LSM: A DSGE Model for Luxembourg[1]

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Abstract

Luxembourg is a small open economy with a set of particular features, including rather limited competition in the domestic goods market, strong union power, and a segmented labour market for resident and non-resident workers. In this paper we develop a medium scale DSGE model that captures these features, calibrate it to mimic the actual behaviour of the key macroeconomic aggregates, and use it to conduct policy experiments aimed at relaxing some of the existing rigidities in the goods and labour market.

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1 Introduction

The key features of models based on the New Open Economy Macroeconomics and Dynamic Stochastic General Equilibrium (NOEM-DSGE) approaches are an optimization-based dynamic general-equilibrium approach; the presence of sticky prices and/or wages in at least some sectors of the economy; the incorporation of stochastic shocks; and the evaluation of economic (typically monetary) policy based on household welfare, with results robust to the Lucas (1976) critique. As in closed-economy DSGE models, early NOEM-DSGE models were highly theoretical and provided only a very stylized representation of the economy, see e.g. Obstfeld and Rogoff (1995). Later developments, such as Ghironi (1999), Bergin (2003), Lubik and Schorfheide (2005), and Justiniano and Preston (2004), estimated small-scale NOEM-DSGE models, usually by Bayesian techniques. Current research, often conducted in policy institutions, aims at further extending NOEM-DSGE models to provide a tool for policy analysis.

We follow this approach and build a medium-scale NOEM-DSGE model for Luxembourg, named LSM (Luxembourg Structural Model). LSM aims at assessing the effects of policy reforms such as greater product and labour market competition (as advocated, e.g., by the OECD (2010) and the IMF (2009)). We pay particular attention to modelling the real side of the economy, combining some original theoretical features with modelling choices aimed at capturing specific characteristics of the Luxembourg economy. In particular, we adopt an overlapping generations approach for households, and combine it with Heijdra and Ligthart (2007) style investment decisions and a right-to-manage specification of a segmented labour market, with both resident and non-resident workers.

The equilibrium conditions resulting from the optimization problems at the cohort and firm level are aggregated analytically. The resulting model is calibrated to match specific features of the Luxembourg economy and solved using a nonlinear local solution method.

There already exist three macroeconometric models for Luxembourg: the STATEC model Modux (Adam (2004, 2007)), the model of the Banque Central du Luxembourg (Guarda (2005)), and the STATEC multi-sector model LuxMod (STATEC (2006)), each developed for specific purposes but none belonging to the NOEM-DSGE class. This is the distinctive feature of our model, LSM, as will clearly emerge from its description in the following sections. With respect to the Modux and BcL models, LSM is substantially more theory-based, but less detailed in terms of the dynamics. Hence, it is more suitable than these models for policy simulations, but perhaps less adapted to short and medium-term forecasting. With respect to LuxMod, the underlying economic theory is also more developed and coherent, but there is no sectoral disaggregation. Hence, LSM should be more appropriate than any of the existing models to evaluate the aggregate effects of changes in economic policy.

To conclude, while some features of LSM are tailored to the specificities of the Luxembourg economy, its overall structure could be easily adapted to assess economic policy in other small open economies.

The paper is structured as follows. In Section 2 we describe the different sectors of LSM. In Section 3 we briefly discuss the equilibrium conditions, with full details provided in Appendix A. In Section 4 we discuss the calibration of LSM, with full details in Appendix B. In Section 5 we use LSM to analyze the effects of increasing competition in the Luxembourg product and labour markets. Finally, in Section 6 we summarize the main results and propose directions for further development.

2 The structure of LSM

In the specification of LSM we follow the Bank of England model BEQM (Harrison et al. (2005)), the Bank of Belgium model NONAME (Jeanfils and Burggraeve (2005)), and the Bank of Finland model AINO (Kilponen and Ripatti (2006)). However, we also introduce a set of technical refinements, mostly needed to tackle the additional complications introduced by the OLG structure when deriving the aggregation equations in closed form, to introduce sufficient flexibility in the dynamics of the model, and to model the specificities of the Luxembourg economy. In the following subsections we will describe in detail the behavior of the different types of agents in LSM, namely: Households, Government, Firms and Unions.

2.1 Households

We provide a detailed description of the household problem at the cohort level in the first subsection. In the second subsection we focus on aggregation. In the third subsection we consider investment and capital accumulation. In the final subsection we discuss the determination of the net foreign asset position.

2.1.1 The consumer's problem at the cohort level

Following the discrete time version of Blanchard (1985), in period t, the representative consumer of generation z maximizes her expected lifetime utility:

$$\mathrm{u}_{z,t} = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u\left(x_{z,s}\right) \right] = \sum_{s=t}^{\infty} \left(\varphi\beta\right)^{s-t} u\left(x_{z,s}\right),$$

where $\varphi \in (0, 1)$ represents the constant *survival rate*, i.e. the share of individuals that survive in each period, β the subjective discount factor, $x_{z,t} \equiv \{c_{z,t}, d_{z,t}\}$ with c_t denoting non-durable consumption (from now on, consumption *tout court*) and d_t the end-of-period desired stock of durable consumption goods (from now on, durables).

The utility function, $u(x_{z,t})$, is of the constant relative risk aversion (CRRA) type, with CES preferences over consumption and durables:

$$u(x_{z,t}) \equiv \frac{\left\{\left[\phi c_{z,t}^{\upsilon} + (1-\phi) d_{z,t}^{\upsilon}\right]^{\frac{1}{\upsilon}}\right\}^{1-\sigma} - 1}{1-\sigma}. \tag{1}$$

In (1), ϕ is related to the expenditure shares of consumption and durables. If we define by σ^c the (constant) intertemporal elasticity of substitution and by σ^m the elasticity of substitution between consumption and durables, then:

$$\sigma = \frac{1}{\sigma^c}, \quad \upsilon = \frac{\sigma^m - 1}{\sigma^m}.$$

The period-by-period budget constraint for the representative agent in generation z can be written as

$$a_{z,t} = \frac{R_t}{\varphi} a_{z,t-1} + \omega_t - (1+\tau_C) p_t \left[c_{z,t} + \varkappa_t^d \left(d_{z,t} - \frac{1-\delta_D}{\varphi} d_{z,t-1}\right)\right], \tag{2}$$

where

$$R_t \equiv 1 + (1-\tau_K) i_t.$$

The variables are defined as follows: a_t is the end-of-period asset stock, R_t is gross rate of return common across assets, τ_K is the tax rate on financial asset returns, i_t the *exogenous* (small open economy assumption) gross-of-tax interest rate, ω_t is current non-financial income, p_t is the price of the final good, τ_C is the tax rate on consumption, δ^D is the depreciation rate of durables, and $\varkappa_t^d$ is an exogenous shock to the relative price for durables. Note that we are assuming that the final consumption good can be transformed into durables at a rate $\varkappa_t^d$. Furthermore, note that $a_{t,t-1} = 0$, for $t \geq z$, meaning that new generations have no endowments.

Following Schmitt-Grohe and Uribe (2004), we assume the existence of a debt-elastic interest-rate premium, i.e. an interest rate that is increasing in the country's net foreign debt:

$$i_t = \bar{\imath} + \xi_i \left[\exp\left(\bar{f} - \frac{F_t}{GDP_t}\right) - 1\right] + \varepsilon_{it},$$

where F_t represents the country's net foreign asset position, $\bar{\imath}$ the constant and exogenous long-run interest rate if the country runs its steady-state net foreign asset position $(\bar{f})$, and ε_{it} an interest-rate shock.

