Monopolistic Competition: CES Redux?*

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Abstract

We study competitive, selection and reallocation effects in monopolistic competition trade models. To this purpose, we extend the Melitz (2003) setup with heterogeneous firms and fixed and variable trade costs to the case of additive non-CES preferences. We find that Melitz-type selection effects are robust to relaxing the CES assumption. Competitive and reallocation effects are instead fragile. In particular, we show that average markups are non-monotonically related to trade costs. We also consider alternative monopolistic competition settings featuring non-additive preferences, strategic interaction and consumers' preference for an ideal variety. We find that none of the these setups delivers a strong and robust trade-induced pro-competitive mechanism.

JEL Classification: F1; *Keywords*: Monopolistic Competition; CES Preferences; International Trade; Competitive, Selection and Reallocation Effects.

1 Introduction

In this paper, we explore the robustness of competitive, selection and reallocation effects in monopolistic competition. Our study is motivated by the fact that the Dixit and Stiglitz (1977, henceforth D-S) setup with constant elasticity of substitution (CES) preferences,

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so far the workhorse of trade economists, has been put under fire by a recent literature.[1] The traditional critique that CES preferences make monopolistic competition little interesting, due to the implied invariance of firm size and markups to market size, has been recently revived by Neary (2004, 2009a, 2010), which proposes to study competitive effects in oligopoly. Dhingra and Morrow (2012) and Zhelobodko, Kokovin, Parenti and Thisse (2012, henceforth ZKPT) propose instead to relax the CES assumption in the standard D-S setup. In particular, ZKPT shows that CES preferences are a knife-edge between cases yielding opposite competitive and selection effects, thereby concluding that a theory of monopolistic competition cannot be built on CES preferences, due to the "peculiarity" of its implications.[2] We argue instead that the critique of the CES assumption in monopolistic competition has gone too far. First, because although CES preferences are clearly peculiar, relaxing the CES assumption does not obviously lead to less peculiar competitive effects. Second, because selection effects *à la* Melitz (2003) are indeed robust to relaxing the CES assumption.

In Section 2, we explore the implications of a variable elasticity of substitution in a D-S setup with additive preferences, as in ZKPT and Dhingra and Morrow (2012), which we generalize by studying the case of a reduction in trade costs in addition to the case of pure globalization (which in this setup is formally equivalent to an increase in market size). We begin by analyzing competitive effects in the case of symmetric firms. We show that, with non-CES preferences, a reduction in trade costs leads firms to charge higher markups in either the domestic or the foreign market. An important, and so far unnoticed, implication is that a reduction in trade costs has a non-monotonic impact on average markups. We argue that these and other peculiar results arise from the fact that, in this setup, markup changes may not be due to the action of competitive forces. As a consequence, a genuinely pro-competitive shock, such as a reduction in trade costs, may be associated with higher markups.

Next, we study competitive and selection effects in the presence of heterogeneous firms. To this purpose, we extend the Melitz (2003) model beyond the CES to the case of additively separable utility functions. We show that the Melitz approach of relating the behavior of any firm to that of the cutoff firm proves useful to simplify the analysis

[1] In this paper, we refer to monopolistic competition as a market structure characterized by product differentiation, a downward sloping average cost curve and free entry. By Dixit-Stiglitz assumption we refer instead to the absence of strategic interaction among firms.

[2] Another recent paper by Arkolakis, Costinot and Rodriguez-Clare (2012) argues that, provided that certain conditions that crucially exploit the properties of CES preferences are satisfied, gains from trade are invariant to the microeconomic details of the underlying model (conditional on observed trade flows). It also shows, however, that with CES preferences two statistics (the expenditure share on domestic goods and the elasticity of substitution) are sufficient to compute the welfare gains from a reduction in trade costs.

also in this setup. This allows us to obtain some of the results independently found by ZKPT and Dhingra and Morrow (2012) and to address new important issues, such as the reallocation effects associated with an increase in market size and the effects of a reduction in trade costs. Our most important result is that, when trade costs are large enough to induce a partitioning of firms into exporters and non-exporters (arguably, the empirically relevant case), Melitz-type selection effects are robust to relaxing the CES assumption. We also show, however, that selection effects are unambiguously associated with aggregate productivity gains only when preferences are CES.

Having shown that, unlike selection effects, competitive effects are not robust to relaxing the CES assumption in a D-S setup with additive preferences, in Section 3 we explore alternative monopolistic competition environments. We first consider the example of quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008), thereby relaxing the assumption of additive preferences. Next, we relax the D-S assumption that firms do not interact strategically by considering Bertrand and Cournot extensions of the basic setup with additive preferences and a variable elasticity. Finally, we reconsider Lancaster's (1979) ideal variety approach to monopolistic competition. Surprisingly, we find that none of these setups delivers a strong and robust trade-induced pro-competitive mechanism.

Section 4 briefly concludes. Our paper is related to the vast theoretical literature on monopolistic competition and international trade, initiated by Dixit and Stiglitz (1977), Krugman (1979, 1980), Lancaster (1979), Helpman (1981), and whose early contributions are systematized in Helpman and Krugman (1985).[3] It is also closely related to the recent heterogeneous-firm extensions of the monopolistic competition trade model, and in particular to Melitz (2003), whose methodology and main results we generalize to a framework encompassing additively separable utility functions. Moreover, our paper is closely related to a recent literature which independently studies some implications of non-CES preferences in a D-S monopolistic competition setup with heterogeneous firms, and in particular to ZKPT, Dhingra and Morrow (2012), Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012, henceforth ACDR) and Mrázová and Neary (2012). Our setup is similar but more general than that of ZKPT, which leads us to reach different conclusions. In particular, ZKPT does not study how trade costs affect competitive and selection effects, nor the impact of reallocations on aggregate productivity and the case of strategic interactions. Dhingra and Morrow (2012) represents the first study of the welfare effects of an increase in market size in a D-S setup with heterogeneous firms, and is therefore complementary to our paper, as we do not focus on welfare issues. ACDR is

[3] More recent contributions include Bertoletti (2006) and Behrens and Murata (2007, 2012), which discuss a specific functional form for DES preferences.

also complementary to our work, as it shows that, in the presence of a Pareto distribution of firm productivity and preferences featuring log concavity of demand and a choke-off price, the welfare effects of a reduction in trade costs are lower than those predicted by models with constant markups. This result seems at odds with Krugman's (1979) original insight on the pro-competitive gains from pure globalization. We view our result on the non-monotonicity of markups with respect to trade costs as a possible way to reconcile these seemingly contrasting implications. Finally, Mrázová and Neary (2012) complements our analysis by introducing a distinction between first-order and second-order selection effects (the former involve a binary choice between performing or not a certain activity, whereas the latter concern how to perform a certain activity) and showing that the former are more robust than the latter.

2 Dixit-Stiglitz Monopolistic Competition with Additive Preferences

We first consider the case of symmetric firms, as in Krugman (1979), then extend the analysis to a setup with heterogeneous firms, as in Melitz (2003). In both cases, we study pure globalization (an increase in market size) and a reduction in trade costs.

2.1 Symmetric Firms

We first illustrate the key features of the baseline model, then study the competitive effects associated with a reduction in trade costs.

2.1.1 Baseline Model

Consider an economy populated by L workers, whose wage is $w = 1$. Consumers share the same additive and symmetric preferences, represented by the following utility function:

$$U = \sum_{i=1}^{N} u(c_i), \tag{1}$$

where c_i is consumption of variety i, defined over a large number N of potential varieties. Only varieties indexed by $i = 1, ..., n$, with $n < N$, are actually produced. The sub-utility function $u(\cdot)$ is strictly increasing and concave, and is at least thrice continuously differentiable. In particular, we assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u(0) = 0$.[4]

[4] Note that $U = nu(c)$ at a symmetric consumption pattern, with $nu(c) > u(nc)$: thus, U embeds a Chamberlinian "taste for variety". Moreover, U is well defined over the positive orthant of the relevant Euclidean space: according to standard results, this implies regular and well-behaved demand functions for strictly positive prices and income.

Producing a differentiated variety involves a fixed cost α and a marginal cost β, both in terms of labor. Firms are symmetric on the cost and demand side, and therefore solve the same problem. In the following, we therefore omit the variety index i.

Utility maximization subject to a budget constraint implies $u'(c) = \lambda(\mathbf{p})p$, where p is the price charged by an individual firm, $\lambda > 0$ is the marginal utility of income, and $\mathbf{p} = [p_1, ..., p_n]$ is the price vector. Preference additivity implies that the elasticity of λ with respect to each price p is of the same order of magnitude as $1/n$, provided that prices are not disproportionate (see, e.g., Deaton and Muellbauer, 1980, Section 5.3). Thus, under the D-S assumption that n is large enough to induce each firm to treat λ as a constant, the inverse individual demand for a variety is

$$p = \frac{u'(c)}{\lambda}, \tag{2}$$

and the perceived price elasticity of demand for an individual firm is

$$\varepsilon(c) = -\frac{p(c)}{p'(c)c} = -\frac{u'(c)}{u''(c)c} = \sigma(c), \tag{3}$$

where $\sigma(c)$ is the elasticity of substitution between any two varieties when they are consumed in the same amount c (for the derivation and discussion of this result, see the Appendix).[5] The assumption $\sigma'(c) = 0$ is the well-known CES case. Krugman (1979) assumed $\sigma'(c) < 0$, i.e., a decreasing elasticity of substitution (henceforth, DES preferences), whereas $\sigma'(c) > 0$ corresponds to the case of an increasing elasticity of substitution (henceforth, IES preferences).

Using (2), the revenue of an individual firm can be written as $pcL = r(c)(L/\lambda)$, where $r(c) = u'(c)c$ and cL is the output. Therefore, the marginal revenue and the derivative of marginal revenue equal, respectively, $r'(c)/\lambda$ and $r''(c)/(\lambda L)$, where

$$r'(c) = u'(c) + u''(c)c, \tag{4}$$
$$r''(c) = 2u''(c) + u'''(c)c. \tag{5}$$

The first-order condition for profit maximization implies:

$$r'(c) = \lambda\beta. \tag{6}$$

To obtain a unique and well-behaved solution to the problem of profit maximization, we

[5]Note also that $\varepsilon(c)$ is independent of λ and L for given c, and that it equals the reciprocal of the elasticity of marginal utility with respect to individual consumption.

assume that the marginal revenue is everywhere positive and decreasing. That is, we assume that $r'(c) > 0$ and $r''(c) < 0$ for all c, with $\lim_{c\to\infty} r'(c) = 0$ and $\lim_{c\to 0} r'(c) = \infty$.

Equations (2), (3), (4) and (6) imply that the profit maximizing price can be written as

$$p = \frac{u'(c)}{r'(c)}\beta = \frac{\sigma(c)}{\sigma(c) - 1}\beta = m(c)\beta, \tag{7}$$

where $\sigma > 1$, m is the price-marginal cost markup, and

$$m'(c) = -\frac{\sigma'(c)}{[\sigma(c) - 1]^2}. \tag{8}$$

Evidently, markups are increasing in individual consumption c with DES preferences, decreasing in c with IES preferences, and constant with CES preferences. Differentiating (3) and using (4) and (5) yields the following expression for σ':

$$\sigma'(c) = \frac{u'(c)r''(c) - u''(c)r'(c)}{[u''(c)c]^2}, \tag{9}$$

which implies that

$$\sigma'(c) \gtrless 0 \Leftrightarrow \frac{r'(c)u''(c)}{r''(c)u'(c)} = \frac{\eta(c)}{\sigma(c)} \gtrless 1, \tag{10}$$

where

$$\eta(c) = -\frac{r'(c)}{r''(c)c} > 0 \tag{11}$$

is (the absolute value of) the reciprocal of the elasticity of marginal revenue. Note that $\eta = \sigma$ when preferences are CES. With non-CES preferences, instead, $\eta \neq \sigma$ and both quantities are variable. Below we show that η plays a critical role for our results.

