Monetary Policy and Bond Prices with Drifting Equilibrium Rates *

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Abstract

We propose a framework that reconciles drifting Treasury bond prices with stationary and predictable bond returns. Bond prices are drifting because they reflect the drift in average expected monetary policy rates over the life of the bonds. In our framework, deviations of bond prices from their drift should be stationary and can originate from term premia or temporary deviations from rational expectations in a behavioral framework. Empirically, modeling the drift in monetary policy rates using demographics and productivity trends, plus long-term inflation expectations, leads to stationary deviations of bond prices from their drift that predict future bond returns.

JEL codes: E43, E52, G12, J11.
Keywords: Monetary Policy Rule, Treasury Bond Yields, Term Structure, Drifting Equilibrium Rate Drivers, Bond Return Predictability.

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1 Introduction

Bond prices are drifting: they are non-stationary. This fact has important implications for modeling monetary policy, the term structure of interest rates and holding period excess bond returns. However, these implications have been so far overlooked since both standard factor models for the term structure and (empirical models built on) monetary policy rules are designed for stationary variables. To address this shortcoming, we introduce a novel and simple framework that reconciles the observed drift in the term structure of Treasury yields with stationary and predictable bond returns.[1]

Our approach specifies a monetary policy rule with a drifting equilibrium rate. It follows that the drift in bond prices of any maturity reflects the drift in the monetary policy rate over the life of the bonds. When the (non-stationary) drivers of the monetary policy rates have been correctly specified, deviations of bond prices from their estimated drifts should be stationary. From a theoretical perspective, these deviations could be related to term premium within a no-arbitrage framework (Bauer and Rudebusch, 2020), and/or to expectation errors (about the short-term rate) in a behavioral model where the hypothesis of Full Information Rational Expectations (FIRE) does not hold (Piazzesi et al., 2015; Cieslak, 2018).

Consistently with these different interpretations of the stationary component of Treasury bond prices, we do not impose no-arbitrage restrictions when estimating our term structure model with drifting equilibrium rates. Despite not imposing no-arbitrage restrictions, we show that a framework with drifting bond prices implies a battery of mis-specification tests such as parametric restrictions on yields and their drifts that are analogous to the restriction between prices and dividends in the Campbell and Shiller (1988) present-value model. In

[1] The implications of modeling the drifts in the term structure of yields are not restricted to Treasury bonds. For example, van Binsbergen (2020) finds that accounting for secular trends in interest rates is fundamental for assessing long duration dividend risk.

the data, our proposed model passes all these (mis-specification) tests.

More specifically, we start by showing that there is a drift in monetary policy rates which can be successfully modeled by fluctuations in productivity, demographics and long-term inflation expectations. By being explicit about the non-stationary drivers of rates, our model is purposely transparent and simple (i.e., not involving any filtering). In our first contribution, we show that our monetary policy rule tracks well the evolution of the short-term rate both in- and out-of-sample. Furthermore, through the lens of our modeling approach, monetary inertia is just the manifestation of omitted drivers of the drifting equilibrium policy rates.

Then, we derive the implications of our monetary policy rule specification for the entire term structure of Treasury bond yields. Our approach decomposes bond yields into a drifting component, the average expected sequence of monetary policy rates over the life of the bond, and a residual cyclical component (namely, the deviation of yields from the drifts). In our second contribution, we show that deviations of bond prices from their drifts are indeed cyclical, and comove strongly with state-of-the-art term premium estimates like the one proposed by Bauer and Rudebusch (2020). This is interesting since our framework does not impose no-arbitrage. Thus, a large fraction of what is deemed risk premium may instead simply reflect temporary deviations from rational expectations.

Finally, in our third and last contribution, we show that a framework with drifting bond prices implies the presence of bond predictability. Specifically, we formally show that (stationary) deviations of bond prices from their drift should predict excess bond returns. Empirically, our model generates large R^2 of about 30% (10%) when it is used to predict the one-year (one-quarter) ahead excess returns on bond with maturities ranging from 2 to 10 years.

To sum up, our paper proposes a general framework that reconciles, in a parsimonious way, drifting bond prices with stationary and predictable holding period returns and it is consistent both with a no-arbitrage approach and a behavioral model in which temporary deviations from the FIRE hypothesis are allowed.

Related Literature. Our evidence that bond prices are drifting is in line with several papers documenting a slow-moving component common to the entire term structure (see, for example, Balduzzi et al., 1998 and Fama, 2006).

Stationarity of returns and non-stationarity of prices is common to many asset classes. In the equity space, standard factor models focus on returns and leave prices undetermined. In a related paper focusing on stock prices, Favero et al. (2020) show that modeling the drift in stock prices leads to an equilibrium correction term in a model relating returns to factors; however, this term is invariably omitted in standard factor model of stock prices. Interestingly, in the fixed income space, standard factor models concentrate on bond prices rather than on holding period returns but ignore their drifts. The evidence in this paper shows that a stationary (factor) framework cannot be adopted for yields-to-maturity. In this regard, our analysis supports the literature that models Treasury yields using shifting endpoints (Kozicki and Tinsley, 2001), vector autoregressive models (VAR) with common trends (Negro et al., 2017), and slow-moving averages of inflation (Cieslak and Povala, 2015) and consumption (Jørgensen, 2018).

Our paper does not present a fully-fledged affine term structure models (ATSM). However, standard ATSMs for bond yields assume stationarity, thus ruling out (by design) the drift in bond prices. Hence, our evidence is in line with Bauer and Rudebusch (2020) who propose a term structure model for interest rates with four state variables, one of which being an (unobserved) stochastic trend common across Treasury yields. Importantly, none of the above cited papers explores the implications of drifting equilibrium rates for monetary policy, Treasury yields, and bond returns predictability within a cohesive framework.[2]

Finally, our paper fits into the literature that studies the role played by (shifts in) the monetary conduct in determining the dynamics of bond yields. Berardi et al. (2020) show that the stance of monetary policy—as proxied by the difference between the natural rate of interest and the current level of short term rate—contains valuable information for bond predictability. Ang et al. (2011) show that the evolution of the Fed's response to inflation affect long-term yields. Similarly to Ang et al. (2011), we propose to model monetary policy and the term structure of interest rates jointly. However, our modeling of the policy rule with a drifting equilibrium rate is different from their model with time-varying policy coefficients. In turn, our approach has implications for interest rates comovement and bond returns predictability induced by deviations of bond prices from their drift. These testable implications are unique to our framework and not shared by Ang et al. (2011).

[2] Also, in our framework stationarity of bond returns naturally co-exists with non-stationary bond prices. Bond returns are predicted by the stationary deviations of bond prices from their drift. Interestingly Bauer and Rudebusch (2020) note that, even when no-arbitrage is imposed, the loading of returns on the unobserved common stochastic trend is an order of magnitude smaller than the loading of prices. They also report that predictive regressions of yields on de-trended yields and trend proxies lead to coefficients on the trend that are not significantly different from zero.

2 Modeling Monetary Policy

Monetary policy rates are drifting. This fact is overlooked in standard specification of the monetary policy rules.

In general, monetary policy rules specify the dynamics of the short-term rate, $y_t^{(1)}$, as follows:

$$\begin{aligned} y_t^{(1)} &= y_t^* + \beta' X_t + u_t^{(1)} \\ u_t^{(1)} &= \rho u_{t-1}^{(1)} + \varepsilon_t^{(1)}, \end{aligned} \tag{1}$$

where y_t^* is the equilibrium monetary policy rate,[3] X_t is a vector of stationary monetary policy drivers, and ρ is a parameter usually interpreted as monetary policy persistence. Arguably, the most famous special case of this specification is the Taylor (1993) rule. In this case, the vector X_t is composed of the output gap and the percentage deviation of inflation from its target. Furthermore, the Taylor (1993) rule assumes a constant equilibrium policy rate and, thus, provides a natural benchmark for our analysis.

With a constant equilibrium rate, the typical estimate of ρ is often close to one. This is to be expected since, if monetary policy rates are drifting, any attempt to model them only by means of stationary factors such as the output and inflation gaps naturally leads to a (close to) unit root process for u_t. What it is commonly interpreted as a monetary policy smoothing parameter can very well measure the mis-specification generated by modeling a drifting variable as mean reverting around a constant. Furthermore, if (1) ρ is high but

[3]The "natural" level of real interest rates is often referred to as the "natural", "equilibrium" or "neutral" real rate of interest. Interestingly, the possibility of a non-stationary equilibrium rate is rarely entertained in the traditional literature. See Giammarioli and Valla (2004) and Kiley (2015) for a review of the various concepts and estimation methods adopted in the literature.

smaller than one, and (2) only stationary factors are employed in the policy rule, then long-term forecast of monetary policy rates with a Taylor rule will inevitably (slowly) converge to the sample mean over the estimation period.[4]

Interest rates are sometimes modeled in first-difference which removes the stochastic trend in policy rate at the cost of leaving the equilibrium level of the policy rate undetermined (e.g., Orphanides, 2003). The model in first-difference is a special case of our general specification when $\rho = 1$. Specifying the monetary policy rule in first-difference comes with benefits and costs.[5] The benefit of making the rule independent from the challenging estimation of the level of the equilibrium rate of interest has to be traded-off against the cost of accepting that any monetary policy shock (i.e., any deviation from the rule) leaves an infinite memory on the time series of policy rates. Indeterminacy is a major concern for long-term forecasting, because as the unconditional distribution of policy rates is not defined, the long-run policy rate is also left undetermined.