Current non-financial income is defined as

$$\omega_t \equiv (1-\tau_L)\left[w_{1,t} n_{1,t} + \bar{w}_{1,t}(1-n_{1,t})\right] + (1-\tau_K)\pi_t + tr_t, \tag{3}$$

where $n_{1,t}$ is the employment rate of resident workers (at the individual level, the unemployment rate $1 - n_{1,t}$ can be interpreted as the probability of being unemployed), $w_{1,t}$ their wage rate, τ_L the tax rate on labour related income, $\bar{w}_{1,t}$ the unemployment benefits for resident former workers (to be defined more precisely below), π_t the exogenous, individual share of total firm profits, and tr_t the net government transfer. Note that the expression for labour income reflects the assumption of perfect unemployment insurance, and distinguishes two types of labour, resident and non-resident.

In each period the consumer can use available resources (current income, assets and durables), or borrow in the financial market to finance consumption or to increase her asset stock (which includes claims on the physical capital stock).[1] The intertemporal budget constraint is the following:

$$(1+\tau_C)\sum_{s=t}^{\infty}\mathrm{R}_{t,s}p_s\left[c_{z,s}+\varkappa_t^d\left(d_{z,s}-\frac{1-\delta_D}{\varphi}d_{z,s-1}\right)\right]=\frac{R_t}{\varphi}a_{z,t-1}+\sum_{s=t}^{\infty}\mathrm{R}_{t,s}\omega_s, \quad (4)$$

where $\mathrm{R}_{t,t}\equiv 1$ and, for $s\geq t+1$, $\mathrm{R}_{t,s}\equiv\prod_{j=t+1}^{s}\frac{\varphi}{R_j}$.

As usual, the representative consumer maximizes intertemporal utility subject to the budget constraint, taking the sequence of prices as given. The resulting first-order conditions can be combined to yield the two Euler equations:

$$u_c\left(x_{z,t+1}\right)\beta R_{t+1}\frac{p_t}{p_{t+1}} = u_c\left(x_{z,t}\right), \quad (5)$$

$$u_d\left(x_{z,t}\right)+\beta\left(1-\delta_D\right)\varkappa_{t+1}^d u_c\left(x_{z,t+1}\right) = \varkappa_t^d u_c\left(x_{z,t}\right), \quad (6)$$

where:

$$u_c\left(x_{z,t}\right)=\left[\phi c_{z,t}^{\upsilon}+(1-\phi)\,d_{z,t}^{\upsilon}\right]^{\frac{1-\upsilon-\sigma}{\upsilon}}\phi c_{z,t}^{\upsilon-1}. \quad (7)$$

Combining (5)-(6) and (7), we can express the optimal level of durables in terms of optimal consumption as:

$$d_{z,t}=\xi_t c_{z,t}, \quad (8)$$

where:[2]

$$\xi_t\equiv\left\{\frac{\phi}{1-\phi}\left[\varkappa_t^d-\frac{\varkappa_{t+1}^d\left(1-\delta_D\right)}{R_{t+1}\frac{p_t}{p_{t+1}}}\right]\right\}^{\frac{1}{\upsilon-1}}.$$

Therefore, according to (8), the desired stock of durables increases when their user cost decreases, when ϕ decreases (the "consumption share" in the utility function), and when

[1] Notice that, even if the life expectancy of the consumer decreases exponentially, she could still live for an infinite number of periods. Therefore, it is important to impose as an additional constraint the no-Ponzi game condition (NPG): $\lim_{T\to\infty}\prod_{s=0}^{T}\varphi\frac{a_{z,t+s}}{R_{t+s}}=0$, which prevents overborrowing. This constraint simply ensures that the market will never allow an individual to finance consumption indefinitely via new debt: sooner or later, financial liabilities of any kind have to be honored.

[2] The expression $\varkappa_t^d-\frac{\varkappa_{t+1}^d(1-\delta_D)}{R_{t+1}\frac{p_t}{p_{t+1}}}$ can be considered the user cost of durables, while $\frac{1}{\upsilon-1}=-\sigma^m$.

the elasticity of substitution between consumption and durables decreases.

For optimal consumption, from (5) we obtain:

$$c_{z,t+1} = \mathcal{E}_{t+1} c_{z,t}, \tag{9}$$

where:

$$\mathcal{E}_{t+1} \equiv \left\{ \left[\frac{\phi + (1-\phi)\,\xi_{t+1}^{\upsilon}}{\phi + (1-\phi)\,\xi_t^{\upsilon}} \right]^{\frac{1-\upsilon-\sigma}{\upsilon}} \beta R_{t+1} \frac{p_t}{p_{t+1}} \right\}^{\frac{1}{\sigma}}.$$

As usual, consumption is postponed when current prices are high relative to future prices and/or interest rates are high. An interesting original element is that the intertemporal path of consumption also depends on the user cost of durables trough the ξ terms.

After some manipulations, equations (8) and (9) imply that

$$\sum_{s=t}^{\infty} \mathrm{R}_{t,s} \left(1+\tau_C\right) p_s \left[c_{z,s} + \varkappa_s^d \left(d_{z,s} - \frac{1-\delta_D}{\varphi} d_{z,s-1} \right) \right] = \zeta_t c_{z,t},$$

where:

$$\zeta_t \equiv \sum_{j=0}^{\infty} \mathcal{Z}_{t+j} \varphi^j \prod_{s=1}^{j} \frac{\mathcal{E}_{t+s}}{R_{t+s}},$$

and:

$$\mathcal{Z}_t \equiv \left(1+\tau_C\right) p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1-\delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right].$$

Note that $\mathcal{Z}_t c_{z,t}$ represents the total value of current consumption *and* net investment in durables for generation z in period t, as the demand for durables is related to the demand for consumption goods via (8). The term $\zeta_t c_{z,t}$, instead, represents the total *discounted* flow of future consumption levels and net investment in durables. Note also that ζ_t can be defined recursively as:

$$\zeta_t = \mathcal{Z}_t + \mathcal{E}_{t+1} \frac{\varphi}{R_{t+1}} \zeta_{t+1},$$

Multiplying both sides by $c_{z,t}$, we can easily provide a simple interpretation:

$$\zeta_t c_{z,t} = \mathcal{Z}_t c_{z,t} + \frac{\varphi}{R_{t+1}} \zeta_{t+1} \underbrace{(\mathcal{E}_{t+1} c_{z,t})}_{c_{z,t+1}}.$$

The discounted flow of future "consumption" $\zeta_t c_{z,t}$ (i.e. consumption plus net investment in durables) equals the current value of "consumption," $\mathcal{Z}_t c_{z,t}$, plus the discounted value of the one-period-ahead flow, $\zeta_{t+1} c_{z,t+1}$.

Using the intertemporal budget constraint in (4), we can therefore write optimal current consumption as:

$$c_{z,t} = \zeta_t^{-1} \left(\frac{R_t}{\varphi} a_{z,t-1} + m_t \right), \tag{10}$$

where:

$$m_t \equiv \sum_{s=t}^{\infty} \mathrm{R}_{t,s}\omega_s, \tag{11}$$

represents human wealth.

Notice that both ζ_t in (10) and ξ_t in (8) are independent of z, which simplifies aggregation.

