Section IV

Bewely models: Applications

Pure credit

- Huggett (1993) studies the simplest version of this framework
- Households have access to a loan market in which they can borrow or lend at a risk-free interest rate r; no other assets are available
- Given φ, a stationary equilibrium is an interest rate $r > 0$, a policy function $a' = g(a,s)$, and a distribution $\lambda(a,s)$, such that:
  - The policy function $g(a,s)$ solves the household's problem;
  - $\lambda(a,s)$ is the stationary distribution induced by Π and $g(a,s)$
  - The loan market clears:

$$\sum_{h=1}^{n}\sum_{z=1}^{m}\lambda(a_h, s_z)\,\underbrace{g(a_h, s_z)}_{a'} = 0$$

- Numerical algorithm:
  - Choose an initial guess for r, say $r_j>0$ where $j=0$
  - Given r_j, solve the household problem for $g_j(a,s)$ and $\lambda_j(a,s)$
  - Check whether the loan market clears by computing the excess demand (or supply) of loans:

$$\sum_{h=1}^{n}\sum_{z=1}^{m}\lambda_j(\mathrm{a}_h,\mathrm{s}_z)g_j(\mathrm{a}_h,\mathrm{s}_z) = E_j$$

  - If $E_j > 0$, then set $r_{j+1} < r_j$
  - If, instead, $E_j < 0$, then set $r_{j+1} > r_j$
  - Iterate until convergence

Productive capital

- Following Aiyagari (1994), we will now study a more developed version of the previous model
- Assume that households are allowed to invest in a single, homogenous, capital good, and denote k_t the household's capital holdings
- No other assets exist, in particular households are not allowed to borrow or lend on a loan market. Note that in this case the borrowing constraint is redundant since $k \geq 0$ by assumption
- The individual capital stock evolves according to the following accumulation equation:

$$k_{t+1} = (1 - \delta + \tilde{r})k_t + ws_t - c_t$$

• Denote $\lambda(k,s)$ the stationary distribution of capital across households; the aggregate per-capita steady-state capital stock and the aggregate employment rate are respectively equal to:

$$K = K' = \sum_{h=1}^{n} \sum_{z=1}^{m} \lambda(\mathrm{k}_h, \mathrm{s}_z) \underbrace{g(\mathrm{k}_h, \mathrm{s}_z)}_{k'}$$

$$N = \sum_{h=1}^{n} \sum_{z=1}^{m} \lambda(\mathrm{k}_h, \mathrm{s}_z)\mathrm{s}_z = \xi'_\infty S$$

where ξ_∞ is the invariant distribution associated with Π and S the corresponding state space

• A representative competitive firm combines capital and labour to produce the single consumption/investment good via the following aggregate "Cobb-Douglas" production function:

$$Y \equiv F(K, N) = K^{\alpha} N^{1-\alpha}$$

• The first order conditions for the problem of the firm imply that:

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

$$\tilde{r} = \alpha\left(\frac{K}{N}\right)^{\alpha-1}$$

- A stationary equilibrium is a policy function $k' = g(k,s)$, a distribution $\lambda(k,s)$, and a triple of positive real numbers $\{K,r,w\}$, such that:

  - $g(k,s)$ solves the household's problem
  - $\lambda(k,s)$ is the stationary distribution induced by Π and $g(k,s)$
  - The factor prices satisfy conditions:

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

$$\tilde{r} = \alpha\left(\frac{K}{N}\right)^{\alpha-1}$$

  - The aggregate capital stock K is implied by the households individual decisions:

$$K = \sum_{h=1}^{n}\sum_{z=1}^{m} \lambda(k_h, s_z) g(k_h, s_z)$$

- Numerical algorithm:
  - Choose an initial guess for K, say $Kj > 0$ where j=0
  - Compute w_j and r_j from the FOCs for the firm
  - Solve the household problem for $g_j(k,s)$ and $\lambda_j(k,s)$
  - Compute the aggregate capital stock:

$$\hat{K}_j = \sum_{h=1}^{n} \sum_{z=1}^{m} \lambda_j(\mathrm{k}_h, \mathrm{s}_z) g_j(\mathrm{k}_h, \mathrm{s}_z)$$

  - Given a fixed "relaxation" parameter $0<\kappa<1$, compute a new estimate of K from:

$$K_{j+1} = \kappa K_j + (1-\kappa)\hat{K}_j$$

  - Iterate until convergence

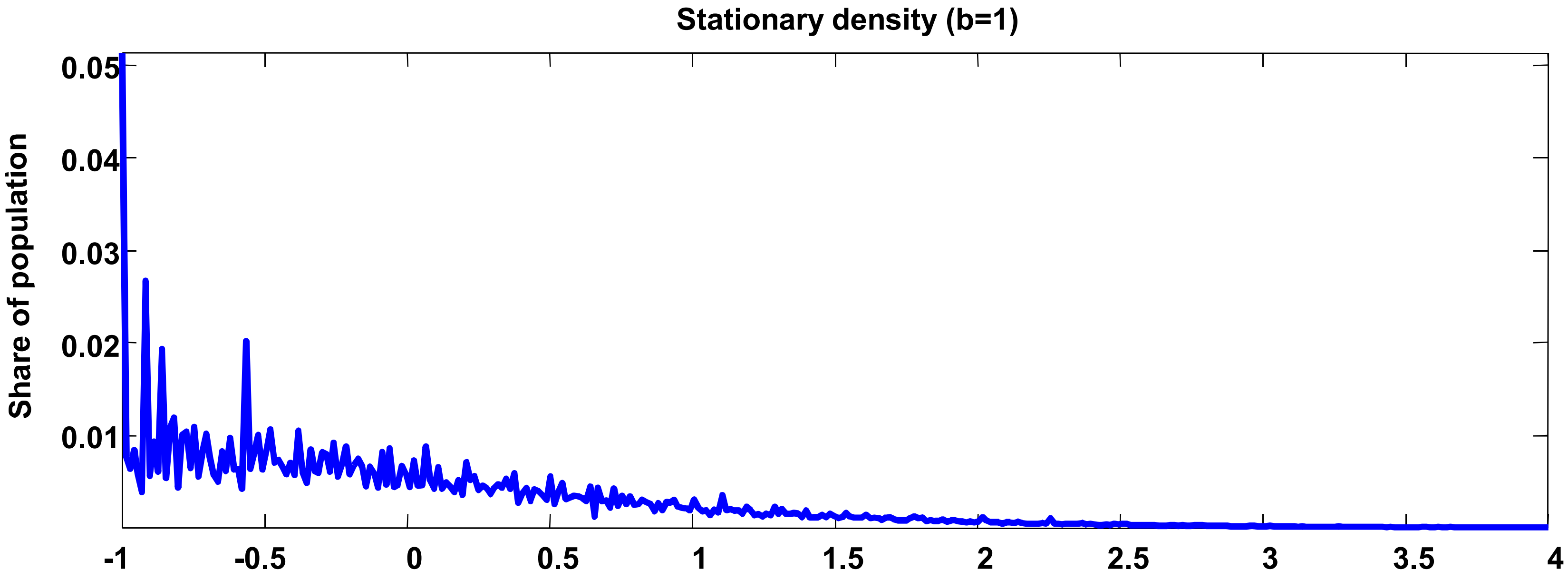
- Following Huggett (1993), assume a CES form for the Bernoulli function, $u(c)=c^{1-\mu}/(1-\mu)$
- Set $\mu=2$, $\beta=0.97$, $w=1$, and $\varphi=1$
- Following Heaton and Lucas (1996), assume that employment status follows a stationary autoregressive process:

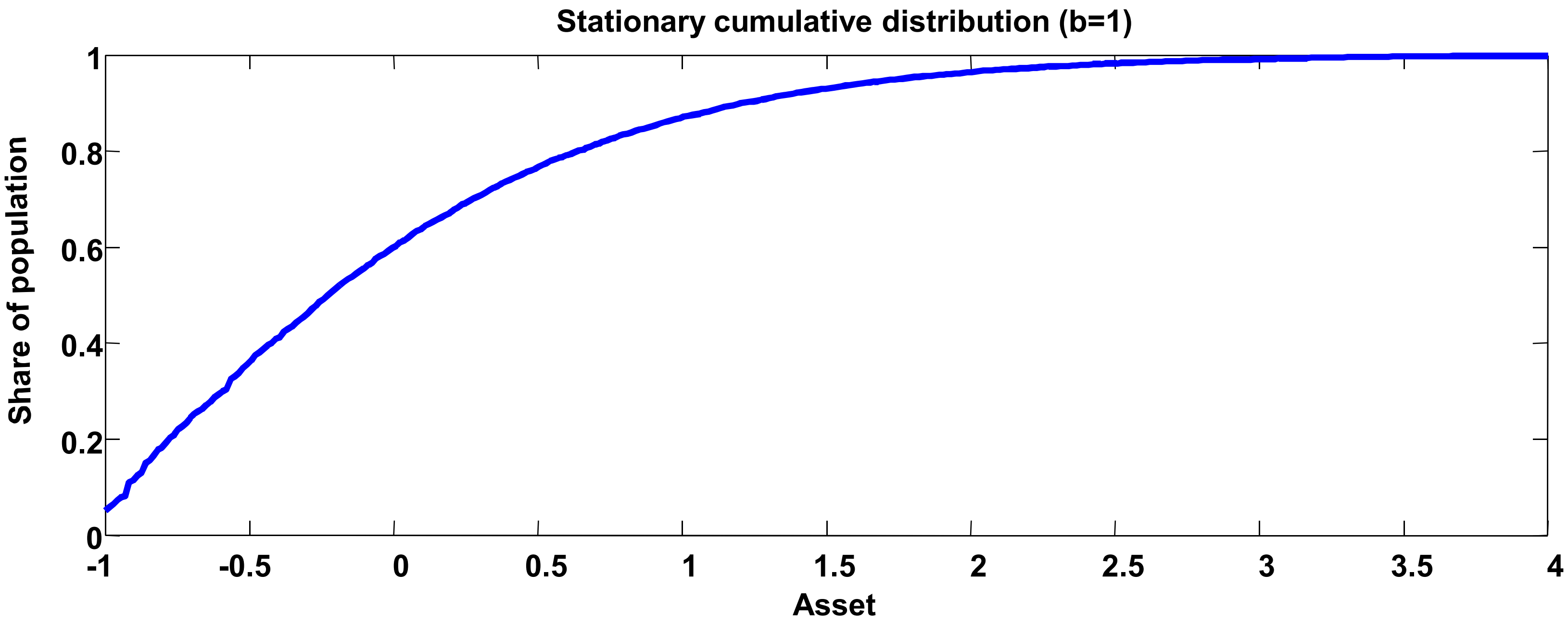
$$\ln s_{t+1} = \rho \ln s_t + \sigma \sqrt{(1-\rho^2)}\, \varepsilon_t$$