Free entry implies zero equilibrium profits, namely, $\pi_v - \alpha = (p - \beta)cL - \alpha = 0$, where π_v denotes variable profits. Using (4) and (7), the free-entry condition can be written as

$$\pi_v = [m(c) - 1]\,\beta Lc = \left[\frac{u'(c)}{r'(c)} - 1\right]\beta Lc = \alpha. \tag{12}$$

Eq. (12) implicitly defines the level of individual consumption consistent with profit maximization and free entry as a function $c(L, \beta, \alpha)$ of market size L, marginal cost β, and the fixed cost α. Differentiating π_v and using (11) yields:

$$\frac{\partial \pi_v}{\partial c} = \beta L\left[\frac{u''(c)r'(c) - u'(c)r''(c)}{r'(c)^2}c - \frac{u''(c)}{r'(c)}c\right] = \beta L\frac{m(c)}{\eta(c)} > 0. \tag{13}$$

Thus, π_v is monotonically increasing in c, which ensures that the equilibrium is unique.

Finally, full employment implies that labor demand, $n(\alpha + \beta cL)$, equals labor supply,

L (equivalently, equilibrium in the goods market implies that individual expenditure, npc, equals income, $w = 1$). Thus, using (12) and (7) yields:

$$n = \frac{L}{\alpha + \beta L c} = \frac{L}{\alpha \sigma(c)} = \frac{1}{m(c) c \beta}, \tag{14}$$

which determines recursively the equilibrium number of firms, $n(L, \beta, \alpha)$.

We can now show how individual consumption depends on the model's parameters. Differentiating (12) with respect to L, β and α yields:

$$\frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial L} + \frac{\partial \pi_v}{\partial L} = \frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial \beta} + \frac{\partial \pi_v}{\partial \beta} = 0 \text{ and } \frac{\partial \pi_v}{\partial c}\frac{\partial c}{\partial \alpha} = 1.$$

Noting that $\frac{\partial \pi_v}{\partial L} L = \frac{\partial \pi_v}{\partial \beta} \beta = \pi_v = \alpha$, we obtain that individual consumption is decreasing in market size L and marginal cost β, and increasing in the fixed cost α, with:

$$\frac{\partial \ln c}{\partial \ln L} = \frac{\partial \ln c}{\partial \ln \beta} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c)} \lesseqgtr -1 \Leftrightarrow \sigma'(c) \gtreqless 0. \tag{15}$$

Eqs. (15) and (8) immediately imply that with IES (DES) preferences markups are increasing (decreasing) in $\beta L/\alpha$, and firm size cL is decreasing (increasing) in market size L.[6]

2.1.2 Costly Trade

We now consider two identical countries separated by a symmetric iceberg trade cost $\tau \geq 1$. From now on, variables related to the foreign market will be indexed by an x, whereas variables related to the domestic market will not be indexed. Total profits now equal $\pi_v(c) + \pi_v^x(c_x) - \alpha$, where $\pi_v(c)$ is still given by (12), and

$$\pi_v^x(c_x) = [m(c_x) - 1]\, \tau \beta c_x L. \tag{16}$$

By (6), profit maximization implies $r'(c) = \lambda\beta$ and $r'(c_x) = \lambda\tau\beta$. Using the former in the latter yields:

$$r'(c_x) = \tau r'(c). \tag{17}$$

[6] As far as we know, the anti-competitive effect of an increase in market size in the case of IES preferences was first shown by Bertoletti, Poletti and Fumagalli (2008), and then further elaborated by ZKPT.

Eq. (17) implicitly defines $c_x = c_x(c, \tau)$; differentiating and using (11) yields:

$$\frac{\partial \ln c_x}{\partial \ln c} = \frac{\eta(c_x)}{\eta(c)} > 0, \quad \frac{\partial \ln c_x}{\partial \ln \tau} = -\eta(c_x) < 0. \tag{18}$$

The zero-profit condition, which implicitly defines $c = c(L, \beta, \alpha, \tau)$, can now be written as

$$\{[m(c) - 1]\, c + [m(c_x) - 1]\, \tau c_x\}\, \beta L - \alpha = 0. \tag{19}$$

Applying the implicit function theorem to (19) and using (13), (17) and (18) yields:

$$\frac{\partial \ln c}{\partial \ln \tau} = \frac{\eta(c) c_x \tau}{cm(c) + c_x \tau m(c_x)} > 0. \tag{20}$$

Evidently, an increase in trade costs τ raises individual domestic demand for domestic firms. It follows that, with DES (IES) preferences, trade liberalization leads firms to charge lower (higher) markups in the domestic market.

Consider now foreign sales. Totally differentiating (17) and using (18) and (20) yields:

$$\frac{d \ln c_x}{d \ln \tau} = \eta(c_x) \left[\frac{c_x \tau}{cm(c) + c_x \tau m(c_x)} - 1 \right] < 0. \tag{21}$$

Thus, an increase in trade costs reduces individual demand for foreign varieties. It follows that, with DES (IES) preferences, trade liberalization leads firms to charge higher (lower) markups in the foreign market.

Finally, evaluating (20) and (21) around the free trade, i.e., for $\tau = 1$ (which implies $c = c_x$), yields:

$$\left[\frac{\partial \ln c}{\partial \ln \tau} + \frac{d \ln c_x}{d \ln \tau} \right]_{\tau=1} = -\frac{\eta(c)}{\sigma(c)} < 0.$$

Consequently, a small increase in trade costs around the free trade equilibrium reduces consumption of each foreign variety by more than it increases consumption of each domestic variety, thereby reducing average individual consumption. Therefore, average markups fall (increase) with DES (IES) preferences.[7] Moreover, given that an increase in trade

[7] To see this, denote by $\widetilde{m}(\tau)$ the average markup in each market. We have:

$$\widetilde{m}(\tau) = \omega(\tau) m(c(\tau)) + (1 - \omega(\tau)) m(c_x(\tau)),$$

where $\omega(\tau) \in [0, 1]$ is the weight on domestic markups (e.g., in terms of output or revenue). Differentiating yields:

$$\widetilde{m}'(\tau) = \omega'(\tau) \left[m(c(\tau)) - m(c_x(\tau)) \right] + \omega(\tau) \left[m'(c) c'(\tau) - m'(c_x) c_x'(\tau) \right] + m'(c_x) c_x'(\tau).$$

Note that, in $\tau = 1$, $c(\tau) = c_x(\tau)$ and $\omega(\tau) = 1/2$. It follows that:

$$\widetilde{m}'(\tau)\big|_{\tau=1} = \left[\frac{m'(c(\tau))}{2} \left(c'(\tau) + c_x'(\tau) \right) \right]_{\tau=1} \lesseqgtr 0 \Leftrightarrow m' \gtreqless 0 \Leftrightarrow \sigma' \lesseqgtr 0.$$

costs from $\tau = 1$ to $\tau = \infty$ is equivalent to a reduction in market size, and thus leads, by (15), to a rise (fall) in markups with DES (IES) preferences, it follows that non-CES preferences imply a non-monotonic relationship between trade costs and average markups.

We record our main results in the following

Proposition 1 *With non-CES preferences, trade liberalization leads firms to charge higher markups in either the domestic market (IES preferences) or the foreign market (DES preferences), and average markups are non-monotonically related to trade costs.*

2.1.3 Discussion

When preferences are IES, larger markets lead to smaller firms and higher markups. These peculiar implications are the opposite of the pro-competitive and de-fragmentation effects implied by Krugman's (1979) model, and are perhaps surprising also because the IES assumption is *prima facie* no less reasonable than the DES assumption.[8] With DES preferences, instead, although an increase in market size L is pro-competitive, a reduction in the marginal cost β is anti-competitive, which implies, e.g., that productivity growth leads to a generalized increase in markups.[9,10] Finally, and more importantly, with non-CES preferences a reduction in trade costs leads firms to charge higher markups in either the domestic or the foreign market, and may be associated with higher average markups.

[8] A possible rationalization of IES preferences is that, by their very nature, differentiated varieties of some product can be used to perform either generic or more specific tasks. For instance, a blue pencil can be used either to write down a laundry list (for which a red pencil would be equally appropriate) or, jointly with a red pencil, to mark different types of comments on an exam paper. Hence, a fall in the symmetric endowment of varieties, by reducing the opportunity to use varieties to perform specific tasks, may also reduce their substitutability. Moreover, consider a situation in which you are endowed with two red pencils and two blue pencils, and compare it with a situation in which you are endowed instead with ten red and ten blue pencils. The key question is: do you perceive a red and a blue pencil as more substitutable in the former or in the latter situation? If you think that varieties become less substitutable when consumption of each shrinks, then preferences are IES and Krugman's assumption is violated. A counter-argument in support of DES preferences, suggested by Mrázová and Neary (2012), is that in a D-S setup with additive preferences the DES assumption is consistent with the so-called Marshall's Second Law, according to which the elasticity of demand is decreasing along a given demand curve (which captures the idea that, as consumption increases, demand becomes less responsive to price changes). Note, however, that a change in the monopolistic competition equilibrium involves a shift in the individual demand curve, implying that Marshall's Second Law may be insufficient to characterize the behavior of the elasticity of demand with non-additive preferences.

[9] Note also, from (14), that both an increase in L and a reduction in β lead to entry and a reduction in firms' market shares $1/n$. Thus, markups may be unrelated to the number of firms with DES preferences.

[10] Note, from (12), that when technical change leads to an equiproportional reduction in both marginal and fixed costs, markups are unchanged. However, in monopolistic competition trade models endogenizing technology it is standard to assume that a lower marginal cost requires a higher fixed cost α, e.g., in terms of R&D expenditures (see the discussion in Mrázová and Neary, 2012, and references therein). This type of technical change would imply even stronger anti-competitive effects with DES preferences.

To interpret the above results note that, in a D-S setup with additive non-CES preferences, markups are entirely driven by the level of individual consumption c.[11] This turns out to be restrictive, first, because the relationship between individual consumption and markups is not robust to the assumptions about preferences. Second, because c is naturally decreasing in the marginal cost, which leads to a pass-through effect whereby markups vary with marginal costs. For instance, DES preferences imply that markups are decreasing in marginal costs (incomplete pass-through), which explains why productivity growth is anti-competitive and a reduction in the marginal cost of exporting leads to higher markups in the foreign market. More importantly, it explains why a genuinely pro-competitive shock, such as a reduction in trade costs, may be associated with an increase in average markups, thus making matters worse, in a sense, relative to the CES case of constant markups.

2.2 Heterogeneous Firms

We now relax the assumption that all firms share the same cost parameters. We begin by illustrating our generalization of Melitz (2003) to the case of a variable elasticity of substitution. Then we use our model to study separately the cases of pure globalization and of a reduction in trade costs.