Finally, changing the arguments in the utility function does not usually change the structure of the optimal solution for consumption, as long as the expression for ζ_t^{-1} is properly modified. For example, Harrison et al. (2005) include external habit formation in the model, while Jeanfils and Burggraeve (2005) exclude durables to make utility dependent on consumption only. Similarly, adding other assets to the model, such as money or foreign bonds, only changes the budget constraint and the expression for wealth.

2.1.2 Aggregation

Let us assume that the size of each new-born generation is z_t, where $z_t = \eta^t z_{-\infty}$ and $z_{-\infty}$ is normalized to one. Then, Z_t the total population at any date t, is equal to:

$$Z_t = \eta^t \sum_{j=0}^{\infty} \left(\frac{\varphi}{\eta}\right)^j = \frac{z_t}{1-\frac{\varphi}{\eta}},$$

and $Z_{t+1} = \eta Z_t$.

The expressions for the aggregate variables can be obtained by linear aggregation of those at the cohort level. Let us start with aggregate assets. We have

$$A_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} a_{z_{t-j},t}. \tag{12}$$

Aggregating the budget constraint in (2) over cohorts, we obtain an equation describing the evolution of aggregate assets:

$$A_t = R_t A_{t-1} + \mathrm{W}_t - \mathcal{Z}_t C_t, \tag{13}$$

where

$$\mathrm{W}_t \equiv \omega_t Z_t,$$

since ω_t is not cohort-dependent, and $\mathcal{Z}_t C_t$ represents the total aggregate value of current consumption and net investment in durables. Equation (13) can be considered as the budget constraint at the aggregate level.

Next, let us consider aggregate net human wealth, where cohort-level human wealth,

m_t, is defined in equation (11). We have:

$$M_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} m_t = m_t Z_t. \tag{14}$$

The evolution of aggregate net human wealth is given by:

$$M_{t+1} = \frac{\eta}{\varphi} R_{t+1} \left(M_t - \mathrm{W}_t \right). \tag{15}$$

For aggregate consumption, aggregating equation (10) over cohorts yields:

$$C_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} c_{z_{t-j},t} = \zeta_t^{-1} \left[R_t A_{t-1} + M_t \right], \tag{16}$$

where aggregate assets, A_t, are defined in (12) and aggregate human wealth, M_t, in (14). The evolution of aggregate consumption is governed by the aggregate Euler equation

$$C_{t+1} = \eta \mathcal{E}_{t+1} \left(C_t - \frac{\eta - \varphi}{\eta} \frac{A_t}{\zeta_t - \mathcal{Z}_t} \right). \tag{17}$$

For aggregate durables we have

$$D_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} d_{z_{t-j},t} = \xi_t C_t, \tag{18}$$

and the dynamics of D_t can be determined from that of C_t.

Finally, aggregate financial wealth can be decomposed into government bonds, foreign bonds, and claims to physical capital. Hence,

$$A_t = B_t + F_t + V_t, \tag{19}$$

where B_t represents the value of the end-of-period stock of government bonds, F_t the value of the end-of-period stock of foreign assets, and V_t the value of the end-of-period stock of claims to physical capital, all measured in consumption units. By assuming assets to be perfect substitutes in the household's portfolio, they earn the same (exogenous) real rate of return in equilibrium. We will now analyze in detail the different types of assets.

2.1.3 Physical capital accumulation

Following Heijdra and Ligthart (2007), we assume that households as a whole, which can be considered as a representative "investment firm," are in charge of investment. More specifically, investment is determined by maximizing the cash flow from investing in physical capital, conditional on the law of motion of physical capital.[3]

The cash flow from investing in physical capital is given by:

$$\sum_{s=t}^{\infty} \tilde{\mathrm{R}}_{t,s} \left\{ \left[(1-\tau_K) \frac{r_s}{p_s} + \tau_K \delta_K \right] K_{s-1} - I_s \right\}, \tag{20}$$

where $\tilde{\mathrm{R}}_{t,s} \equiv \prod_{j=t+1}^{s} \left[R_j \left(p_{j-1}/p_j \right) \right]^{-1}$ is the aggregate discount factor,[4] r_t is the rental rate on capital, and I_t denotes investment. Note that the investment firm can deduct all depreciation from its taxable income. Physical capital evolves according to:

$$K_t = (1-\delta_K) K_{t-1} + \Xi\left(\frac{I_t}{K_{t-1}} \right) K_{t-1}, \tag{21}$$

where δ_K is the depreciation rate of capital and the term $\Xi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$ denotes the adjustment costs. Following Jermann (1998), we assume

$$\Xi\left(\frac{I_t}{K_{t-1}} \right) = \frac{\Xi_1}{\varsigma} \left(\frac{I_t}{K_{t-1}} \right)^{\varsigma} + \Xi_2.$$

The two parameters Ξ_1 and Ξ_2 are designed to make the adjustment cost vanish in the steady state.

The investment firm maximizes the cash flow subject to the accumulation equation for physical capital; the first order conditions are:

$$\nu_t = \Xi'\left(\frac{I_t}{K_{t-1}} \right)^{-1}, \tag{22}$$

$$R_{t+1} = \frac{(1-\tau_K) r_{t+1} + p_{t+1}\left(\tau_K \delta_K - \frac{I_{t+1}}{K_t} \right) + p_{t+1}\nu_{t+1}\left[1 - \delta_k + \Xi\left(\frac{I_{t+1}}{K_t} \right) \right]}{p_t \nu_t}. \tag{23}$$

Equation (23) corresponds to the standard no-arbitrage condition, where the last term on the right-hand side represents the future marginal contribution of capital to lower installation costs. In other words, the future net-of-tax gross return on claims to physical capital has to be equal to the future return of holding a unit of capital for one period (i.e. the future rental rate plus the future shadow price corrected for depreciation plus the future decrease in installation costs) divided by the current shadow price of the same unit of capital. Thus ν_t corresponds to the well-known Tobin q.

[3] Introducing adjustment costs to physical capital accumulation in our Blanchard-Yaari framework the investment decisions taken at the individual level are highly non linear and cannot be easily aggregated. We borrow the elegant solution in Heijdra and Ligthart (2007).

[4] We can show that $\tilde{\mathrm{R}}_{t,s} \equiv \prod_{j=1}^{s} \left[R_{t+j} \frac{p_{t+j-1}}{p_{t+j}} \right]^{-1} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{p_t}{p_{t+s}}$, where λ_t is the aggregate shadow value of firms' profits in the household budget constraint.

It can be easily shown that:

$$\nu_t K_t = \frac{\left[(1-\tau_K)\frac{r_{t+1}}{p_{t+1}} + \tau_K \delta_K\right] K_t - I_{t+1} + \nu_{t+1} K_{t+1}}{R_{t+1}\frac{p_t}{p_{t+1}}}. \tag{24}$$

Hence, iterating on the previous expression and imposing the TVC yields:

$$\nu_t K_t = \sum_{s=t+1}^{\infty} \tilde{\mathrm{R}}_{t,s} \left\{ \left[(1-\tau_K) r_s + \tau_K \delta_K p_s\right] K_{s-1} - p_s I_s \right\}. \tag{25}$$

The right-hand side in (25) represents the discounted flow of future cash flows in real terms, i.e. the stock market value of claims to physical capital. This implies that:

$$V_t = p_t \nu_t K_t. \tag{26}$$

2.1.4 Net foreign asset position

Combining (19), (13), (26), (24), and (32), we get the following law of motion for net foreign assets:

$$F_t = R_t F_{t-1} + \mathrm{W}_t + \left[(1-\tau_K) r_t + \tau_K \delta_K p_t\right] K_{t-1} - \mathcal{Z}_t C_t - p_t I_t - (G_t - T_t).$$

2.2 Firms and Unions

In LSM firms produce intermediate and final goods.[5] We assume that there is a single representative firm producing the final good Y under perfect competition. This firm combines $\mathcal{N}$ intermediate goods using a CES production function, possibly with increasing returns in the variety of intermediate inputs.