where $\varepsilon_t \sim N(0,1)$, $\rho=0.53$, and $\sigma=0.296$

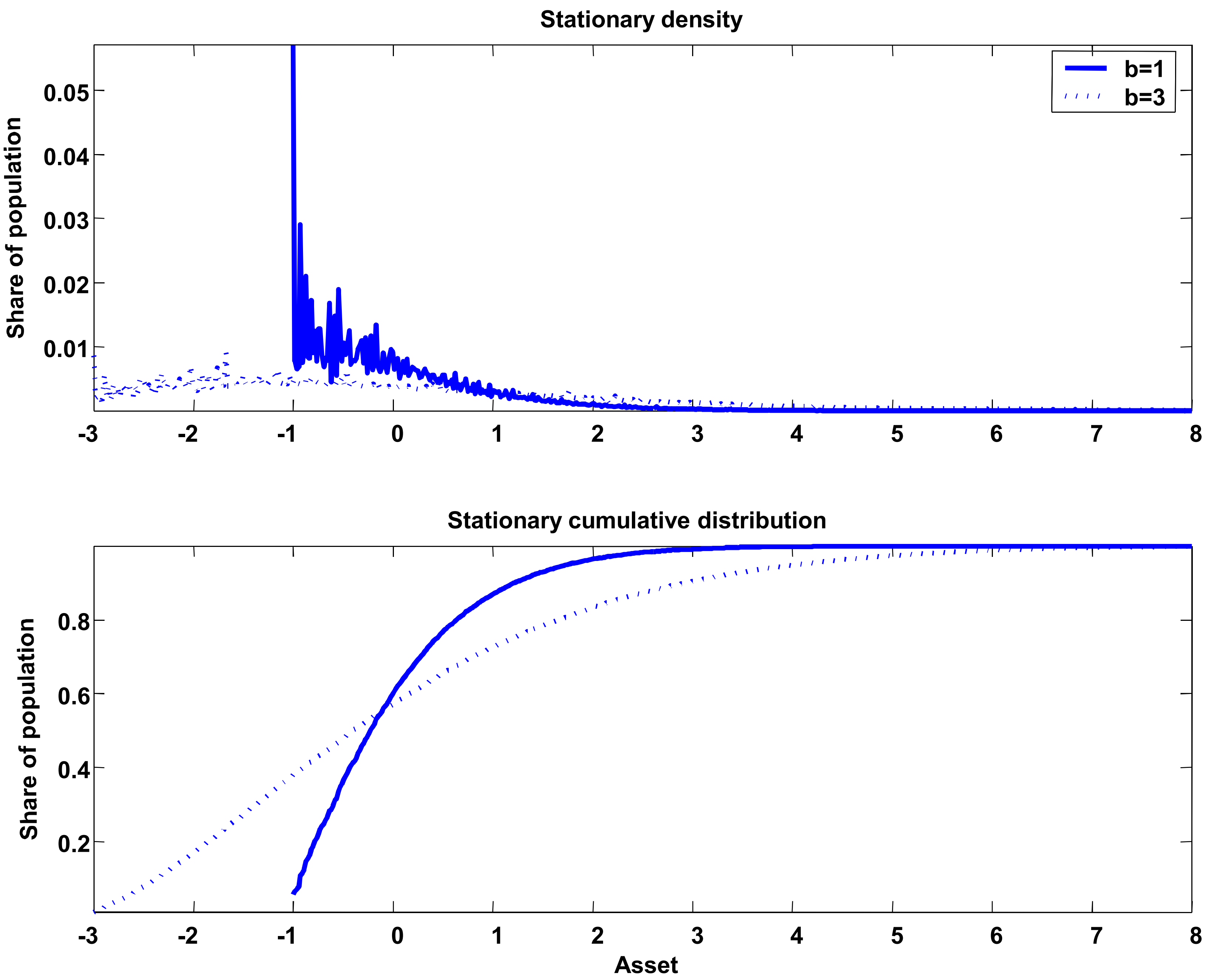
- Using Tauchen's or Rouwenhorst’s methods, we can approximate the continuous-state autoregressive process with a finite-state Markov chain
- Finally, following Aiyagari (1994), se set $\alpha=0.36$ and $\delta=0.08$

