Internet Appendix for "Implications of Return Predictability for Consumption Dynamics and Asset Pricing"

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A Data

We consider a set of quarterly equity returns over the period 1952Q2 to 2012Q4. Our choice of the start date is dictated by the availability of data for our predictors. Real returns are computed by deflating nominal returns by the Consumer Price Index inflation.

1. Stock returns: Return data on the value-market index are obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. We use the NYSE/Amex value-weighted index with dividends as our market proxy, R_{t+1}. Quarterly returns are constructed by compounding their monthly counterparts. The h-horizon return is calculated as $R_{t+h} = \exp(r_{t,t+h}) = \exp(r_{t+1} + \ldots + r_{t+h})$ where $r_{t+j} = \ln(R_{t+j})$ is the 1-period log stock return between dates $t + j - 1$ and $t + j$ and R_{t+j} is the simple gross return.
2. Portfolio returns: With regard to value portfolios, we form quintiles and use the returns on the High, Medium, and Low portfolios. We measure value using three alternative signals, which gives us a total of nine portfolios. The first measure of value is standard, and it is based on the ratio of the book value to the market value of equity, as in Fama and French (1992). Book values are observed each June and refer to the previous fiscal year-end in December to ensure data availability to investors at the time of portfolio formation. The most recent market values are used to compute the ratios following Asness and Frazzini (2013). Consistent with previous literature, we exclude financial firms: a

given book-to-market ratio might indicate distress for a non-financial firm, but not for a financial firm (see Fama and French (1995)). We denote this measure $BM_{i,t,Ex.fin.}$. However, because many financial firms are large and in the investment opportunity set of most investors, we also consider a second set of industry-adjusted book-to-market ratios: $BM_{i,t,Ind.adj.}$, which subtract from each $BM_{i,t}$ the value-weighted average book-to-market ratio of the industry to which stock i belongs. Finally, we also perform a sort across 17 industries, using for each industry the average book-to-market ratio as value signal (denoted Industry$BM_{i,t}$).

3. Short-term rate: The nominal short-term rate ($R_{f,t+1}$) is the annualized yield on the 3-month Treasury bill taken from the Center of Research in Security Prices (CRSP) at the University of Chicago. Longer (than 1Q) returns are constructed by rolling over the three-month T-bill. As an alternative, we have also considered the yield-to-maturity on a zero-coupon bond with maturity matching the horizon h. Results were basically unchanged and particularly so at the 1-year horizon. Since in all cases one needs to subtract the inflation realized over the given horizon h from the return on each strategy, for robustness we have also considered the case in which real yields on Treasury Inflation Protected Securities (TIPS) were used instead of nominal yields minus realized inflation. Once again results were unaffected.

4. Stock market predictors: consumption-wealth ratio, cay_t, see Lettau and Ludvigson (2001); the dividend-price ratio, dp_t, see Campbell and Shiller (1988a) and Campbell and Shiller (1988b).

5. Portfolio predictors: in addition to cay_t and dp_t, we also use the book-to-market ratio of the portfolio. Specifically, we use $BM_{i,t}$ for the classical value portfolios that exclude financial firms, the industry-adjusted book-to-market ratios, $BM_{i,t,Ind.adj.}$, for the portfolios that include financials, and the Industry$BM_{i,t}$ for the industry-based portfolios. Throughout, $i = \{High, Medium, Low\}$.

6. Short-rate predictors: the lagged yield spread, spr_t; the one month maturity US treasury-bill rates, y_t (see, e.g., Campbell (1987) and Fama and French (1989)). The term spread, spr_t, is the difference between the long term 5-year yield on government bonds and the

Treasury-bill (see, e.g., Campbell (1987) and Fama and French (1989)).

7. Inflation: we use the seasonally unadjusted CPI from the Bureau of Labor Statistics.

B Model Estimation and Parameters Value

B.1 Long-Run Risks

B.1.1 Bansal, Kiku, and Yaron (2016)

The model is estimated using annual data from 1930 till 2015. The estimation procedure is based on the Generalized Method of Moments (GMM). To account for a potential discrepancy between the sampling frequency of the data and the decision interval, the vector of model parameters is estimated simultaneously with the decision interval of the agent. That is, estimating the model entails searching jointly for the best parameter set and the decision frequency that fit the data.