2.2.1 Setup

Following Melitz (2003) we assume that, upon paying a fixed entry cost α_e, firms draw their marginal cost $\beta \in \left[\underline{\beta}, \infty\right)$ from a common, continuous cumulative distribution $G(\beta)$ with density $g(\beta)$ and $\underline{\beta} > 0$. We also assume a continuum of varieties, we index firms by their marginal cost β and denote individual demand for a β-firm by $c(\beta)$. Otherwise, the setup is the same as in the previous Section, from which we recall the following definitions:

$$r(c) = u'(c)c, \ \sigma(c) = -\frac{u'(c)}{u''(c)c}, \ m(c) = \frac{u'(c)}{r'(c)}, \ \eta(c) = -\frac{r'(c)}{r''(c)c}.$$

Firm productivity, size and markups Denote by $p(\beta) = m(c(\beta))\beta$ and $\pi_v(\beta) = [p(\beta) - \beta]\, c(\beta)L$ the price and variable profit of a β-firm. The first-order and second-order

[11] In Section 3 we will consider alternative monopolistic competition settings in which the number of firms can exert a direct impact on markups.

conditions for profit maximization, $r'(c) = \lambda\beta$ and $r''(c) < 0$, imply that:

$$\frac{d\ln c(\beta)}{d\ln\beta} = \frac{\lambda\beta}{r''(c(\beta))c(\beta)} = -\eta(c(\beta)) < 0, \tag{22}$$

$$\frac{d\ln \pi_v(\beta)}{d\ln\beta} = -c(\beta)L\frac{\beta}{\pi_v(\beta)} = 1 - \sigma(c(\beta)) < 0, \tag{23}$$

$$\frac{d\ln p(\beta)}{d\ln\beta} = \frac{\eta(c(\beta))}{\sigma(c(\beta))} \gtreqless 1 \Leftrightarrow \sigma'(c(\beta)) \gtreqless 0. \tag{24}$$

Evidently, high-productivity (low-β) firms are larger (22) and more profitable (23), as in Melitz (2003). Unlike the Melitz model, however, where preferences are CES and markups are constant, with IES (DES) preferences more productive firms charge lower (higher) markups (24). Note also that η and σ govern, respectively, relative firm size and relative profitability.

Zero cutoff profit condition Denote by β^* the marginal cost cutoff, namely, the value of β satisfying the zero cutoff profit condition $\pi(\beta^*) = 0$:

$$\pi_v(\beta^*) = [m(c^*) - 1]\,\beta^* c^* L = \alpha. \tag{25}$$

Eq. (25) implicitly defines the individual demand for the cutoff firm, $c^* = c^*(\beta^*, L, \alpha)$. Differentiating yields:

$$\frac{\partial \ln c^*}{\partial \ln L} = \frac{\partial \ln c^*}{\partial \ln \beta^*} = -\frac{\partial \ln c^*}{\partial \ln \alpha} = -\frac{\eta(c^*)}{\sigma(c^*)} < 0. \tag{26}$$

Individual demand for a β-firm Following Melitz (2003), we can now relate the behavior of a β-firm to that of the cutoff firm by eliminating the marginal utility of income λ (as in Krugman, 1979) from the relevant first-order conditions for profit maximization. Formally, profit maximization by the cutoff firm implies $r'(c^*) = \lambda\beta^*$. Solving for λ and substituting into $r'(c) = \lambda\beta$ yields:

$$r'(c) = r'(c^*)\frac{\beta}{\beta^*}. \tag{27}$$

Eq. (27) is key to the characterization of the equilibrium, as it implicitly defines the individual demand for a β-firm, $c\,(\beta) = c(\beta; \beta^*, c^*)$. Using $c^*(\beta^*, L, \alpha)$, we can now show how $c\,(\beta) = c(\beta; \beta^*, c^*(\beta^*, L, \alpha)) = c(\beta; \beta^*, L, \alpha)$ varies with β^*, L and α. Differentiating (27) with respect to β^* yields:

$$r''(c)\frac{\partial c}{\partial \beta^*} = \frac{\beta}{\beta^{*2}}\left[r''(c^*)\frac{\partial c^*}{\partial \beta^*}\beta^* - r'(c^*)\right].$$

Rearranging terms and using (26) gives:

$$\frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0.$$

Hence, as in Melitz (2003), individual demand is increasing in the cutoff marginal cost β^*. Similarly, differentiating (27) with respect to L and using (26) yields:

$$\frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c^*)} < 0.$$

Thus, for given β^*, individual demand is decreasing in market size L and increasing in the fixed cost α. Note also that, with IES preferences, $\sigma(c^*) < \sigma(c)$ for $c^* < c$; it follows that

$$\frac{\partial \ln c}{\partial \ln L} < -\frac{\eta(c)}{\sigma(c)} < -1,$$

where the latter inequality follows from (10). Thus firm size, cL, is decreasing in L for given β^*. In contrast, with DES preferences $\sigma(c^*) > \sigma(c)$ for $c^* < c$, and hence firm size is increasing in L.

The following lemma summarizes

Lemma 1 *Individual demand for a β-firm is increasing in β^* and, for given β^*, it is increasing in α and decreasing in L, with*

$$\frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0, \quad \frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c^*)} \lesseqgtr -1 \Leftrightarrow \sigma'(c) \gtreqless 0. \tag{28}$$

Free-entry condition Free entry implies that expected profits,

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} \pi(\beta) dG(\beta),$$

equal the sunk entry cost α_e. Integrating π^E by parts using (23) and noting that $\pi(\beta^*) = G(\underline{\beta}) = 0$ yields:

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} c(\beta) L G(\beta) d\beta = \alpha_e. \tag{29}$$

Differentiating π^E with respect to β^* yields:

$$\frac{\partial \pi^E}{\partial \beta^*} = c(\beta^*) L G(\beta^*) + \int_{\underline{\beta}}^{\beta^*} \frac{\partial c(\beta)}{\partial \beta^*} L G(\beta) d\beta > 0, \tag{30}$$

where the inequality follows from Lemma 1. Hence, as in Melitz (2003), expected profits are increasing in β^* and the free-entry condition (29) uniquely pins down β^*, thereby defining the equilibrium value of $c(\beta) = c(\beta; L, \alpha, \alpha_e)$.[12]

2.2.2 Competitive, Selection and Reallocation Effects of Pure Globalization

We can now study the effects of pure globalization, which in this setup is formally equivalent to an increase in market size.

Selection effects The impact of an increase in market size L on the marginal cost cutoff β^* can be computed by applying the implicit function theorem to (29):

$$\frac{d\beta^*}{dL} = -\frac{\partial \pi^E / \partial L}{\partial \pi^E / \partial \beta^*}, \tag{31}$$

where $\partial \pi^E / \partial \beta^* > 0$ by (30) and, using Lemma 1,

$$\frac{\partial \pi^E}{\partial L} = \int_{\underline{\beta}}^{\beta^*} \left[\frac{\partial c(\beta)}{\partial L} L + c(\beta) \right] G(\beta) d\beta = \int_{\underline{\beta}}^{\beta^*} \left(1 - \frac{\eta(c(\beta))}{\sigma(c^*)} \right) c(\beta) G(\beta) d\beta \lesseqgtr 0 \Leftrightarrow \sigma' \gtreqless 0. \tag{32}$$

Thus, with DES preferences, $d\beta^*/dL < 0$: an increase in market size leads to a standard selection effect. Conversely, with IES preferences an increase in market size leads to an increase in β^* and a consequent *anti-selection* effect, whereby less productive firms can survive in a larger market.[13] Therefore, as also noted by ZKPT and Dhingra and Morrow (2012), selection effects seem to crucially depend on the assumptions about preferences. Moreover, it is implicitly argued in this recent literature that aggregate productivity gains or losses depend only on whether β^* rises or falls. Below we show, however, that both conclusions are fragile.

Competitive effects The competitive effects of pure globalization depend on how it affects the behavior of inframarginal firms and the selection of firms through entry or exit. The impact on inframarginal firms can be obtained by simply studying how an increase

[12] The measure of active firms n is determined by the goods market equilibrium condition (or, equivalently, by the full-employment condition), requiring average expenditure to equal $1/n$, and thus:

$$n = \left[\int_{\underline{\beta}}^{\beta^*} p(\beta) c(\beta) \frac{dG(\beta)}{G(\beta^*)} \right]^{-1}.$$

[13] Proceeding as above, it is also possible to show that, independent of consumer preferences, an increase in the fixed production cost α yields a selection effect, whereas an increase in the sunk entry cost α_e yields an anti-selection effect.

in L affects individual consumption. Totally differentiating π^E with respect to L yields

$$\int_{\underline{\beta}}^{\beta^*} \frac{\pi_v(c(\beta))}{L} dG(\beta) + \int_{\underline{\beta}}^{\beta^*} \frac{m(c(\beta))}{\eta(c(\beta))} \frac{dc}{dL} \beta L dG(\beta) = 0, \tag{33}$$

where $dc/dL = \partial c/\partial L + (\partial c/\partial \beta^*)(d\beta^*/dL)$ represents the total impact of L on c. Note that the first term of (33) is positive, thereby implying that the second term is negative. Moreover, using Lemma 1,

$$\frac{d\ln c}{d\ln L} = -\frac{\eta(c(\beta))}{\sigma(c^*)} \left[1 - (\sigma(c^*) - 1)\frac{d\ln \beta^*}{d\ln L}\right], \tag{34}$$

where the sign of the term in square brackets is independent of β. It follows that the sign of dc/dL is the same for all firms, and must therefore be negative according to (33). Thus, as in the baseline model with symmetric firms, an increase in market size leads inframarginal firms to charge lower (higher) markups with DES (IES) preferences.[14]

Finally, note that the anti-competitive effect associated with IES preferences is strengthened by the concomitant anti-selection effect, leading to the *entry* of low-productivity, *high*-markup firms, and which tends to further increase average markups. In contrast, with DES preferences there is a countervailing reduction in average markups due to the *exit* of low-productivity, *low*-markup firms.

Reallocation Effects We now show that, with non-CES preferences, an increase in market size not only changes the marginal cost cutoff, it also affects relative firm ouput, revenue and profits. The impact on relative output can be seen directly from (34), which implies that an increase in L reduces individual demand for low-β firms relatively less only if $\eta' < 0$. Similarly, it can be shown that the impact of L on relative revenue is governed by

$$\chi(c) = 1 + \eta(c) - \frac{\eta(c)}{\sigma(c)},$$

where $\chi = \eta = \sigma$ with CES preferences and, more generally, we have:

$$\chi'(c) = \frac{\sigma(c) - 1}{\sigma(c)} \eta'(c) + \frac{\eta(c)}{\sigma^2(c)} \sigma'(c).$$

Note that the sign of χ' is unambiguously equal to the sign of σ' and η' only when the two derivatives agree in sign. Finally, it can be shown that the elasticity of variable profits with respect to L depends on the sign of σ'. It follows that reallocations are unambiguously in

[14] Similarly, it is possible to show that a rise in α or in α_e are pro-competitive with IES preferences and anti-competitive with DES preferences.

favor of more productive firms only if $\sigma', \eta' < 0$, as in this case an increase in L affects low-β firms relatively less. Instead, if $\sigma', \eta' > 0$, reallocations are unambiguously in favor of less productive firms.