Huggett 1

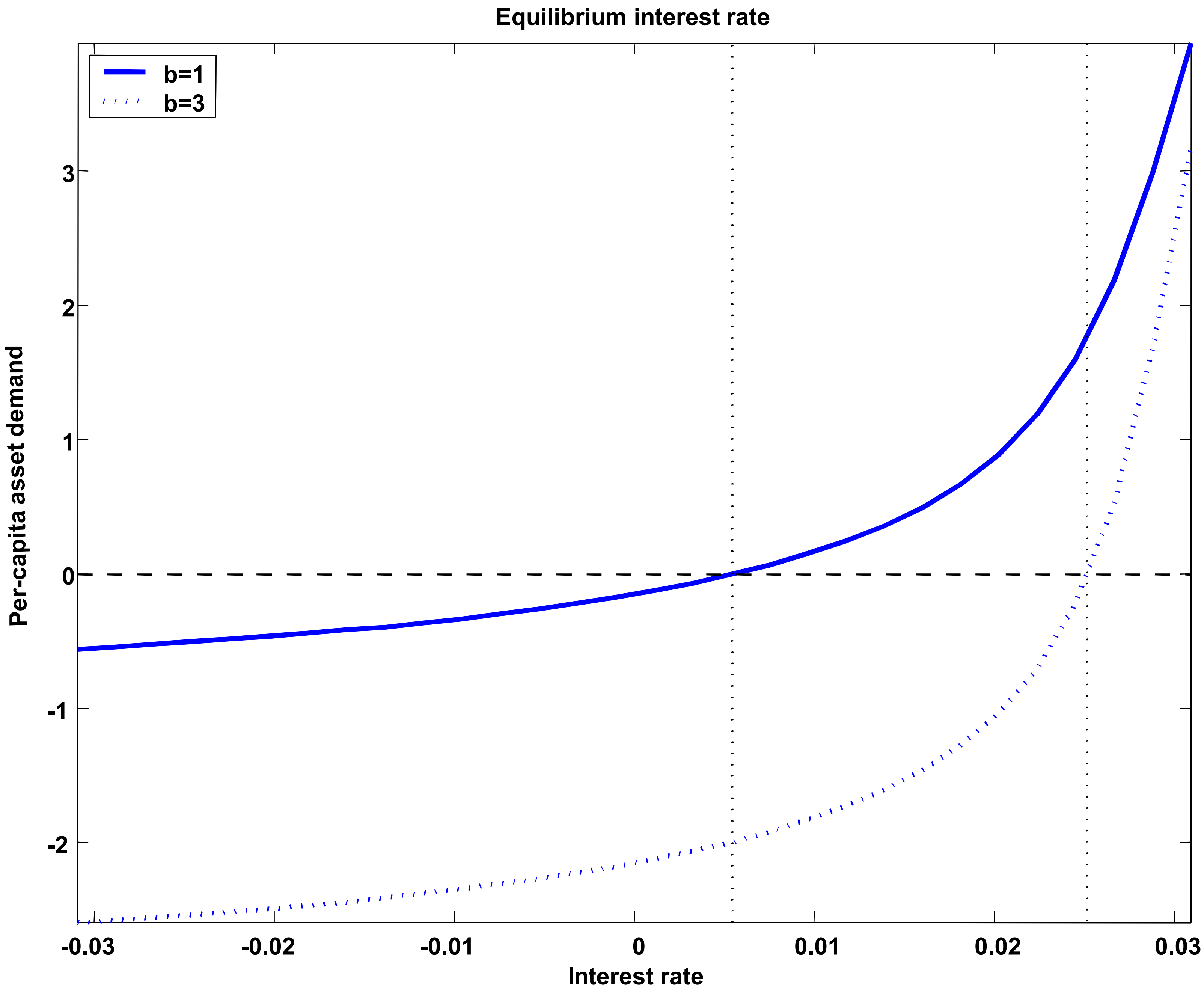

Stationary density (b=1)
Share of population
0.05
0.04
0.03
0.02
0.01
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4


Stationary cumulative distribution (b=1)
Share of population
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
Asset