We propose a "cointegrating" approach to drifting policy rates, where the stationarity of residuals of the monetary policy reaction function is taken as an indication of a valid specification for y_t^*. Equivalently, a valid specification for the equilibrium rate requires that y_t^* is the stochastic trend that drives drifting policy rates.[6]

[4] Rudebusch (2002) highlights the contradiction between apparent high-persistence and low-predictability of policy rates.

[5] Cochrane (2007) provides a thorough discussion on the effects of specifying a model in level vs. first-difference to compute long-term yield-curve decomposition.

[6] Our approach is in line with, e.g., Woodford (2001) who observed that the optimal policy response to real disturbances requires including a time-varying real rate in monetary policy rules.

In particular, we propose to model drifting policy rates as follows:[7]

$$\begin{aligned} y_t^{(1)} &= y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ y_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} + \gamma_3 \pi_t^* \\ u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \varepsilon_t^{(1)} \end{aligned} \tag{2}$$

where $y_t^{(1)}$ is the one-period (three-month) yield, y_t^* is the equilibrium nominal rate, π_t is the percentage annual log change in Personal Consumption Expenditures (PCE), π_t^* is the Fed perceived target rate (PTR), and x_t is the output gap (log percentage difference between real GDP and potential GDP). The drivers of the equilibrium real rate are the age structure of population and potential output growth.[8] We obtain the nominal equilibrium rate by adding the central bank inflation target π_t^*. Appendix A provides details on the data source.

Following Geanakoplos et al. (2004) and Favero et al. (2016), the age structure of the population is described by the ratio of middle-aged (40-49) to young (20-29) population in the U.S. (labelled as MY). Potential output growth is the percentage annual log change in potential output. Both MY_t, and Δx_t^{pot} are highly persistent variables with an AR(1) coefficient of 0.99 at quarterly frequency (see also Appendix Figure A.1).

Finally, in all our tests, we always compare the results from our baseline (drifting) model to the results of a restricted model that, inspired by the large body of literature on the classical Taylor (1993) rule, does not model the drift in monetary policy:

$$\begin{aligned} y_t^{(1)} &= y^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \varepsilon_t^{(1)}. \end{aligned} \tag{3}$$

[7] We consider a forward-looking version of the policy rule as, for example, in Clarida et al. (2000).

[8] See **?** for a comprehensive work on the drivers of the U.S. real equilibrium rate.

2.1 Empirical Results

Panel A of Figure 1 displays the realized nominal short-term rate, the fitted rates from our cointegrated monetary rule (c.f. Equation (2)), and the fitted monetary policy rates from a version of our model which restricts the equilibrium rate to be constant (c.f. Equation (3)). Panel B plots the monetary policy residuals implied by our proposed monetary policy rule and its restricted version. Table 1 reports the estimation results for these two rules.[9]

Figure 1–Panel A shows that our monetary rule with a drifting equilibrium rate tracks well the short-term rate movements throughout the sample. Indeed, the R^2 for the cointegrated specification is about 95% whereas that of a model with constant equilibrium rate is just 11% (c.f. Table 1).[10,11] Figure 1–Panel B shows that the residuals implied by our drifting monetary policy rule are mean reverting. On the other hand, the residuals from a rule with constant equilibrium rates display a close-to-unit root behavior. This is confirmed in Table 1: the residuals from the rule with drifting (constant) equilibrium rates have an autoregressive coefficient equal to 0.67 (0.95).

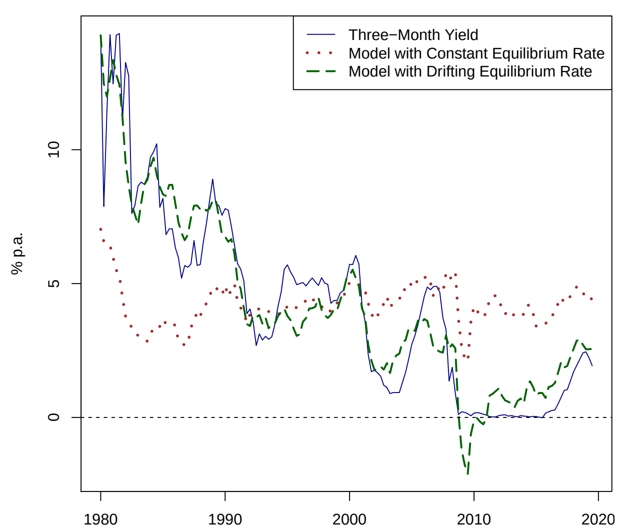
[9]Our estimate of the loading on π_t^* is in line with parameter values reported in Bauer and Rudebusch (2020, Table 1) despite the difference in the maturity of the bond analyzed (their Table 1 analyzes the 10-year bond, whereas we focus on the 3-month Treasury bill).

[10]Furthermore, a regression of the three-month yield on the fitted values implied by the two monetary rules (dotted and dashed lines in Figure 1–Panel A) delivers an estimate of zero on the rule with constant equilibrium rates (3), and a statistically significant estimate not different from one on the drifting rule (2).

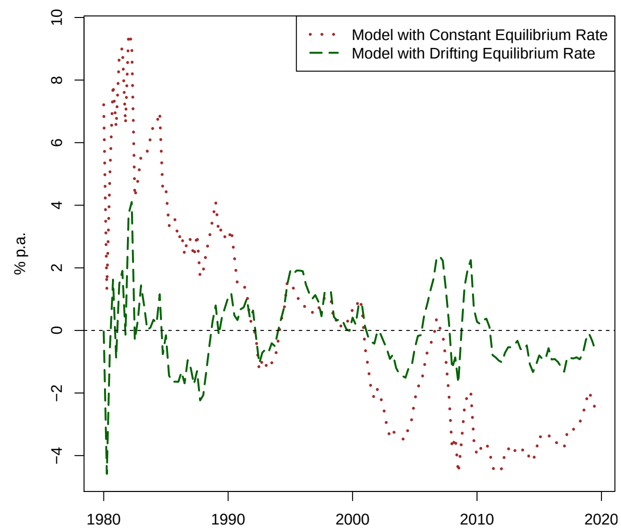
[11]Positing the following cointegration framework where the equilibrium real rate r_t^* is estimated first, i.e.,

$$\begin{aligned} y_t^{(1)} &= \alpha_1 r_t^* + \alpha_2 \pi_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ r_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} \\ u_t^{(1)} &= \rho u_{t-1}^{(1)} + \epsilon_t^{(1)} \end{aligned}$$

leaves our conclusions unaltered. See Appendix Figure B.1.



(a) Rates and fitted values from policy rules.



(b) Residual $u_t^{(1)}$ from policy rules.

Figure 1: Actual vs Fitted Short-Term Rate. Panel (a) shows actual three-month yield and fitted values for our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); see green dashed line) as well as for a model that restricts the equilibrium rate to be constant (c.f. equation (3); see brown dotted line). Panel (b) shows the differences between actual three-months yield and the fitted values. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Figure 2 displays the forecasts implied by the two monetary rules. The rule with constant equilibrium rates generates forecasts that converge fast to the unconditional mean. On the other hand, the drifting monetary policy rule tracks well the future evolution of the short rate for each of the three out-of-sample periods considered in the figure. Appendix Figure B.2 confirms that allowing for inertia in the restricted rule would not alter our conclusion.

Table 1: Short-term rate models with and without drifting equilibrium rate

This table reports the estimates for our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); see column (2)) as well as estimates for a model that restricts the equilibrium rate to be constant (c.f. equation (3); see column (1)). We estimate the two rules by instrumental variables, where the instruments are lags of inflation gap and output gap. The last row reports OLS estimates for the monetary policy residuals' persistence. Values in parenthesis are GMM standard errors that correct for autocorrelation in the residuals. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	Three-Month Yield	
	(1)	(2)
MY		-2.652^{***}
		(0.726)
Δx_t^{pot}		0.932^{***}
		(0.317)
π_t^*		1.656^{***}
		(0.177)
$E_t(\pi_{t+1} - \pi_{t+1}^*)$	0.721	0.709^{***}
	(0.519)	(0.244)
$E_t(x_{t+1})$	0.086	0.389^{***}
	(0.481)	(0.137)
Constant	4.656^{***}	
	(1.036)	
Observations	160	160
Adjusted R^2	0.036	0.950
ρ	0.949^{***}	0.673^{***}
	(0.022)	(0.110)