A full list of moment conditions is presented in Table 3 of Bansal, Kiku, and Yaron (2016). Three sets of moments are exploited:

1. moments that characterize the joint dynamics of consumption and dividend growth rates;
2. moments that characterize the level and volatility of asset returns;
3. moments that characterize the predictability of asset returns and consumption.

The first set of moments comprises the mean, volatility and autocorrelation of consumption and dividend growth rates as well as their correlation. The second set of moments consists of the mean and volatility of the equity returns, the risk-free rate, and the price-dividend ratio, thus confronting the model with both the equity premium (Mehra and Prescott (1985)) and the volatility puzzles (Shiller (1982)). The estimation also uses the autocorrelation of the price-dividend ratio. To account for predictability of consumption growth and equity returns, Bansal et al. (2016) use the correlations of the price-dividend ratio with future consumption growth and with future market returns.

To simulated the Bansal et al. (2016) model-implied SDF, we use all the estimated parameters reported in Table 2 of Bansal et al. (2016), and reported in Table B1 for reader's convenience.

Table B1 Parametrization of Bansal et al. (2016) long-run risks model. The estimated values and corresponding standard errors (in parentheses) are taken from (Bansal et al., 2016) Table 2. The model is simulated at the monthly frequency.

	Parameter	Bansal-Kiku-Yaron (BKY)
Preferences		
Time preference	δ	0.9990 (0.0001)
Risk aversion	γ	9.67 (1.44)
EIS	ψ	2.18 (0.21)
Consumption growth dynamics, g_t		
Mean	μ	0.0016 (0.0005)
Long-run risk, x_t		
Persistence	ρ	0.9762 (0.0035)
Volatility parameter	φ_e	0.0318 (0.0053)
Consumption growth volatility, σ_t		
Mean	σ_0	0.0070 (0.0009)
Persistence	v	0.9984 (0.0007)
Volatility parameter	σ_w	2.12×10^{-6} (5.32×10^{-7})
Aggregation	h	11 (2.16)

B.1.2 Schorfheide, Song, and Yaron (2018)

The model is estimated using a particle MCMC approach that exploits the conditional linear structure of the approximate equilibrium in the endowment economy. All model parameters and latent stochastic volatilities are estimated jointly using full likelihood.

The vector of observables contains the consumption growth rate, the dividend growth rate, the observed market return and the risk-free rate. To estimate the model, Schorfheide, Song, and Yaron (2018) use data sampled at different frequencies. In particular they use annual consumption growth data from 1930 to 1959, and monthly data from 1960:M1 to 2014:M12; monthly dividend annual growth data from 1930:M1 to 2014:M12; monthly returns and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ from 1930:M1 to 2014:M12; finally, the ex-ante real risk-free rate constructed as the fitted value from a projection of the ex-post real rate on the current nominal yield and inflation over the previous year from 1930:M1 to 2014:M12.

To simulate the Schorfheide et al. (2018) model-implied SDF, we use the 50% posterior values of the estimated parameters reported in Table 5 of Schorfheide, Song, and Yaron (2016), and reported in Table B2 for reader convenience.

Table B2 Parametrization of Schorfheide et al. (2018) long-run risks model. The estimated values are taken from Schorfheide et al. (2016), Table 5. The model is simulated at the monthly frequency.

	Distr.	Schorfheide-Song-Yaron (SSY) Posterior		
		5%	50%	95%
Household Preference				
δ	*B*	–	0.999	–
ψ	*G*	1.134	1.935	3.416
γ	*G*	5.441	8.598	12.969
Preference Risk				
ρ_λ	*U*	0.9157	0.9559	0.9818
σ_λ	*IG*	0.0003	0.0005	0.007
Consumption Growth Process				
ρ	*U*	0.9486	0.9872	0.9995
φ_x	*U*	0.1388	0.2315	0.5058
σ	*IG*	0.0020	0.0032	0.0044
ρ_{h_c}	N^T	0.9733	0.9914	0.9958
$\sigma^2_{h_c}$	*IG*	0.0074	0.0088	0.0100
ρ_{h_x}	N^T	0.9874	0.9943	0.9988
$\sigma^2_{h_x}$	*IG*	0.0027	0.0039	0.0061
Consumption Measurement Error				
σ_ϵ	*IG*	0.0006	0.0010	0.0016
σ^a_ϵ	*IG*	0.0061	0.0231	0.0423