Huggett 2

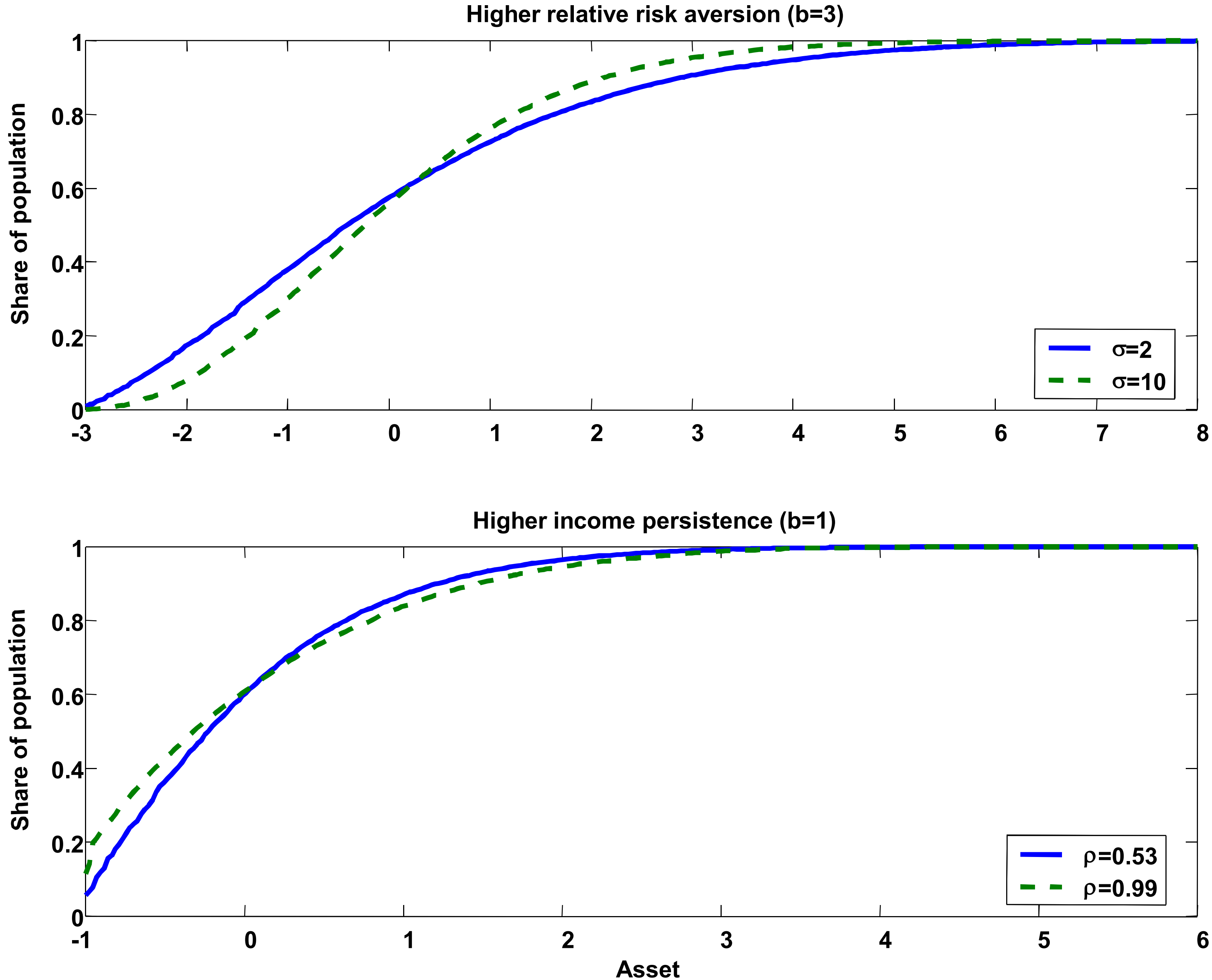

Stationary density
Share of population
0.05
0.04
0.03
0.02
0.01
b=1
b=3
-3 -2 -1 0 1 2 3 4 5 6 7 8
Stationary cumulative distribution
Share of population
0.8
0.6
0.4
0.2
-3 -2 -1 0 1 2 3 4 5 6 7 8
Asset

Huggett 3

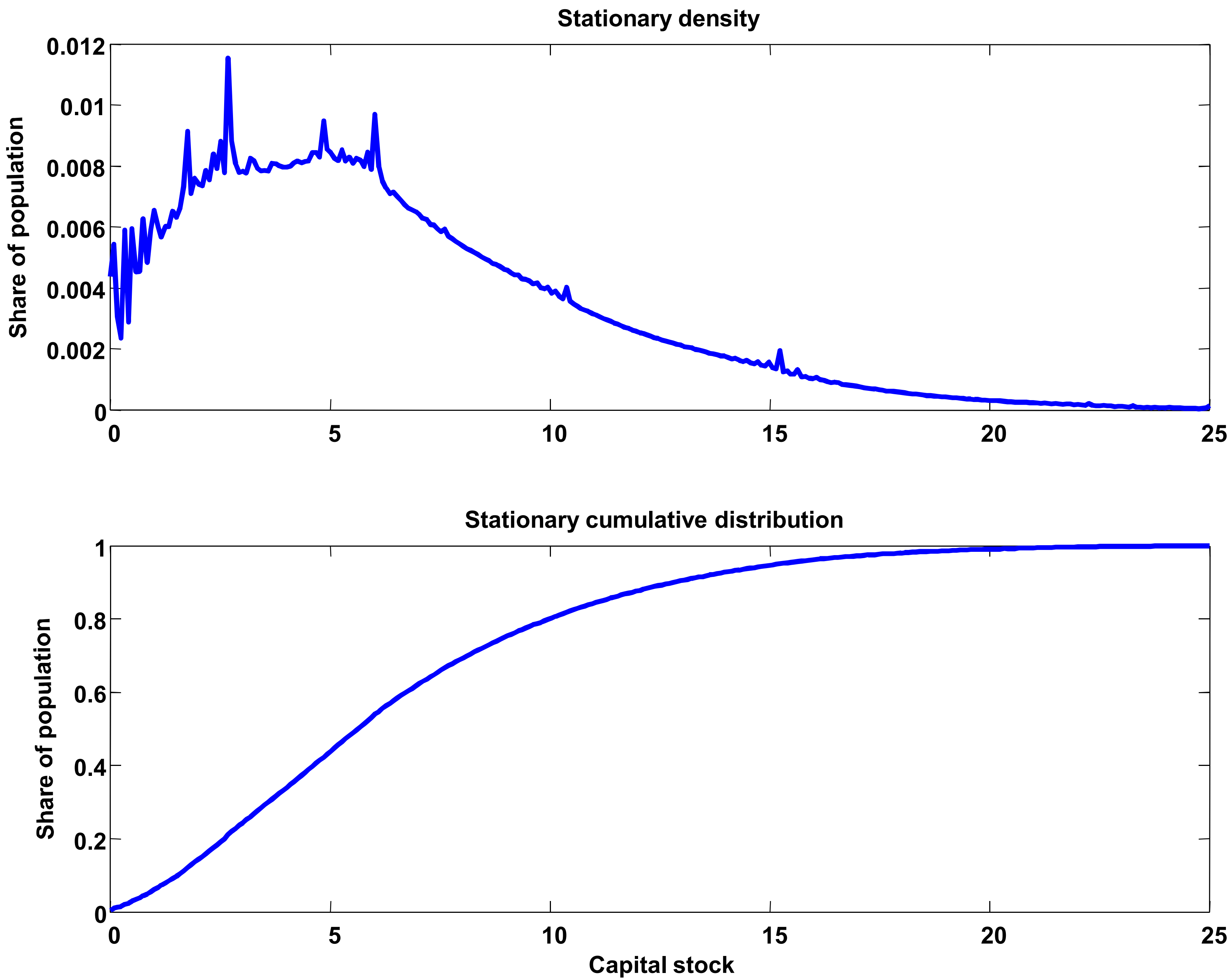


Equilibrium interest rate
b=1
b=3
Per-capita asset demand
Interest rate
3
2
1
0
-1
-2
-0.03
-0.02
-0.01
0
0.01
0.02
0.03

