Applied Numerical Finance
ANF - Code 20247

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joint with Francesco Rotondi (LAB classes)

Optional Courses presentation
Bocconi MSc. Finance

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Applied Numerical Finance: Contents Overview

Course objectives: providing tools to understand and solve important computational issues in financial engineering.

Contents overview:

1. Trees for American and path-dependent options.
2. Basket and Quanto options in continuous time.
3. Asset prices with jumps and European call/put options.
4. Monte Carlo methods in financial engineering.

All topics covered by lecture notes/slides.

Lab Classes: VBA.

Why Visual Basic for Applications (VBA)?

- Flexible, easily available, appreciated by employers.
- Lab classes: *from* `how to access the VBA editor` *to* `managing arrays in VBA`.
- Option: You can use your alternative favorite programming language for the assignment (e.g. MatLab).

Exam Mark : 30% Assignment + 70% Final Written Exam

- Assignment: Write the code, present the preliminary results, submit the assignment at the exam date only.
- Assignments can be shared. *Presentations are mandatory for attending students only*, and are **not** graded.

B. Bignoli, RBS, London (Graduation Date: April 2015):

```
"As a structured credit trader, my knowledge of the
multidimensional Black-Scholes model, instantaneous
correlation and jump diffusions is fundamental to
correctly price derivatives and appropriately manage
and hedge the risk of the trading book."
```

Thanks for the attention!

Students who pass the Applied Numerical Finance exam qualify for writing their MSc Thesis under my supervision

Applied Numerical Finance: Detailed Contents Overview I

1. Trees for American and path-dependent options
   - Easy to build for European/American plain vanilla options. Flexible to incorporate barrier features $\Rightarrow$ Widely used in practice.
   - BUT: If the option payoff is path-dependent usual trees do *not* work $\Rightarrow$ *Solution?* Forward shooting grid method (see the example, slide 7)
2. Basket and Quanto options in continuous time
   - Pricing and hedging derivatives on several underlying assets: The multidimensional Black-Scholes market.
   - Foreign markets and quantization: risk premia, domestic/foreign risk-neutral measure, covered/uncovered interest rate parity.

Applied Numerical Finance: Detailed Contents Overview II

3. Asset prices with jumps and European call/put options.
   - Stock returns exhibit fat tails $\longrightarrow$ simple Black-Scholes does not work $\longrightarrow$ add **jumps** to the stock price.
     Black-Scholes + Jumps = the Jump-Diffusion Model (JDM)
   - The JDM is the simplest model with jumps (reduced-form credit risk models).
   - European call and put options in the JDM
4. Monte Carlo methods in financial engineering.
   - Simulation of asset prices (Euler discretization)
   - Pricing European derivatives.
   - Enhancing the accuracy of the estimate: Variance reduction.
   - Barrier options via importance sampling.
   - Monte Carlo techniques to compute Greeks

Self-contained lecture notes/slides distributed via BlackBoard.

If you are curious...

In the next slides you find a *simple* example of how to evaluate a European lookback call option in the binomial model by means of the forward shooting grid method.

The familiar binomial model

$$
\begin{array}{ccccc}
n = 0 & & n = 1 & & n = 2 \\
 & & & & S_0 u^2 \\
 & & & \nearrow & \\
 & & S_0 u & & \\
 & \nearrow & & \searrow & \\
S_0 & & & & S_0 ud \\
 & \searrow & & \nearrow & \\
 & & S_0 d & & \\
 & & & \searrow & \\
 & & & & S_0 d^2
\end{array}
$$

where
r interest rate
σ volatility
q dividend yield

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = u^{-1} = e^{-\sigma\sqrt{\Delta t}}$$

$$q_{up} = \mathbb{Q}\left[S\left(t + \Delta t\right) = S\left(t\right) \cdot u\right] =$$
$$= \frac{e^{(r-q)\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}$$