Note also that:

$$\eta'(c) = -\frac{r''(c)^2 c - [r'''(c)c + r''(c)]\, r'(c)}{(r''(c)c)^2} \Rightarrow \eta'(c) \gtrless 0 \Longleftrightarrow -\frac{r'''(c)c}{r''(c)} \gtrless \frac{1+\eta(c)}{\eta(c)}, \quad (35)$$

where $r''' = u''''c + 3u'''$, thus suggesting that the behavior of η depends on properties of preferences which are hard to pin down and do not appear to be directly related to the sign of σ' (which governs the selection effects). In particular, although $\sigma' > 0$ if and only if $\eta > \sigma$, no general relation can be inferred between the sign of σ' and η'. As a consequence, if σ' and η' do not agree in sign, selection and reallocation effects may point in opposite directions. As shown in the Appendix, an important implication is that an increase in market size may have an ambiguous overall impact on a measure of aggregate productivity.[15] For instance, the selection effect implied by DES preferences ($\sigma' < 0$) may not be associated with aggregate productivity gains if $\eta' > 0$. The following proposition summarizes

Proposition 2 *An increase in market size is associated with unambiguous reallocation effects only if η' and σ' agree in sign.*

2.2.3 Costly Trade

Consider now two identical countries separated by symmetric trade costs. Specifically, assume that selling in an identical foreign market involves a fixed cost of exporting $\alpha_x > 0$ and a variable iceberg trade cost $\tau \geq 1$. As in Melitz (2003), we focus on the more interesting case in which trade costs are large enough to induce a partitioning of firms into exporters and non-exporters. This requires $\beta^* > \beta_x^*$, where β_x^* is the marginal cost cutoff for exporting firms.[16]

Using the same notation as in Section *2.1.2*, a β-firm's profits in the domestic and foreign markets are given, respectively, by $\pi(\beta) = \pi_v(\beta) - \alpha$ and $\pi_x(\beta) = \pi_v(\tau\beta) - \alpha_x$, where $\pi_v(\beta) = [m(c(\beta)) - 1]\, \beta c(\beta) L$ and L is the size of each market.[17] Thus, the marginal

[15] Although aggregate productivity changes do not directly measure welfare changes, they provide the standard way of quantifying empirically the gains from trade due to reallocations across heterogeneous firms: see, e.g., Melitz and Trefler (2012, p. 109) on this point.

[16] Given that we consider two identical countries, balanced trade requires wages to be equalized. Thus we can choose the wage as the numeraire, as in the Melitz model and in Section *2.1.2*.

[17] Note that a domestic firm producing *only* for the foreign market would incur an overall fixed cost equal to $\alpha + \alpha_x$, which implies that this case cannot arise in equilibrium. For analytical convenience, we can therefore apportion the fixed cost α to domestic profits, in this following the heterogeneous-firm literature.

cost cutoffs for exporters and domestic firms, β_x^* and β^*, are implicitly given by:

$$\pi_x(\beta_x^*) = [m(c_x^*) - 1] \tau \beta_x^* c_x^* L - \alpha_x = 0, \tag{36}$$

$$\pi(\beta^*) = [m(c^*) - 1] \beta^* c^* L - \alpha = 0, \tag{37}$$

where $c_x^* = c(\tau\beta_x^*)$, $c^* = c(\beta^*)$, and $\pi_v^{-1}(\alpha_x) < \tau\pi_v^{-1}(\alpha)$ in order for the condition $\beta^* > \beta_x^*$ to be satisfied. Finally, the free-entry condition is now given by:[18]

$$\pi^E = \int_{\underline{\beta}}^{\beta^*} \pi(\beta) dG(\beta) + \int_{\underline{\beta}}^{\beta_x^*} \pi_x(\beta) dG(\beta) = \alpha_e. \tag{38}$$

We use (36) to define $\beta_x^* = \beta_x^*(\beta^*, \tau, \alpha_x)$ and, as in the previous Section, (37)-(38) to pin down β^*.

Selection effects Consider first a fall in the variable trade cost τ. Applying the implicit function theorem to (38) yields:

$$\frac{\partial \beta^*}{\partial \tau} = -\frac{\partial \pi^E / \partial \tau}{\partial \pi^E / \partial \beta^*}. \tag{39}$$

As for the numerator of (39), note that:

$$\frac{\partial \pi^E}{\partial \tau} = \int_{\underline{\beta}}^{\beta_x^*} \frac{\partial \pi_v(\tau\beta)}{\partial \tau} dG(\beta) + \frac{\partial \beta_x^*}{\partial \tau} \pi_x(\beta_x^*) G(\beta_x^*) = -\int_{\underline{\beta}}^{\beta_x^*} c(\tau\beta) \beta L dG(\beta) < 0,$$

where the latter equality follows from $\pi_x(\beta_x^*) = 0$ by (36) and $\partial \pi_v(\tau\beta)/\partial\tau = -c(\tau\beta)\beta L$ by (23). Similarly, as for the denominator of (39), we can write:

$$\begin{aligned} \frac{\partial \pi^E}{\partial \beta^*} &= \int_{\underline{\beta}}^{\beta^*} \frac{\partial \pi_v(\beta)}{\partial c} \frac{\partial c(\beta)}{\partial \beta^*} dG(\beta) + \int_{\underline{\beta}}^{\beta_x^*} \frac{\partial \pi_v(\tau\beta)}{\partial c} \frac{\partial c(\tau\beta)}{\partial \beta^*} dG(\beta) \\ &= \int_{\underline{\beta}}^{\beta^*} \frac{m(c(\beta))}{m(c^*)} \frac{\beta L c(\beta)}{\beta^*} dG(\beta) + \int_{\underline{\beta}}^{\beta_x^*} \frac{m(c(\tau\beta))}{m(c^*)} \frac{\tau\beta L c(\tau\beta)}{\beta^*} dG(\beta) > 0, \end{aligned}$$

[18] The measure of active firms is given by:

$$n = \left[\int_{\underline{\beta}}^{\beta^*} p(\beta) c(\beta) \frac{dG(\beta)}{G(\beta^*)} + \int_{\underline{\beta}}^{\beta_x^*} p(\tau\beta) c(\tau\beta) \frac{dG(\beta)}{G(\beta_x^*)} \right]^{-1}.$$

where the latter equality follows from $\partial\pi_v/\partial c = \beta L\,[m(c)/\eta(c)] > 0$ by (13), and from $\partial \ln c/\partial \ln \beta^* = \eta(c)/m(c^*) > 0$ by Lemma 1. Thus:

$$\frac{\partial \ln \beta^*}{\partial \ln \tau} = \frac{\displaystyle\int_{\underline{\beta}}^{\beta_x^*} m(c^*)c\,(\tau\beta)\,\tau\beta dG(\beta)}{\displaystyle\int_{\underline{\beta}}^{\beta^*} p(\beta)c(\beta)dG(\beta) + \int_{\underline{\beta}}^{\beta_x^*} p(\tau\beta)c(\tau\beta)dG(\beta)} > 0. \tag{40}$$

Evidently, a reduction in τ leads to a reduction in β^*, a standard selection effect.

Consider now a reduction in the fixed cost of exporting α_x. Proceeding as above yields:

$$\frac{\partial\beta^*}{\partial\alpha_x} = -\frac{\partial\pi^E/\partial\alpha_x}{\partial\pi^E/\partial\beta^*},$$

where $\partial\pi^E/\partial\beta^* > 0$ and

$$\frac{\partial\pi^E}{\partial\alpha_x} = \int_{\underline{\beta}}^{\beta_x^*} \frac{\partial\pi_x(\beta)}{\partial\alpha_x} dG(\beta) = -G(\beta_x^*) < 0.$$

Hence $\partial\beta^*/\partial\alpha_x > 0$, implying that a reduction in α_x leads to a fall of β^*. We record our results in the following

Proposition 3 *When fixed (α_x) and variable (τ) trade costs induce a partitioning of firms into exporters and non-exporters, a reduction in either α_x or τ leads to a selection effect (a reduction in β^*) independently of the sign of σ'.*

Thus, Melitz-type selection effects are robust to the sign of $\sigma'(c)$. We view these results as reassuring.

Competitive effects To study how trade liberalization affects the markups of domestic and foreign inframarginal firms, we just have to study how $c(\beta)$ and $c(\tau\beta)$ vary with trade costs. Consider first a reduction in α_x. Note, from (36)-(37), that α_x affects individual consumption only through β^*, and that β^* is increasing in α_x by Proposition 3. Moreover, individual consumption is increasing in β^* by Lemma 1. It follows that a reduction in α_x leads to a fall of both $c(\beta)$ and $c(\tau\beta)$, thereby reducing (increasing) the markups of inframarginal firms with DES (IES) preferences.

Consider now a reduction in τ. By Proposition 3 and Lemma 1, it leads to a reduction in β^* and thus in individual consumption of domestic varieties $c(\beta)$:

$$\frac{d\ln c(\beta)}{d\ln\tau} = \frac{\partial \ln c(\beta)}{\partial \ln \beta^*}\frac{\partial \ln \beta^*}{\partial \ln \tau} = \frac{\eta(c(\beta))}{m(c^*)}\frac{\partial \ln \beta^*}{\partial \ln \tau} > 0.$$

It follows that a reduction in τ reduces (increases) the markups of inframarginal domestic firms with DES (IES) preferences. As for individual consumption of foreign varieties, $c(\tau\beta)$, a reduction in τ has both a positive direct effect and a negative indirect effect through β^*. The net effect is given by:

$$\frac{d\ln c(\tau\beta)}{d\ln\tau} = \frac{\partial \ln c(\tau\beta)}{\partial\ln\tau} + \frac{\partial c(\tau\beta)}{\partial\beta^*}\frac{\partial\beta^*}{\partial\tau}\frac{\tau}{c(\tau\beta)} = \eta(c(\tau\beta))\left[\frac{1}{m(c^*)}\frac{\partial\ln\beta^*}{\partial\ln\tau} - 1\right] < 0,$$

where the latter equality follows from $\partial\ln c(\tau\beta)/\partial\ln\tau = -\eta(c(\tau\beta)) < 0$ by (22), and from $\partial\ln c/\partial\ln\beta^* = \eta(c)/m(c^*) > 0$ by Lemma 1, whereas the inequality follows directly from (40). Hence, as in the symmetric-firm case, a lower variable trade cost, by increasing $c(\tau\beta)$, leads inframarginal exporting firms to charge higher (lower) markups in the foreign market with DES (IES) preferences.

However, as also discussed in ACDR, the distribution of markups is also affected by variation in the cutoffs induced by changes in α_x and τ, possibly leading to countervailing effects on average markups. For instance, as in the case of pure globalization, a reduction in τ with DES preferences forces the exit of some low-productivity firms in the domestic market. Given that these firms charge lower markups, their exit tends to increase the average markup charged by domestic firms. Conversely, the entry of new low-productivity, low-markup exporters may countervail the increase in the average markup charged by inframarginal exporters.

Finally, note that

$$\left[\frac{d\ln c(\beta)}{d\ln\tau} + \frac{d\ln c(\tau\beta)}{d\ln\tau}\right]_{\tau=1} = \left[\eta(c(\beta))\left(\frac{\partial\ln\beta^*}{\partial\ln\tau}\frac{2}{m(c^*)} - 1\right)\right]_{\tau=1} < 0,$$

where the inequality follows from (40), which implies that $\partial\ln\beta^*/\partial\ln\tau < m(c^*)/2$ when evaluated at $\tau = 1$. Thus, a small increase in τ around $\tau = 1$ leads all inframarginal exporting firms to reduce average sales, and therefore to charge lower (higher) average markups with DES (IES) preferences. If in addition $\alpha_x = \alpha$ (namely, if all firms export), the above result applies to the average markup charged by all inframarginal firms. Finally, given that the size of marginal firms entering or exiting the domestic and foreign markets after the increase in trade costs is smaller than the average, the overall impact on the weighted average of markups is likely to be negative (positive) with DES (IES) preferences.