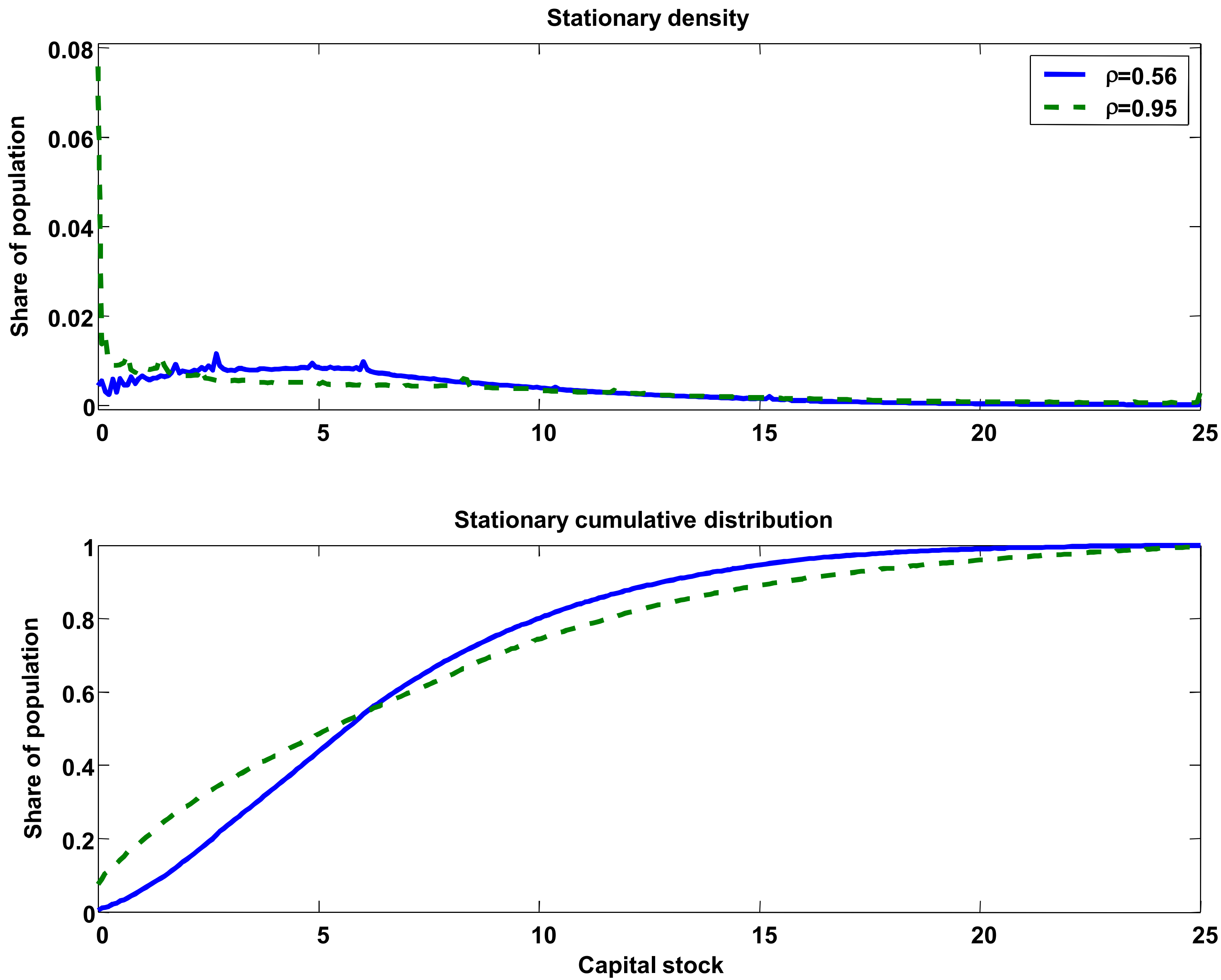
Huggett 4


Higher relative risk aversion (b=3)
Share of population
1
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3
4
5
6
7
8
σ=2
σ=10
Higher income persistence (b=1)
Share of population
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
Asset
ρ=0.53
ρ=0.99