A final comment is in order. Schorfheide et al. (2018) also allow for stochastic volatility in dividends, $\sigma^2_{d,t}$. This additional volatility is important to reproduce asset pricing moments, but irrelevant in our context since the SDF in Eq. (10) in the main text is solely determined by consumption dynamics. This is why we report only the parameters for $\sigma^2_{c,t}$ and $\sigma^2_{x,t}$ in Table B2.

B.2 Rare disasters

B.2.1 Nakamura, Steinsson, Barro, and Ursúa (2013)

The Nakamura, Steinsson, Barro, and Ursúa (2013) model is estimated using annual data from 1890 to 2006. The estimation is carried out in two steps. The first step consists in estimating (using Bayesian Markov-Chain Monte-Carlo methods) the model parameters solely related to the consumption dynamics. This step relies on annual data from the Barro and Ursúa (2008) dataset. In the second step, the preference parameters are calibrated to match asset pricing moments. In particular Nakamura et al. (2013) set the IES to 2. The value of the coefficient of relative risk aversion (CRRA) is chosen to match the equity premium in the data (when IES$=2$). In the baseline case, CRRA equals to 6.4. Finally, the discount factor β is chosen to match the risk-free rate in the data for the baseline values of CRRA and IES. This procedure yields a value of $\beta = \exp(-0.034)$.

In the main text of the paper we also consider three alternative specifications of the disaster model. In particular we consider (a) the case in which disasters are completely permanent but unfold over several years; (b) the Rietz (1988)-Barro (2006) model of permanent and instantaneous disasters (i.e. a disaster occurs in a single period, and the drop is permanent); (c) the case of No Disasters, i.e. a long sample in which agents expect disasters to occur with their normal frequency but none actually occur. For these alternative cases we use the value of parameters reported in Table 7 of Nakamura et al. (2013).

To simulate the Nakamura et al. (2013) model-implied SDF, we use the estimated posterior mean of the disaster parameters reported in Table 1; the estimated posterior mean of the post-1973 mean growth rate of potential consumption (united states) reported in Table 3; and the estimated posterior mean of of the variances of the permanent (for U.S.) and transitory shocks (post-1946, for U.S.) to consumption reported in Table 4 of Nakamura et al. (2013). We report this parameters in Table B3 for the reader's convenience.

Table B3 Parametrization of asset pricing model incorporating rare disasters by Nakamura et al. (2013). The value of the parameter estimates and corresponding standard errors (in parentheses) are taken from Nakamura et al. (2013). The model is simulated at the annual frequency.

	Parameter	Annual
Preferences		
Time preference	δ	0.967
Risk aversion	γ	6.4
Elasticity of intertemporal substitution	ψ	2
Potential consumption dynamics, g_t, only for US		
Mean of potential consumption growth, x_t	μ	0.022 (0.003)
Volatility parameter	σ_ϵ	0.003 (0.002)
Volatility parameter	σ_η	0.018 (0.002)
Disaster parameters		
Probabilities of		
a world-wide disaster	p_W	0.037 (0.016)
a country will enter a disaster when a world disaster begins	p_{CbW}	0.623 (0.076)
a country will enter a disaster "on its own."	p_{CbI}	0.006 (0.003)
a country will stay at the disaster state	$1 - p_{Ce}$	0.835 (0.027)
Disaster gap process, z_t		
Persistence	ρ_z	0.500 (0.034)
a temporary drop in consumption caused by shock, ϕ_t		
Mean	ϕ	−0.111 (0.008)
Volatility parameter	σ_θ	0.121 (0.015)
a permanent shift in consumption caused by shock, θ_t		
Mean	θ	−0.025 (0.007)
Volatility parameter	σ_ϕ	0.083 (0.006)

B.2.2 Gabaix (2012)

Gabaix (2012) incorporates a time-varying severity of disasters into the baseline disasters model of Rietz (1988) and Barro (2006).

