-period and permanent - is sufficient for the model to satisfy the bounds. Finally, the bounds point to multiple volatility components in consumption as a promising dimension for long-run risks models.

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1 Introduction

This paper shows that the Stochastic Discount Factor (SDF, henceforth) variation implied by return predictability is a useful moment to discriminate among leading asset pricing models that feature the same preference specification, but different consumption dynamics. This is important since multiple frameworks (notably long-run risks, rare disasters, and habit) have emerged as leading contenders to explain the equity premium puzzle (Mehra and Prescott, 1985), the volatility puzzle (Shiller, 1982), and other features of the aggregate stock market. The surge of several models that fit essentially the same aggregate moments begs the question as to which tools are suitable to disentangle the proposed asset pricing frameworks.

This paper exploits the predictability of asset returns to construct predictors-based variance bounds, i.e. bounds on the variance of those SDF that price a given set of returns conditional on the information contained in a vector of returns predictors. We focus our analysis on models based on recursive utility a-la Epstein and Zin (1989) and Weil (1990). This restricts our attention to two classes of models: long-run risks and rare disasters. While focusing on a single functional form for the SDF, our analysis investigates a wide range of specification for the state dynamics. We explore differences across frameworks - the emphasis in the long run risks model is on the first two conditional moments of consumption, while the rare disasters model puts emphasis on higher-order moments - and within the same framework - for example we look at multiperiod disasters with partial recovery against the case in which disasters are completely permanent. Using the requirement for the variance of the model-implied SDF to be larger than the variance dictated by the predictors-based variance bounds, we are able to discriminate between different specifications for the state variables. Formal statistical tests favor rare disasters over long run risks. The essential ingredient for the success of the rare disaster is the multi-period nature of disasters. The presence of recoveries after disasters is, instead, less important. Within the long-run risks framework, instead, our bounds favor a specification featuring multiple sources of stochastic volatility in the state dynamics.

We compute predictors-based bounds as in Gallant, Hansen, and Tauchen (1990) and Bekaert and Liu (2004). To make the bounds operational, we specify the set of asset returns to be forecast, the predictors, and the forecasting horizons. We construct predictor based bounds for three alternative investment horizons - namely $h = 1, 4, 20$ quarters, and two alternative sets of returns: a first set that includes the equity market plus the returns from rolling a 3-month Treasury Bills over the horizon h, and a second set that adds the returns from equity value-growth portfolios to the first set. We employ as stock market predictors the dividend-price and consumption-wealth ratios, and as predictors of value strategies the portfolio's book-to-market ratios. Despite selecting only predictors that are economically motivated by accounting identities (see Campbell and Shiller (1988) for the dividend-price ratio, Campbell and Mankiw (1989) and Lettau and Ludvigson (2001) for the consumption-wealth ratio, and Vuolteenaho (2002) for the book-to-market ratio), our bounds are sharp enough to discriminate the state dynamics across, and within, models. Also, looking at multiple horizons is important: whereas the intermediate $1-$year horizon allows us to emphasize the importance of the multi-period nature of disasters versus the presence of recoveries, the long-horizon dilutes these differences.

Within the long-run risk framework we investigate both the Bansal, Kiku, and Yaron (2016) model, where the state variables are the first two conditional moments of log consumption growth, and the recent specification described in Schorfheide, Song, and Yaron (2018), in which the set of state variables is expanded to account for three separate volatility components: one governing the dynamics of the persistent cash flow growth component, and the other two controlling temporally independent shocks to consumption and dividend volatility. Within the rare disasters class we concentrate on the framework of Nakamura, Steinsson, Barro, and Ursúa (2013) since this specification allows for the possibility of recoveries after disasters, and the notion that disasters may unfold over several years, thus addressing recent critiques (see Constantinides and Ghosh, 2008; and Gourio, 2008a). Noteworthy, by suitably choosing the parameters space the Nakamura et al. (2013) model embeds the classical formulation of Rietz (1988) and Barro (2006) where disasters are permanent and occur in a single period.

Broadly speaking our work is related to the recent literature in asset pricing that looks at which of the many models that fit essentially the same aggregate moments is the "right" description of the underlying risk. Hansen and Scheinkman (2009) and Borovicka, Hansen, Hendricks, and

Scheinkman (2011) propose to compare structural models by computing the term structure of (model-implied) exposures of cash flows to shocks, as well as the compensation for these exposures. Zviadadze (2017) builds on their results, and propose to compare models using impulse response function (IRF) of expected returns to economic shocks. She concludes that volatility shocks are important for the long-run risks model to replicate the shape and level of expected equity returns impulse responses in the data. This result is consistent with our finding that multiple volatility components (like in Schorfheide et al. (2018)) are needed for a model to satisfy comfortably the restrictions on the variance of the SDF dictated by our predictors-based bounds at multiple horizons. Since both our predictors-based bounds and the IRFs proposed by Zviadadze (2017) rely on the predictability of asset returns, it is not surprising that we achieve similar conclusions. Importantly, whereas our bounds are model free, Zviadadze (2017) uses information from the model to identify the IRFs in the data. Entropy bounds provide another tool to discriminate across asset pricing models. For example, Backus, Chernov, and Zin (2014), Bakshi and Chabi-Yo (2012) and Ghosh, Julliard, and Taylor (2011) build on the recent information-theoretic literature to restrict the admissible regions for the SDF, and its permanent and transitory components. Interestingly, the one-period entropy restricts high-order cumulants of the *log* SDF. We instead restrict first and second moments of the (raw, not log) SDF. Also, whereas the multiperiod entropy exploits information from bond yields, we exploit information only from equity returns to discriminate across models.

Our work is related to Cochrane and Hansen (1992), who have been the first to look systematically at Hansen and Jagannathan (1991) bounds across horizons to ascertain the relative performance of different asset pricing models. The present paper extends their analysis in several directions. First, we account explicitly for the effect of returns predictability at different horizons. Moreover, we highlight the interplay between preferences, dynamics and horizons in a wider variety of models. From this standpoint, in particular, we extend the comparative horizons analysis of Cochrane and Hansen (1992) to models that explicitly contain a low frequency component, such as the long-run risk and rare disaster models, and investigate if and how different specifications of this components can make asset pricing puzzles less pronounced at longer horizons. Finally, we account for estimation uncertainty in both the bounds and the model-implied SDFs.

Kirby (1998) provides an explicit link between linear predictability and the Hansen and Jagannathan (1991) bounds. Whereas Kirby (1998) investigates whether the ability of predictors to forecast a given set of return is correctly priced by *some* rational asset pricing model, in the sense that there exist SDFs that price correctly those dynamic strategies which condition on the predictors, our interest here is different: we want to exploit the informational content of *a given set of predictors* to investigate the potential of *a given asset pricing model* to price *a given set of returns*.

Kan and Robotti (2016) compare the long-run risks, the Campbell and Cochrane (1999) habit, and disaster risk models by means of unconditional SDF bounds as an empirical example to illustrate the importance of reporting confidence intervals for the Hansen and Jagannathan (1991) bounds instead of only presenting their point estimates. Their analysis is silent on the use of the bounds to discriminate alternative state dynamics for a fixed set of preferences (in particular Kan and Robotti (2016) do not investigate the Schorfheide et al. (2018) specification with multiple volatility components, nor they compare multiperiod disasters against one-period, permanent disaster models). Most importantly, the Kan and Robotti (2016) analysis is unconditional, and it is not informative about the role of predictability in asset returns, and thus about the role of the forecasting horizon, for constructing the bounds and separating models.

The rest of this paper is organized as follows. Sections 2 discusses the interplay between predictability, bounds and asset pricing models from the standpoint of the predictors-based variance bounds. Section 3 documents the existence of significant predictable variation in aggregate stock market and value-growth equity portfolios, and shows how conditioning information plays an important role in the construction of the bounds at different horizons. We then assess whether various SDF specifications are consistent with the predictors-based bounds. Section 4 concludes. The online Appendix reports details on the estimation of the different asset pricing models and investigates the impact of misspecification of the conditional moments on our results.

2 Predictability, Variance Bounds, and Asset Pricing

In this section we briefly summarize the bounds implied by asset returns on the volatility of SDFs, in both their unconditional and conditional form. Assuming that conditioning is based on a set of return predictors, we show how to use the conditional variance bounds to assess two leading classes of asset pricing models, long run risks and rare disasters. For completeness, we briefly review the main characteristics of the consumption dynamics and the SDFs associated with these two classes of models.