Local firms in the intermediate goods sector produce N varieties of differentiated goods, operating under monopolistic competition. A share Θ of these N locally produced varieties cannot be traded (exported). The remaining $(1-\Theta)$ can be exported.[6]

Furthermore, $(1-\Theta^*)\,\mathrm{N}^*$ other varieties can be imported from abroad, where N^* indicates the total number of foreign produced varieties, and Θ^* the share of them that can be imported in Luxembourg. Hence, the total number of varieties of differentiated intermediate goods in Luxembourg is given by $\mathcal{N} = \mathrm{N} + (1-\Theta^*)\,\mathrm{N}^*$.

Each firm in the local intermediate sector adopts a nested CES production function with capital and two different types of labour as inputs. The different types of labour are introduced to capture the segmented labour market in Luxembourg, and represent resident

[5] The split between final and intermediate goods is common in the literature. See for instance the seminal works by Christiano et al. (2005) and Smets and Wouters (2003).

[6] Also the split between tradables and non-tradables is relatively common in the literature: Justiniano and Preston (2004) discuss the issue in some detail.

and non-resident workers. The firm chooses the optimal demand of capital and each type of labour by maximizing profits subject to the production function constraint, taking wages and the cost of capital as given. The cost of capital is determined endogenously in order to match demand and supply of capital. For the sake of exposition, we present all the derivations for a generic production function, and then specialize the results to the nested CES case in Appendix A, which requires a more cumbersome notation.

Wages are determined by the interaction between the intermediate sector firms and the unions, which represent the workers (the so-called "right to manage" model). In particular, we assume that there is a union for each type of worker, and that bargaining with the firm takes place in a Nash setting. We assume that there is a separate union for each firm, but this is not a restrictive hypothesis since in symmetric equilibrium firms will make the same choices in terms of demand for labour and capital.[7]

Technically, the interaction between the production and labour markets is represented as a game in two stages, where wage bargaining takes place in the first stage and production in the second. As in Lockwood (1990), the second stage is solved first, and its solution is used in the first stage. Therefore, after discussing the final good sector, we will first describe the problem of intermediate sector firms (second stage), and then the firm-union bargain (first stage). We will deal, in turn, with producers of non-tradable goods, tradable goods, and importers of foreign intermediate goods.

2.2.1 Final good sector

The cost function for the final good producing firm is:[8]

$$\begin{aligned} \mathcal{C}_F\left(\{p_j\}, Y\right) &\equiv \min_{\{y_j\}} \sum_{j=1}^{\mathcal{N}} p_j y_j, \\ &\quad \text{s.t. } \mathcal{N}^{\rho-\mu}\left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}}\right)^{\mu} \geq Y. \end{aligned} \tag{27}$$

where y_j is the amount of the j^{th} intermediate good used for production of the final good Y, $j = 1, ..., \mathcal{N}$; $\mu > 1$ is indirectly related to the elasticity of substitution between goods and directly related to the mark-up in the intermediate goods sector; and $\rho \geq 1$ is a parameter that captures increasing returns to variety; see Kim (2004) for details. Cost minimization leads to the usual conditional demand for intermediate good j:

$$y_j = \left(\frac{p_j}{p}\right)^{\frac{\mu}{1-\mu}} Y \mathcal{N}^{\frac{\rho-\mu}{\mu-1}}.$$

[7] We assume that given the current wage the supply of non-resident workers adjusts to meet demand. This is of course a limitation of the model, but helps to simplify the framework.

[8] From now on, we drop the time index for the sake of notational simplicity.

2.2.2 Intermediate goods sector - Non-tradable goods: $j \in [1, \Theta\mathrm{N}]$

Second stage: profit maximization The problem of a generic firm in the intermediate goods sector producing a non-tradable good can be formulated as

$$\begin{aligned} \max_{\left\{h_{zj}^{NT}, k_j^{NT}\right\}} \pi_j^{NT} &\equiv p_j^{NT}\left(y_j^{NT}\right) y_j^{NT} - r k_j^{NT} + \\ & - \left(1+\tilde{\tau}_L\right) \sum_{z=1}^{2} w_{zj}^{NT} h_{zj}^{NT} - \psi_j, \end{aligned}$$

where $p(y_j^{NT})$ indicates the price of the j^{th} non-tradable intermediate good; h_{zj}^{NT}, $z = 1, 2$, the amount of the two types of labour (resident and non-resident) and k_j^{NT} the amount of capital services; ψ_j is a fixed financial cost to enter the market (the fixed cost generates economies of scale and therefore justifies monopolistic competition; see Kim, 2004); and $\tilde{\tau}_L$ represents taxes on labour paid by firms; labour income taxes paid by workers will be taken into account later. In addition:

$$\begin{aligned} p_j^{NT}\left(y_j^{NT}\right) &= \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{y_j^{NT}}{Y}\right)^{\frac{1-\mu}{\mu}} p, \\ y_j^{NT} &= f\left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}\right), \end{aligned}$$

where the specific functional form for the production function will be discussed later.

The first order conditions are:

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT}\right) \frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} = \left(1+\tilde{\tau}_L\right) w_{zj}^{NT}, \tag{28}$$

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT}\right) \frac{\partial y_j^{NT}}{\partial k_j^{NT}} = r, \tag{29}$$

where $z \in \{1, 2\}$.

Note that, thanks to the Envelope Theorem, (28) implies:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}}\right)^2 \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} + p_j^{NT} \frac{\partial^2 y_j^{NT}}{\left(\partial h_{zj}^{NT}\right)^2} \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \mu \left(1+\tilde{\tau}_L\right).$$

Hence:

$$\frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \frac{1}{w_{zj}^{NT}} \left[(1-\mu) \frac{\left(1+\tilde{\tau}_L\right) w_{zj}^{NT}}{p_j^{NT} y_j^{NT}} + \frac{\partial^2 y_j^{NT}}{\left(\partial h_{zj}^{NT}\right)^2} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}}\right)^{-1}\right]^{-1},$$

since:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} = \frac{1-\mu}{\mu} \frac{p_j^{NT}}{y_j^{NT}}.$$

First stage: firm-union bargaining (Labour market) The loss function of the union representing type z workers in the j^{th} non-tradable sector is

$$\widetilde{V}_{U,zj}^{NT} = (1-\tau_L) \times \left[\frac{w_{zj}^{NT}}{P} h_{zj}^{NT}\left(w_{zj}^{NT}\right) + \frac{w_{zj}^{T}}{P} h_{zj}^{T}\left(w_{zj}^{T}\right) + \frac{\overline{w}_z}{P}\left(M_{zj} - h_{zj}^{NT}\left(w_{zj}^{NT}\right) - h_{zj}^{T}\left(w_{zj}^{T}\right)\right)\right], \quad (30)$$

where $\sum_j M_{1j}$ represents the total working age population of Luxembourg (Z_1), while $\sum_j M_{2j}$ represents total union membership among non resident workers, which is equal to number of employed non-resident workers, and unemployment benefits paid abroad are $\bar{w}_2$. Therefore, the union cares about the total resident population (workers and unemployed) since the resident population coincides with the home labour force, and about the non-resident union members (workers and unemployed), but takes the level of unemployment benefits as given.