We record our main results in the following

Proposition 4 *With DES preferences, a reduction in the fixed cost of exporting α_x leads inframarginal firms to charge lower markups in both the domestic and the foreign market. Instead, a reduction in the variable trade cost τ leads inframarginal firms to charge lower* *markups in the domestic market and higher markups in the foreign market. A small increase in variable trade costs around $\tau = 1$ and $\alpha_x = \alpha$ leads inframarginal firms to charge higher average markups. The opposite results hold with IES preferences.*

To conclude, our results suggest that, although selection effects are robust to relaxing the CES assumption, competitive effects are not. In the next section, we therefore discuss the robustness of competitive effects in alternative monopolistic competition environments.

3 Alternative Environments

One possible explanation for the lack of robust competitive effects in a D-S monopolistic competition setup with additive preferences is that the latter implies that markups do not directly depend on the number of firms n. This is because the Dixit-Stiglitz assumption that firms do not interact strategically implies that the perceived demand elasticity equals the elasticity of substitution, and the latter is independent of n when preferences are additive.[19] Moreover, in this setup the number of product characteristics equals the number of varieties/firms and thus an increase in n does not "crowd" the variety space.[20] It follows that a pro-competitive effect can arise only through variation in c, which does not fully capture the operation of competitive forces.

In this Section, we therefore consider monopolistic competition environments in which the number of firms may directly affect the elasticity of demand. We first consider the case of quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008),[21] then study the implications of strategic interaction *à la* Bertrand and Cournot, and finally discuss Lancaster (1979)'s ideal variety approach to monopolistic competition. Perhaps surprisingly, we find that none of the above setups seems to yield a robust pro-competitive mechanism in monopolistic competition.

3.1 Quasi-Linear Quadratic Preferences

Consider a D-S monopolistic competition setup with symmetric firms and a utility function $U(c_0, \underline{u}(\mathbf{c}_{-0})) = c_0 + \underline{u}(\mathbf{c}_{-0})$, where c_0 is consumption of an outside good, $\mathbf{c}_{-0} =$

[19] With non-additive preferences, the elasticity of substitution between any two varieties also depends on the consumption of other varieties (see the Appendix). At a symmetric equilibrium, this may translate into a direct impact of n on the elasticity of substitution.

[20] See Pettengill (1979) on this point. He claims (p. 960) that "one's normal idea of monopolistic competition is that as the number of products in the industry increases, they become closer and closer substitutes". He therefore argues that Lancaster's (1975) ideal variety approach to monopolistic competition is more realistic in this respect.

[21] Their influential monopolistic competition model with heterogeneous firms is widely perceived as an alternative to the original Melitz (2003) model, as it yields variable markups.

$[c_1, c_2, .., c_n]$ is consumption of n varieties of some product and

$$\underline{u}(\mathbf{c}_{-0}) = a\sum_{j=1}^{n} c_j - \frac{b}{2}\sum_{j=1}^{n} c_j^2 - \frac{\gamma}{2}\left(\sum_{j=1}^{n} c_j\right)^2, \tag{41}$$

with $a, b, \gamma > 0$.[22] The implied demand function for variety i is

$$c_i(\mathbf{p}) = \frac{a}{b + n\gamma} - \frac{p_i}{b} + \frac{n\gamma}{b + n\gamma}\frac{1}{b}\overline{p}, \tag{42}$$

where $\overline{p} = \frac{1}{n}\sum_{j=1}^{n} p_j$ is the average price of a variety. Under the D-S assumption that firms take n and $\overline{p}$ as given, firm i's perceived demand is linear in p_i. Thus, at a symmetric equilibrium the perceived demand elasticity (which equals the elasticity of substitution σ: see (71) in the Appendix) is

$$\varepsilon = \frac{p}{bc} = \frac{1}{b}\left[\frac{a}{c} - b - \gamma n\right]. \tag{43}$$

Note that, perhaps surprisingly, for given c the number of firms n has a *negative* direct impact on the elasticity of demand. This example shows that, even if non-additive preferences introduce a direct channel whereby n affects ε, the resulting effect is not obviously positive.[23]

Eq. (43) implies:

$$m(c) = 1 + \frac{bc}{\beta}, \quad p = m\left(c\right)\beta = bc + \beta. \tag{44}$$

Using (44) in the free-entry condition ($\pi_v = (m-1)\,\beta Lc = \alpha$) and in the goods market equilibrium condition ($pcn = 1$) to solve for c, p, m and n yields:

$$c = \sqrt{\frac{\alpha}{bL}}, \quad p = \beta + \sqrt{\frac{b\alpha}{L}}, \quad m = 1 + \frac{1}{\beta}\sqrt{\frac{b\alpha}{L}}, \quad n = \frac{\alpha/L}{1 + \beta\sqrt{\frac{L}{b\alpha}}}. \tag{45}$$

Evidently, m is decreasing in market size L/α. However, given the negative direct impact of n on ε for given c, this pro-competitive effect is entirely driven by the fact that, just as in the case of DES preferences, the elasticity of substitution is decreasing in c. Note, also, that m is decreasing in β, which implies the incomplete pass-through of marginal costs to prices.

Consider now two identical countries separated by a symmetric iceberg trade cost

[22] We assume that $c_0 > 0$, i.e., that an internal solution arises in equilibrium.

[23] Another example of non-additive preferences which may yield an elasticity of substitution that is *decreasing* in n is provided by the so-called Leontief-Diewert expenditure function (Varian, 1992, p. 209).

$\tau \geq 1$. Indexing variables related to the foreign market by an x and using (44) gives:

$$m_x(c_x) = 1 + \frac{b}{\beta}\frac{c_x}{\tau}, \quad p_x = m_x(c_x)\,\tau\beta = bc_x + \tau\beta. \tag{46}$$

To compute the free-entry condition ($\pi_v + \pi_v^x = \alpha$), note first that, using (44) and (46), variable profits in the domestic and foreign market can be written as:

$$\begin{aligned} \pi_v(c) &= [m(c) - 1]\,\beta Lc = bLc^2, \\ \pi_v^x(c_x) &= [m_x(c_x) - 1]\,\tau\beta Lc_x = bLc_x^2 = \pi_v(c_x)\,. \end{aligned} \tag{47}$$

Next, note that (42) and the expressions for p and p_x in (44) and (46) imply that

$$c - c_x = \frac{p_x - p}{b} = \frac{\beta(\tau - 1)}{2b}, \tag{48}$$

which defines c_x as a function of c and τ, i.e., $c_x(c, \tau) = c - \beta(\tau - 1)/2b$. Using (47) and (48) in the free-entry condition, which implicitly defines c, yields:

$$\pi_v(c) + \pi_v(c_x(c, \tau)) = bLc^2 + bL\left(c - \frac{(\tau - 1)\beta}{2b}\right)^2 = \alpha. \tag{49}$$

Differentiating (48) and (49) yields:

$$\frac{\partial c}{\partial \tau} = \frac{\beta}{2b}\frac{c_x}{c + c_x} > 0, \quad \frac{dc_x}{d\tau} = \frac{\beta}{2b}\left(\frac{c_x}{c + c_x} - 1\right) < 0.$$

Thus, using (44) and (46):

$$\begin{aligned} \frac{dm}{d\tau} &= \frac{b}{\beta}\frac{\partial c}{\partial \tau} = \frac{1}{2}\frac{c_x}{c + c_x} > 0, \\ \frac{dm_x}{d\tau} &= \frac{b}{\beta\tau^2}\left(\frac{dc_x}{d\tau}\tau - c_x\right) = \frac{1}{2\tau}\left(\frac{c_x}{c + c_x} - 1\right) - \frac{bc_x}{\beta\tau^2} < 0. \end{aligned}$$

Note that, as in the case of DES preferences, the incomplete pass-through implies that trade liberalization leads to opposite competitive effects in the domestic and foreign market.[24]

[24] Similarly, when preferences are translog, as in Feenstra (2003), it can be shown that a reduction in trade costs leads firms to charge higher markups in the foreign market. As shown by ACDR, this may lead to smaller welfare gains from trade liberalization than in the CES case. However, Feenstra's preferences imply that at a symmetric equilibrium the elasticity of substitution is linearly increasing in n and the markup is independent of the level of marginal cost β.

Finally, computing the above derivatives in $\tau = 1$ yields:

$$\left.\frac{\partial c}{\partial \tau}\right|_{\tau=1} = \frac{\beta}{4b} = -\left.\frac{dc_x}{d\tau}\right|_{\tau=1},$$

$$-\left.\frac{dm_x}{d\tau}\right|_{\tau=1} = \frac{1}{4} + \frac{bc}{\beta} > \frac{1}{4} = \left.\frac{dm}{d\tau}\right|_{\tau=1}.$$

Consequently, as in the case of DES preferences, a small increase in trade costs around the free trade causes markups on foreign sales to fall by more (in absolute value) than the increase in markups on domestic sales, thus leading to a reduction in average markups.

The following proposition summarizes

Proposition 5 *As in the case of DES/IES preferences, quasi-linear quadratic preferences imply that average markups are non-monotonically related to trade costs.*

3.2 Strategic Interaction

The Dixit-Stiglitz assumption that each firm treats λ (the marginal utility of income) as a constant removes a direct channel whereby an increase in the number of firms may raise the perceived demand elasticity ε. Instead, in a model with strategic interaction firms properly treat λ as a function of the price vector (i.e., $\lambda = \lambda(\mathbf{p})$) and the elasticity of demand no longer coincides with the elasticity of substitution.

Consider first our baseline setting with Bertrand competition, in which firms set prices. In a symmetric Bertrand-Nash equilibrium, firms perceive the actual demand elasticity, given by (see (70) in the Appendix):

$$\varepsilon^b(c,n) = \sigma(c) - \frac{\sigma(c) - 1}{n}. \tag{50}$$

Note that $\partial\varepsilon^b/\partial n > 0$, which captures the direct pro-competitive channel induced by strategic interaction. Thus, the pricing equation is:

$$p = m^b(c,n)\beta = \frac{(n-1)\,\sigma(c) + 1}{(n-1)\,\sigma(c)} m(c)\beta, \tag{51}$$

where $\partial m^b/\partial n < 0$, and $\partial m^b/\partial c \gtrless 0 \Leftrightarrow m' \gtrless 0 \Leftrightarrow \sigma' \lesseqgtr 0$.