2.1 Predictability of returns and variance bounds

Suppose that N assets are traded at a given time t, with returns vector R_{t+h}, where h is the investment horizon. We denote by μ the vector of unconditional mean returns, and by Σ their unconditional covariance matrix. Recall that an SDF that prices the returns R_{t+h} is a random variable m_{t+h} that satisfies the pricing equation

$$E\left(m_{t+h}R_{t+h}\right) = \mathbf{1} \ , \tag{1}$$

where $\mathbf{1}$ is the unit vector. Letting then $E\left(m_{t+h}\right) = \nu$, in their seminal paper Hansen and Jagannathan (1991), showed that the variance $Var\left(m_{t+h}\right)$ of any SDF with mean ν satisfies the following relationship:

$$Var\left(m_{t+h}\right) \geqq \left(\mathbf{1} - \nu\mu\right)^{T} \Sigma^{-1} \left(\mathbf{1} - \nu\mu\right) \equiv \sigma^{2}_{HJ}\left(\nu\right) \ , \tag{2}$$

where the quadratic form in ν appearing at the right-hand-side takes the name of unconditional Hansen-Jagannathan bound, or HJ bound for short. To establish this fact Hansen and Jagannathan (1991) show that

$$m^{HJ}_{t+h} \equiv \nu + \left(\mathbf{1} - \nu\mu\right)^{T} \Sigma^{-1} \left(R_{t+h} - \mu\right) \ , \tag{3}$$

is not only a legitimate SDF, but it is in fact the SDF with minimum variance among all the SDFs with mean ν.

Next, we consider the case in which a vector Z_t predicts the returns R_{t+h}, i.e. $Var(\mu_t) > 0$ where $\mu_t \equiv E\left[R_{t+h}\left|Z_t\right.\right]$. We concentrate our attention on the SDFs that price returns conditionally on the realizations of the predictors Z_t, that is

$$E\left(m_{t+h}R_{t+h}\left|Z_t\right.\right) = \mathbf{1} \ . \tag{4}$$

Importantly, while the law of iterated expectations guarantees that any such SDF also prices the returns R_{t+h} unconditionally, in general an SDF that satisfies the unconditional pricing equation fail to price correctly the returns when the information in the predictors Z_t is accounted for. Therefore, the set of SDFs that price returns conditionally on Z_t is a subset of the set of SDFs that satisfy the unconditional pricing equation.

We fix now an SDF m_{t+h} that prices returns conditional on Z_t, we let $\nu_t \equiv E\left[m_{t+h}\left|Z_t\right.\right]$ and $\nu = E\left(\nu_t\right)$, and we characterize the SDF m^Z_{t+h} with minimum variance among all the SDFs that price returns conditionally and whose mean is ν. The solution to this problem is supplied by Gallant et al. (1990), who show that m^Z_{t+h} is obtained by replacing in (3) the unconditional moments with the conditional ones, i.e.

$$m^Z_{t+h} = \nu_t + \left(\mathbf{1} - \nu_t\mu_t\right)^T \Sigma_t^{-1}\left(R_{t+h} - \mu_t\right) \tag{5}$$

where Σ_t is the conditional variance-covariance matrix.

The predictors-based bound $\sigma^2_Z(v)$ is then defined as the function that maps the mean of m^Z_{t+h} into its variance. In fact, to make this relationship more transparent, Bekaert and Liu (2004) work on the expression of $Var\left(m^Z_{t+h}\right)$ to show that

$$\begin{aligned} \sigma^2_Z(v) \equiv Var\left(m^Z_{t+h}\right) = \ & E\left[\left(\mathbf{1} - \tfrac{\nu-b}{1-d}\mu_t\right)^T\left(\mu_t\mu_t^T + \Sigma_t\right)^{-1}\left(\mathbf{1} - \tfrac{\nu-b}{1-d}\mu_t\right)\right] \\ & -E\left[\left(\mathbf{1} - \tfrac{\nu-b}{1-d}\mu_t\right)^T\left(\mu_t\mu_t^T + \Sigma_t\right)^{-1}\mu_t\right] \end{aligned} \tag{6}$$

where $b = E\left[\mu_t^T \left(\mu_t\mu_t^T + \Sigma_t\right)^{-1} \mathbf{1}\right]$ and $d = E\left[\mu_t^T \left(\mu_t\mu_t^T + \Sigma_t\right)^{-1} \mu_t\right]$. Given v, therefore, $\sigma_Z^2(v)$ is a quadratic form in ν that yields the minimum variance across all SDFs that price the returns conditionally on the information in Z_t and whose unconditional mean is v. It is readily seen that the quadratic form $\sigma_Z^2(v)$ lies above $\sigma_{HJ}^2(\nu)$ for any value of v. As mentioned above, in fact, the law of iterated expectation implies that any SDF which prices returns conditionally on Z_t does so unconditionally as well, hence $E\left(m_{t+h}^Z R_{t+h}\right) = \mathbf{1}$. Since m_{t+h}^{HJ} is the SDF with minimum variance among all the SDFs with mean ν, it follows immediately that $\sigma_Z^2(v) \equiv Var\left(m_{t+h}^Z\right) \geq Var\left(m_{t+h}^{HJ}\right) \equiv \sigma_{HJ}^2(\nu)$, an inequality that in fact holds strictly banning degenerate cases.

To illustrate the effect of predictability on the bounds we report in Panel A of Figure 1 the unconditional bounds along with the conditional bounds based on two predictors, the dividend-price ratio and the consumption-wealth ratio. Interestingly the gap between the unconditional and conditional bounds increases with the holding period as a consequence of the fact that predictability increases with the horizon. In Section 3.3 we will exploit these bounds to compare alternative asset pricing models.

2.2 Predictors-based bounds, consumption dynamics and asset pricing

In a nutshell, a representative-agent asset pricing model is characterized by a couple $\left(X_t, m_{t+h}^X\right)$ where $X_t \subset \Omega_t$ denotes the set of state variables of the model, Ω_t the information set available to the representative agent, and m_{t+h}^X denotes the SDF of the model. Ideally, the state variables should represent a sufficient statistic of the information set Ω_t, in the sense that the following Euler condition should hold

$$E\left(m_{t+h}^X R_{t+h} \middle| \Omega_t\right) = E\left(m_{t+h}^X R_{t+h} \middle| X_t\right) = \mathbf{1} \ . \tag{7}$$

In words, if the state variables are a sufficient characterization of the information set Ω_t then the model SDF m_{t+h}^X should price the returns from managed portfolios that condition on all the available information.

Consider now the situation in which a set $Z_t \subset \Omega_t$ of observable predictors of the returns is available to the optimizing agent. The law of iterated expectation applied to (7) imposes on any model-based SDF m^X_{t+h} the necessary condition

$$E\left(m^X_{t+h}R_{t+h}\middle|\, Z_t\right) = \mathbf{1}\ . \tag{8}$$

Since the predictors-based bound $\sigma^2_Z(v)$ defined in equation (6) identifies a lower bound on the variance of any SDF that prices returns conditional on Z_t, then any model-based SDF must satisfy

$$Var\left(m^X_{t+h}\right) \geq \sigma^2_Z(v)\ . \tag{9}$$

In the empirical part of the paper we use equation (9) to discriminate among leading asset pricing models. We focus on models where the representative consumer has preferences of the type developed by Epstein and Zin (1989) and Weil (1990). For this preference specification, Epstein and Zin (1989) show that the SDF takes the form

$$\ln\left(m^X_{t+h}\right) = A + B\ g_{t+h} + C\ r_{a,t+h} \tag{10}$$

where $r_{a,t+h}$ denotes the (continuously compounded) return on an asset that delivers a dividend equal to aggregate consumption, and A, B, C, are functions of the subjective discount factor, risk-aversion coefficient and intertemporal elasticity of substitution of the representative investor (see e.g. Bansal and Yaron (2004)).

In this paper we analyze models that, while having the same functional form for the SDF given in Eq. (10), encompass a variety of specifications for the state variables X_t generating the SDF.