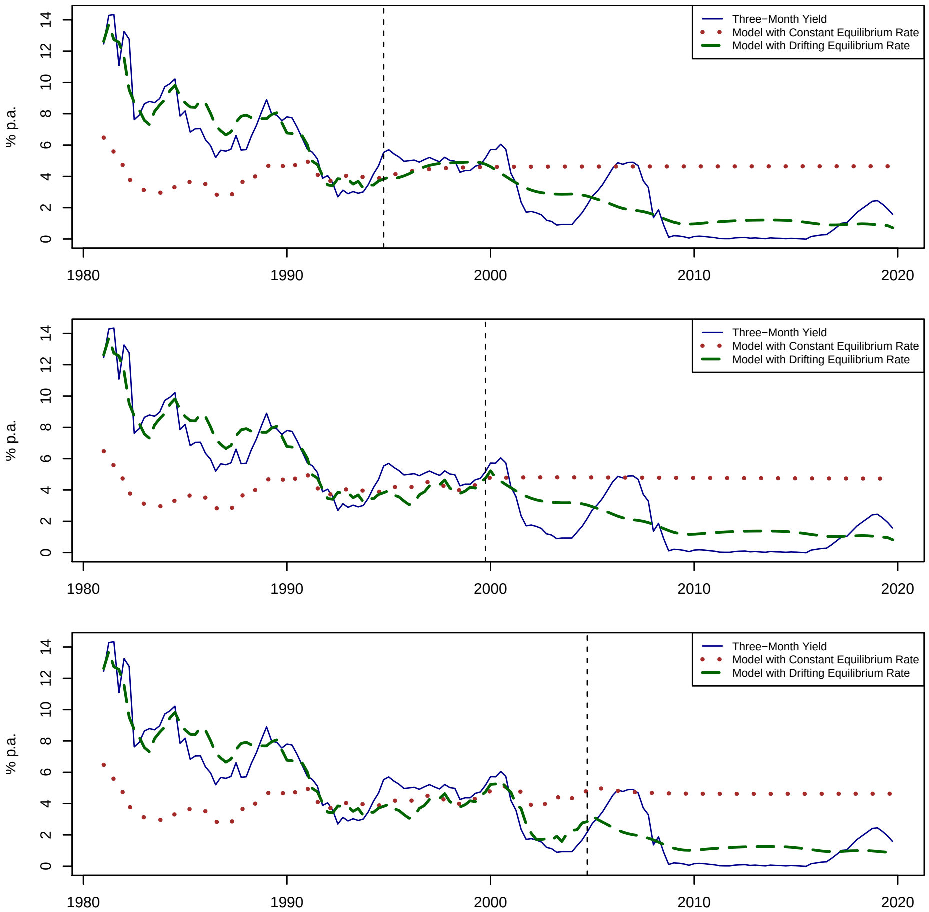


Figure 2: Short-Term Rate Forecasts. This figure shows actual three-month yield and predicted rates implied by our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); green dashed line) and by a model that restricts the equilibrium rate to be constant (c.f. Equation (3); brown dotted line). The forecast of the drifting rule exploits the exogeneity of the demographic variable (MY) and of potential output (Δx^{pot}). In particular, the rule is estimated until 1995, 2000, and 2005 in the top, mid, and bottom panels, respectively. We then use the coefficients estimates, the projections of MY and Δx^{pot} (see also Appendix A), and the forecast of inflation and output gap from a VAR(1) as in equations (8) and (9). π^* is modeled as a random walk. Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

3 Modeling a Drifting Term Structure

The entire term structure is drifting. Models that parsimoniously describe the term structure by projecting rates on a set of factors and by modeling the dynamics of the factors with a VAR will be inevitably confronted with the problem generated by the presence of unit roots in the VAR. Highly persistent VAR generate imprecise forecasts at long-horizons (e.g., Giannone et al., 2019). This feature can explain mixed results from the forecasting performance of affine term structure models (see, for example, Duffee, 2002; Sarno et al., 2016). Remarkably, this problem has not been fully acknowledged until very recently (see Bauer and Rudebusch (2020), Cieslak and Povala (2015), Favero et al. (2016)). We use the drift in monetary policy rates to model the drift in the entire term structure:

$$\begin{aligned} y_t^{(n)} &= y_t^{(n),*} + \delta_0 + u_t^{(n)} \\ u_t^{(n)} &= \rho_n u_{t-1}^{(n)} + \epsilon_t^{(n)} \\ y_t^{(n),*} &= \left(\frac{1}{n}\right) \sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] \end{aligned} \tag{4}$$

Yields at all maturities are decomposed into a trend, $y_t^{(n),*}$, and a cyclical component, $\delta_0 + u_t^{(n)}$. The trend is the average of expected monetary policy rates over the duration of the bond, while the cyclical component is the stationary residuals from the $(1, -1)$ cointegrating relationship between yields and their drift. This decomposition is consistent with the stylized facts reported in the literature that document a slow-moving component common to the entire term structure (e.g., Balduzzi et al., 1998; Fama, 2006; Cieslak and Povala, 2015; Bauer and Rudebusch, 2020).

We consider as valid any model of the term structure that delivers cointegration between

$y_t^{(n)}$ and $y_t^{(n),*}$ with a $(1,-1)$ cointegrating vector and therefore a stationary $u_t^{(n)}$, i.e., $\mid \rho_n \mid < 1$. We do not impose any restrictions on the stationary cyclical component. Under a Rational Expectations-No Arbitrage (RE-NA) approach this component would be identified with the term premium of the n-period bond. Consistently with our approach, Dai and Singleton (2002) argues that it is not plausible to consider the risk premium as a non-mean reverting component. However, a stationary $u_t^{(n)}$ does not necessarily provide support for the RE-NA framework. In fact, a stationary $u_t^{(n)}$ is also consistent with, e.g., temporary deviations from Rational Expectations generated within a Diagnostic Expectations framework (see Gennaioli and Shleifer, 2018) where long rates over-react relative to change in expectations about short rates. Following Bordalo et al. (2018) and d'Arienzo (2020), diagnostic expectations about policy rates can be represented as follows:

$$E^D\left[y_{t+i}^{(1)} \mid I_t\right] = E\left[y_{t+i}^{(1)} \mid I_t\right] + \theta\left(E\left[y_{t+i}^{(1)} \mid I_t\right] - E\left[y_{t+i}^{*} \mid I_t\right]\right) \; . \tag{5}$$

Diagnostic expectations, $E^D\left[y_{t+i}^{(1)} \mid I_t\right]$, differ from rational expectations, $E\left[y_{t+i}^{(1)} \mid I_t\right]$, by a shift in the direction of the information received at time t on deviations of monetary policy from its (stochastic) trend. Under the diagnostic expectations hypothesis agents over-react to the stationary deviations of monetary policy from its trend. Thus, the (stationary) component $u_t^{(n)}$ can in principle be explained by this over-reaction: this component can be observed also when term premia are constant or even absent. Deviations from rational expectations depend on the parameter θ and on the persistence of deviations of monetary policy rates from the rule, $u_t^{(1)}$. The stationarity of $u_t^{(1)}$ implies the progressive convergence at long future horizons of diagnostic expectations for monetary policy rates towards rational expectations. The empirical relevance of overreaction has been recently documented by

Cieslak (2018) for the short end of the curve. Similarly, Piazzesi et al. (2015) provide evidence that realized survey (interest rates) forecast errors as well as forecast differences relative to VAR-based measure may be responsible for the time-variation in bond premia from statistical models.

Consistently with the different possible interpretations of the stationary component of yields we do not impose NA restrictions when estimating our model. Thus, our estimation strategy runs the cost of losing efficiency if NA holds to gain consistency in the case NA is violated.[12] Our full term structure model is specified as follows:

$$
\begin{aligned}
y_t^{(1)} &= y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} && (6)\\
y_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} + \gamma_3 \pi_t^* \\
u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \epsilon_t^{(1)} \\
y_t^{(n)} &= y_t^{(n),*} + \delta_0 + u_t^{(n)} && (7)\\
u_t^{(n)} &= \rho_n u_{t-1}^{(n)} + \epsilon_t^{(n)} \\
y_t^{(n),*} &= \left(\frac{1}{n}\right) \sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] \\
(\pi_t - \pi_t^*) &= \theta_{1,1}\left(\pi_{t-1} - \pi_{t-1}^*\right) + \theta_{1,2} x_{t-1} + \theta_{1,3}\left(y_{t-1}^{(1)} - y_{t-1}^*\right) + v_{1,t} && (8)\\
x_t &= \theta_{2,1}\left(\pi_{t-1} - \pi_{t-1}^*\right) + \theta_{2,2} x_{t-1} + \theta_{2,3}\left(y_{t-1}^{(1)} - y_{t-1}^*\right) + v_{2,t} && (9)\\
& Cov(v_{1,t}, u_t^{(1)}) = Cov(v_{2,t}, u_t^{(1)}) = 0
\end{aligned}
$$

Projections of the equilibrium policy rates depend on productivity and demographics that we take as exogenous. The U.S. Census Bureau and the U.S. Congressional Budget Office

[12] Joslin et al. (2013) find that the estimated joint distribution within a macro-finance term structure model with NA is nearly identical to the estimate from an economic-model-free factor vector-autoregression. Our evidence in Section 3.1.1 suggests that this conclusion is likely to hold true also in a model of drifting term structure.

provide ready-to-use projections respectively for MY and potential output. Equations (8) and (9) are used to compute the projections of inflation and output gaps. The dynamics of these two stationary variables depend on their own lags and on a third stationary variable: the deviation of the short-term rate from its trend. This cycle in monetary policy enters the dynamics of output and inflation gaps with a one-quarter lag; this is consistent with the delay with which monetary policy affects these variable in our specification of the forward looking policy rule (6).