Consider now the case of Cournot competition, in which firms set production levels. Multiplying both sides of the first-order conditions for utility maximization ($u'(c_i) = \lambda p_i$) by c_i, adding up and using the budget constraint ($\sum_j p_j c_j = w = 1$) yields:

$$\lambda = \sum\nolimits_j u'(c_j)c_j. \tag{52}$$

Using (52) in the first-order conditions yields the following inverse demand system:

$$p_i(\mathbf{c}) = \frac{u'(c_i)}{\sum_j u'(c_j)c_j}, \quad i = 1, ...n. \tag{53}$$

Firm i's revenue is $p_i(\mathbf{c})c_iL$; marginal revenue is therefore given by

$$\frac{\partial\left(p_i(\mathbf{c})c_i\right)}{\partial c_i} = \frac{r'(c_i)\left(\sum_{j\neq i} u'(c_j)c_j\right)}{\left(\sum_j u'(c_j)c_j\right)^2}, \tag{54}$$

and is decreasing in c_i under our assumptions that $r'' < 0$ and $r' > 0$. In a Cournot-Nash equilibrium, each firm chooses its quantity to satisfy the first-order condition $\partial\left(p_i(\mathbf{c})c_i\right)/\partial c_i = \beta$ under a correct conjecture about the quantities produced by its competitors. Then (54) implies that, in any symmetric Cournot-Nash equilibrium,

$$c = \frac{(n-1)\,r'(c)}{n^2u'(c)\beta} = \frac{n-1}{n^2m(c)\beta}. \tag{55}$$

By (53) and (55), the pricing equation is thus

$$p = m^c(c,n)\beta = \frac{n}{n-1}m(c)\beta, \tag{56}$$

where $\partial m^c/\partial n < 0$, and $\partial m^c/\partial c \gtrless 0 \Leftrightarrow m' \gtrless 0 \Leftrightarrow \sigma' \lesseqgtr 0$. Accordingly, Cournotian firms perceive a demand elasticity equal to[25]

$$\varepsilon^c(c,n) = \frac{\sigma(c)n}{\sigma(c)+n-1}. \tag{57}$$

We can use the pricing equations, (51) or (56), the free-entry condition ($p = \beta + \alpha/cL$) and goods market equilibrium condition ($p = 1/nc$) to characterize the symmetric Bertrand and Cournot equilibria, summarized by the two-equation system

$$m^s(c^s,n^s)c^s\beta = \frac{1}{n^s} = \frac{\alpha}{L} + c^s\beta, \tag{58}$$

where $s = b, c$. Differentiating (58) yields:

$$\begin{bmatrix} \left(\frac{\partial m^s}{\partial c}c^s + m^s\right)\beta & \frac{\partial m^s}{\partial n}c^s\beta + \frac{1}{(n^s)^2} \\ \beta & \frac{1}{(n^s)^2} \end{bmatrix} \begin{bmatrix} \frac{\partial c^s}{\partial(\alpha/L)} & \frac{\partial c^s}{\partial\beta} \\ \frac{\partial n^s}{\partial(\alpha/L)} & \frac{\partial n^s}{\partial\beta} \end{bmatrix} = \begin{bmatrix} 0 & -m^sc^s \\ -1 & -c^s \end{bmatrix}. \tag{59}$$

[25]Note that $\varepsilon^c(c,n) < \varepsilon^b(c,n)$, i.e., Cournotian firms perceive a lower demand elasticity relative to Bertrand firms.

Thus, applying Cramer's rule,

$$\left[\begin{array}{cc} \frac{\partial c^s}{\partial(\alpha/L)} & \frac{\partial c^s}{\partial\beta} \\ \frac{\partial n^s}{\partial(\alpha/L)} & \frac{\partial n^s}{\partial\beta} \end{array}\right] = \frac{1}{D^s}\left[\begin{array}{cc} \frac{\partial m^s}{\partial n}c^s\beta + \frac{1}{(n^s)^2} & -\frac{m^s c^s}{(n^s)^2} + c^s\left(\frac{\partial m^s}{\partial n}c^s\beta + \frac{1}{(n^s)^2}\right) \\ -\left(\frac{\partial m^s}{\partial c}c^s + m^s\right)\beta & -\beta\frac{\partial m^s}{\partial c}\left(c^s\right)^2 \end{array}\right], \quad (60)$$

where D^s is the determinant of the first matrix on the LHS of (59). As shown in the Appendix,

$$\frac{\partial c^s}{\partial\left(\alpha/L\right)} \geq 0,\ \frac{\partial c^s}{\partial\beta} < 0,\ \frac{\partial n^s}{\partial\left(\alpha/L\right)} < 0 \text{ and } sign\left\{\frac{\partial n^s}{\partial\beta}\right\} = sign\left\{\sigma'\right\}, \quad (61)$$

where $\partial c^s/\partial(\alpha/L) = 0$ only for $s = c$ and $n = 2$.[26]

Thus, the comparative statics results delivered by the baseline model hold also in the presence of Bertrand and Cournot competition. This allows us to study the competitive effects of an increase in market size (i.e., a reduction in α/L) and of a reduction in the marginal cost β. Note first that

$$\frac{\partial m^s}{\partial\left(\alpha/L\right)} = \frac{\partial m^s}{\partial c}\frac{\partial c^s}{\partial\left(\alpha/L\right)} + \frac{\partial m^s}{\partial n}\frac{\partial n^s}{\partial\left(\alpha/L\right)}, \quad (62)$$

where $\partial m^s/\partial n < 0$ and (61) imply that the second term on the RHS is always positive. It follows that $\partial m^s/\partial\left(\alpha/L\right) > 0$ for $\sigma' \leq 0$, as in this case $\partial m^s/\partial c \geq 0$. Consequently, an increase in market size is pro-competitive when preferences are DES or CES. Instead, as shown in the Appendix, $\partial m^s/\partial\left(\alpha/L\right) < 0$ for $\sigma' > 0$, unless n is small. Thus, as in the baseline model, an increase in market size is generally anti-competitive with IES preferences.

Finally, note that

$$\frac{\partial m^s}{\partial\beta} = \frac{\partial m^s}{\partial c}\frac{\partial c^s}{\partial\beta} + \frac{\partial m^s}{\partial n}\frac{\partial n^s}{\partial\beta}, \quad (63)$$

implying that the change in individual consumption and in the number of firms induced by a change in the marginal cost β affect markups in opposite directions. In the Appendix we show, however, that in all cases $sign\left\{\frac{\partial m^s}{\partial\beta}\right\} = sign\left\{\sigma'\right\}$, and hence that a reduction in the marginal cost is anti-competitive (pro-competitive) with DES (IES) preferences.

We summarize our main results in the following

Proposition 6 *a) When firms interact strategically, an increase in market size is pro-competitive with DES or CES preferences; with IES preferences, instead, a pro-competitive effect may arise only when the number of firms is small. b) A reduction in the marginal*

[26] Although we treat n as a real number (thereby ignoring the integer problem), consistent with the assumption of strategic interaction we focus on values of n equal or greater than 2.

cost is anti-competitive with DES preferences, neutral with CES preferences, and pro-competitive with IES preferences.

3.2.1 Discussion

Strategic interaction with additive preferences seems to yield the same competitive effects illustrated in Section 2 for IES/DES preferences. In particular, an increase in market size is still generally anti-competitive when preferences are IES. Moreover, the competitive effects of a reduction in the marginal cost are still generally the opposite of those of an increase in market size. This implies, e.g., that with DES preferences markups increase when marginal costs fall, and thus the pro-competitive effect of a reduction in trade costs is still contaminated by the increase in markups on foreign sales due to the incomplete pass-through.

Finally note that, by (50) and (57), independent of the assumptions about $\sigma(c)$, the competitive effects arising from strategic interactions can be relevant only when the number of firms is small, or else the impact on markups of an increase in n becomes negligible and the model's predictions boil down to those of the baseline model in Section 2. However, as forcefully argued by Dixit and Stiglitz (1993), assuming that n is small is problematic in a monopolistic competition setting. In particular, if the number of firms is small enough to induce them to interact strategically, it is unclear why they do not also engage in collusion and entry deterrence, thereby preventing the free entry of firms. By the same token, it is actually unclear why we should expect significant competitive effects in a market situation in which the number of firms is large enough to make sense of the free-entry condition postulated in monopolistic competition models.

3.3 Ideal Variety Approach to Monopolistic Competition

In a Dixit-Stiglitz setting, the introduction of new varieties does not crowd the variety space, as the number of characteristics/varieties is the same as the number of firms. One may argue, however, that a pro-competitive effect may naturally arise in a framework in which an increase in the number of available varieties reduces their distance in a fixed space of characteristics, thereby increasing their substitutability. In closing this Section we show that, surprisingly, this need not be the case.

To make the point, we reconsider Lancaster's (1975, 1979) ideal variety approach to monopolistic competition. In this setting, consumer preferences are heterogeneous and the aggregate demand for each variety arises from diversity of tastes. In particular, each consumer has a most preferred ("ideal") variety. As described in Helpman and Krugman

(1985, pp. 120-21), ideal variety means that "when the individual is offered a well-defined quantity of the good but is free to choose any potentially possible variety, he will choose the ideal variety independently of the quantity offered and independently of the consumption level of other goods. Moreover, when comparing a given quantity of two different varieties, the individual prefers the variety that is closest to his ideal product".

These assumptions are formalized by assuming that each variety is represented by a point ω on the unit length circumference Ω of a circle, and that preferences for the ideal product are uniformly distributed over Ω across consumers. L is the size (and density) of the population. The utility function of a consumer with ideal variety $\widetilde{\omega}$ is assumed to be:

$$U = \sum_{\omega \in \Omega} \frac{c(\omega)}{h(\delta(\omega, \widetilde{\omega}))}, \tag{64}$$

where $\delta(\omega, \widetilde{\omega})$ is the shortest arc distance between ω and $\widetilde{\omega}$, and $h(\delta)$ is the so-called Lancaster's compensation function, assumed to be positive, non decreasing and generally normalized so that $h(0) = 1$ (see Lancaster, 1975). Moreover, it is generally assumed (see Helpman and Krugman, 1985) that $h(\delta)$ is strictly increasing and convex, and that $h'(0) = 0$.

The above assumptions imply that the marginal rate of substitution between $\widetilde{\omega}$ and ω is given by $MRS(\omega, \widetilde{\omega}) = h(\delta(\omega, \widetilde{\omega}))$, and is thus an increasing convex function of δ. Following Helpman (1981) and Helpman and Krugman (1985), it is possible to show that, at a symmetric equilibrium, the price elasticity of total demand equals

$$\varepsilon = 1 + \frac{1}{2\epsilon_h(1/2n)},$$

where $\epsilon_h = \frac{h'(1/2n)}{2nh(1/2n)}$ is the elasticity of the compensation function. Accordingly, an increase in market size L, by increasing the number of firms n, also increases ε if $\epsilon_h' > 0$, thus yielding a pro-competitive effect.[27] Instead, if $\epsilon_h' < 0$, an increase in market size reduces ε and is therefore anti-competitive. Finally, if $\epsilon_h' = 0$ (i.e., if $h(\cdot)$ is isoelastic), ε is independent of n, just as in the "love for variety" approach of Section 2 when preferences are CES. Thus, a pro-competitive effect is unwarranted even in a framework in which an increase in the number of firms crowds the variety space. There are two main reasons for this result. First, given (64), varieties are of the "perfect substitute" type, their elasticity of substitution is constant, and a crowding of the variety space cannot affect it. In this respect, the model fails to capture a potentially genuine pro-competitive effect of the

[27] For instance, in a generalized model of ideal variety with income effects, Hummels and Lugovskyy (2012) assume, without discussion, a functional form for the compensation function that implies $\epsilon_h' > 0$ and thus delivers pro-competitive effects.

crowding of the variety space.

Second, the ideal variety approach does not impose sufficient restrictions on $h(\cdot)$ to pin down the properties of ϵ_h, and therefore the relationship between ε and n.[28] Specifically, is the assumption $\epsilon_h' > 0$ plausible? Note that $MRS(\widetilde{\omega}, \omega) = h(\delta(\omega, \widetilde{\omega}))$ implies that, in order for Lancaster's model to deliver a pro-competitive effect, consumer preferences must feature an ever increasing distance elasticity of the marginal rate of substitution between $\widetilde{\omega}$ and ω. It is hard to provide a rationale for this assumption, which seems no more plausible than the opposite assumption of a decreasing distance elasticity, which would however deliver an anti-competitive effect.