The first model is the Bansal et al. (2016) model of long-run risks, where the consumption growth has the following dynamics:

$$g_{t+1} = \mu + x_t + \sigma_t\eta_{t+1} \tag{11}$$

$$x_{t+1} = \rho x_t + \varphi_e\sigma_t e_{t+1} \tag{12}$$

$$\sigma^2_{t+1} = \sigma_0 + \nu\left(\sigma^2_t - \sigma^2_0\right) + \sigma_w w_{t+1}$$

with the three shocks, η, e, and w being i.i.d Normal and uncorrelated. In this model the state variables are the first two conditional moments of log consumption growth g_t, that is $X_t = (x_t, \sigma_t^2)$.

We also consider the specification of the long-run risks model described in Schorfheide et al. (2018). Whereas in Bansal et al. (2016) the time-varying volatility in realized and expected consumption growths is common, the Schorfheide et al. (2018) specification allows for two separate volatility components. Specifically, consumption evolves as follow:

$$g_{t+1} = \mu + x_t + \sigma_{c,t}\eta_{c,t+1} \tag{13}$$

$$x_{t+1} = \rho x_t + \sqrt{1-\rho^2}\sigma_{x,t}e_{t+1} \tag{14}$$

and

$$\sigma_{i,t} = \varphi_i \sigma \exp\left(h_{i,t}\right) \ , \quad h_{i,t+1} = \rho_{h_i} h_{i,t} + \sigma_{h_i,t} w_{t+1} \ \text{ for } i = c, x \ .$$

Therefore, in this model $X_t = (x_t, \sigma_{c,t}^2, \sigma_{x,t}^2)$.

The last model is the Rare Disaster model of Nakamura et al. (2013). In this model the log consumption c_t is the sum of three unobserved components:

$$c_t = x_t + z_t + \epsilon_t \tag{15}$$

where x_t denotes "potential" consumption at time t; z_t denotes the "disaster gap" – i.e., the amount by which actual consumption differs from potential consumption due to current and past disasters, with ϵ_t an i.i.d. normal shock. The occurrence of disasters is governed by a Markov process I_t. $I_t = 0$ denotes "normal times" and $I_t = 1$ denotes times of disaster. Disasters affect consumption in two ways. First, a disaster at time t generate a one-off permanent shift in the level of potential consumption, denoted by θ_t. Second, a disaster at time t causes a temporary drop in consumption, denoted by ϕ_t. Specifically we have

$$\Delta x_t = \mu + \eta_t + I_t\theta_t \tag{16}$$

$$z_t = \rho_z z_{t-1} - I_t\theta_t + I_t\phi_t \tag{17}$$

where μ is the average growth rate of trend consumption and η_t is an i.i.d. normal shock to the growth rate of trend consumption. We also study two restricted version of the Nakamura et al. (2013) model, a version of the model with "permanent disasters"and a version of the model with "permanent and one-period disasters". Both cases require that $\phi_t = \theta_t$. On top of this, the model with permanent and one-period disasters sets the probability of exiting a disaster to one. Finally, in this case the state variables X_ts are (I_t, z_t).

3 Empirical Results

Our empirical investigation is articulated in two steps. First, we review the predictability of raw returns within a standard, linear forecasting model (see Section 3.1). Given the evidence of significant predictability, we compute the predictors-based bound $\sigma_Z^2(\nu)$ for different sets of assets and horizons, and compare them with the unconditional bounds (see Section 3.2). Importantly, we establish that the predictors employed in our linear forecasting model are indeed useful in sharpening the diagnostic efficacy of variance bounds with respect to the unconditional case.

Second, we compare the variability of model-implied SDFs against the predictors-based bounds (see Section 3.3). We show that, by exploiting the first conditional moment of returns across horizons, the predictors-based bounds provide a useful model-selection tool that permits either to set models apart, or to select the common behavior among apparently different models.

3.1 The predictive model

To conduct our empirical analysis, we consider three horizons and two alternative sets of assets for each horizon. Specifically, we concentrate on horizons of $h = 1, 4, 20$ quarters, and for each horizon we consider alternatively the following two sets of returns:

1. SET A: the equity market returns plus the returns from rolling a three-month Treasury Bill over the holding period h;
2. SET B: the returns from SET A plus the returns on nine equity portfolios based on value strategies.

In particular the market return is the (gross) return on the value weighted portfolio of all stocks traded in the NYSE, the AMEX, and NASDAQ. With regard to value portfolios, we form quintiles and use the returns on the High, Medium, and Low portfolios. We measure value using three alternative signals, which gives us a total of nine portfolios. The first measure of value is standard, and it is based on the ratio of the book value to the market value of equity, as in Fama and French (1992). Following Asness and Frazzini (2013) we use the most recent market values to compute the ratios. Also consistent with previous literature, we exclude financial firms: a given book-to-market ratio might indicate distress for a non-financial firm, but not for a financial firm (see Fama and French, 1995). We denote this value signal $BM_{i,t,Ex.fin.}$. Since many financial firms are large and in the investment opportunity set of most investors, we also consider a second set of industry-adjusted book-to-market ratios: $BM_{i,t,Ind.adj.}$, which subtract from each $BM_{i,t}$ the value-weighted average book-to-market ratio of the industry to which stock i belongs. Finally, we also perform a sort across 17 industries, using for each industry the average book-to-market ratio as value signal (denoted Industry$BM_{i,t}$). All returns are gross and deflated using the consumer price index (CPI).

These two sets correspond to a universe of equity assets whose return properties are the subject of much scrutiny in the empirical asset pricing research. In particular SET A relates our findings to the classical unconditional HJ (1991) bounds, as well as to the vast literature that tackles the equity premium puzzle of Mehra and Prescott (1985). We expand SET A using value strategies for three reasons. First, the (unconditional) premium of value over growth portfolios is considered as one of the most prominent anomalies in the cross sectional asset pricing literature (see Cochrane, 2011). Second, the returns on value strategies should be informative about cash flows and discount rate news since short-duration assets like value stocks are more sensitive to cash flow shocks, whereas long-duration assets like growth stocks are more sensitive to discount rate shock (see e.g. Lettau and Wachter (2007)). Finally, and most important for our purpose, value strategies are predictable in the time series, as documented by Asness, Friedman, Krail, and Liew (2000), Cohen, Polk, and Vuolteenaho (2003), and Baba Yara, Boons, and Tamoni (2018).

Table 2 presents full-sample statistics of the quarterly stock returns on the market and on the classical value portfolio based on the $BM_{High,t,Ex.fin.}$ signal.

[Insert Table 2]

In contrast to the simple random walk view, stock returns do seem predictable, and markedly more so the longer the horizon. To review this claim we use a typical specification that regresses rates of return on lagged predictors. In particular we consider the following linear predictive system:

$$R^i_{t+h} = \beta^i_{0,h} + \beta^i_{1,h} Z^i_t + u^i_{t+h} \tag{18}$$

where $i = M, V$ stands for Market and Value portfolios, respectively, and $Z_t = \left(Z^M_t, Z^V_t\right)$ denotes the vector of returns' predictors, potentially different for the aggregate market and the value portfolios. As mentioned above, the holding period ranges from one quarter to five years, i.e. $h = 1, 4, 20$ quarters. The stock market return predictors are the dividend-price ratio, dp_t, and the consumption-wealth ratio, cay_t, i.e.

$$Z^M_t = [dp_t,\ cay_t]\ .$$

The value portfolio returns predictors are dp_t, cay_t, and the book-to-market ratio of the portfolio $i = 1, \ldots, 9$, generically denoted $BM_{i,t}$, i.e.

$$Z^V_t = [Z^M_t,\ BM_{i,t}]\ .$$

Finally, we forecast the real return on the T-bill using the lagged yield spread and the level of the US treasury-bill rates. This is consistent with the literature on term structure which uses principal components to forecast bond returns (see Duffee, 2013).

The choice of dp_t as a stock market predictor is motivated by the present value logic, see Campbell and Shiller (1988), while the choice of cay_t follows from a linearization of the accumulation equation for aggregate wealth in a representative agent economy, see Campbell and Mankiw (1989) and Lettau and Ludvigson (2001). Similarly, the book-to-market ratio $BM_{i,t}$ can be justified as the natural predictor for value portfolios using the present value model of Vuolteenaho (2002).