3.1 Empirical Results

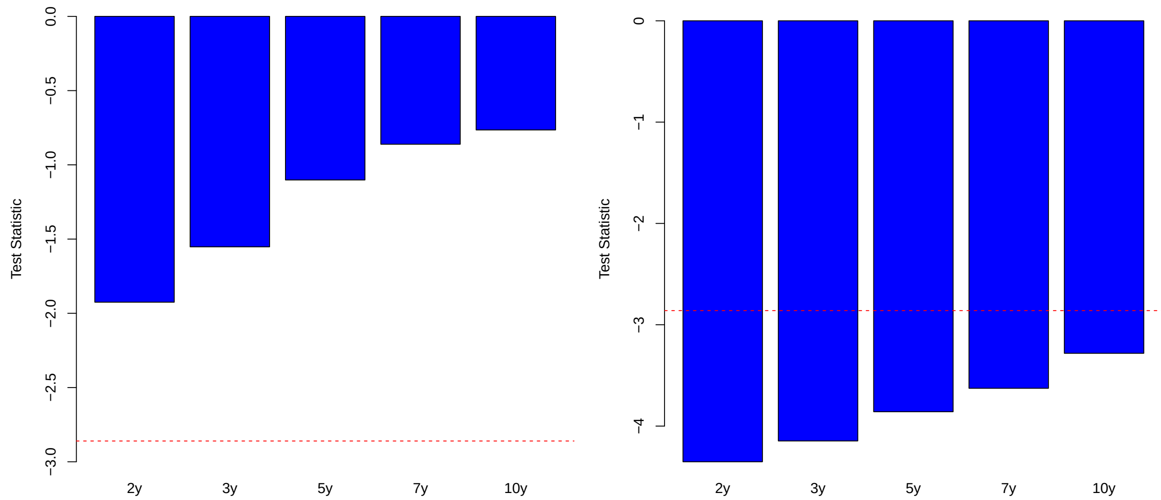
3.1.1 Misspecification test for term structure models

The validity of a model with drifting monetary policy rates and bond prices can be assessed by checking the existence of cointegrating relationships with parameters $(1, -1)$ between $y_t^{(n)}$ and $y_t^{(n),*}$ (see equation (7)). Thus, in this section we investigate the strength of the cointegrating relationship, the $(1, -1)$ parametric restriction, and the behavior of the residuals for our baseline model (see equations (6)–(9)) as well as for its restricted version where the drift in monetary policy is assumed away (i.e., $y_t^* = y^*$).

Figure 3 reports the results for the (strength of the) cointegration relationship for five maturities ranging from 2- ($n = 8$ quarters) to 10-years ($n = 40$ quarters). The left panel is for the restricted model whereas the right panel is for our model with drifting equilibrium policy rates.

Our model provides overwhelming evidence to reject the null hypothesis of absence of cointegrating relation between $y_t^{(n)}$ and $y_t^{(n),*}$ for all the considered maturities.

Next we study the behavior of the residual. Specifically, Figure 4 shows the decomposition of the 10-year yield $y_t^{(40)}$ into $y_t^{(40),*}$ and $\delta_0 + u_t^{(40)}$, as per equation (7). As before,



(a) Model with constant equilibrium rate.

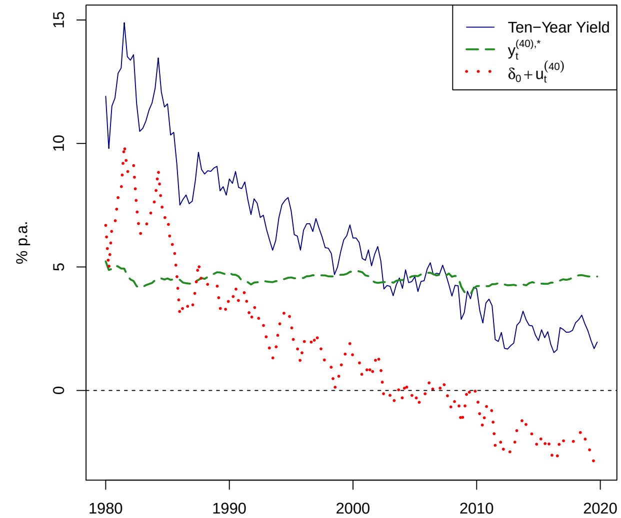
(b) Model with drifting equilibrium rate.

Figure 3: Engle and Granger (1987) Cointegration Test.: This figure shows results for the Engle and Granger (1987) cointegration test for the residuals from regressing $y_t^{(n)}$ on $y_t^{(n),*}$ for different maturities. Panel (a) reports test statistics for a model that restricts the equilibrium rate to be constant (c.f. equation (3)). Panel (b) reports test statistics for our (cointegrated) model with drifting equilibrium rates (c.f. equations (6)–(9)). The null hypothesis is absence of cointegration. The dashed red line is the critical value at 5% level of significance as suggested by MacKinnon (2010). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

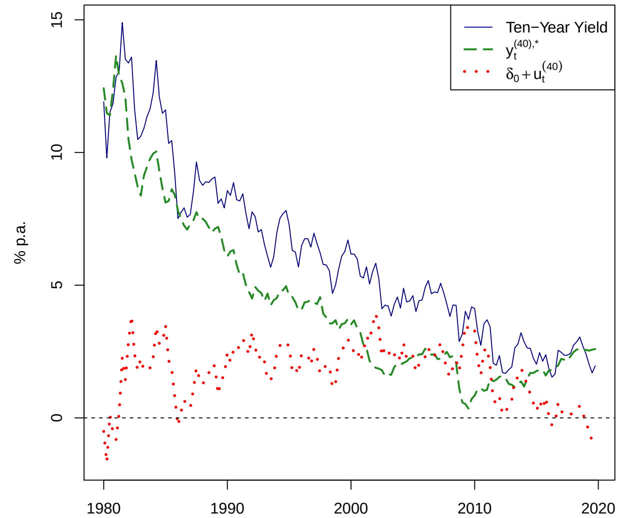
the left panel refers to the restricted model whereas the right panel refers to our benchmark model with drifting equilibrium policy rates. It is obvious that the two models have opposite implications: the residuals (dotted line) follow a random walk under the classical model with constant equilibrium rates, but are stationary in our model with drifting rates.[13,14] Importantly, Appendix Figure B.3 shows that our estimated deviations of bond prices from their

[13]Replacing the perceived target rate π_t^* with a fixed target rate at 2% as in the classical Taylor rule, leaves our conclusion unchanged: the 10-year residual is close to a random walk with an AR(1) coefficient of 0.98.

[14]Wright (2011) argue for term premiums to decline internationally over the sample 1990–2007. Bauer et al. (2014) and Wright (2014) discuss the extent to which small-sample bias in maximum likelihood estimates of affine term structure models alters the conclusions about term premia and its (a)cyclical properties. Our evidence is complementary: we do not focus on statistical biases but we stress the importance of modeling the economic determinants of equilibrium rates. Furthermore, our framework is flexible and allows, without imposing, to interpret the (stationary) deviations of bond prices from their drifts as term premia.



(a) Model with constant equilibrium rates.



(b) Model with drifting equilibrium rates.

Figure 4: Decomposing long-term rates. Panel (a) shows the decomposition of the ten-year yield implied by a model which assumes away drifting monetary policy rates (i.e., $y_t^* = y^*$). Panel (b) shows the decomposition of the ten-year yield implied by our model with drifting equilibrium rates (see equations (6)–(9)). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

drifts comove strongly with state-of-the-art term premium estimates like the one proposed by Bauer and Rudebusch (2020). However, our framework does not impose no-arbitrage and, thus, hints to the possibility that a large fraction of what is deemed to be risk premium is a mere reflection of temporary deviations from rational expectations.

Finally, Appendix Table B.1 confirms that, within our framework with drifting policy rates, the parametric restriction $(1, -1)$ on the cointegrating relationship between yields and their drift is supported in the data for every maturities ranging from 2- to 10-years.

3.1.2 The dynamics of cyclical yields components and further discussions

Despite not having presented a fully-fledged factor term structure model, our evidence that bond yields and their drifts can be naturally described via cointegration, once the drift in the short-term policy rate is properly modeled, has important implications for factor models of interest rates.

First, principal components model blindly applied to bond yields as if they were stationary cannot work. Affine term structure models should follow the lead of Bauer and Rudebusch (2020) and allow for "falling (i.e., drifting) stars". Interpolated models based on the projection of Nelson-Siegel factors, following the strategy first proposed by Diebold and Li (2006), need to be complemented with a model for drifts in factors. In fact, forecasts based on stationary VARs would inevitably miss the dynamics induced by the drift.

Second, we need both drivers (to model prices) and factors (to model returns). In our benchmark model, we adopted three drivers (MY_t, Δx_t^{pot}, and π_t^*) to describe the drift in monetary policy and, consequently, in the entire term structure. The cyclical component of monetary policy rates is then modeled with three stationary factors: output gap, inflation gap, and deviations of monetary policy rates from their long-term drift.

Third, our evidence contributes to the debate of spanned versus unspanned factors. A factor is defined as unspanned by the yield curve if it is not deterministically related to the cross-section of interest rates. Factors are taken as stationary variables. We have shown that—because of the drift in interest rates—both non-stationary variables (drivers) and stationary variables (factors) are needed to model properly the term structure of Treasury yields. We separate interest rates of all maturities into two components: a drift component and a cyclical component. The trend component in monetary policy is unspanned by the yield curve. This is because this long-run trend does not affect yields directly but only trough the drift in monetary policy rates. Differently from Bauer and Rudebusch (2020), our trend component is observable and related to productivity, demographics, and inflation trends. The cyclical component of the short-term rate depends instead on monetary policy shocks, deviation of inflation from its long-run target and the output gap. The (spanned/unspanned) nature of the cyclical component depends on its impact on expected monetary policy and term premia.[15] If the impact is equal but with opposite sign, then the cyclical component is unspanned; otherwise, it becomes spanned. A test of spanning can be constructed once the two components of yields at all maturities are identified and the impact of factors on these two components is identified and estimated.

Since the seminal contribution of Joslin et al. (2014), a vast literature has thought of output (gap) and inflation (gap) as unspanned factors. A contribution of our analysis is to point to the importance of decomposing yields into trend and cycle before evaluating the nature of (non-stationary) drivers and (stationary) factors as spanned or unspanned. In fact, the question on the nature of spanned or unspanned of factors can be properly answered only after modeling of the drift in the term structure with non-stationary drivers.