Each firm-union pair bargains over type-z wage, maximizing the following Nash objective function, taking the firms' labor demand curve into account:

$$\max_{\left\{w_{zj}^{NT}\right\}} \Omega_{zj}^{NT} \equiv \left(\widetilde{V}_{U,zj}^{NT} - V_{U,zj}^{NT}\right)^{\theta_z} \left[\tilde{\pi}^{NT}\left(w_{zj}^{NT}\right) - \pi^{NT}\right]^{1-\theta_z}, \quad (31)$$

where θ_z is a parameter describing the relative bargaining power of the union for type z workers (constant across sectors); and $V_{U,zj}$ and π represent the outside options if the negotiation fails:

$$\begin{aligned} V_{U,zj}^{NT} &= (1-\tau_L)\frac{\overline{w}_z}{p}\left[M_{zj} - h_{zj}^{T}\left(w_{zj}^{T}\right)\right] + (1-\tau_L)\frac{w_{zj}^{T}}{p} h_{zj}^{T}\left(w_{zj}^{T}\right), \\ \pi^{NT} &= -\left(r k_j^{NT} + \phi_j\right). \end{aligned}$$

Combining (30) and (31), the problem of the union can be rewritten as

$$\max_{\left\{w_{zj}^{NT}\right\}} \Omega^{NT} \equiv \left[(1-\tau_L)\left(\frac{w_{zj}^{NT}}{p} - \frac{\overline{w}_z}{p}\right) h_{zj}^{NT}\right]^{\theta_z} \left[\frac{\tilde{\pi}\left(w_{zj}^{NT}\right)}{p}\right]^{1-\theta_z},$$

where:

$$\tilde{\pi}^{NT}\left(w_{zj}^{NT}\right) = p^{NT}\left[f\left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}\right)\right] f\left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}\right) - (1+\tilde{\tau}_L)\sum_{z=1}^{2} w_{zj}^{NT} h_{zj}^{NT}.$$

For $j = 1, 2$, the first order conditions can be written as:

$$\theta_z \left(1 + \frac{w_{zj}^{NT} - \overline{w}_z}{w_{zj}^{NT}} \epsilon_{zj}^{NT}\right) \frac{\tilde{\pi}_j^{NT}}{h_{zj}^{NT}} = (1 + \tilde{\tau}_L)(1 - \theta_z)\left(w_{zj}^{NT} - \overline{w}_z\right),$$

where:

$$\epsilon_{zj} \equiv \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} \frac{w_{zj}^{NT}}{h_{zj}^{NT}}.$$

Several factors affect real wages in LSM. First, as usual, labour productivity. Second, the characteristics of the labour market, such as union power θ_2 and the replacement ratios $\bar{w}_j/w_j$. Third, the profit rate, since unions extract some of the producer surplus. Fourth, the relative productivity of the two types of labour, the relative size of the labour forces, and the unemployment rates. Finally, the relative productivity with respect to capital and the amount of capital per worker.

2.2.3 Intermediate goods sector - Tradable goods: $j \in [\Theta\mathrm{N}, \mathrm{N}]$

Let us now consider the problem of a generic firm in the intermediate goods sector producing tradable goods, y_j^T, such that $y_j^H = s_j^H y_j^T$ is sold at home and $y_j^F = s_j^F y_j^T$ is exported ($s_j^F = 1 - s_j^H$, and $0 \leq s_j^H \leq 1$), with corresponding prices given by p_j^H and p_j^F. The firm should choose the amount of labour and capital to be used for the production of y_j^T (h_{zj}^T and k_j^T, respectively, $z = 1, 2$), and the share of y_j^T sold at home, s_j^H, to optimize the following problem:

$$\max_{\left\{h_{zj}^T, k_j^T, s_j^H\right\}} \pi_j^T \equiv p_j^T\left(y_j^T\right) y_j^T - r k_j^T - (1 + \tilde{\tau}_L) \sum_{z=1}^{2} w_{zj}^T h_{zj}^T - \psi_j,$$

where:

$$\begin{aligned}
p_j^T &= s_j^H p_j^H + s_j^F p_j^F, \\
s_j^F &= 1 - s_j^H, \\
y_j^T &= f\left(k_j^T, h_{1j}^T, h_{2j}^T\right), \\
p_j^H &= \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^H y_j^T}{Y}\right)^{\frac{1-\mu}{\mu}} p, \\
p_j^F &= \left(1 - t^F\right)\left(\mathcal{N}^*\right)^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^F y_j^T}{Y^*}\right)^{\frac{1-\mu}{\mu}} p^*.
\end{aligned}$$

Note that Y^* and p^* represent foreign output and the foreign aggregate price. Furthermore, note that the elasticity of substitution between intermediate goods is the same at home and abroad, i.e. $\mu^* = \mu$: this assumption is maintained for notational simplicity, but the model can be easily generalized.[9] As in the non-tradable sector, ψ_j is a fixed financial cost to enter the market that generates economies of scale and therefore provides a basis for monopolistic competition; see Kim, 2004.

Since the technical aspects of the problem of the firms and of the unions in the tradable sector are similar to those analyzed in detail above for the non-tradable sector, we do not present these derivations (but they are available upon request).

2.2.4 Intermediate goods sector - Imported goods

The importing firms buy goods abroad at the price p_M^* and resell them in the internal market at the price p_j^M. Their problem is

$$\max_{\{y_j^M\}} \pi_j^M \equiv \left[p_j^M\left(y_j^M\right) - \left(1+t^M\right)p_M^*\right]y_j^M - \psi_j,$$

where:

$$p_j^M = \mathcal{N}^{\frac{\rho-\mu}{\mu}}\left(\frac{y_j^M}{Y}\right)^{\frac{1-\mu}{\mu}}p.$$

The first order condition is given by

$$p_j^M = \mu\left(1+t^M\right)p_M^*,$$

and the resulting profits are

$$\pi_j^M \equiv (\mu-1)\left(1+t^M\right)p_M^* y_j^M - \psi_j.$$

2.3 Government

The Government budget constraint is:

$$B_t = R_t B_{t-1} + G_t - T_t, \tag{32}$$

where G and T indicate, respectively, total expenses and revenues, while B is government debt.

The Government collects revenues from taxes on the returns on financial assets, on profits, and on labour income (H_1 and H_2 are, respectively, resident and non-resident workers, whose wages are w_1 and w_2, unemployment benefits are $\overline{w}$; workers pay taxes

[9] The distinction between local and foreign elasticities is important to study shocks to local markups that do not transmit to markups in foreign markets. In this case, we obviously use the generalized version of the model.

at the rate τ_L and firms pay social contributions at the rate $\tilde{\tau}_L$). Furthermore, the government collects taxes on consumption and on imports. Therefore, total revenues in period t amount to:

$$\begin{aligned} T_t &= \tau_K \left[i_t F_{t-1} + \left(r_t - \delta_K p_t \right) K_{t-1} + \Pi_t \right] + \\ &+ \left(\tau_L + \tilde{\tau}_L \right) \left(w_{1,t} H_{1,t} + w_{2,t} H_{2,t} \right) + \tau_L \bar{w}_{1,t} \left(1 - H_{1,t} \right) + \\ &+ \tau_C p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] C_t + \\ &+ t_M \left(1 - \Theta^* \right) \mathrm{N}^* p_M^* y^M . \end{aligned}$$

where t_M, Θ^*, N^*, p_M^* , and y^M represent respectively the import tariff, the share of foreign varieties that can be traded, the total number of foreign varieties, the price of these foreign varieties, and the quantity imported (this is discussed in more detail in the following sections).