They are all “noisy” predictors of future asset returns. According to the present value model, the dividend yield should either forecast dividend growth or returns, or both. Empirically, the dividend yield *does* forecast excess returns and not dividend growth. Table 3-Panel A presents regressions of the real stock returns R^s_{t+h} onto Z^A_t. Although the $R^2 = 4.7\%$ at quarterly horizon is low, it then rises with the horizon, reaching a value of about 51%, at the 5 years horizon. Each variable has an important impact on forecasting long horizon returns: using the price-dividend ratio as the sole forecasting variable, for instance, would lead to an R^2 of “only” 22% at the 5 years horizon. Table 3-Panel B presents regressions of the real returns on the classical value portfolio onto $Z^V_t = [Z^M_t,\ BM_{High,t,Ex.fin.}]$, and it confirms the pattern of predictability observed for market returns. Overall, these results are consistent with much of the recent empirical research on the predictability of aggregate stock returns (see Campbell (1987), Cochrane (2001; 2008), and Lewellen (2004) among others) and of value returns (see Asness et al. (2000), Cohen et al. (2003), Baba Yara et al. (2018)).

[Insert Table 3]

3.2 Predictors-based bounds across horizons

It seems apparent from Table 3 that expected returns vary over time. To evaluate by how much the predictors sharpen the diagnostic ability of the variance bounds, we compare the predictors-based bounds to the unconditional HJ bounds.

To compute the predictors-based bounds we use Eq. (6). In particular, we use the linear predictive model in equation (18) to obtain the first and second conditional moments of asset returns, μ_t and Σ_t. The conditional covariance matrix for returns is estimated as the variance matrix of residuals in the forecasting regressions.

Figure 1(a) presents results for SET A for the investment horizons $h = 1, 4, 20$ quarters. In each panel we report the efficient bounds generated with conditioning information (solid lines) along with the unconditional HJ bounds (dashed lines) that make no use of conditioning information. Similar to Cochrane and Hansen (1992), Figure 1(a) shows that the bottom of the mean standard deviation frontier shifts up and to the left as we increase the investment horizon. Importantly, the picture shows that the predictability across horizons documented in Table 3 translates into a tighter lower bound on the variance of the SDF. In particular Figure 1(a) shows that the predictor-based bounds are sharper relative to the unconditional ones. For instance, the minimum point of the frontier at the 1-year (5-year) horizon obtained using conditioning information is about 1.98 (1.79, respectively) times sharper than the unconditional lower bound, thereby substantiating the incremental value of conditioning information in asset pricing applications. The difference between the bounds with and without conditioning information across horizons reflects the predictability documented in Table 3.

Figure 1(b) presents the same analysis for SET B. In this case, the use of conditioning information yields a bound that is about 1.47 (1.34) times the unconditional HJ bound at the 1-year (5-year, respectively) horizon. Upon comparing Figures 1(a) with 1(b), moreover, we observe that expanding the number of assets, i.e. moving from SET A to SET B, leads to a bound that is tighter than the one obtained using only returns from SET A.

[**Insert Figures 1 about here**]

Figures 1 imparts two conclusions. First, these figures highlight the three effects that are at work simultaneously: the conditioning information embedded in moments of returns, the horizon at which this information becomes relevant and the set of assets available for investment. The tightening of the volatility bounds is the combination of these three forces simultaneously at work. Second, although in the predictive regressions the role of the information contained in the predictors becomes more apparent as we lengthen the investment horizon, the predictors-based bounds reveal the important role played by conditioning information even at short horizons.

3.3 Predictors-based bounds and asset pricing models

Predictor-based bounds can be used to evaluate alternative specifications of the dynamics of the state variables under the same structure for preferences. In this paper we restrict our attention to models that rely on Epstein-Zin-Weil recursive preferences.

The models we investigate have been described in Section 2.2. We focus on estimated version of these models since we want to quantify the impact of parameter uncertainty on the mean and volatility of their SDFs. Deep parameters of all three classes of models are estimated using a long span of the data to better capture the overall low frequency variations in asset prices and macroeconomic data and to reduce the measurement errors that arise from seasonalities and other measurement problems (see e.g. Wilcox, 1992). Finally, all models have been solved by well established methods that facilitate the computation of the first and second unconditional moments of their SDFs. Specifically, for the the rare disasters model, we solve numerically for a fixed point for the price of the consumption claim as a function of the state(s) of the economy. This method has been used by Campbell and Cochrane (1999), Nakamura et al. (2013), and Wachter (2013), among others. For the long-run risks models, we follow Bansal and Yaron (2004) and use a linearized solution method based on the Campbell and Shiller (1988) present-value relation.

The online Appendix B reports details on the estimation of the different asset pricing models considered in the paper. For the reader's convenience, Tables B1, B2, and B3 in the online Appendix report, for each model, the complete specification of the parameter values for preferences and exogenous state dynamics, along with the standard errors of the estimated parameters.

It is worth noting that a number of recent papers (see e.g. Gourio (2008b), Gabaix (2012), Wachter (2013)) propose versions of the rare disasters model that employ time-varying probability and/or severity of disasters to explain the predictability and volatility of stock returns, among other anomalous features of asset returns. Unfortunately the effect of estimation uncertainty on the moments of a model-implied SDF cannot be evaluated in any of these papers since they rely solely on calibration. Hence, we relegate to Section B.2.2 of the online Appendix an investigation of their performances.

Finally, it is important to stress that the models under consideration have all been estimated using information from the joint dynamics of consumption, dividends and aggregate equity market prices. Since all these models achieve a very good fit, it is not surprising that judging them using the very same cash flows and equity market moments used in the estimation step is not very informative. We show this fact in Table 1: all models feature low risk aversion, they all have low volatility of consumption, and yet they all deliver an (unconditional) equity risk premium comparable to the data. The main purpose of this paper is to show that the distance between the model-implied SDF and the predictors-based bound is a statistics effective for discriminating across these models.

[Insert Table 1]

3.3.1 Model-implied SDFs and predictors-based bounds

To provide some intuition about the ability of the model-implied SDFs to satisfy the predictors-based bounds, first we abstract from parameter and small-sample uncertainty, and we use a single simulation run to infer the (population) mean and volatility of the SDF of each model. Specifically, we simulate 600,000 monthly observations (50,000 years) of the model-implied SDF for the long-run risks model, and 50,000 annual observations for the rare disaster model. We return to the issue of parameter and small-sample uncertainty in Section 3.3.3.

Figure 1 displays the predictors-based bounds and the SDFs generated by the different models for different horizons and for different sets of test assets. The predictors-based bound are displayed along with the classical HJ bound. For the long-run risks, we report with a circle the SDF for the Bansal et al. (2016) specification, and with a star the SDF for the Schorfheide et al. (2018). For the disasters model of Nakamura et al. (2013), the triangle and square correspond to the SDF in the baseline case and a "No Disaster" case, respectively. The "No Disasters" case features a long sample in which agents expect disasters to occur with their normal frequency but none actually occur. In all cases, the mean and the standard deviation of the SDFs reported in the figures represent population values obtained using estimated parameters.

A first observation that emerges clearly from Figures 1 concerns the importance of jointly considering conditioning information and horizons for the equity premium puzzle. Figure 1(a) shows that the SDFs of all models satisfy the unconditional HJ bounds at the 1-year horizon. This is essentially a graphical representation of the fit of these models, see Table 1. However, the conclusions are different when we incorporate conditioning information. In this case the classical long-run risks model with one time-varying volatility process struggles to meet the bounds, whereas the specification with multiple volatility components lies exactly on top of the predictors-based bound at the 1-year horizon, and satisfies the 5-year horizon bound comfortably. As expected, using SET B (see Figure 1(b)), i.e. exploiting the additional predictable information from value strategies, exacerbates these results: the classical long-run risks models fall largely below the predictors-based bounds (while still marginally satisfying the 1- and 5-years unconditional bounds), whereas the Schorfheide et al. (2018) specification lies slightly outside of the predictors-based bound at the 1-year horizon, but it is still within the bounds at the 5-year horizon. The baseline specification of the disasters model makes it comfortably within the bounds at all relevant horizon, while the no-disaster case is always outside.

Importantly, had we considered the 1-year bounds with no conditioning information, we would have concluded that the equity premium puzzle, and to a lesser extent the unconditional value premium, could be resolved as long as sufficient time-nonseparability is incorporated in preferences. However, the predictors-based bounds highlight that what really matters is the interaction between preferences *and* the consumption dynamics.