The following proposition summarizes

Proposition 7 *When preferences are of the ideal variety type, as in Lancaster (1979), an increase in market size does not affect markups when the compensation function is isoelastic. An increase in market size is instead pro-competitive (anti-competitive) when the compensation function features an increasing (decreasing) distance elasticity.*

4 Conclusion

We have studied the competitive, selection and reallocation effects of trade liberalization in monopolistic competition. We have shown that, in a Dixit-Stiglitz setup with heterogeneous firms and fixed and variable costs of exporting, Melitz-type selection effects are robust to relaxing the CES assumption, whereas competitive and reallocation effects are fragile. We have also shown that allowing for non-additive preferences, for strategic interactions and for the crowding of the variety space does not obviously lead to stronger and more robust competitive effects in monopolistic competition.

Our results suggest that the effects of trade liberalization delivered by non-CES preferences in monopolistic competition are more complex and articulated than generally believed. Consistently, one should carefully look for empirical evidence in support, e.g., of the implied anti-competitive effects. Alternatively, one can invoke a sort of second-best argument in support of CES preferences, according to which relaxing one out of a number of restrictive assumptions does not necessarily lead to a better model. We think that the latter option is not equivalent to assuming that competitive effects are little relevant, but rather that they should be studied in somehow richer trade models, among which oligopoly settings seem a prominent alternative, as suggested by Neary (2003, 2009b, 2010), who

[28] Only in the limit case in which n goes to infinity, and due to the (rather ad hoc) assumptions $h(0) = 1$ and $h'(0) = 0$, the aggregate demand elasticity is increasing in n (and goes to infinite). This requires a situation, unfeasible under a positive fixed cost, in which the circumference of the circle is full and the aggregate demand for each firm is infinitesimal.

also warns us about the difficulties of dealing with them in general equilibrium. We conclude that more creative theoretical work is still needed to better our understanding of the effects of globalization.

5 Appendix

5.1 Elasticity of Demand, Elasticity of Substitution and the Number of Firms

In this Section we show how the elasticity of demand, the elasticity of substitution, and the number firms/varieties can be related at a symmetric equilibrium. Consider a general setting in which consumers share the same preferences, represented by the utility function $U(c_0, \underline{u}(\mathbf{c}_{-0}))$, where c_0 is consumption of an "outside" good, $\mathbf{c}_{-0} = [c_1, c_2, ..., c_n]$ is the consumption vector of n varieties of some good, and $\underline{u}(\cdot)$ is symmetric and concave in its arguments. Denote the associated system of compensated (Hicksian) demand by $\widetilde{\mathbf{c}} = [\widetilde{c}_0, \widetilde{\mathbf{c}}_{-0}] = \widetilde{\mathbf{c}}(\mathbf{p}, U)$, where $\mathbf{p} = [p_0, \mathbf{p}_{-0}]$ is the price vector. Moreover, denote by $\widetilde{c}_{ij} = \partial \widetilde{c}_i / \partial p_j$ $(i, j = 0, 1, ...n)$ the compensated demand-price derivatives, and by $\widetilde{\varepsilon}_{ij} = \partial \ln \widetilde{c}_i / \partial \ln p_j$ the corresponding elasticities. The elasticity of substitution between goods i and j $(i \neq j)$ measures the substitutability between any two goods at a given consumption vector $\mathbf{c} = [c_0, \mathbf{c}_{-0}]$, and is given by:[29]

$$\sigma_{ij} = \widetilde{\varepsilon}_{ji} - \widetilde{\varepsilon}_{ii}. \tag{65}$$

Specifically, σ_{ij} measures how the marginal rate of substitution between i and j, $MRS_{ij} = (\partial U / \partial c_i) \, / \, (\partial U / \partial c_j)$, varies with the consumption ratio c_i / c_j. The Slutsky equation, which decomposes the price effect on the Marshallian (uncompensated) demand $c_i(\mathbf{p}, Y)$ into a substitution and an income effect, is given by:

$$c_{ji} = \widetilde{c}_{ji} - c_{jY} c_i, \tag{66}$$

where Y is income, $c_{ij} = \partial c_i / \partial p_j$ and $c_{iY} = \partial c_i / \partial Y$. In elasticity terms, (66) can be written as:

$$\varepsilon_{ji} = \widetilde{\varepsilon}_{ji} - \varepsilon_{jY} \theta_i, \tag{67}$$

where $\theta_j = p_j c_j / Y$, $\varepsilon_{jY} = \partial \ln c_j / \partial \ln Y$, and $\varepsilon_{ji} = \partial \ln c_j / \partial \ln p_i$ are, respectively, good j's expenditure share, income elasticity, and cross-price demand elasticity with respect to

[29] See Blackorby and Russell (1989) for a discussion of the elasticity of substitution.

good i.[30] Using (65) and (67) yields:

$$\varepsilon_{ii} = -\sigma_{ij} + \widetilde{\varepsilon}_{ji} - \varepsilon_{iY}\theta_i. \tag{68}$$

We now show that, at a symmetric equilibrium, (68) implies a simple relationship between the elasticity of demand, the elasticity of substitution and the number of firms. In the case in which $p_i = p$ (which implies $c_j = c$) for all $i \neq 0$ (i.e., the case of an entirely symmetric consumption pattern over $\mathbf{c}_{-0}$), by the symmetry of preferences it must be the case that the price and income effects involving varieties are identical and symmetric (i.e., in particular, $\sigma_{ij} = \sigma_{ih}$ and $\widetilde{\varepsilon}_{ji} = \widetilde{\varepsilon}_{ij}$, $i, j, h = 1, ..., n$, $j \neq i \neq h$). Moreover, since $\sum_{j=1}^{n} \widetilde{\varepsilon}_{ij} = -\widetilde{\varepsilon}_{i0}$ $(i = 1, ...n)$, where $\widetilde{\varepsilon}_{i0} = \partial \ln \widetilde{c}_i / \partial \ln p_0$, and $\sum_{j=1}^{n} \varepsilon_{jY}\theta_j + \varepsilon_{0Y}\theta_0 = 1$, "averaging" (68) over j yields:

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{1}{n}\left[1 + \widetilde{\varepsilon}_{i0} - \varepsilon_{0Y}\theta_0\right], \qquad i, j = 1, ..., n;\ i \neq j, \tag{69}$$

which shows that, for n large, the elasticity of demand for a variety is approximately equal to its (common) elasticity of substitution with the other varieties.

In particular, when there is no outside good (as in Krugman, 1979 and in Section 2), (69) simplifies to:[31]

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{1}{n}, \tag{70}$$

which generalizes the well-known formula for the CES to the case of non-additive preferences. Another simple case arises when preferences are quasi-linear, i.e., when $U(c_0, \underline{u}(\mathbf{c}_{-0}) = c_0 + \underline{u}(\mathbf{c}_{-0})$, as in Section 3.1. In this case, the demand for variety i exhibits no income effect and, accordingly,

$$\varepsilon_{ii} = -\frac{n-1}{n}\sigma_{ij} - \frac{\widetilde{\varepsilon}_{i0}}{n}. \tag{71}$$

Finally, we can show that, if $u(\cdot)$ is additive, namely, if $\underline{u}(\mathbf{c}_{-0}) = \sum_{i=1}^{n} u(c_i)$, then for $c_i = c_j$ $(i, j = 1, ..., n,\ i \neq j)$ the elasticity of substitution is independent of the number of firms and is given by:

$$\sigma_{ij} = -\frac{u'(c_i)}{u''(c_i)c_i}. \tag{72}$$

Eq. (72) implies that the demand elasticity perceived under the Dixit-Stiglitz assumption is always equal to the elasticity of substitution at a pairwise symmetric consumption. To

[30] For expositional purposes, in this Appendix it proves convenient to define the demand elasticity as $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$, rather than $\varepsilon_i = -\partial \ln c_i / \partial \ln p_i$, as in the main text.

[31] The same expression holds in the presence of an outside good, provided that preferences are Cobb-Douglas, i.e., when $U(c_0, \underline{u}(\mathbf{c}_{-0})) = c_0^{\alpha}\underline{u}(\mathbf{c}_{-0})^{1-\alpha}$, where $0 < \alpha < 1$ and $\underline{u}(\cdot)$ is homogeneous. The reason is that in this case homotheticity implies $\varepsilon_{jY} = 1$ $(j = 0, 1, ..., n)$, and by (67) $\varepsilon_{i0} = \partial \ln c_i / \partial \ln p_0 = \widetilde{\varepsilon}_{i0} - \theta_0 = 0$ $(i = 1, ..., n)$.

prove (72), note that differentiating the first-order conditions

$$p_i = \mu U_{\underline{u}}(c_0, \sum_i u(c_i))u'(c_i),$$

where μ is the relevant Lagrangian multiplier and $U_{\underline{u}} = \partial U/\partial \underline{u}$, yields:

$$\begin{aligned} 1 &= \mu_i U_{\underline{u}} u'(c_i) + \mu U_{\underline{u}} u''(c_i)\widetilde{c}_{ii} + \mu u'(c_i)\left[U_{\underline{u}0}\widetilde{c}_{0i} + U_{\underline{uu}}\sum_{h=1}^{n} u'(c_h)\widetilde{c}_{hi}\right], \\ 0 &= \mu_j U_{\underline{u}} u'(c_i) + \mu U_{\underline{u}} u''(c_i)\widetilde{c}_{ij} + \mu u'(c_i)\left[U_{\underline{u}0}\widetilde{c}_{0j} + U_{\underline{uu}}\sum_{h=1}^{n} u'(c_h)\widetilde{c}_{hj}\right], \end{aligned}$$

where $U_{\underline{u}0} = \partial^2 U/(\partial \underline{u} \partial c_0)$, $U_{\underline{uu}} = \partial^2 U/\partial \underline{u}^2$ and $\mu_i = \partial \mu/\partial p_i$. Subtracting the second expression from the first, exploiting the symmetry of price effects implied by the compensated demand functions (i.e., $\widetilde{c}_{ij} = \widetilde{c}_{ji}$) and manipulating yields:

$$1 = p_i\left[\frac{\mu_i - \mu_j}{\mu} + \frac{U_{\underline{u}0}}{U_{\underline{u}}}(\widetilde{c}_{0i} - \widetilde{c}_{0j}) + \frac{U_{\underline{uu}}}{U_{\underline{u}}}\sum_{h=1}^{n} u'(c_h)(\widetilde{c}_{hi} - \widetilde{c}_{hj})\right] + \frac{u''(c_i)c_i}{u'(c_i)}(\widetilde{\varepsilon}_{ii} - \frac{c_j}{c_i}\widetilde{\varepsilon}_{ji}). \quad (73)$$

Note that, for $p_i = p_j$, then, due to symmetry of preferences over $\mathbf{c}_{-0}$, $c_i = c_j$, $\mu_i = \mu_j$, $\widetilde{c}_{hi} = \widetilde{c}_{hj}$ $(h = 0, ..., n,\ i \neq h \neq j)$ and $\widetilde{c}_{ii} = \widetilde{c}_{jj}$. Thus (73) boils down to (72).