The consumption growth has the following dynamic

$$\frac{C_{t+1}}{C_t} = e^{g_C} \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1 \end{cases}$$

where g_C is the normal-time growth rate of log consumption and $B_{t+1} > 0$ is the recovery rate of consumption after disaster and assumed to be a random variable. At each period $t+1$, a disaster may happen with a probability p_t. p_t is a random variable with mean μ_p, and standard deviation σ_p.

Furthermore, the notion of "resilience" H_{it} of asset i is introduced to model the time variation in the asset's recovery rate. The resilience of a consumption claim

$$H_{c,t} = p_t E_t \left[B_{t+1}^{-\gamma} - 1 \right]$$

can be split into a constant part H_{c*} and a variable part $\widehat{H}_{c,t}$. As shown in the online appendix of Levintal (2017), the steady state value H_{c*} is

$$H_{c*} = e^{-\gamma\mu_p \log\left(\overline{B}\right) + \mu_p \log\left(\overline{B}\right)} - 1 \ ,$$

and $\widehat{H}_{c,t}$ satisfies the condition

$$\widehat{H}_{c,t+1} = \frac{1 + H_{c*}}{1 + H_{c,t}} e^{-\phi_H} \widehat{H}_{c,t} + \varepsilon^H_{c,t+1} \ ,$$

where $\frac{1+H_{c*}}{1+H_{c,t}}$ is close to 1 and $E_t \left[\varepsilon^H_{c,t+1} \right] = 0$.

To be consistent with the asset pricing models that we diagnosed in the main part of the paper, we restrict our attention to the extension of the Gabaix (2012) model to an Epstein-Zin economy. Within the Epstein-Zin setup, the SDF is given approximately (see Theorem 3 in Gabaix (2012)) by

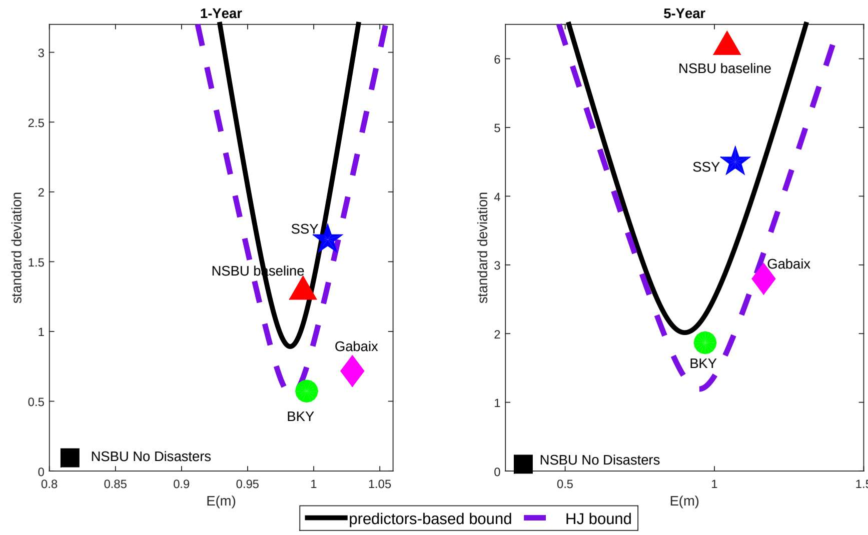
$$m_{t+1} = e^{-\delta}\left(1+(\chi-1)H_{ct}+\varepsilon^M_{t+1}\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1 \end{cases}$$

where $\delta = \rho + g_C/\psi$, $\chi = \frac{1-1/\psi}{1-\gamma}$, $\varepsilon^M_{t+1} = (1-\chi)\frac{\varepsilon^H_{c,t+1}}{\delta_C+\phi_H}$ and $\delta_C = \delta - g_C - \chi H_{c*}$.