The figures shed also some light on the long-horizon behavior of the SDFs from the competing models. This is interesting since frictions or measurement errors can disrupt the link between returns and consumption growth at high frequencies, while leaving the relation intact at low frequencies and long horizons. Hence, one would expect for asset pricing puzzles to be less pronounced at longer horizons (see e.g. Daniel and Marshall (1997)). With the visual aid of Figure 1 we can see that this expectation is fulfilled when no conditioning information is incorporated: the 5-year unconditional HJ bound is satisfied with good margin by all models, with the only exception of the no-disaster case. However, this changes significantly at the light of the predictors-based bound at the 5-year horizon. Focusing on SET B, we observe that the rare disasters model satisfies the predictor based bounds with a good margin, the long-run risk model with multiple volatility stands at the bounds, while the standard long-run risk model finds it onerous to satisfy the bounds at such a long horizon. Moreover, we show in Section 3.3.3 that accounting for statistical uncertainty sharpens the discrimination across models that we have highlighted so far.

The results in this section point to an interesting fact about the role played by preferences and state dynamics. Although the rare disasters and long-run risks models share the same preferences for early resolution of uncertainty, the two model-implied SDFs have very different behaviors.

This can be explained by the different ways the long-run risks and the rare disasters decompose consumption. Both models assume that the level of log consumption includes a deterministic trend and a stochastic trend. In the long-run risk model (see Eqs. (11) and (12), or (13) and (14) for the specification with two stochastic volatilities, one in realized and one in expected consumption growth), the growth rate of the stochastic trend captures expected consumption growth and contains (i) a persistent component, (ii) long-run variation in volatility. In the rare disaster model (see Eqs. (15) and (16)), on the other hand, the growth rate of the stochastic trend captures potential consumption and follows a jump process. Differently from the long-run risk, the rare disaster model of Nakamura et al. (2013) incorporates also a transitory component in the log consumption level - labeled disaster gap, see (17) - which allows for partial recoveries after disasters. Our predictors-based bounds favor a specification of the growth rate of the stochastic trend in consumption with jumps and recoveries compared with a specification with a persistent component and stochastic volatility. Within the long run risk models, instead, the predictors-based bounds favor a specification where the i.i.d. and persistent components of consumption growth feature separate stochastic volatilities.

3.3.2 The rare disaster model under the magnifying lens

A natural question is whether the superior performance of the Nakamura et al. (2013) model stems from the multiperiod nature of disasters (recall that the indicator I_t in Eq. (16) may take the value of 1 over multiple years), or from the partial recoveries after disasters, see Eq. (17), or from both.

To address this point, in Figure 2 we consider two specifications of the Nakamura et al. (2013) rare disasters model in addition to the baseline economy analyzed so far. While all specifications have recursive preferences, they allow for different disaster dynamics. In particular we compare the model-implied SDF of Nakamura et al. (2013) (red triangle), with the SDF of a model in which disasters are completely permanent but unfold over several years (black triangle), and with the SDF of a model with permanent-and-one-period disasters of the type analyzed in Barro (2006) (blue triangle). Importantly, both the permanent, and permanent-and-one-period disasters model have been re-calibrated to deliver the same (observed) equity premium of 4.8%. Whereas the baseline model matches the equity premium with a coefficient of relative risk aversion $\gamma = 6.4$, this value for γ yields an equity premium of 10% when disasters are permanent, and an implausible 47% in the permanent-and-one-period disasters case. Thus, to match the equity premium in the data we set $\gamma = 4.4$ in the permanent disasters case, and $\gamma = 3.0$ in the permanent-and-one-period disasters case (see Table 1).

By comparing the baseline model with the permanent disasters model we can assess the importance of allowing for recoveries after disasters. In turn, by comparing the baseline model with the permanent-and-one-period disasters model we can assess the role of the multi-period nature of disasters. Looking at the one-year horizon, we observe that the SDF from the permanent model satisfies the predictors-based bounds (solid line), while it struggles when we incorporate returns on value strategies, whereas the SDF from the permanent-and-one-period disasters model is below the predictors-based bound independently from the test assets used. This fact highlights the important role played by the multi-period nature of disasters at the one-year horizons. Turning to the longer, five-years horizon, we observe that all specifications meet comfortably the predictors based bounds (the only exception is the permanent-and-one-period disasters model laying right on the bounds when SET B is considered). This fact highlights the importance of rare disasters for long-run dynamics independently from whether the disasters unfold over several years, or whether recoveries are allowed.

[**Insert Figure 2 about here**]

Putting together the evidence in Figures 1 and 2, we conclude that disaster dynamics are important to meet the predictors-based bounds over the long run but the exact specification - e.g. whether disasters unfold over multiple years or whether they are systematically followed by recoveries - is less relevant. At shorter horizon, in order to meet the predictors-based bounds it is instead key to have multiperiod disasters, whereas partial recovery matters to a lesser extent.

The evidence so far points to the ability of the predictors-based bounds to discriminate across models. We next quantify the robustness of these conclusions when we account for estimation uncertainty.

3.3.3 How uncertain is the distance from the bounds?

In this section we evaluate whether the difference between the estimated model-implied variance of the SDF, $Var\left(m_t^X\right)$, and the estimated predictors-based bound, $\sigma_Z^2(v)$, is large in a statistical sense. To properly compare the moments of the model-implied SDF with the variance bounds we must account for two sources of uncertainty. First, the computation of the mean and variance of a model-implied SDF relies on the estimates of the exogenous state dynamics, and hence it reflects the uncertainty in the parameters describing these dynamics. Second, since the volatility bounds are estimated from the data, they must reflect the uncertainty surrounding the linear predictive model used to compute the conditional moments of returns, see Eq. (18). Hereafter, we account for these sources of uncertainty to obtain the finite sample distribution of the difference $\Delta = Var\left(m_t^X\right) - \sigma_Z^2(v)$ (see Cecchetti, Lam, and Mark (1994) and Burnside (1994) for a related approach). For the models under scrutiny we draw the parameters for the state dynamics using the values given in the online Appendix, see Tables B1, B2, and B3. We draw the coefficients of each asset return predictors analogously. Given these parameters, for each model we simulate an SDF of length equal to our dataset, i.e. 742 months. Finally, we compute the model-implied variance of the SDFs, the predictors-based bounds, and their difference. We repeat this exercise 10000 times.

The results are summarized in Table 4, for SET A and SET B. We start by looking at the rightmost block. The results reflect the estimation uncertainty in both the predictors-based bounds and the moments of the model-implied SDF. The first conclusion that emerges from Table 4 is that independently from the horizon and test assets considered, the SDF of the rare disaster model satisfies comfortably the predictors-based bound even after accounting for estimation uncertainty. Interestingly, within the long-run risks class, the novel specification of Schorfheide et al. (2018) with multiple volatility components constitutes a noticeable improvement upon the classical long-run risks model based on a single time-varying volatility. Indeed, the Schorfheide et al. (2018) model now satisfies the 1-year and 5-year predictor-based bounds constructed from SET A, although it still fails the bounds constructed from SET B.

[Insert Table 4 about here]

The remaining two blocks of Table 4 report a decomposition of the uncertainty in the comparison of $Var\left(m_t^X\right)$ with $\sigma_Z^2(v)$. Following Cecchetti et al. (1994), we think of this uncertainty as arising from three basic sources. Given the expected value of the SDF, v, there is uncertainty in both the location of the bound, $\sigma_Z^2(v)$, and the standard deviation implied by the model, $Var\left(m_t^X\right)$. In addition, there is uncertainty induced by the fact that the mean of the SDF for the model, v, must be estimated. The leftmost block reports the estimated distance for a fixed mean of the SDF. The middle block of the table reports the uncertainty in Δ that arises solely from randomness in v. If no uncertainty in the mean of the SDF is considered then all models, except for the no-disasters model, meet the volatility restriction at the 1- and 5-year horizons, for both SET A and SET B, with a 5% confidence level (see leftmost panel). Thus, uncertainty surrounding the volatility bounds is sufficient to make the SDFs satisfy the restrictions *if* one is fully confident about the location of the mean of the SDF. However, once the uncertainty about the mean of the SDF is accounted for, the conclusions are reversed (see the middle block) and, when using SET B, all models fail to meet the volatility restrictions with the sole exception of the rare disaster model. As one would expect, re-introducing uncertainty about the bounds helps the model: moving from the middle to the rightmost block shows that Δ gets closer to positive values. However, the uncertainty on the bounds is not large enough to undo the uncertainty in the mean of the SDF. Hence, by comparing the distance of a model-implied SDF across the different columns in Table 4, it is clear that the main source of uncertainty lies in the fact that v must be estimated.