Government expenditure is composed of unemployment benefits for residents ($SUBS$), transfers to non-resident workers (TRF), and core expenditure ($\bar{G}$), where the latter can be further split into other transfers to resident households (TR), public investment in infrastructures ($INFR\_INV$), and general government consumption ($GCON$). Overall, we have:

$$\begin{aligned} G_t &= SUBS_t + TRF_t + \bar{G}_t, \\ SUBS_t &= \bar{w}_{1,t} \left(1 - H_{1,t} \right), \\ TRF_t &= TR_t^F \left(\tau_L + \tilde{\tau}_L \right) w_{2,t} H_{2,t}, \\ TR_t &= \varrho_1 \bar{G}_t, \\ GCON_t &= \varrho_2 \bar{G}_t, \\ INFR\_INV_t &= \left(1 - \varrho_1 - \varrho_2 \right) \bar{G}_t. \end{aligned}$$

where $\varrho \in (0,1)$ represents the share of transfers to resident households from core government expenditure. Note that TRF is modelled as a percentage (TR_t^F) of total labour taxes on non-resident workers. Unemployment benefits for type-j workers are equal to a replacement rate rep_j times the net factor income of the resident workers: $\bar{w}_{j,t} = rep_j \cdot NETINC_t$, where:[10]

$$NETINC_t = \left(1 - \tau_L \right) \left[w_{1,t} H_{1,t} + \bar{w}_{1,t} (1 - H_{1,t}) \right] + \\ \left[(1 - \tau_K) r_t + \tau_K \delta_K p_t \right] K_{t-1} + (1 - \tau_K) \Pi_t .$$

[10] Alternatively, unemployment benefits can be defined in terms of a share of gross wages. This unfortunately leads to indeterminacy in our model. Our formulation guarantees determinacy.

The stock of public infrastructures evolves according to the following accumulation equation:

$$INFR_t = (1 - \delta_{INFR})\, INFR_{t-1} + INFR\_INV_t, \tag{33}$$

and affects Total Factor Productivity via a purely external effect (see Section 4.1 for further details). Note that δ_{INFR} represents the depreciation rate for public infrastructures.

We further assume that core government expenditure is persistent and depends on the part of the (primary) deficit which excludes core government expenditure, $T_t - (G_t - \bar{G}_t)$:

$$\bar{G}_t = \vartheta \bar{G}_{t-1} + (1 - \vartheta)\, d^{LR} \left[T_t - \bar{w}_{1,t} (1 - H_{1,t}) - TR_t^F (\tau_L + \tilde{\tau}_L)\, w_{2,t} H_{2,t} \right]. \tag{34}$$

This specification of the Government sector implies a zero public debt and deficit in steady state when $d^{LR} = 1$. Otherwise, a value of $d^{LR} > 1$, combined with that of the other variables and parameters in (34), determines the equilibrium level of debt and deficit. Note that the parameter ϑ measures the persistence of core government expenditure.

2.4 Other variables of interest

Finally, we report the equations for GDP, GNP, net trade, terms of trade, imports, and exports (of intermediate goods):

$$\begin{aligned} GDP_t &= (1 + \tilde{\tau}_L)\, w_{1t} H_{1t} + (1 + \tilde{\tau}_L)\, w_{2t} H_{2t} + \\ &\quad r_t K_t + \left[\Pi_t + (1 - \Theta^*)\, \mathrm{N}^* t_M p_{t,M}^* y_t^M \right], \\ GNP_t &= GDP_t + i_t F_{t-1} - \left[TR_t^F (\tau_L + \tilde{\tau}_L) + 1 - \tau_L \right] w_{2,t} H_{2,t}. \end{aligned}$$

We can easily recover the national accounting identity:

$$\begin{aligned} GDP_t &= p_t C_t + p_t C_t \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) + \\ &\quad p_t I_t + GCON_t + INFR\_INV_t + NX_t. \end{aligned}$$

where net trade, NX_t, equals the change in the country's net foreign position plus the wages of non-resident workers, as implied by the definition of the balance of payments:

$$NX_t = F_t - (1 + i_t)\, F_{t-1} + \left[TR_t^F (\tau_L + \tilde{\tau}_L) + 1 - \tau_L \right] w_{2,t} H_{2,t}.$$

Focusing on intratemporal trade in goods (produced in the intermediate-good sector, but considered final because either exported or imported):

$$\begin{aligned}IMP_t^{IG} &= (1-\Theta^*)\,\mathrm{N}^* p_{t,M}^* y_t^M,\\ EXP_t^{IG} &= (1-\Theta)\,\mathrm{N} p_t^F y_t^F,\\ ToT_t &= \frac{p_t^F}{p_t^M}.\end{aligned}$$

3 Symmetric equilibrium

In a symmetric equilibrium, for all firms in a given sector the prices charged for the differentiated goods and the quantities produced are the same, i.e., $p_j^i = p^i$ and $y_j^i = y^i$, where $i = NT, H, F, M$. Furthermore, the equilibrium is characterized by the optimality conditions for households and government.[11] In Appendix A we provide a detailed derivation of the symmetric equilibrium of LSM. In particular, we specialize the analysis of the production sector and labour market to the case of a CES production function. For the sake of clarity, we do not distinguish between tradable and non-tradable goods, but the same production function is assumed in both sectors:

$$\begin{aligned}y &= A\left[\alpha k^{\lambda} + (1-\alpha)(\Lambda h)^{\lambda}\right]^{\frac{1}{\lambda}},\\ h &= \left[\varkappa_1 (a_1 h_1)^{\kappa} + \varkappa_2 (a_2 h_2)^{\kappa}\right]^{\frac{1}{\kappa}},\end{aligned}$$

with $\varkappa_2 = 1 - \varkappa_1$. Note that Λ represents a labour-augmenting productivity parameter.[12] We allow for a (purely external) effect of the stock of public infrastructure ($INFR_t$) on Total Factor Productivity A. In particular, we model A as:

$$A = (INFR_t)^{\varpi} \cdot \gamma \cdot t,$$

where $0 < \varpi < 1$, and $\gamma \cdot t$ represents exogenous technical progress growing at a constant rate γ.

The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but variables are measured in efficiency units.

[11] We set the numeraire as the price of the non-traded goods: $p^{NT} = 1$.

[12] We use this nested CES specification since it clearly distinguishes the elasticity of substitution between aggregate labour and capital, and that between the two types of labours.

4 Calibration

Due to the complexity of LSM and the availability of only 15 years of quarterly observations for Luxembourg, the model cannot be estimated and we have to fully calibrate it. In this section we summarize the calibration procedure for the model parameters. Appendix B lists all the parameters of LSM, summarizes their meaning, and discusses their calibration in more detail.