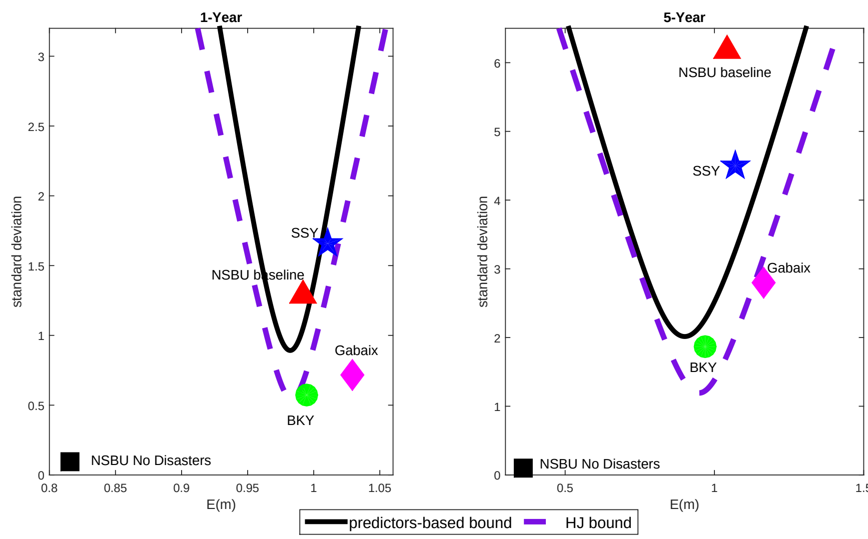
This model is calibrated at annual frequency. The parameter values are obtained directly from Gabaix (2012), and its online appendix. For completeness, we report them in Table B4. Figure B.1 shows the SDF obtained from Gabaix (2012)'s model. We also report the SDFs generated by the different models discussed in the main part of the paper. The figure highlights that the Gabaix (2012) SDF satisfies the 5-year unconditional bounds but it fails to meet our predictors-based bounds at all horizons. Since there is no estimated parameters with related standard deviation values, we only include the results of this model in this appendix.

Table B4 Gabaix (2012) Variable Rare Disasters model. The parameters' values are calibrated as in Gabaix (2012). The model is simulated at the annual frequency.

	Parameter	Gabaix (2012)
Preferences		
Time preference	ρ	0.048
Risk aversion	γ	4
Elasticity of intertemporal substitution	ψ	2
Consumption dynamic		
Growth rate of consumption	g_C	0.025
Recovery rate of consumption after disaster	$\overline{B}$	0.66
Probability of disaster	μ_p	0.0363
	σ_p	0.004
Resilience		
Speed of recovery	ϕ_H	0.13
Volatility	σ_H	0.010
Normal-times volatility in SDF	σ_{ε^M}	0.064



(a) SET A



(b) SET B

Figure B.1 Predictors-based bound ($\sigma_z(v)$), Hansen-Jagannathan (1991) bound ($\sigma_{HJ}(v)$), and model-implied SDFs.- SET A and SET B. The figure displays the Hansen-Jagannathan (1991) bounds (dashed violet line) and the predictors-based bounds (solid black line). We follow Bekaert and Liu (2004) to construct the predictors-based bounds. To construct the bounds we use data from 1952Q2 to 2012Q3. The green circle and blue star correspond to the (average mean and standard deviation of the) SDF obtained from 10 simulation runs of 600,000 months of the Bansal et al. (2016) (BKY model) and Schorfheide et al. (2018) (SSY model) long-run risks models, respectively. The red triangle corresponds to the SDF obtained from 10 simulation runs of 50,000 years of the (baseline case of) Nakamura et al. (2013) rare disasters model. The black square corresponds to the no disasters case, i.e. it refers to a long sample in which agents expect disasters to occur with their normal frequency but none actually occur. The magenta diamond corresponds to the SDF obtained from 10 simulation runs of 50,000 years of the Gabaix (2012) variable rare disasters model.