5.2 Aggregate Productivity

We now show that selection effects due to an increase in market size are unambiguously associated with a measure of aggregate productivity changes only if η' and σ' agree in sign. Define the average marginal cost as

$$\widetilde{\beta} = \int_{\underline{\beta}}^{\beta^*} \beta d\Gamma(\beta), \quad (74)$$

where

$$\Gamma(\beta) = \int_{\underline{\beta}}^{\beta} \frac{c(z)dG(z)}{\int_{\underline{\beta}}^{\beta^*} c(s)dG(s)} \quad (75)$$

is the distribution of β weighted by the corresponding production levels. Note that $\widetilde{\beta}$ satisfies the property of invariance of total cost, i.e.,

$$n\left[\int_{\underline{\beta}}^{\beta^*} \widetilde{\beta} c(\beta) L \frac{dG(\beta)}{G(\beta^*)} + \alpha\right] \equiv n\left[\int_{\underline{\beta}}^{\beta^*} \beta c(\beta) L \frac{dG(\beta)}{G(\beta^*)} + \alpha\right].$$

As a consequence, $\widetilde{\beta}$ is a proper cost measure and is thus relevant from a welfare standpoint.

Our measure of aggregate productivity is therefore $1/\widetilde{\beta}$; it is determined by $\Gamma(\beta)$, which has support $[\underline{\beta}, \beta^*]$ and depends on $G(\beta)$ and $c(\beta)$. Let $\gamma = \Gamma'$ be the density function associated with Γ, denote by $h = d \log \Gamma / d\beta = \gamma/\Gamma$ the corresponding so-called reverse hazard rate, and recall that, if two distributions have the same reverse hazard rate, they are identical. We have:

$$h(\beta) = \frac{c(\beta)g(\beta)}{\int_{\underline{\beta}}^{\beta} c(z)dG(z)} = \frac{g(\beta)}{\int_{\underline{\beta}}^{\beta} \frac{c(z)}{c(\beta)}dG(z)}, \tag{76}$$

which implies that the reverse hazard rate corresponding to a β-firm depends on its output relative to that of all other firms with a lower marginal cost. Thus, by governing relative firm size, η affects how market size impacts on aggregate productivity.

For instance, if $\eta', \sigma' < 0$, a simple "reverse hazard rate dominance" argument (see, e.g., Shaked and Shanthikumar, 1994) shows that an increase in market size from L to $\widehat{L}$ leads to a rise in aggregate productivity. To see this, denote by Γ_L and $\Gamma_{\widehat{L}}$ the corresponding distributions, with supports $[\underline{\beta}, \beta_L^*]$ and $\left[\underline{\beta}, \beta_{\widehat{L}}^*\right]$, and $\beta_{\widehat{L}}^* < \beta_L^*$ due to the selection effect implied by DES preferences ($\sigma' < 0$). For all firms active in both market situations, $h_{\widehat{L}}(\beta) \leq h_L(\beta)$ due to $\eta' < 0$, thereby implying

$$\int_{\underline{\beta}}^{\beta} \frac{c_L(z)}{c_L(\beta)} dG(z) \leq \int_{\underline{\beta}}^{\beta} \frac{c_{\widehat{L}}(z)}{c_{\widehat{L}}(\beta)} dG(z).$$

Moreover, since $h_{\widehat{L}}(\beta) = 0 \leq h_L(\beta)$ for all $\beta \in \left(\beta_{\widehat{L}}^*, \beta_L^*\right]$, it follows that $h_{\widehat{L}}(\beta) \leq h_L(\beta)$ for all $\beta \in \left[\underline{\beta}, \beta_{\widehat{L}}^*\right]$ and thus $\widetilde{\beta}_{\widehat{L}} < \widetilde{\beta}_L$. By the same reasoning, if $\eta', \sigma' > 0$, a rise in market size leads to an unambiguous reduction in aggregate productivity. On the contrary, if η' and σ' do not agree in sign, the selection effect (governed by σ) and the reallocation effect (governed by η) point in opposite directions, and the impact of an increase in market size on aggregate productivity is therefore ambiguous, as it will in general depend on the properties of the distribution G.

5.3 Comparative Statics of the Bertrand and Cournot Equilibria

We now prove the results in Proposition 6.

Bertrand competition. Differentiating (51) yields:

$$\begin{aligned}\frac{\partial m^b}{\partial c} &= \frac{n}{n-1}m', \\ \frac{\partial m^b}{\partial n} &= -\frac{1}{(n-1)^2(\sigma-1)} = -\frac{m}{(n-1)^2\sigma} = -\frac{m^b}{(n-1)[(n-1)\sigma+1]}.\end{aligned} \tag{77}$$

Next, using (77) in (60) yields:

$$\begin{bmatrix} \frac{\partial c^b}{\partial(\alpha/L)} & \frac{\partial c^b}{\partial\beta} \\ \frac{\partial n^b}{\partial(\alpha/L)} & \frac{\partial n^b}{\partial\beta} \end{bmatrix} = \frac{1}{D^b}\begin{bmatrix} \overbrace{\frac{1}{n^b}\left(\frac{1}{n^b} - \frac{m}{\sigma(n^b-1)^2 m^b}\right)}^{>0} & \overbrace{\frac{c}{(n^b)^2}\left(1-m^b-\frac{n^b}{(n^b-1)[(n^b-1)\sigma+1]}\right)}^{<0} \\ \underbrace{-\frac{\beta}{n^b-1}\left[n^b\left(m'c^b+m\right)-1\right]}_{}^{<0} & -\frac{n^b\beta(c^b)^2}{n^b-1}m' \end{bmatrix}, \tag{78}$$

where

$$D^b = \frac{\beta}{n^b(n^b-1)}\left[m'c^b+m-1+\frac{m}{\sigma(n^b-1)m^b}\right] > 0,$$

and the inequality follows from (13), implying that $m'c+m-1>0$. Similarly, the inequalities in (78) are straightforward implications of $m'c+m-1>0$ and of the definition of m^b. This proves that the inequalities in (61) hold for $s=b$.

Next, using (78) in (62) yields:

$$\begin{aligned}\frac{\partial m^b}{\partial(\alpha/L)} &= \frac{n^b m'}{n^b-1}\frac{\partial c^b}{\partial \frac{\alpha}{L}} - \frac{m^b}{(n^b-1)[(n^b-1)\sigma+1]}\frac{\partial n^c}{\partial\frac{\alpha}{L}} \\ &= \frac{1}{D^b}\left\{\frac{m'}{n^b-1}\left[\frac{1}{n^b}-\frac{m}{\sigma(n^b-1)^2 m^b}\right] + \frac{m^b\beta\left[n^b\left(m'c^b+m\right)-1\right]}{(n^b-1)^2[(n^b-1)\sigma+1]}\right\} \\ &= \frac{m+m'c^b g(n^b)-\frac{1}{n^b}}{c^b(n^b-1)^2[\sigma(n^b-1)+1]D^b},\end{aligned} \tag{79}$$

where $g(n)=\frac{(n-1)[(n-1)\sigma+1]}{n}>0$ is a monotonically increasing function, with $g(1+\frac{1}{\sqrt{\sigma}})=1$. Note that $\partial m^b/\partial(\alpha/L)>0$ if and only if $m+m'c^b g\left(n^b\right)>1/n^b$, a condition that is always satisfied for $m'\geq 0$ (i.e., with DES or CES preferences). Instead, for $m'<0$ (namely, with IES preferences), $\partial m^b/\partial(\alpha/L)<0$ unless n^c is small, i.e., sufficiently close to $1+\frac{1}{\sqrt{\sigma}}$.

Finally, using (78) in (63) yields:

$$\begin{aligned}\frac{\partial m^b}{\partial \beta} &= m' \frac{n^b}{n^b-1}\frac{\partial c^b}{\partial \beta} - \frac{m^b}{(n-1)\left[(n-1)\sigma+1\right]}\frac{\partial n^b}{\partial \beta} \\ &= \frac{1}{\left(n^b-1\right)D^b}\left\{m'\frac{c^b}{n^b}\left(1-m^b-\frac{n^b}{(n^b-1)\sigma+1}\right)+\frac{m^b m' n^b \beta\left(c^b\right)^2}{\left(n^b-1\right)\left[(n-1)\sigma+1\right]}\right\} \\ &= \frac{m'c^b\left(1-m^b\right)}{n^b\left(n^b-1\right)D^b}, \qquad (80)\end{aligned}$$

which implies that $sign\left\{\frac{\partial m^c}{\partial \beta}\right\} = -sign\left\{m'\right\} = sign\left\{\sigma'\right\}$.

Cournot competition. Differentiating (56) yields:

$$\frac{\partial m^c}{\partial c} = \frac{n}{n-1}m', \; \frac{\partial m^c}{\partial n} = -\frac{m}{(n-1)^2} = -\frac{m^c}{n\left(n-1\right)}. \qquad (81)$$

Next, using (81) in (60) yields:

$$\begin{bmatrix} \frac{\partial c^c}{\partial(\alpha/L)} & \frac{\partial c^c}{\partial \beta} \\ \frac{\partial n^c}{\partial(\alpha/L)} & \frac{\partial n^c}{\partial \beta}\end{bmatrix} = \frac{1}{D^c}\begin{bmatrix} \overbrace{\frac{n^c-2}{(n^c)^2\left(n^c-1\right)}}^{\geq 0} & \overbrace{\frac{c^c}{n^c\left(n^c-1\right)}\left(1-m-\frac{2}{n^c}\right)}^{<0} \\ \overbrace{-\frac{n^c\beta}{n^c-1}\left(m'c^c+m\right)}^{<0} & -\frac{n^c\beta(c^c)^2}{n^c-1}m' \end{bmatrix}, \qquad (82)$$

where

$$D^c = \frac{\beta}{(n^c)^2}\left[\frac{n^c\left(m'c^c+m\right)}{n^c-1}-1+\frac{1}{n^c-1}\right] > 0. \qquad (83)$$

Note that $\frac{\partial c^c}{\partial(\alpha/L)} = 0$ only for $n^c = 2$, and that the inequalities in (82) and (83) are straightforward when recalling that $m + m'c - 1 > 0$. Thus, the inequalities in (61) hold also for $s = c$.

Next, using (82) in (62) yields:

$$\frac{\partial m^c}{\partial(\alpha/L)} = \frac{n^c m'}{n^c-1}\frac{\partial c^c}{\partial(\alpha/L)} - \frac{m}{(n^c-1)^2}\frac{\partial n^c}{\partial(\alpha/L)} = \frac{m+m'c^c(n^c-1)}{n^c\left(n^c-1\right)^2 D^c c^c}. \qquad (84)$$

Note that $\partial m^c/\partial\left(\alpha/L\right) > 0$ if and only if $m + m'c^c(n^c-1) > 0$, a condition that is always satisfied for $m' \geq 0$ (i.e., with DES or CES preferences). Instead, for $m' < 0$ (IES preferences), $\partial m^c/\partial\left(\alpha/L\right) < 0$ unless n^c is small, i.e., sufficiently close to 2.

Finally, using (82) in (63) yields:

$$\frac{\partial m^c}{\partial \beta} = \frac{n^c m'}{n^c-1}\frac{\partial c^c}{\partial \beta} - \frac{m^c}{n^c\left(n^c-1\right)}\frac{\partial n^c}{\partial \beta} = \frac{c^c m'\left(1-m-\frac{1}{n^c}\right)}{\left(n^c-1\right)^2 D^c}, \qquad (85)$$

implying that $sign\left\{\frac{\partial m^c}{\partial \beta}\right\} = -sign\{m'\} = sign\{\sigma'\}$.

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