

Applied Numerical Finance

ANF - Code 20247

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joint with Francesco Rotondi (LAB classes)

Optional Courses presentation
Bocconi MSc. Finance

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Applied Numerical Finance: Contents Overview

Course objectives: providing tools to understand and solve important computational issues in financial engineering.

Contents overview:

1. Trees for American and path-dependent options.
2. Basket and Quanto options in continuous time.
3. Asset prices with jumps and European call/put options.
4. Monte Carlo methods in financial engineering.

All topics covered by lecture notes/slides.

Lab Classes: VBA.

Why Visual Basic for Applications (VBA)?

- ▶ Flexible, easily available, appreciated by employers.
- ▶ Lab classes: *from* how to access the VBA editor *to* managing arrays in VBA.
- ▶ Option: You can use your alternative favorite programming language for the assignment (e.g. MatLab).

Exam Mark : 30% Assignment + 70% Final Written Exam

- ▶ Assignment: Write the code, present the preliminary results, submit the assignment at the exam date only.
- ▶ Assignments can be shared. *Presentations are mandatory for attending students only*, and are **not** graded.

B. Bignoli, RBS, London (Graduation Date: April 2015):

"As a structured credit trader, my knowledge of the multidimensional Black-Scholes model, instantaneous correlation and jump diffusions is fundamental to correctly price derivatives and appropriately manage and hedge the risk of the trading book."

Thanks for the attention!

Students who pass the Applied Numerical Finance exam qualify for writing their MSc Thesis under my supervision

Applied Numerical Finance: Detailed Contents Overview I

1. Trees for American and path-dependent options

- ▶ Easy to build for European/American plain vanilla options. Flexible to incorporate barrier features \Rightarrow Widely used in practice.
- ▶ BUT: If the option payoff is path-dependent usual trees do *not* work \Rightarrow *Solution?* Forward shooting grid method (see the example, slide 7)

2. Basket and Quanto options in continuous time

- ▶ Pricing and hedging derivatives on several underlying assets: The multidimensional Black-Scholes market.
- ▶ Foreign markets and quantization: risk premia, domestic/foreign risk-neutral measure, covered/uncovered interest rate parity.

Applied Numerical Finance: Detailed Contents Overview II

3. Asset prices with jumps and European call/put options.

- ▶ Stock returns exhibit fat tails → simple Black-Scholes does not work → add **jumps** to the stock price.
Black-Scholes + **Jumps** = the **Jump-Diffusion Model (JDM)**
- ▶ The **JDM** is the simplest model with jumps (reduced-form credit risk models).
- ▶ European call and put options in the **JDM**

4. Monte Carlo methods in financial engineering.

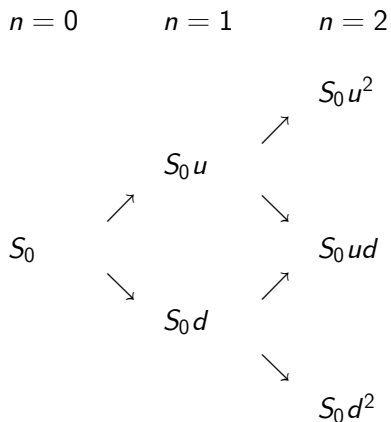
- ▶ Simulation of asset prices (Euler discretization)
- ▶ Pricing European derivatives.
- ▶ Enhancing the accuracy of the estimate: Variance reduction.
- ▶ Barrier options via importance sampling.
- ▶ Monte Carlo techniques to compute Greeks

Self-contained lecture notes/slides distributed via BlackBoard.

If you are curious...

In the next slides you find a *simple* example of how to evaluate a European lookback call option in the binomial model by means of the forward shooting grid method.

The familiar binomial model



where

r interest rate

σ volatility

q dividend yield

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = u^{-1} = e^{-\sigma\sqrt{\Delta t}}$$

$$\begin{aligned} q_{up} &= \mathbf{Q}[S(t + \Delta t) = S(t) \cdot u] = \\ &= \frac{e^{(r-q)\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \end{aligned}$$

A simple example: European lookback call option

Two-period binomial model with $S_0 = 10$, $\sigma = 0.15$, $r = 0.02$, $q = 0$, $\Delta t = 1$.

These values imply $q_{up} = 0.53$, $u = 1.162$ and $d = 0.861$.