A simple example: European lookback call option

Two-period binomial model with $S_0 = 10$, $\sigma = 0.15$, $r = 0.02$, $q = 0$, $\Delta t = 1$.
These values imply $q_{up} = 0.53$, $u = 1.162$ and $d = 0.861$.
The terminal payoff of a European **lookback call option** with *fixed* strike $K = 9$ and maturity $T = 2$ is

$$X(2) = (M(2) - K)^{+}$$

where

$$M(2) = \max_{s=0,1,2} S(s)$$

is the terminal *running maximum of* S.
To evaluate the option we need at T *not only* S *but also* M $\rightarrow$ *the standard binomial model does not work!* $\rightarrow$ *introduce the* auxiliary variable M $\rightarrow$ **Forward Shooting Grid Method**.

The Forward Shooting Grid Method

Main idea in two phases:
1st Phase (Forward):

Attach to all binomial realizations of $S(t)$ *the auxiliary vector of representative values for* $M(t)$, $\mathbf{M}(t)$, *of (fixed) dimension* $\mathbf{d}$.

In the example $\mathbf{M}$ *is the representative vector of running maxima of* S.

2nd Phase (Backward):

Compute the price of the derivative at time t *for any value of* $S(t)$ *and for any representative value of* $M(t)$.

You get a $\mathbf{d}-$*dimensional vector of representative prices for the derivative* $\mathbf{V}(t)$.

The dimension $\mathbf{d}$ of the vector $\mathbf{M}(t)$ may increase with t (since more values are possible for $M(t)$ as t increases).

European lookback call option (continued) I

Next slide:

- binomial tree for S (black),
- auxiliary vector of running maxima $\mathbf{M}$
- option price vector $\mathbf{V}$.

In the two-period binomial model an auxiliary vector of dimension $\mathbf{d} = 2$ covers all possible values for the terminal running maximum $\mathbf{M}(2)$ at time $T = 2$.

European lookback call option (continued) II

Legenda:

S, $\mathbf{M}$, $\mathbf{V}$

$$
\begin{array}{lll}
 & & 13.5 \longrightarrow [13.5, 13.5 \\
 & & \qquad [4.5, 4.5] \\
 & \nearrow & \\
 & 11.62 \longrightarrow [11.62, 11.62] & \\
 & \qquad [3.54, 3.54] & \\
\nearrow & & \searrow \\
10 \longrightarrow [10, 10] & & 10 \longrightarrow [10, 11.62] \\
\qquad [2.29, 2.29] & & \qquad [1, 2.62] \\
\searrow & & \nearrow \\
 & 8.61 \longrightarrow [10, 10] & \\
 & \qquad [0.98, 0.98] & \\
 & & \searrow \\
 & & 7.41 \longrightarrow [10, 10] \\
 & & \qquad [1, 1]
\end{array}
$$

$\Downarrow$

$\mathbf{V}(0) = 2.29$

Initial price of the lookback call option

So simple? I

When the number of periods is large we face many issues:

- The fixed maximal dimension of the auxiliary vector becomes binding (the number of entries of the representative vector is smaller than the number of possible realizations of the running maximum).
- **1st Phase:** we have to *make approximations for* $\mathbf{M}$. The algorithm to select representative approximated values for $\mathbf{M}$ depends on the payoff structure.
- **2nd Phase:** Due to the approximation in the 1st Phase, there are nodes $S(t), \mathbf{M}(t)$ that do *not* have an *exact predecessor* among $S(t-\Delta t), \mathbf{M}(t-\Delta t)$. How do we select the more suitable one?

So simple? II

- How do we find the price of $\mathbf{V}(t)$, associated to $S(t), \mathbf{M}(t)$?
- If the early exercise is allowed before maturity, i.e. if the option is *American*, how do we compute the value of waiting to exercise until the next period *(continuation value)*? What do we do if there is no exact match for the successor of $S(t), \mathbf{M}(t)$ because of previous approximations?
- How do we compute the price of the American option?

Discover the solution (and more) in the ANF classes!