We can divide the model parameters into three groups according to how we set their values. The parameters in the first group are set directly to standard values in the DSGE literature. In particular, we fix the subjective discount rate (β) to 0.995, the elasticity of intertemporal substitution to unity (i.e. $\sigma = 1$ which implies that preferences are logarithmic), the weight of capital in the production function (α) to 0.36 (the implied capital share in production is 25%), the persistence of core government expenditure (ϑ) to 0.9, the returns to variety to zero (which implies that $\rho = 1$), the elasticity of substitution among intermediate goods to 6 (so that $\mu = 1.2$), the relative bargaining power of the unions (θ_z) to 0.5 and the elasticity of substitution between the two labour types in the CES labour aggregator to 1.5 (so that $\kappa = 1/3$).[13]

We follow Backus, Henriksen, and Storesletten (2008) in setting the depreciation rate on physical capital (δ_K) to 8.5%, on durables (δ_D) to 1.5%, and on the stock of public infrastructure (δ_{INFR}) to 4.15%. We set the elasticity of the international interest rate with respect to the national debt/GDP ratio (ξ_i) to 0.000742 based on Schmitt-Grohe and Uribe (2004). Following Boldrin, Christiano, and Fisher (2001) we assume that the elasticity of the adjustment cost with respect to the investment-capital ratio is 0.23 (so that $\varsigma = -3.348$).

We set the parameter related to the elasticity of substitution between durables and non-durables in the utility function (υ) in order to reproduce an elasticity of substitution equal to 1.5. The percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers (TR_t^F) is chosen to be 0.6: this allows the model to closely reproduce the observed ratio of social security transferred abroad over wages paid to non-resident workers in 2008, equal to 0.18. We choose a small value for the fixed cost to enter the market of intermediate good j (ψ_j) and set it equal to 0.00001. The parameter related to the elasticity of TFP with respect to public infrastructure (ϖ) is chosen to be equal to 0.01.

Next, we normalize the foreign aggregate price level (P^*), the labour-augmenting productivity parameter (Λ) and the parameters augmenting type-1 (a_1) and type-2 (a_2)

[13] Guarda (2000) actually found evidence of complementarity between these labour types in Industry and in Services, but he was using a Translog production function with gross output (instead of value added) and intermediate consumption. In addition, his sample covered 1984-1996 using unpublished national accounts data prior to the introduction of ESA95. We prefer to use values based on published data for the sake of reproducibility.

labour in the labour CES aggregator to unity. We also assume that Luxembourg and the rest of the world are symmetric in terms of the the share of non-traded varieties, both Θ and Θ^* are equal to 0.5. We normalize the number of traded varieties to unity, which implies that we set both N and N^* equal to 2, again for the sake of symmetry.

For the second parameter group, some values are directly observable or can be estimated. Average life expectancy at birth in Luxembourg was 79.18 years in 2008 (CIA factbook) which implies that the individual survival rate in our model (φ) is 0.987. The average value of net foreign position ($\bar{f}$) was 85% of GDP at the end of 2007 an 2008 (according to the BcL bulletin). The population growth rate in Luxembourg is 1.2% (data from CIA factbook, year 2008) which implies that η equals to 1.012.

Guarda (1997) estimates the elasticity of substitution between capital and labour in a CES production function to be 1.012 in the tradables sector in Luxembourg (implies that $\lambda = 0.012$). We set the share of type-1 labour in the labour CES aggregator (χ_1) to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.

We set the tax rates in LSM according to the latest values reported in *Taxation trends in the EU*, European Commission, 2008. In particular, the tax rate on consumption (τ_C) equals to 25.1%. The total average effective tax rate on labour related income is 29.6%, but only 67.9% of this amount is paid by the employee while the remaining part is paid by the employer. Thus, we set the tax rate on labour related income (τ_L) to 20.1% and the social contribution rate on labour related income ($\tilde{\tau}_L$) to 9.5%. Estimates of the tax rate on capital income (τ_K) are not reported in the mentioned source due to data availability problems, so we take the adjusted statutory tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.

The average TFP growth rate (γ) in Luxembourg over 1995-2009, as reported in the Annual Report of the Luxembourg Central Bank (2006, p. 54) was 0.6%.

We use the Overall Trade Restrictiveness Index for the European Union to set the tariffs in the model at 6.6%. The ad-valorem equivalent of all tariff and non-tariff barriers that the European Union imposed against foreign imports was equal to 6.6% in 2006. However, in Luxembourg 94.5% of all imported goods were originated from countries within the EEA in 2007 and no tariffs were applied on them. Thus, the average effective tariff on imported goods was 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares. Similarly, 88.2% of all exported goods from Luxembourg in 2007 were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate of 9%, which is the MA-OTRI in 2006 for the European Union. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.

In the third group there are nine model parameters that we calibrate jointly so that the resulting steady state matches values observed in the data.

The relative weight of durables and non-durables in the utility function (ϕ) is calibrated in order to reproduce the share of durables consumption in household final consumption expenditure (average annual share between 1995-2008) 0.116. The constant and exogenous long-run interest rate equals $\bar{\imath}$ if the country settles down to a net foreign position equal to its steady-state value. We calibrate its value to match the observed net asset foreign position at 85% of GDP in Luxembourg (represented by $\bar{f}$). The implied value is 2.035%. The parameter related to the long-run debt/GDP ratio (d^{LR}) is calibrated in order to reproduce the observed debt/GDP ratio of Luxembourg equal to 0.069.

The share of transfers to resident households (ϱ_1) and the share of public investment in infrastructures (ϱ_2) in core (government) expenditure are calibrated in order to make the model replicates the share of government transfers in total government expenditure (data from OECD annual national accounts, years 2003-2007) and the share of government investment in total government expenditure (data from OECD annual national accounts, years 2003-2007). The replacement ratio of unemployment benefit for domestic workers ($REP1$) and the replacement ratio of unemployment benefit for foreign workers ($REP2$), are both expressed as a share of the total gross income of employed domestic workers. These are calibrated in order to replicate a 5% unemployment rate of type-1 workers and a ratio of type-1 to type-2 workers equal to 1.4238. The calibrated parameter values are reported in Appendix B.

Finally, the foreign real output level (Y^*) and the price of imported goods (p_M^*) are calibrated to match a net exports to GDP ratio equal to 0.35.

5 LSM at work

We now discuss the steady state of the model, which reflects the calibration choices introduced in the previous section. Next, to illustrate the capabilities of LSM, we assess the consequences of an increase in the replacement rate, a measure often proposed to improve the conditions of the unemployed, and of a decrease in the mark-up, associated with liberalization in the product market, another measure often advocated in the policy debate. For each of the mentioned policy measures, we focus on the effects on a set of key variables: changes in wages of resident and non-resident workers, in employment of resident and non-resident workers, in the total wage bill for resident and non-resident workers, in overall firms' profits, in the private demand components (Consumption, Investment, Net exports), in gross domestic product (GDP), in the government deficit, and in total factor productivity (TFP). We focus on the changes in each variable with respect to its starting value, and use +, ++ and +++ to denote an increase in the range of, respectively, 0-0.5%, 0.5-1% or larger than 1%. The symbols -, - -, and - - - have a similar interpretation for negative changes. More detailed results and findings for other variables are available upon request.

5.1 Steady state

The steady state values for the endogenous variables of LSM are determined by the interaction of model specification and parameter calibration.

In terms of final demand, the shares of consumption, investment and public expenditure in GDP are about 37.5%, 30.3% and 32.831%, respectively. This leaves a share for net exports smaller than the actual value for Luxembourg: in order to improve this ratio it would be necessary to introduce re-export services and explicitly model the financial sector, which are left for future research.