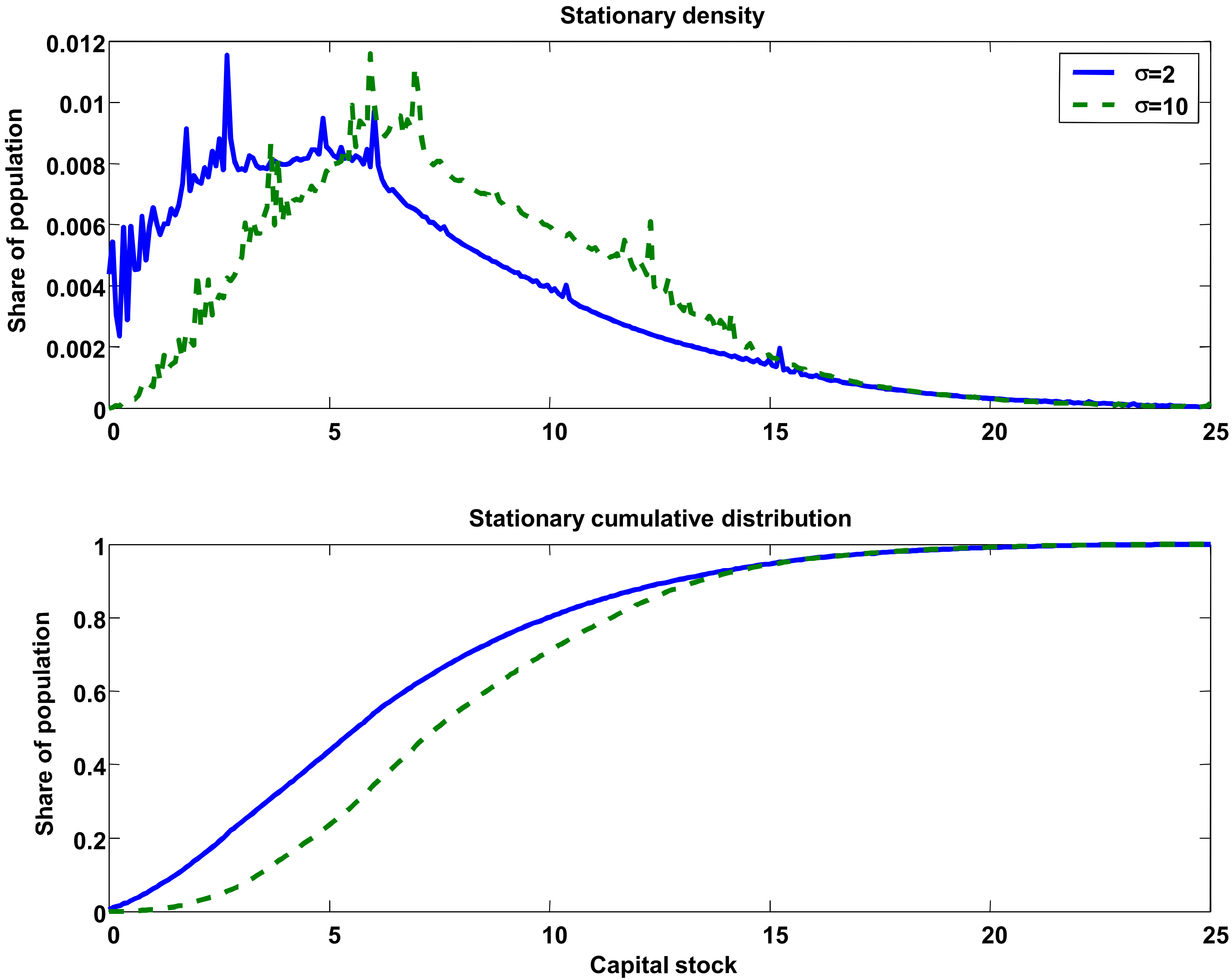
Aiyagari 1


Stationary density
Share of population
0.012
0.01
0.008
0.006
0.004
0.002
0
0
5
10
15
20
25
Stationary cumulative distribution
Share of population
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Capital stock