In sum, this section shows that by incorporating conditioning information from a well-established set of stock predictors, the predictors-based bounds are a useful tool to assess the performance of asset pricing models at multiple horizons. It is worth noting that each asset pricing model parametrization approximates quite reasonably the (annual) unconditional equity premium and the real risk-free return, while simultaneously calibrating closely to the first two unconditional moments of consumption growth (see Table 1). The rare disaster model handily meets the predictor-based bounds across horizons and asset classes, even after accounting for estimation uncertainty. The classical version of the long-run risks model with a single common source of stochastic volatility (see Bansal et al. (2016)) fails to meet the restrictions imposed by the predictor-based bounds at the 1-year horizon, with the standard deviation of their SDFs never approaching the bounds. The version of the long-run risks that accommodates multiple volatility components (see Schorfheide et al. (2018)) represents an improvement in the sense that it satisfies the 1-year bound constructed from SET A. At long horizons, finally, while the classical long-run risks model with one time-varying volatility process generates an SDF not volatile enough independently from the test assets considered, the modified long-run risks model with multiple volatility components satisfies the predictors-based bound when using SET A but fails when we consider our largest set of equity returns.

3.4 Model-implied SDF, predictors-based bounds and sensitivity to risk aversion

In this section, we inspect the sensitivity of model-implied SDFs to the coefficient of relative risk aversion (γ). To this end, we simulate and compare the model-implied SDFs with both high and low values of γ in addition to the baseline specification considered so far (see Figure 1).

Figure 3 displays the predictors-based bounds, and the SDFs generated by the Schorfheide et al. (2018) (stars) and the Nakamura et al. (2013) (triangles) models for different values of γ. For the Schorfheide et al. (2018) model, we simulate the model using the 5%, 50% and 95% posterior values of the estimated risk aversion reported in Table 5 of Schorfheide, Song, and Yaron (2016). Specifically, in the baseline specification γ equals 8.60, while it equals 5.44 and 12.97 for the low and high specifications, respectively. These values imply an equity premium of about 8.2% in the baseline case (see Table 1), and 4.24% and 13.36% for the low and high γ values. For the Nakamura et al. (2013) model, the baseline specification features a risk aversion of 6.4, which corresponds to an equity premium of 4.8% (see Table 1). We also analyze the case when γ is set to 4.4 and 8.4. These low and high values imply an equity premium of 2% and 8.3%, respectively (see Table 7 in Nakamura et al. (2013)).

[Insert Figure 3 about here]

The figure shows that increasing risk aversion improves substantially the performance of the long-run risks model at the 1-year horizon, and that of the rare disasters model at the 5-year horizon. However, an important observation is in order.

Risk aversion is not a free parameter, and the higher levels of γ in the two models generate equity premia that are not in line with the data. In particular, setting $\gamma = 8.4$ in the Nakamura et al. (2013) model delivers an equity premium of 8.3%, which is higher than any of the estimates in Table 1. Similarly, a baseline level of $\gamma = 8.6$ generates in the Schorfheide et al. (2018) model an equity premium of 8.2%. This evidence is even more important at the light of the recent literature documenting a decline in the equity premium (see e.g. Guvenen, 2003; Lettau, Ludvigson, and Wachter, 2007; and McGrattan and Prescott, 2005).

In sum, increasing risk aversion tends to deliver implausible results for the equity premium, and the dynamics of consumption become the key ingredient to fix the volatility of the SDF. This observation reinforces the usefulness of tools that discriminate between competing models of consumption dynamics and asset pricing. We believe that our predictors-based bounds give a positive contribution in this direction.

4 Conclusions

We have investigated the capability of asset pricing models that rely on recursive utility a-la Epstein and Zin (1989) and Weil (1990) to capture the Stochastic Discount Factor variation implied by return predictability. The models we picked fit essentially the same aggregate macro and asset-pricing moments, and thus a natural question arises as to which model provides the "right" description of the underlying sources of risks in the economy.

Our predictors-based bounds favor rare disasters dynamics over specifications which are based on long-run risks. Interestingly, the horizon over which we compute the bounds represents an important choice that conveys useful, discriminatory information. For instance, within the rare disaster class, the 1-year predictor-based bounds provide insights about the nature of disasters: a key ingredient for the model to fit the bounds is the presence of disasters that unfold over multiple years, whereas the role of recoveries after disasters is less relevant. Our bounds computed over a longer 5-year horizon show instead that the sole presence of disasters - even if permanent and one-period - suffices to make the model-implied SDF able to meet the bounds. Within the long-run risks class, our predictors-based bounds point to a specification that features multiple volatility components as a promising avenue for future research. However, the predictability of value strategies, once incorporated into the bounds, represent a hurdle that even a long-run risks specification with multiple volatility components finds hard to overcome. Rare, multiperiod disasters prove instead successful even when confronted with bounds that embed information from value portfolios.

We conclude that our predictors-based bounds provide a fruitful new way of comparing different models to the data. A natural extension of our work would be to compare models featuring habits preferences, but featuring different dynamics such as Campbell and Cochrane (1999) and its variant proposed by Bekaert and Engstrom (2010).

SUPPLEMENTARY MATERIAL

Internet Appendix: The Internet Appendix contains a detailed description of the: (A) Data set used for the empirical analysis; (B) Estimation approach and parameter values for each asset pricing model; (C) Method to compute the difference between the estimated model-implied standard deviation of the SDF, $\sigma\left(m_t^X\right)$, and the estimated predictors-based bound, $\sigma_Z\left(v\right)$; (D) Impact of mispecified conditional moments in constructing the predictors-based bounds.

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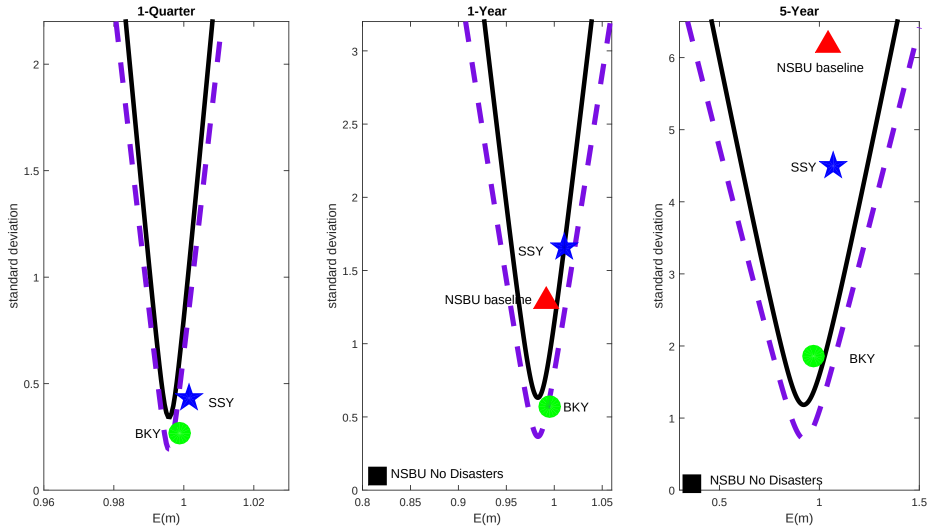
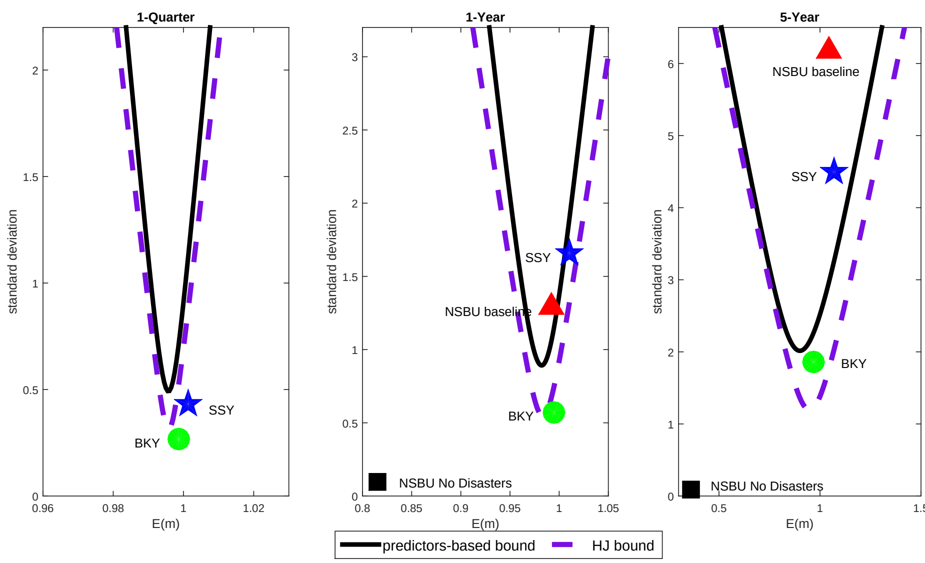
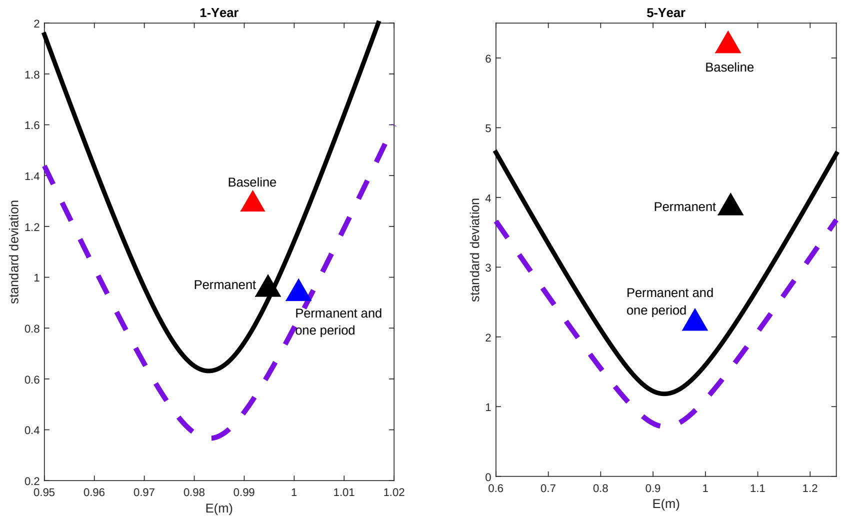