GDP can be also decomposed into wages, profits and returns on capital. In this case, the respective shares of GDP are about 44%, 28% and 28%.

In terms of production factors, employment of resident workers is about 95% of the labour force, and about 94% of employment in the tradable sector. Similarly, about 94% of capital is in the tradable sector, and the overall capital to GDP ratio is about 2.9. Employment of the non-resident workers can be interpreted as the percentage of people who would be willing to work in Luxembourg, and the value in this case is about 67%, much smaller than for the resident population but still considerable and in line with the segmented labour market. The wages of the non-resident workers are about 15% lower than those of the resident workers.

Finally, for the public sector, the deficit is very low (due to a comparable level of tax receipts and expenditures) and the public debt is about 7% of GDP, in line with actual values.

5.2 Higher replacement rate

In order to illustrate the mechanisms of LSM, one will firstly consider a change in the replacement rate. Such shock is spreading throughout the whole economy via the adjustment on the labour market, which is specific in LSM. For the sake of illustration, we consider a permanent increase of 1% in the replacement rate and report the results in Table 1. Looking at our simulations, it turns out that, in addition to the expected positive income effect for the unemployed, there is also an unexpected positive effect on the wage of workers that are still employed. Due to the bargaining structure of the labour market, if the outside option for workers improves, their wage also has to increase.

The ultimate impact of such changes is to affect employment. Indeed this partially offsets the positive impact of this policy. Therefore, we have higher wages but employment falls, with the latter effect dominating the former so that the total wage bill actually decreases.

Lower wages for resident workers imply a cut in disposable income, so that consumption also decreases. Lower consumption shrinks firm profits, which in turns reduces investment, which further reduces demand and gross domestic product (GDP). The only positive effect is on net trade, since lower consumption decreases imports.

In addition, the higher replacement rate combined with lower employment raises public expenditures for unemployment benefits. Tax receipts decrease due to lower wages, profits and consumption. And the combination of higher expenditures and lower receipts increases the government deficit. Moreover, cuts in government investment (infrastructure, but also research and development, education, etc.) translate into a negative impact on total factor productivity.

In summary, while at first sight desirable, an increase in the replacement rate could have a negative rather than a positive impact on workers, and on the entire economy. The size of the reaction of the economy is indeed dependent on the calibration of the model. Still, the model helps to understand the potential problem with this policy, which is the increase in wages associated with higher unemployment benefits. One potential solution is to break the link between higher benefits and higher wages. If higher benefits for the unemployed are associated with stable wages for the employees, the negative effects on employment could be avoided, as well as those on the total wage bill, income and consumption. Alternatively, a higher replacement rate associated with tighter eligibility conditions or a limited duration would cushion the adverse effects identified here by providing the right incentives to return to work once a transitory decline in activity is over. However, it would still be necessary to finance the increased government expenditure. Since higher taxes could depress income (also profits and investment), a better solution would be to cut non-productive government consumption.

5.3 Lower mark-up

Liberalization of product markets is often considered a tool to increase competition, decrease the mark-up and therefore improve consumer welfare. However, a lower mark-up may lower firm profits, constraining investment and reducing employment. Hence, apriori it is not clear whether the overall effects of a lower mark-up are beneficial.

In Table 2 we present the results of a 1% permanent decrease in the mark-up. The situation is indeed more complex than could be expected. One important reason for this is that profits also depend on sales: price reductions have a favorable effect on sales that more than compensates the fall in goods prices.

As a consequence of the labour market bargain, higher profits translate into higher real wages for (both resident and non-resident) workers. In turn, higher wages reduce labour demand and therefore decrease employment. However, the total wage bill is increased, as well as income and therefore consumption. As said before, higher demand further boosts profits, and therefore also investment, which brings about an additional increase in private demand, which is only in part compensated by higher imports.

Moreover, higher profits, total wages and consumption imply higher tax receipts and lower expenditures in unemployment benefits, thus substantially improving public finances, i.e., the government deficit falls substantially. Hence, an additional expansionary fiscal policy becomes possible, an effect that is generally not taken into account.

In summary, this is an example of a policy measure whose overall effects are uncertain at first sight but turn out to be more beneficial than expected when evaluated in a general equilibrium setting. All the consequences of the policy change should be jointly evaluated, and not only those related to one market or one type of economic agent, and this can only be done in a model like our LSM.

6 Conclusions

In this paper we have developed a structural macroeconometric model for Luxembourg of the NOEM-DSGE type. The model, labeled LSM for Luxembourg Structural Model, is characterized by a careful theory-based specification of the economy, which is represented by households, government, firms and unions, which interact in the product, labour and financial markets.

A properly calibrated version of LSM provides useful qualitative insights on the expected consequences of changes in economic policy, and can also be relevant to assess the effects and propagation of several types of economic shocks.

While LSM includes a set of specific features of the Luxembourg economy, such as a segmented labour market combined with strong union power, its general structure can be of wider interest for modelling small open economies.

To conclude, there is of course scope for additional research in this area, ranging from estimation of a simplified version of the model to the specification of an even more complex model to represent the financial sector more carefully.

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Table 1: Effects on selected variables of a 1% permanent increase in the replacement rate

		Horizon in years after the shock							
LSM mnemonic	*Variable*	*1y*	*2y*	*3y*	*4y*	*5y*	*10y*	*20y*	*50y*
GDP	GDP	--	--	--	--	--	--	---	---
c	Consumption	--	--	---	---	---	---	---	---
x	Investment	-	-	-	-	-	--	--	--
NX_IG	Export share - intermediate goods	+++	+++	+++	+++	+++	+++	+++	+++
govdef	Government deficit	+++	+++	+++	+++	+++	+++	+++	---
n1	Employment, resident	--	--	--	--	--	--	--	--
n2	Employment, non resident	--	--	--	--	--	--	--	--
profit	Profits	---	---	---	---	---	---	---	---
w1	Wages, resident	+	+	+	+	+	+	-	-
w2	Wages, non resident	+	+	+	+	+	+	-	-
wage_bill_1	Total wages, resident	--	--	--	--	--	--	--	--
wage_bill_2	Total wages, non resident	--	--	--	--	--	--	--	--
tfp	Total Factor Productivity	-	-	-	-	-	-	-	-

Table 2: Effects on selected variables of a 1% permanent decrease in the markup

		Horizon in years after the shock							
LSM mnemonic	*Variable*	*1y*	*2y*	*3y*	*4y*	*5y*	*10y*	*20y*	*50y*
GDP	GDP	+	+	+	+	+	+	+	+
c	Consumption	++	++	++	++	++	++	++	++
x	Investment	+	+	+	+	+	+	++	++
NX_IG	Export share - intermediate goods	---	---	---	---	---	---	---	---
govdef	Government deficit	---	---	---	---	---	---	---	+
n1	Employment, resident	-	-	-	-	-	-	-	-
n2	Employment, non resident	-	-	-	-	-	-	-	-
profit	Profits	++	++	++	++	++	++	++	++
w1	Wages, resident	+	+	+	+	+	+	+	+
w2	Wages, non resident	+	+	+	+	+	+	+	+
wage_bill_1	Total wages, resident	+	+	+	+	+	+	+	+
wage_bill_2	Total wages, non resident	+	+	+	+	+	+	+	+
tfp	Total Factor Productivity	+	+	+	+	+	+	+	+

Source: LSM simulations.

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value.

-, - -, and - - - indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.