4 Predicting Holding Period Excess Returns

Predictability of interest rates on the basis of the cointegration between $y_t^{(n)}$ and $y_t^{(n),*}$, also implies predictability of holding period excess returns on the basis of the stationary deviations of bond yields from their drift.

To see this, write the expected excess returns obtained by holding for one period the

[15]Interestingly, the cyclical component of yields, $u_t^{(n)}$, has a long-run unit coefficient on the deviations of monetary policy rates from equilibrium rates, i.e. $u_t^{(1)}$ (see Appendix Figure B.4).

n-period bond as:

$$
\begin{aligned}
E_t(rx_{t+1}^{(n)}) &= y_t^{(n)}n - (n-1)E_t(y_{t+1}^{(n-1)}) - y_t^{(1)} \\
&= y_t^{(n)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n)}\right) - y_t^{(1)} \\
&= y_t^{(n)} - y_t^{(1)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right) - (n-1)\left(y_t^{(n-1)} - y_t^{(n)}\right), \qquad (10)
\end{aligned}
$$

where $y_t^{(n)} - y_t^{(1)}$ is the slope of the term structure, $\left(y_t^{(n-1)} - y_t^{(n)}\right)$ is known as the roll-down, and $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right)$ is the expected change in prices of the $(n-1)$-maturity bond. Since the seminal contributions by Fama and Bliss (1987) and Campbell and Shiller (1991), the slope of the term structure has played a central role for forecasting bond returns. Indeed, it is common to assume away any predictability arising from $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right)$, since the level of the term structure is deemed to be close to unforecastable (see, e.g., Duffee, 2013).

Our proposed "cointegrated" specification of the monetary policy rule and the term structure suggests otherwise. Using Equation (7) and the autoregressive dynamics of the residual, one can express the expected price changes as

$$
E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)} = E_t\left(y_{t+1}^{(n-1),*} - y_t^{(n-1),*}\right) + \left(\rho_{(n-1)} - 1\right)\underbrace{\left(y_t^{(n-1)} - y_t^{(n-1),*} - \delta_0\right)}_{u_t^{(n-1)}}. \qquad (11)
$$

Therefore, in our model, persistent but stationary deviations of bond prices from their drift, $u_t^{(n-1)}$, show up as a natural predictor of excess bond returns.[16] This term has gone unrecognized since standard models start off with stationary factor (within our framework, this is equivalent to assume a constant equilibrium rate). In turn, this leads to a close-to-

[16] More precisely, $u_t^{(n-1)}$ should forecast the price change component in bond returns. However, empirically the correlation between $rx_{t+4}^{(n)}$ and the price change term, $-\left(y_{t+4}^{(n-4)} - y_t^{(n-4)}\right)$, is high at 93%, 95%, 97%, 98%, and 99% for $n = 8, 12, 20, 28, 40$ quarters, respectively.

unit-root residual (c.f., Figure 4(a)), or $\rho_{(n-1)} - 1 \approx 0$ (and the level being a random walk $E_t(y_{t+1}^{(n-1)}) = y_t^{(n-1)}$).[17]

We start the evaluation of the predictive performance of our model with a drifting equilibrium rate by running the following regression:

$$rx_{t+4}^{(n)} = \alpha + \beta E_t(rx_{t+4}^{(n)}) + \epsilon_t \ , \tag{12}$$

where $rx_{t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity n-quarters. We denote with $E_t(rx_{t+4}^{(n)})$ the expected excess return implied by our specification that allows for stationary deviations of bond prices from their drifts.[18] We compare our specification to the classical model with a constant equilibrium rate.[19] Table 2 displays the results for the model with constant equilibrium rate in Panel A, and the results for our model with drifting bond prices in Panel B. We consider maturities ranging from 2 ($n = 8$ quarters) to 10 years ($n = 40$ quarters). The regression of realized excess returns on the expected returns implied by our (cointegrated) model with drifting equilibrium rates delivers statistically significant estimates and coefficients of determination that are greater than 30% at all maturities.[20] On the other hand, a classical model with constant equilibrium rates leads to a coefficient not significantly different from zero and to small explanatory power.

[17] Cieslak and Povala (2015) and Jørgensen (2018) predict bond returns using a de-trended (term structure) level factor. Using their proposed persistence-based Wold decomposition, Ortu et al. (2020) extract a cyclical component from the level of the yield curve and show that it contains information about future excess bond returns. To our knowledge, we are the first to show that a cyclical component of the level of the term structure emerges as a natural predictor within a cointegrated framework of bond prices.

[18] We exploit equations (6)–(9) together with the exogeneity of demographics and potential output to construct the expected change in constant-maturity yield $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1}\right)$ in equation (10).

[19] To make our results comparable to a large literature (e.g., Cochrane and Piazzesi, 2005; Cieslak and Povala, 2015) we focus on one-year excess returns. However, our conclusions are identical when we use one-quarter holding period returns.

[20] The constant is not statistically significant for bond with maturities $n = 8, 12, 20$ quarters.

Table 2: Predictive Regressions across Different Maturities

This table reports OLS estimates for the regression $rx^{(n)}_{t+4} = \alpha + \beta E_t(rx^{(n)}_{t+4}) + \epsilon_t$ where $rx^{(n)}_{t+4}$ is the realized one-year holding period excess return of a bond with maturity n-period and $E_t(rx^{(n)}_{t+4})$ is the expected excess return implied by our specifications. Panel A reports results for the classical model with a constant equilibrium rate. Panel B reports results for our model with drifting equilibrium rates. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Panel A: Model with constant equilibrium rate.

	$rx^{(8)}_{t+4}$ (1)	$rx^{(12)}_{t+4}$ (2)	$rx^{(20)}_{t+4}$ (3)	$rx^{(28)}_{t+4}$ (4)	$rx^{(40)}_{t+4}$ (5)
$E_t(rx^{(8)}_{t+4})$	0.146**				
	(0.069)				
$E_t(rx^{(12)}_{t+4})$		0.107			
		(0.078)			
$E_t(rx^{(20)}_{t+4})$			0.077		
			(0.077)		
$E_t(rx^{(28)}_{t+4})$				0.067	
				(0.075)	
$E_t(rx^{(40)}_{t+4})$					0.065
					(0.075)
Observations	156	156	156	156	156
R^2	0.107	0.062	0.035	0.029	0.028

Panel B: Model with drifting equilibrium rate.

	$rx^{(8)}_{t+4}$ (1)	$rx^{(12)}_{t+4}$ (2)	$rx^{(20)}_{t+4}$ (3)	$rx^{(28)}_{t+4}$ (4)	$rx^{(40)}_{t+4}$ (5)
$E_t(rx^{(8)}_{t+4})$	0.808***				
	(0.150)				
$E_t(rx^{(12)}_{t+4})$		0.788***			
		(0.165)			
$E_t(rx^{(20)}_{t+4})$			0.682***		
			(0.159)		
$E_t(rx^{(28)}_{t+4})$				0.698***	
				(0.162)	
$E_t(rx^{(40)}_{t+4})$					0.614***
					(0.155)
Observations	156	156	156	156	156
R^2	0.360	0.362	0.329	0.360	0.319

We also highlight that the model with constant equilibrium rates performs worse than a (reduced-form) model based just on the slope. This is easily explained. The realized returns $rx_{t+4}^{(n)}$ on the left hand side of (12) are stationary whereas the expected returns $E_t(rx_{t+4}^{(n)})$ from the model with constant equilibrium rate is non-stationary since it inherits the drift from the residual component $u_t^{(n)}$ (c.f. Figure 4).

To further dissect the unique contribution coming from our cointegrated approach, Table 3 shows that the expected change in the $(n-1)$-maturity bond prices drives away the predictability of the slope (column (1)), and that deviations of bond prices from their drift, $u_t^{(n-1)}$, are the most important driver of such predictability (c.f. columns (3) and (4)). Also, the loading on the cyclical component $u_t^{(n-1)}$ is negative as predicted by our framework: if $0 < \rho_{(n-1)} < 1$, then next period returns are negative in times when bond prices are higher than those implied by their drift.[21]

Finally, Appendix Table C.1 shows that the US cyclical component $u_t^{(n)}$ predict UK and Canadian bond returns, even after controlling for the local slope of the term structure.[22] This finding resonates with the evidence in Dahlquist and Hasseltoft (2013). Despite this similarity, Dahlquist and Hasseltoft (2013) attributes the international comovement in bond returns to a global (admittedly, mostly US) bond risk premium; on the other hand, we have not imposed no-arbitrage restrictions so that our cyclical component is also compatible with investors overreacting to deviations of policy rates from its trend leading to overestimation

[21] Appendix Table B.2 shows that the same conclusions apply when we forecast bond returns with maturities ranging from 2- to 7-years. Appendix Table B.3 confirms that stationary deviations of bond prices from their drift also predict quarterly holding period bond returns (i.e., non-overlapping returns).