The terminal payoff of a European **lookback call option** with *fixed* strike $K = 9$ and maturity $T = 2$ is

$$X(2) = (M(2) - K)^+$$

where

$$M(2) = \max_{s=0,1,2} S(s)$$

is the terminal *running maximum* of S .

To evaluate the option we need at T *not only* S *but also* $M \rightarrow$ *the standard binomial model does not work!* \rightarrow *introduce the auxiliary variable* $M \rightarrow$ **Forward Shooting Grid Method**.

The Forward Shooting Grid Method

Main idea in two phases:

1st Phase (Forward):

Attach to all binomial realizations of $S(t)$ the auxiliary vector of representative values for $M(t)$, $\mathbf{M}(t)$, of (fixed) dimension \mathbf{d} .

In the example \mathbf{M} is the representative vector of running maxima of S .

2nd Phase (Backward):

Compute the price of the derivative at time t for any value of $S(t)$ and for any representative value of $M(t)$.

You get a \mathbf{d} -dimensional vector of representative prices for the derivative $\mathbf{V}(t)$.

The dimension \mathbf{d} of the vector $\mathbf{M}(t)$ may increase with t (since more values are possible for $M(t)$ as t increases).

European lookback call option (continued) I

Next slide:

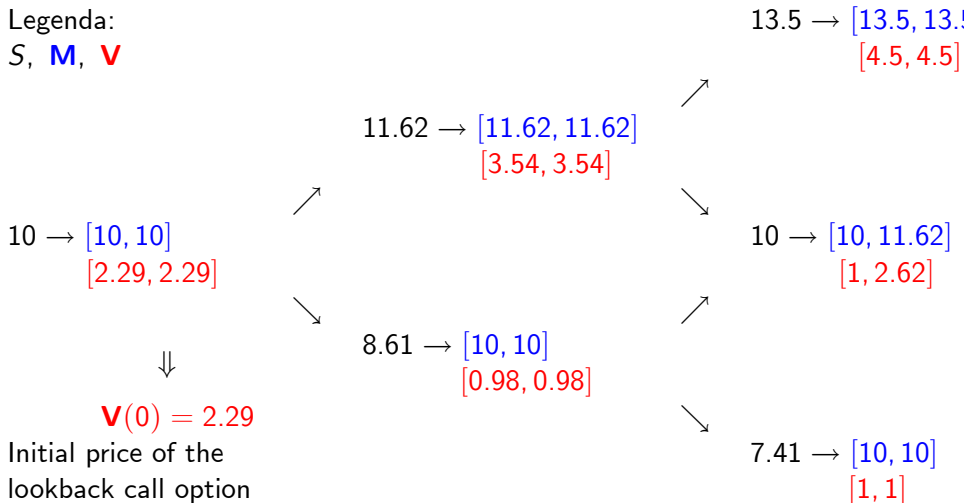
- ▶ binomial tree for S (black),
- ▶ auxiliary vector of running maxima \mathbf{M}
- ▶ option price vector \mathbf{V} .

In the two-period binomial model an auxiliary vector of dimension $\mathbf{d} = 2$ covers all possible values for the terminal running maximum $\mathbf{M}(2)$ at time $T = 2$.

European lookback call option (continued) II

Legenda:

S , M , V



So simple? I

When the number of periods is large we face many issues:

- ▶ The fixed maximal dimension of the auxiliary vector becomes binding (the number of entries of the representative vector is smaller than the number of possible realizations of the running maximum).
- ▶ **1st Phase:** we have to *make approximations for \mathbf{M}* . The algorithm to select representative approximated values for \mathbf{M} depends on the payoff structure.
- ▶ **2nd Phase:** Due to the approximation in the 1st Phase, there are nodes $S(t), \mathbf{M}(t)$ that do *not* have an *exact predecessor* among $S(t - \Delta t), \mathbf{M}(t - \Delta t)$. How do we select the more suitable one?

So simple? II

- ▶ How do we find the price of $\mathbf{V}(t)$, associated to $S(t)$, $\mathbf{M}(t)$?
- ▶ If the early exercise is allowed before maturity, i.e. if the option is *American*, how do we compute the value of waiting to exercise until the next period (*continuation value*)? What do we do if there is no exact match for the successor of $S(t)$, $\mathbf{M}(t)$ because of previous approximations?
- ▶ How do we compute the price of the American option?

Discover the solution (and more) in the ANF classes!