C Quantifying uncertainty

This appendix explains how we account for uncertainty in the evaluation of the difference between the estimated model-implied standard deviation of the SDF, $\sigma\left(m_t^X\right)$, and the estimated predictors-based bound, $\sigma_{Z(v)}$. First, to compute the mean and variance of a given model-implied SDF, we take into account the uncertainty of the parameters of the exogenous state dynamics. Second, since the predictors-based bound are estimated from the data, we also account for the uncertainty surrounding the linear predictive model, which is used to compute the conditional moments of asset returns, see Eq.(12). Finally, we obtain the finite sample distribution of the difference, $\Delta = \sigma\left(m_t^X\right) - \sigma_{Z(v)}$, based on a related approach in Cecchetti, Lam, and Mark (1994) and Burnside (1994).

To make explicit the dependence of the moments of the SDF from the parameters of the model, we denote the model-implied mean and standard deviation of the SDF as

$$\mu_m(\phi, \psi)$$

$$\sigma_m(\phi, \psi)$$

where ϕ denotes the vector of parameters that characterize the preferences, and ψ contains all the parameters associated with the state dynamics. For instance, in the LRR model, $\phi = (\delta, \gamma, \psi)$ and $\psi = (\mu, \mu_d, \phi, \varphi_d, \rho_{dc}, \rho, \varphi_e, \overline{\sigma}, \upsilon, \sigma_\omega)$, (see Table B1). Then, for the Bansal et al. (2016), and the Nakamura et al. (2013) models, we draw the parameters related to state dynamics, i.e. ψ, from normal distributions with mean and standard deviation given in Table B1 and B3, respectively. Similarly, for the Schorfheide et al. (2018) model we use the distributions detailed in B2. In this latter case, we also verify that our results are robust to drawing parameters from normal distributions with 90% intervals covering the maximum among the 5% and 95% values reported in Table B2. Given these parameters, for each model we simulate an SDF of length equal to the size of our data, i.e. 742 months, and compute the model-implied mean, $\mu_m(\phi, \psi)$, and variance, $\sigma_m(\phi, \psi)$.

Moving to the predictors-based bounds, we draw the coefficients in Eq. (18), $\beta_0^S, \beta_1^{S,cay}, \beta_1^{S,dp}$, for stocks, and $\beta_0^B, \beta_1^{B,spr}, \beta_1^{B,y}$, for bonds, from normal distributions. For each parameter, the mean of the normal distribution is set to the sample estimates from the predictive regressions, and the standard deviation is provided by the Newey-West t-stats corrected value (see the regression results in Table 3). Given these parameters, we simulate a series of returns of length equal to 742 months, re-estimate the predictive regressions, and compute the predictors-based bounds using Eq. (6).

Finally, we compute the difference between $\sigma_m(\phi, \psi)$ and $\sigma_{Z(\mu_m)}$ and repeat this exercise 10000 times. We consider three different cases. In the first case (reported in the leftmost block in Table 3) the mean of the SDF, $\mu_m(\phi, \psi)$, is fixed to the long-run mean obtained from a simulations run of 600,000 months, v^{lr}. Given this (population) value of the mean SDF, the only source of uncertainty stems from the location of the bound, $\sigma_Z(v^{lr})$, and the standard deviation implied by the model, $\sigma_m(\phi, \psi)$. The second case (reported in the middle block in Table 3) keeps the parameters of the predictive regressions fixed at their point estimates so that (for a given value of the average SDF, $\mu_m(\phi, \psi)$) there is no uncertainty in the location of the bound, $\sigma_{Z(\mu_m)}$. However, there is uncertainty induced by the fact that the mean of the SDF for the model, $\mu_m(\phi, \psi)$, must be estimated, as well as uncertainty in the standard deviation implied by the model, $\sigma_m(\phi, \psi)$. The last case (reported in the the rightmost block) is the case when there is: (i) uncertainty induced by the fact that the mean of the SDF for the model, $\mu_m(\phi, \psi)$, must be estimated (ii) uncertainty in the location of the bound, $\sigma_{Z(\mu_m)}$ (for a given expected value of the SDF), (iii) and uncertainty in the standard deviation implied by the model, $\sigma_m(\phi, \psi)$ (for a given expected value of the SDF).

D Extensions

The empirical results in the main text have shown that incorporating predictability of asset returns does make the variance bounds tighter and hence it imposes a harder yardstick on asset pricing models that deliver unpredictable discounted returns. In this Section we analyze the robustness of our results to the possibility of misspecification of the model for the conditional moments of returns.