[22] In the spirit of our model, we employ the local slope of the term structure as a proxy for the deviations of non-US yields from their drifts. Controlling for the local cyclical component does not change our conclusion. However, it is worth to emphasize that the lack of an exogenous potential output series, Δx_t^{pot}, and of a perceived target inflation rate, π_t^*, may be responsible for the poor performance of the local cycle in Canada and UK. Further investigation on this topic is on our agenda for future research.

of future short rates (and lower bond returns).[23]

Table 3: Dissecting Predictive Regressions

This table reports OLS estimates for the regression $rx_{t+4}^{(40)} = \alpha + \beta' X_t + \epsilon_t$ where $rx_{t+4}^{(40)}$ is the realized one-year holding period excess return of a bond with maturity 10-year and X_t contains different return predictors. Column (1) exploits equation (10) reported here for reader's convenience:

$$E_t(rx_{t+4}^{(40)}) = y_t^{(40)} - y_t^{(4)} - (40-4)\left(E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)}\right) - (40-4)\left(y_t^{(40-4)} - y_t^{(40)}\right).$$

Column (2) shows that the slope is a significant predictor of excess bond returns when considered in isolation. Columns (3) and (4) exploit the decomposition of expected price changes per equation (11) reported here for reader's convenience:

$$E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)} = E_t\left(y_{t+4}^{(40-4),*} - y_t^{(40-4),*}\right) + \left(\rho_{(40-4)} - 1\right) u_t^{(40-4)}.$$

In columns (3) and (4) we neglect the roll-down term which empirically is found to be insignificant. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+4}^{(40)}$			
	(1)	(2)	(3)	(4)
$y_t^{(40)} - y_t^{(4)}$	3.257	2.320	1.139	1.010
	(1.995)	(1.548)	(1.933)	(1.390)
$-(40-4)\left(E_t(y_{t+4}^{(36)}) - y_t^{(36)}\right)$	0.538***			
	(0.151)			
$-(40-4)(y_t^{(36)} - y_t^{(40)})$	−4.359			
	(3.871)			
$-(40-4)\left(E_t(y_{t+4}^{(36),*}) - y_t^{(36),*}\right)$			0.204	
			(2.071)	
$-(40-4)\ u_t^{(36)}$			−0.607***	−0.615***
			(0.182)	(0.160)
Adjusted R^2	0.341	0.060	0.317	0.321

[23] Our findings are also consistent with the idea that the Fed is the leader among central banks in setting monetary policy (Brusa, Savor and Wilson, 2019). See also One Policy to Rule Them All: Why Central Bank Divergence Is So Slow (Wall Street Journal, 2016) for a recent discussion on the topic.

5 Conclusions

This paper has proposed a general framework to model a common drift in bond prices.

We started by showing that there is a drift in monetary policy rates which can be successfully modeled by fluctuations in productivity, demographics and long-term inflation expectations. Modeling the drift component of monetary policy rates produces monetary policy residuals that are substantially less persistent than those implied by standard policy rules. Thus, through the lens of our modeling approach, monetary inertia is just the manifestation of omitted factors in the estimated rule.

The drift in bond prices is then described by the average of expected monetary policy (drifting) rates over the residual life of the bond. Appropriate modeling of the drift in monetary policy must deliver stationary deviation of yields to maturity from their drift. These stationary deviations of bond prices from their drift could be explained by the presence of term premia in a no-arbitrage framework or by temporary deviations from rational expectations in a behavioral framework.

Our evidence supports the presence of stationary deviations of bond prices from their drift. When these deviations are interpreted as term premia, our finding implies that models without drift in monetary policy and bond prices cannot generate stationary term premia.

Finally, we have shown that persistent but stationary deviations of bond prices from their drift predict excess returns. Next period returns from holding long-term bonds are negative in times when bond prices are higher than those implied by their drift. Once again this predictability could be related either to predictable term premia or to the reversion of temporary overreaction about future monetary policy. Future research should focus on investigating the origins of the bond price deviations from their drift and of predictability of holding period returns documented in this paper.

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Appendix

A Data

We employ quarterly data in our empirical analysis; thus, we proxy for the 1-period bond yields using the end-of-quarter 3-month Treasury bill rates from the Federal Reserve's H.15 release. Our sample period starts with Paul Volcker's appointment as Fed chairman, because of evidence that monetary and macroeconomic dynamics changed at that time (e.g., Gertler et al., 1999).

Zero-coupon Treasury yields with 1- to 10-year maturities are from Gürkaynak et al. (2007).

The Federal Reserve's perceived target rate (PTR) for inflation is a survey-based measure of long-run inflation expectations; PTR is used in the Fed's FRB/US model and available at `https://www.federalreserve.gov/econres/us-models-package.htm`.

MY is available until 2050 and is hand-collected from various past Census reports available at `https://www.census.gov/data.html`. Potential output is available until 2030 and can be downloaded at `https://fred.stlouisfed.org/series/GDPPOT`. See also Figure A.1.

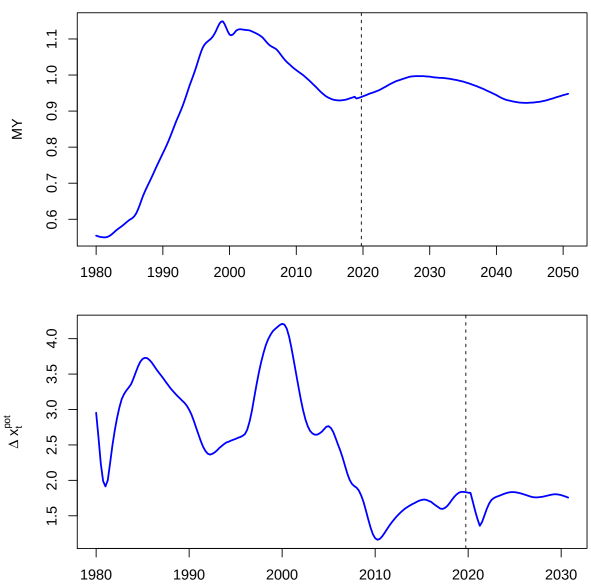

Figure A.1: Demographics and Potential Output Growth. This figure shows the dynamics for the ratio of middle-aged (40-49) to young (20-29) population, MY, and for potential output growth, Δx_t^{pot}. MY is available until 2050 and is hand-collected from various past Census reports available at `https://www.census.gov/data.html`. Potential output is available until 2030 and can be downloaded at `https://fred.stlouisfed.org/series/GDPPOT`. Dotted vertical lines denote the end of our sample, i.e., 2019:Q4. Quarterly observations.

B Additional Results

Table B.1: Testing Parametric Restriction on the Cointegrating Relationship between Yields and Drifting Equilibrium Rates

This table reports OLS estimates for the regression $y_t^{(n)} = \alpha + \beta y_t^{(n),*} + \varepsilon_t$, where $y_t^{(n)}$ is the observed yield at time t of a bond with maturity n-period and $y_t^{(n),*} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E[y_{t+i}^{(1)} \mid I_t]$. Values in parethesis are 95% confidence interval. Costant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$y_t^{(8)}$	$y_t^{(12)}$	$y_t^{(20)}$	$y_t^{(28)}$	$y_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$y_t^{(8),*}$	1.077***				
	(0.940, 1.214)				
$y_t^{(12),*}$		1.056***			
		(0.913, 1.199)			
$y_t^{(20),*}$			1.013***		
			(0.855, 1.171)		
$y_t^{(28),*}$				0.985***	
				(0.815, 1.155)	
$y_t^{(40),*}$					0.962***
					(0.766, 1.158)
Observations	160	160	160	160	160
R^2	0.921	0.913	0.903	0.894	0.882

Table B.2: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression $rx_{t+4}^{(n)} = \alpha + \beta_1(y_t^{(n)} - y_t^{(4)}) + \beta_2(-(n-4)\ u_t^{(n-4)}) + \epsilon_t$, where $rx_{t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity n-period, $y_t^{(n)} - y_t^{(4)}$ is the slope for a n-period bond, and $(-(n-4)\ u_t^{(n-4)})$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+4}^{(8)}$		$rx_{t+4}^{(12)}$		$rx_{t+4}^{(20)}$		$rx_{t+4}^{(28)}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$y_t^{(8)} - y_t^{(4)}$	1.613***							
	(0.540)							
$-(8-4)\ u_t^{(4)}$	−0.930***	−0.851***						
	(0.219)	(0.217)						
$y_t^{(12)} - y_t^{(4)}$			1.637**					
			(0.765)					
$-(12-4)\ u_t^{(8)}$			−0.792***	−0.759***				
			(0.199)	(0.199)				
$y_t^{(20)} - y_t^{(4)}$					1.563			
					(0.994)			
$-(20-4)\ u_t^{(16)}$					−0.726***	−0.744***		
					(0.178)	(0.179)		
$y_t^{(28)} - y_t^{(4)}$							1.356	
							(1.145)	
$-(28-4)\ u_t^{(24)}$							−0.682***	−0.720***
							(0.168)	(0.171)
Observations	156	156	156	156	156	156	156	156
Adjusted R^2	0.323	0.243	0.339	0.270	0.361	0.313	0.357	0.331

Table B.3: Predictive Regressions (quarterly holding period returns): Slope versus Cyclical component