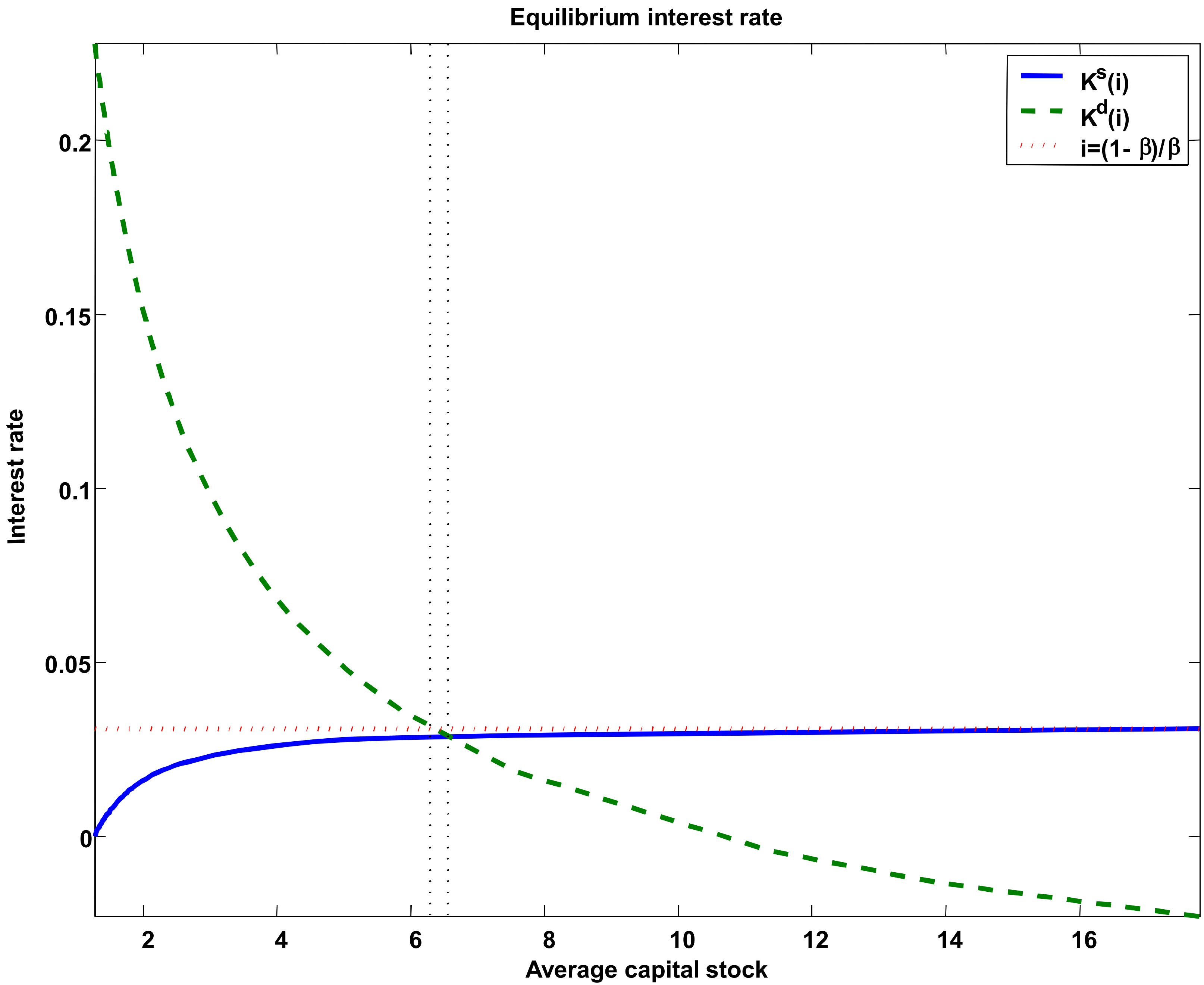
Aiyagari 2


Stationary density
Share of population
0.08
0.06
0.04
0.02
0
ρ=0.56
ρ=0.95
0
5
10
15
20
25
Stationary cumulative distribution
Share of population
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Capital stock

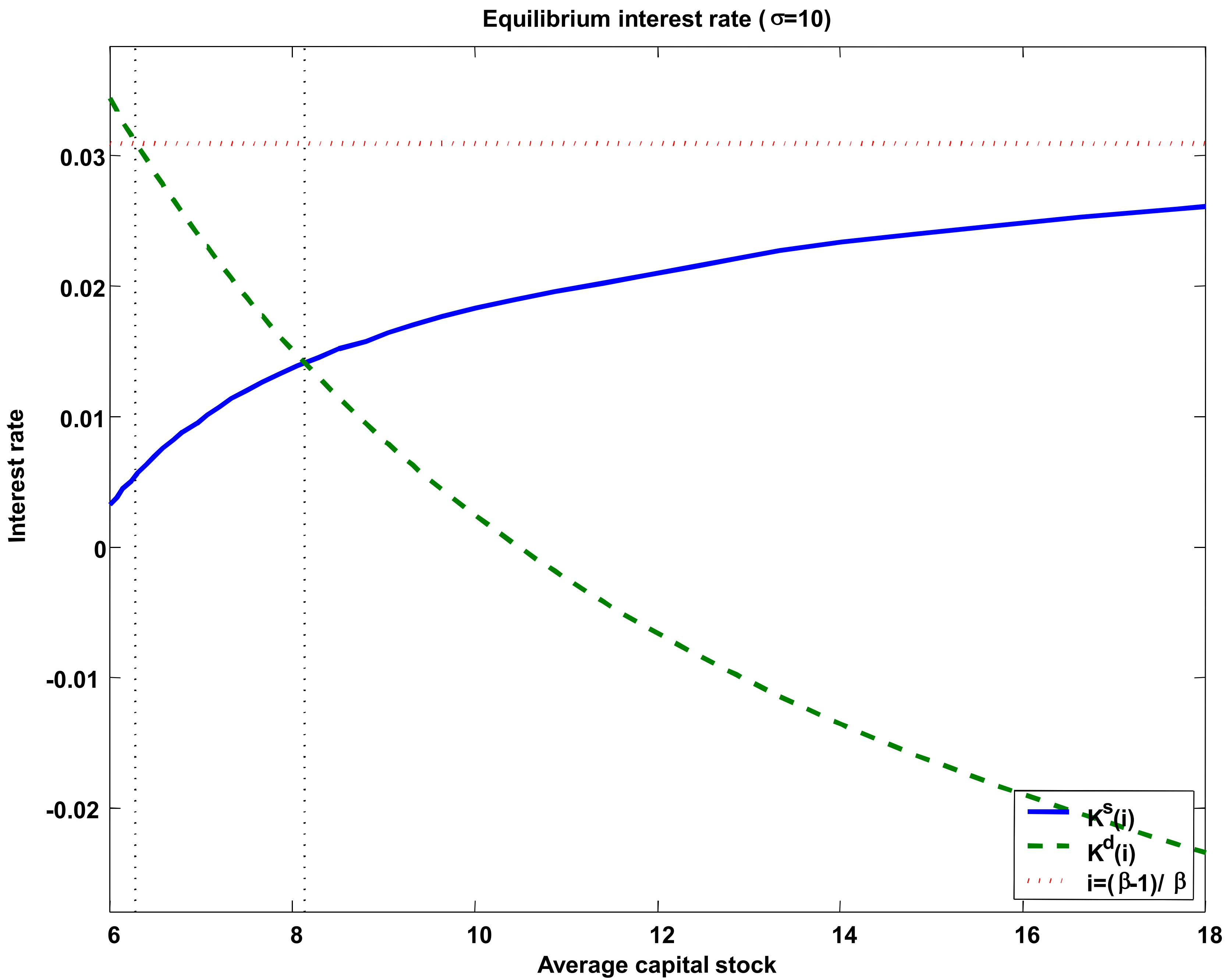
Aiyagari 3


Stationary density
Share of population
0.012
0.01
0.008
0.006
0.004
0.002
0
0
5
10
15
20
25
σ=2
σ=10
Stationary cumulative distribution
Share of population
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Capital stock

Aiyagari 4


Equilibrium interest rate
$K^s(i)$
$K^d(i)$
$i=(1-\beta)/\beta$
Interest rate
0.2
0.15
0.1
0.05
0
2
4
6
8
10
12
14
16
Average capital stock

Aiyagari 5


Equilibrium interest rate (σ=10)
Interest rate
0.03
0.02
0.01
0
-0.01
-0.02
6
8
10
12
14
16
18
Average capital stock
$K^s(i)$
$K^d(i)$
$i=(\beta-1)/\beta$