D.1 Predictability, model mis-specification and variance bounds

Recall that the results presented so far are obtained under the assumptions of a time-invariant variance-covariance matrix for returns and a linear model for their conditional means. To investigate possible mis-specification of the conditional moments and the efficiency of the bound we plot in Figure D.1 alternative implementations of the variance bounds. Specifically, along with the predictors-based bound obtained following Bekaert and Liu (2004) (BL), in this figure we plot the bounds obtained following alternatively Gallant, Hansen, and Tauchen (1990) (GHT) and Ferson and Siegel (2003, 2009) (FS).

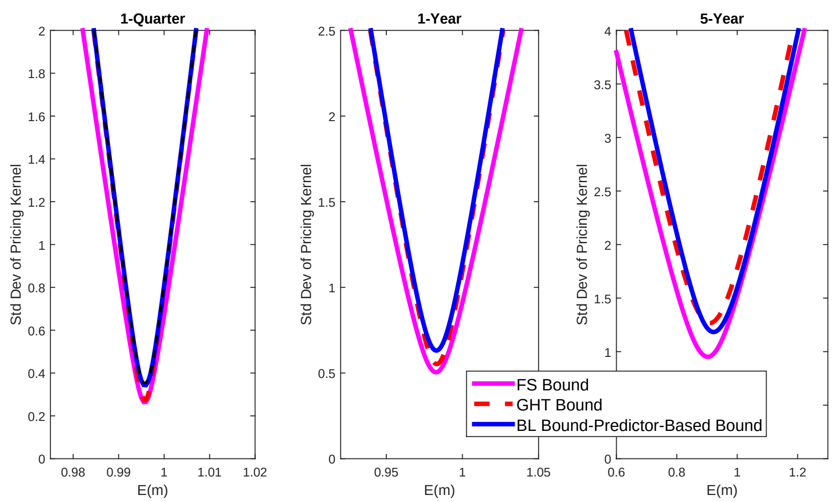


Figure D.1 Alternative implementation of the HJ bounds – SET A. We present the volatility bounds using conditional information based on Ferson and Siegel (2003, 2009) (FS Bound), Bekaert and Liu (2004) (BL-predictors-based bound) and Gallant et al. (1990) (GHT Bound) specifications, respectively. The bounds are generated using asset returns included in SET A. Sample: 1952Q2 - 2012Q3.

Bekaert and Liu (2004) show that their bound, obtained by maximizing the Sharpe ratio over all returns obtained from portfolios that condition on Z_t and that cost 1 on average, must be a parabola under the null of correct moments specification. Figure D.1 shows that in our case we obtain a smooth parabola indeed. The figure, moreover, shows that the GHT bound and the BL bound are virtually on top of each another, i.e. there is no duality gap. This suggests that the BL bound closely approximates the efficient use of conditioning information. Overall the three alternative implementations of the variance bounds that incorporate information from the predictors Z_t generate similar bounds with no visible misspecification. The FS is the lowest bound: this is readily understood by observing that the FS bound collects all those payoffs that are generated by trading strategies that reflect the information available at time t, and that have unit price almost surely equal to one, and not just on average as for the BL case.[1] Although the FS approach yields the most conservative bound, the differences between the three approaches would not change our conclusions. This evidence suggests that misspecification of the conditional moments does not seem to be a driver of our results.

[1]More formally, the FS bound (see Ferson and Siegel (2003)) is defined as

$$\sigma^2_{FS}(v) = \nu^2 \sup_{R^w_{t+h} \in \mathcal{R}^{FS}} \left(\frac{E(R^w_{t+h}) - \nu^{-1}}{Var\left(R^w_{t+h}\right)} \right)^2$$

where $\mathcal{R}^{FS} = \left\{ R^w_{t+h} \in \mathcal{R}^Z \mid w'_t e = 1 \text{ } almost \text{ } surely \right\}$ i.e. the FS variance bound follows from maximizing the Sharpe ratio over the set of returns from portfolios that, while conditioning on Z_t, are required to have unit price almost surely, and not just on average. Therefore, it is evident that $\sigma^2_{FS}(v) \leq \sigma^2_Z(v)$.

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