This table reports OLS estimates for the regression $rx_{t+1}^{(n)} = \alpha + \beta_1(y_t^{(n)} - y_t^{(1)}) + \beta_2(-(n-1)\ u_t^{(n-1)}) + \epsilon_t$, where $rx_{t+1}^{(n)}$ is the realized one-quarter holding period excess return of a bond with maturity n-period, $y_t^{(n)} - y_t^{(1)}$ is the slope for a n-period bond, and $(-(n-1)\ u_t^{(n-1)})$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+1}^{(8)}$	$rx_{t+1}^{(12)}$	$rx_{t+1}^{(20)}$	$rx_{t+1}^{(28)}$	$rx_{t+1}^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$y_t^{(8)} - y_t^{(1)}$	-0.151^{*}				
	(0.084)				
$-(8-1)\ u_t^{(7)}$	-0.146^{***}				
	(0.036)				
$y_t^{(12)} - y_t^{(1)}$		-0.103			
		(0.154)			
$-(12-1)\ u_t^{(11)}$		-0.183^{***}			
		(0.043)			
$y_t^{(20)} - y_t^{(1)}$			-0.026		
			(0.213)		
$-(20-1)\ u_t^{(19)}$			-0.183^{***}		
			(0.046)		
$y_t^{(28)} - y_t^{(1)}$				0.025	
				(0.251)	
$-(28-1)\ u_t^{(27)}$				-0.172^{***}	
				(0.045)	
$y_t^{(40)} - y_t^{(1)}$					0.062
					(0.307)
$-(40-1)\ u_t^{(39)}$					-0.153^{***}
					(0.044)
Observations	159	159	159	159	159
R^2	0.140	0.130	0.116	0.108	0.093

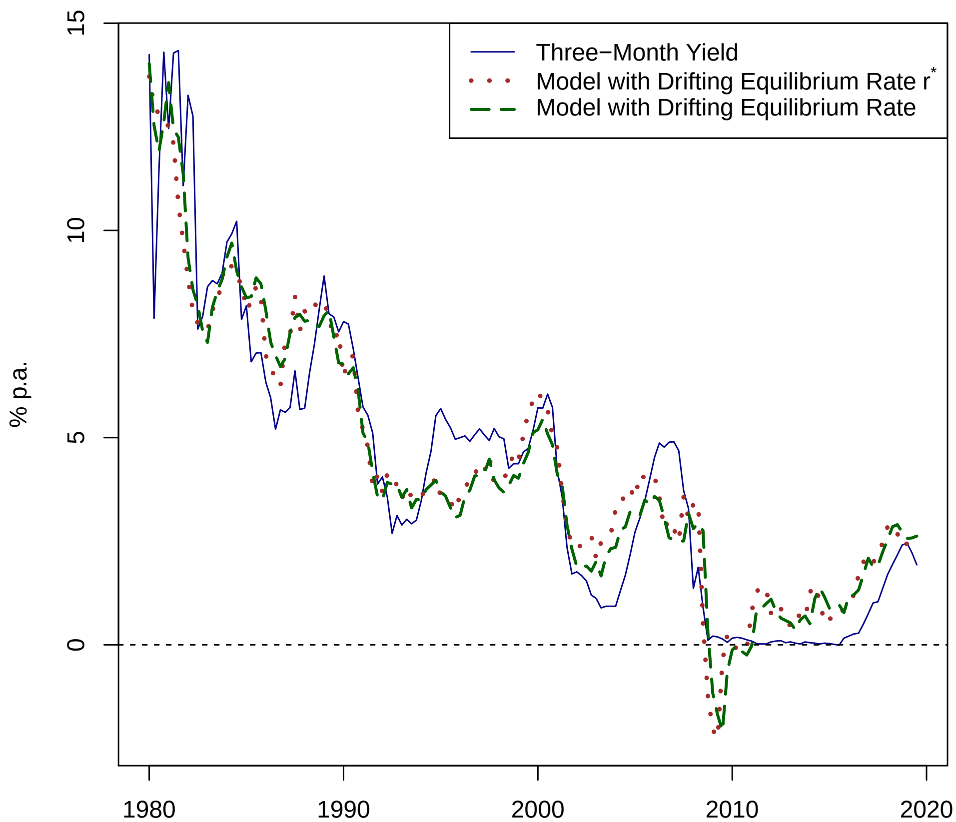

Figure B.1: Actual vs Fitted Short-Term Rate: Additional Results. This figure shows actual three-months yield and fitted values for our baseline (cointegrated) model with drifting equilibrium rates (c.f. equation (2); green dashed line), and for a cointegrated rule with r^* (brown dotted line). The estimated cointegrated policy rule with r^* has the following coefficients (HAC standard errors in parenthesis):

$$y_t^{(1)} = \underset{(0.092)}{0.667^{***}} \; r_t^* + \underset{(0.068)}{1.449^{***}} \; \pi_t^* + \underset{(0.173)}{0.822^{***}} \; E_t(\pi_{t+1} - \pi_{t+1}^*) + \underset{(0.083)}{0.318^{***}} \; E_t(x_{t+1}), \;\; R^2 = 94.3\%$$
.

We denote r^* as the equilibrium real rate. We get an estimate for the equilibrium real rate by regressing the real rate $r_t = y_t^{(1)} - E_t(\pi_{t+4})$ on MY and potential output growth. We use as $E_t(\pi_{t+4})$ the expected one-year ahead inflation from the Survey of Professional Forecasters (SPF). The estimates for r^* are:

$$r_t^* = -\underset{(1.397)}{4.040^{***}} \; MY_t + \underset{(0.309)}{1.812^{***}} \; \Delta x_t^{pot}, \; R^2 = 68\% \; .$$

Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

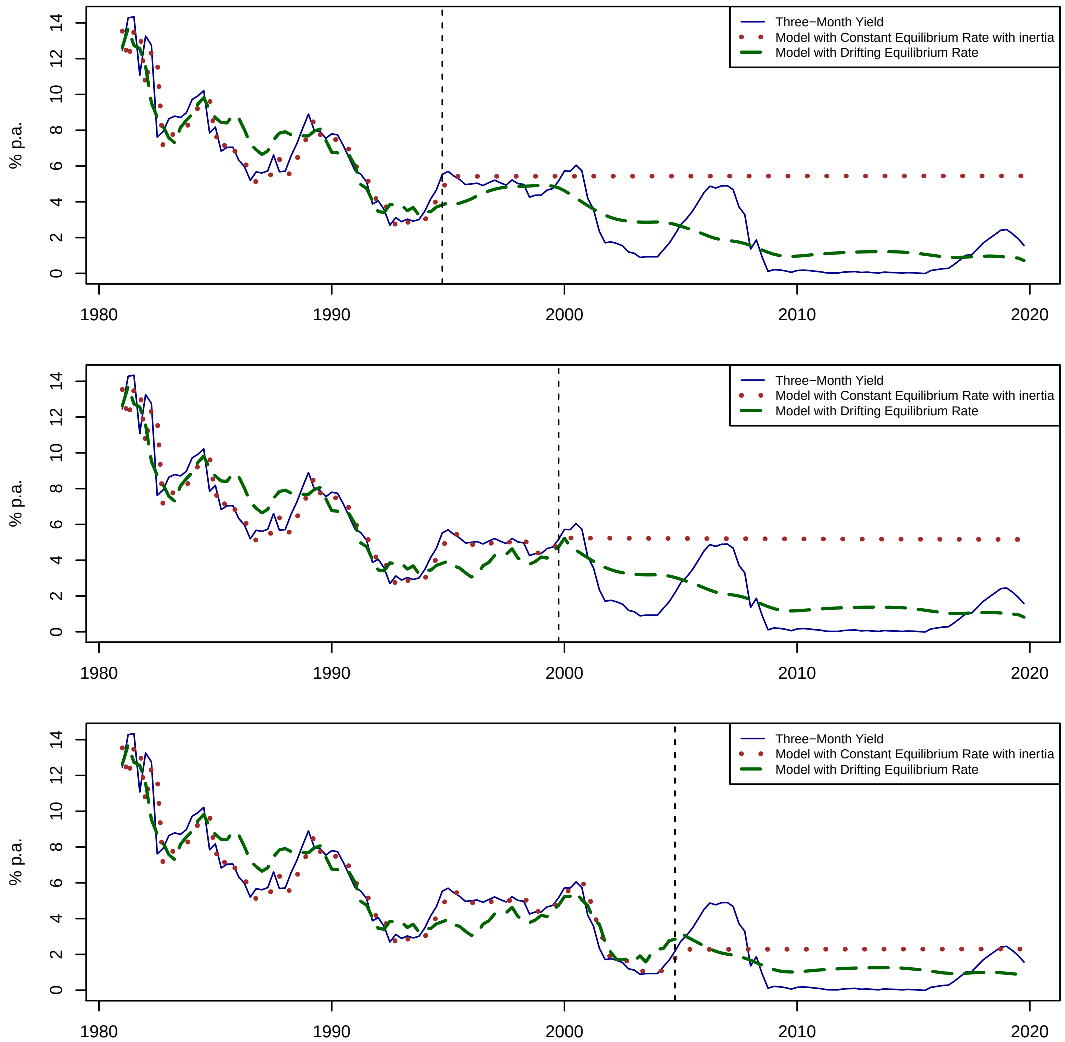

Figure B.2: Short-Term Rate Forecasts: Additional Results. This figure shows actual three-months yield and predicted rates implied by equation (2) in case of the policy rule with constant equilibrium rate and inertia (brown dotted line) or our baseline (cointegrated) model with drifting equilibrium rates (green dashed line). The estimated empirical Taylor rule with inertia has the following coefficients (HAC standard errors in parenthesis):

$$y_t^{(1)} = \underset{(0.108)}{0.310}^{***} + \underset{(0.015)}{0.935}^{***}\, y_{t-1}^{(1)} - \underset{(0.140)}{0.034}\, E_t(\pi_{t+1} - \pi^*_{t+1}) + \underset{(0.028)}{0.070}^{**}\, E_t(x_{t+1}),\ R^2 = 92.7\%.$$

Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

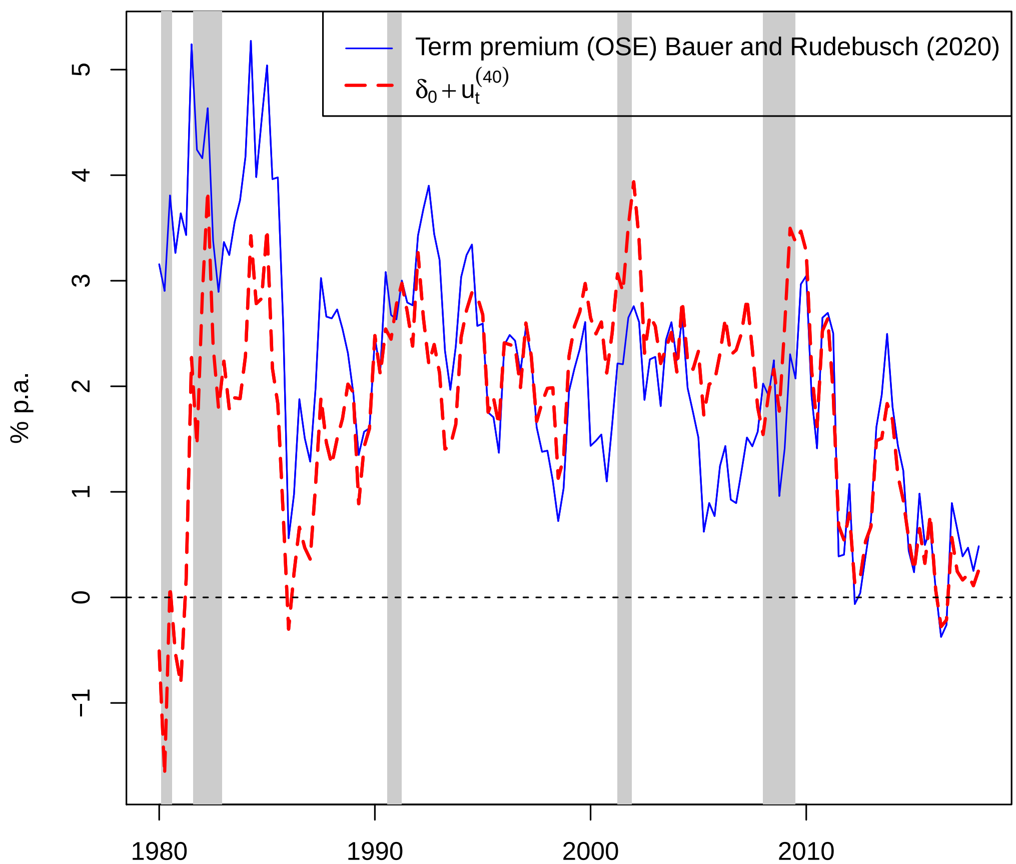

Figure B.3: Cyclical component from model with drifting equilbrium rates vs. term premium estimate: This figure shows the term premium component for a 10-year Treasury bond estimated following the methodology (OSE, observed shifting endpoint) proposed by Bauer and Rudebusch (2020) together with deviations of the 10-year bond yields from their drift, $\delta_0 + u_t^{(40)}$, implied by our (cointegrated) model with drifting equilibrium rates (c.f., equations (6)–(9)). Quarterly observations. The sample period is 1980:Q1 to 2018:Q1.

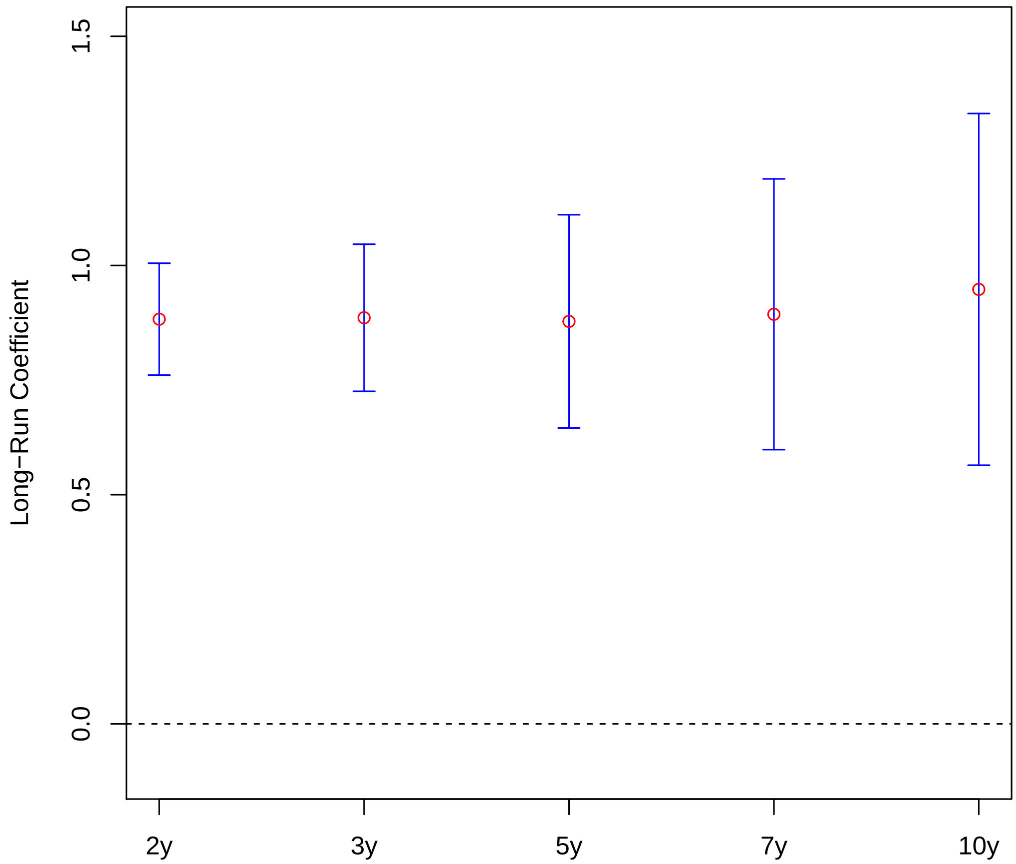

Figure B.4: Long-Run Loadings of Short-Term Cycle. This figure shows the long-run coefficients for the regression $u_t^{(n)} = \phi_1 u_{t-1}^{(n)} + \phi_2 u_t^{(1)} + \varepsilon_t$, where $u_t^{(n)}$ and $u_t^{(1)}$ are defined respectively in equation (6) and (7). Long-run coefficients are computed as $\phi_2/(1-\phi_1)$. The 95% confidence interval is calculated via the delta method. For the 10-year Treasury bond, the estimated regression is (HAC standard errors in parenthesis):

$$u_t^{(40)} = \underset{(0.045)}{0.784^{***}}\; u_{t-1}^{(40)} + \underset{(0.049)}{0.205^{***}}\; u_t^{(1)},\; R^2 = 78.2\%\ .$$

Results are robust to including inflation gap and output gap. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

C International Evidence

Table C.1: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression $rx^{(n)}_{t+4} = \alpha + \beta_1(y^{(n)}_t - y^{(4)}_t) + \beta_2(-(n-4)\ u^{(n-4)}_t) + \epsilon_t$, where $rx^{(n)}_{t+4}$ is the realized one-year holding period excess return of a bond with maturity n-period, $y^{(n)}_t - y^{(4)}_t$ is the slope for a n-period bond, and $(-(n-4)\ u^{(n-4)}_t)$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. For UK, zero-coupon bonds data are from the Bank of England (https://www.bankofengland.co.uk/statistics/yield-curves); the sample period is 1980:Q1 to 2019:Q4. For Canada, zero-coupon bonds data are from the Bank of Canada (https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/); the sample period is 1986:Q1 to 2019:Q4.

Panel A: UK.

	$rx^{(12)}_{t+4}$		$rx^{(20)}_{t+4}$		$rx^{(40)}_{t+4}$	
	(1)	(2)	(3)	(4)	(5)	(6)
$y^{(12)}_t - y^{(4)}_t$	1.159*					
	(0.603)					
$-(12-4)\ u^{(8)}_t$	−0.309**	−0.327**				
	(0.150)	(0.149)				
$y^{(20)}_t - y^{(4)}_t$			1.544*			
			(0.883)			
$-(20-4)\ u^{(16)}_t$			−0.321**	−0.327**		
			(0.150)	(0.150)		
$y^{(40)}_t - y^{(4)}_t$					1.979	
					(1.312)	
$-(40-4)\ u^{(36)}_t$					−0.286**	−0.287**
					(0.131)	(0.131)
Observations	156	156	156	156	156	156
Adjusted R^2	0.124	0.069	0.150	0.081	0.160	0.089

Panel B: Canada.

	$rx^{(12)}_{t+4}$		$rx^{(20)}_{t+4}$		$rx^{(40)}_{t+4}$	
	(1)	(2)	(3)	(4)	(5)	(6)
$y^{(12)}_t - y^{(4)}_t$	1.079					
	(0.784)					
$-(12-4)\ u^{(8)}_t$	−0.421**	−0.450***				
	(0.165)	(0.159)				
$y^{(20)}_t - y^{(4)}_t$			1.384			
			(1.054)			
$-(20-4)\ u^{(16)}_t$			−0.397**	−0.428***		
			(0.158)	(0.151)		
$y^{(40)}_t - y^{(4)}_t$					1.906	
					(1.649)	
$-(40-4)\ u^{(36)}_t$					−0.325**	−0.385***
					(0.135)	(0.119)
Observations	132	132	132	132	132	132
Adjusted R^2	0.208	0.152	0.253	0.189	0.263	0.202