(a) SET A

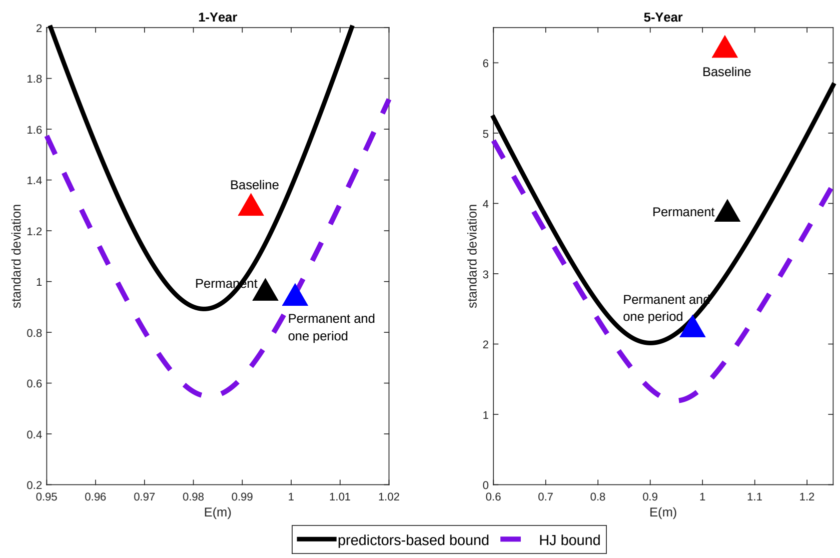


(b) SET B

Figure 1 Predictors-based bound ($\sigma_z(v)$), Hansen-Jagannathan (1991) bound ($\sigma_{HJ}(v)$), and model-implied SDFs – SET A and SET B. The figure displays the Hansen-Jagannathan (1991) bounds (dashed violet line) and the predictors-based bounds (solid black line). We follow Bekaert and Liu (2004) to construct the predictors-based bounds. To construct the bounds we use data from 1952Q2 to 2012Q3. The green circle and blue star correspond to the (average mean and standard deviation of the) SDF obtained from 10 simulation runs of 600,000 months of the Bansal et al. (2016) (BKY model) and Schorfheide et al. (2018) (SSY model) long-run risks models, respectively. The red triangle corresponds to the SDF obtained from 10 simulation runs of 50,000 years of the (baseline case of) Nakamura et al. (2013) rare disasters model. The black square corresponds to the no disasters case, i.e. it refers to a long sample in which agents expect disasters to occur with their normal frequency but none actually occur.

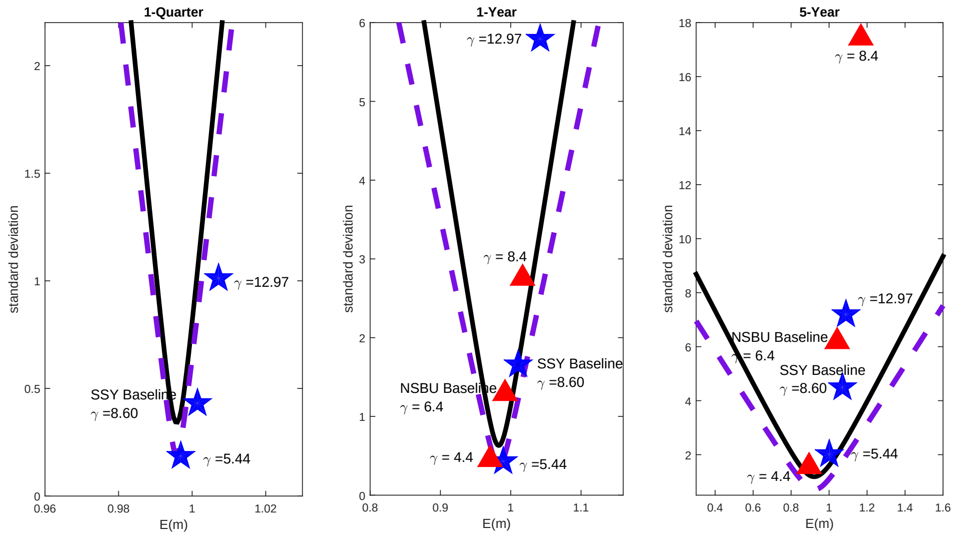


(a) SET A

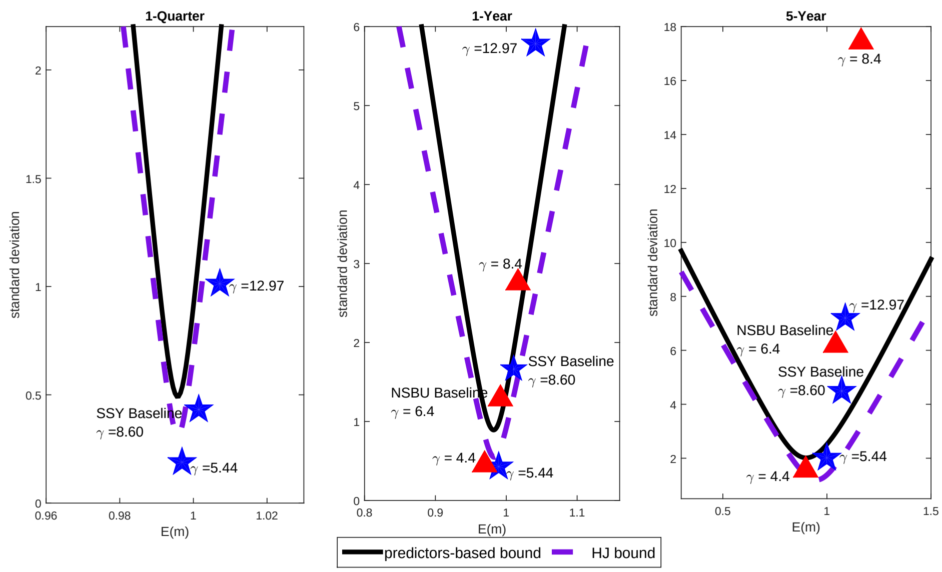


(b) SET B

Figure 2 Rare disaster model-implied SDFs, Hansen-Jagannathan (1991) bound ($\sigma_{HJ}(v)$), predictors-based bounds ($\sigma_z(v)$) – SET A and SET B. The figure displays the Hansen-Jagannathan (1991) bounds (dashed violet line) and the predictors-based bounds (solid black line). We follow Bekaert and Liu (2004) to construct the predictors-based bounds. To construct the bounds we use data from 1952Q2 to 2012Q3. The red triangle corresponds to the (average mean and standard deviation of the) SDF obtained from 10 simulations run of 50,000 years of the baseline Nakamura et al. (2013) case, in which the risk aversion parameter (γ) equals to 6.4 and IES equals to 2. The blue triangle corresponds to the SDF obtained from simulations of the permanent and one-period disasters case, in which γ is set to 3.0 and the IES to 2. The black triangle corresponds to the SDF obtained from simulations of the permanent but multiple disasters case, in which γ is set to 4.4 and the IES to 2.



(a) SET A



(b) SET B

Figure 3 Model-implied SDFs, predictors-based bounds and sensitivity to risk aversion (γ) – SET A and SET B. The figure displays the Hansen-Jagannathan (1991) bounds (dashed violet line) and the predictors-based bounds (solid black line). We follow Bekaert and Liu (2004) to construct the predictors-based bounds. To construct the bounds we use data from 1952Q2 to 2012Q3. The blue stars correspond to the (average mean and standard deviation of the) SDF obtained from 10 simulations run of 600,000 months of the Schorfheide et al. (2018) model, with γ sets to 5.44, 8.60 (Baseline) and 12.97, respectively. The red triangles correspond to the (average mean and standard deviation of the) SDF obtained from 10 simulations run of 600,000 months of the Nakamura et al. (2013) model, with γ sets to 4.4, 6.4 (Baseline) and 8.4, respectively.

Table 1 Moments of consumption and asset returns. We report the data and model-implied (annualized) moments of consumption and asset returns for the three asset pricing models analyzed in the main text. For Bansal et al. (2016) (BKY), we take the values from their paper directly. For Schorfheide et al. (2018) (SSY), we take values from their working paper Schorfheide et al. (2016). For Nakamura et al. (2013) (NSBU), the model-implied moments of consumption growth and price-dividend ratios are computed by the authors. We simulate the model using the posterior mean of estimated values of parameters. Δc is the annual consumption growth rate. pd is the the log of the end of year price over the twelve month trailing sum of dividends. r_f is the logarithm of the annual risk-free rate. $(R_m - R_f)$ is the annual equity risk premium.

Moments	BKY		SSY		NSBU			
Sample	1930-2015		1930.1-2014.12		1890/1914-2006			
Frequency	annual		monthly and annual		annual			
	Data	Model	Data	Model	Data	Baseline	Perm.	Perm. &1-period
Risk Aversion								
γ		9.67		8.598		6.4	4.4	3
Consumption								
$E(\Delta c)$	0.018	0.019	0.019	0.019		0.018	0.012	0.010
$sd(\Delta c)$	0.021	0.022				0.036	0.037	0.070
$AC1(\Delta c)$	0.472	0.395				0.151	0.148	-0.020
Asset Prices								
$E(pd)$	3.404	3.409	3.40	3.14		3.269	3.193	3.364
$E(r_f)$	0.005	0.010	0.004	0.006		0.010	0.007	0.000
$E(R_m - R_f)$	0.075	0.067	0.077	0.082	0.048	0.048	0.048	0.048

Table 2 Statistics of the Data. This table reports sample statistics of quarterly annualized nominal stock market and classical value portfolio returns. Stock market returns are nominal returns on the stock total returns on the value weighted portfolio of all stocks traded in the NYSE, the AMEX, and NASDAQ from CRSP. Classical value portfolio returns are nominal returns on the stock portfolio formed by the $BM_{i,t,Ex.fin.}$ ratio. Sample: 1952Q2−2012Q3.

	Asset	
	Market	Value Portfolio
Mean return (% p.a.)	11.45	13.17
Standard deviation (% p.a.)	16.68	16.55

Table 3 Predictability of stock and portfolio returns. Panel A reports quarterly overlapping regressions of multiple horizon real gross stock market returns onto a constant, cay_t, and the log dividend-price ratio dp_t. Panel B reports quarterly overlapping regressions of multiple horizon real gross return on a classical value portfolio onto a constant, cay_t, dp_t, and the portfolio's book-to-market ratio $BM_{High,t}$. The table reports coefficient estimates, the R^2 of the regression, and, Newey-West t-statistics in parentheses. Following Lazarus et al. (2016), we set the Newey-West truncation parameter to $S_T = \left(\frac{3}{2B}\right) T$, where $B = 8$. We evaluate the test statistic using critical values from the t_B. Critical values of Student's t-distribution with 8 degrees of freedom are 1.860, 2.306 and 3.355 at 10%, 5% and 1% significant level, respectively. Sample: 1952Q2: 2012Q3.

Panel A: Predictive regressions for stock market returns

Horizon (quarters)	cay_t $(t{-}stat)$	dp_t $(t{-}stat)$	$R^2(\%)$
1	0.84 (2.11)	0.03 (3.08)	4.7
4	3.19 (2.61)	0.12 (3.30)	17.0
20	16.06 (4.14)	0.60 (4.71)	51.2

Panel B: Predictive regressions for value portfolio returns

Horizon	BM_t $(t{-}stat)$	cay_t $(t{-}stat)$	dp_t $(t{-}stat)$	$R^2(\%)$
1	0.09 (3.18)	0.48 (1.51)	−0.03 (−1.15)	6.3
4	0.27 (3.21)	1.64 (1.49)	−0.04 (−0.56)	19.5
20	0.71 (3.07)	6.12 (2.54)	0.23 (1.35)	52.7

Table 4 Distance between model-implied SDFs and the predictors-based bounds. The table displays the distance computed as the difference between the model-implied standard deviation of the SDF and the volatility bound, $\Delta = \sigma\left(m_t^X\right) - \sigma_Z(v)$. A positive value means the model cannot be rejected at the five percent level (displayed in bold). We consider two asset pricing models: the long-run risks and the rare disasters models. For the long-run risks class, we report results for the Bansal et al. (2016)(BKY) and Schorfheide et al. (2018)(SSY) models. For the rare disasters class, we report the results of the baseline case and the case where no disasters happen during the sample period. We compute the finite sample distribution of the distance as described in Internet Appendix. The distribution accounts for parameter and small-sample uncertainty. The parameter uncertainty reflects estimation uncertainty in both the asset pricing model and the return predictive regressions. The table reports results for three cases. The leftmost block is the case when the mean of the SDF, $E[m]$, is fixed but there is uncertainty surrounding the predictors-based bounds. In this case, the mean of the SDF is fixed to the long-run mean obtained from a simulations run of 600,000 months. In the second case reported in the middle block, $E[m]$ is random but there is no parameter uncertainty in the predictors-based bounds. Finally, the rightmost block reports the results when both $E[m]$ and the predictors-based bounds are random.

$E[m]$	Fixed		Random		Random	
Bounds	Random		Fixed		Random	
	SET A	SET B	SET A	SET B	SET A	SET B
Horizon(quarters)	Long Run Risk Models – BKY					
1	−0.186	−0.104	−0.138	−0.255	−0.112	−0.124
4	**0.058**	**0.040**	−0.176	−0.351	−0.142	−0.205
20	**1.887**	**1.441**	−0.255	−1.252	−0.092	−0.878
	Long Run Risk Models – SSY					
1	−0.258	−0.242	−0.133	−0.270	−0.064	−0.123
4	**0.640**	**0.509**	−0.108	−0.344	**0.028**	−0.139
20	**2.614**	**2.147**	−0.110	−1.585	**0.142**	−1.013
	Rare Disasters Models – Baseline					
4	**0.821**	**0.642**	**0.327**	**0.065**	**0.389**	**0.172**
20	**4.250**	**3.816**	**2.344**	**1.445**	**2.480**	**1.753**
	Rare Disasters Models – No Disasters					
4	−9.312	−9.587	−8.337	−8.675	−8.339	−8.635
20	−7.658	−8.241	−7.408	−8.277	−7.308	−8.106