Epistemic Uncertainty in the Ranking and Categorization of Probabilistic Safety Assessment Model Elements: Issues and Findings

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Abstract

In this work, we study the effect of epistemic uncertainty in the ranking and categorization of elements of probabilistic safety assessment (PSA) models. We show that, while in a deterministic setting a PSA element belongs to a given category univocally, in the presence of epistemic uncertainty, a PSA element belongs to a given category only with a certain probability. We propose an approach to estimate these probabilities, showing that their knowledge allows to appreciate "*the sensitivity of component categorizations to uncertainties in the parameter values*"[US NRC Regulatory Guide 1.174]. We investigate the meaning and utilization of an assignment method based on the expected value of importance measures. We discuss the problem of evaluating changes in quality assurance, maintenance activities prioritization (etc.) in the presence of epistemic uncertainty. We show that the inclusion of epistemic uncertainly in the evaluation makes it necessary to evaluate changes through their effect on PSA model parameters. We propose a categorization of parameters based on the Fussell-Vesely and Differential Importance (DIM) measures. In addition, issues in the calculation of the expected value of the joint importance measure are present when evaluating changes affecting groups of components. We illustrate that the problem can be solved using DIM. A numerical application to a case study concludes the work.

Keywords: *Epistemic Uncertainty, Importance Measures, Probabilistic Safety Assessment.*

1 Introduction

The purpose of this work is to introduce a methodical approach to the categorization of system structures and components (SSCs) in the presence of epistemic uncertainty.

A wide literature dealing with uncertainty in Risk Assessment problems is available. While its complete consideration is out of the scope of this work, we recall the works of Apostolakis (1990) [3], (1995) [4], Breeding et al (1992) [11], Helton and Breeding (1993) [19], Hoffman and Hammond (1994) [25], Helton (1994) [20], Patè-Cornell (1996) [34], Helton (1997) [21], Helton et al (1999) [22]. These works underline that in probabilistic risk assessment it often emerges the need to distinguish between two types of uncertainties. Aleatory uncertainty, i.e., uncertainty about the occurrence of a random event, and epistemic uncertainty, i.e., uncertainty in the values of the probabilities of such occurrences.

Risk-Informed Decision Making means to make use of the information that is derived from the Probabilistic Safety Assessment (PSA) model. Importance measures are sensitivity indicators that are utilized to identify the safety/risk significance (S/RS) of systems, structures and components (SSCs). Quoting directly from Appendix A of the NRC regulatory guide 1.174 [33]: "*the Fussell-Vesely (FV) Importance, Risk Reduction Worth, and Risk Achievement Worth (RAW) are the most commonly identified measures in the relative risk ranking of SSCs.*" Information derived from these importance measures (Cheok et al, 1998, [14]) is used in the evaluation of risk-informed decisions in applications such as the prioritization of maintenance activities, changes in quality assurance programs, changes in maintenance or testing activities, aging processes, that tend to alter the status quo of the plant[1] (Brewer and Canady, 1999 [12]; Caruso et al, 1998[13]).

Technical issues in the use of importance measures are summarized in Appendix A of Regulatory Guide 1.174: "*(1) ... risk rankings apply only to individual contributions and not to combinations or sets of contributors, and (2) ... risk rankings are not necessarily related to the risk changes that result from those contributor changes*"[33]. The issues are revealed and discussed in Fleming (1996) [17], Vesely (1998) [40], Cheok et al (1998) [14], Vasseur and Llory (1999)[41], Borgonovo and Apostolakis

[1] In the remainder we shall refer to these applications as "changes."

(2001) [6], and Borgonovo (2007) [9].

A further issue associated with the use of importance measures is highlighted in Regulatory Guide 1.174, Appendix A: "*The sensitivity of component categorizations to uncertainties in the parameter values should be addressed by the licensee. Licensees should be satisfied that SSC categorization is not affected by data uncertainties*" [33]. In fact, SSCs ranking and categorization is usually based on values of importance measures estimated when all parameters are at their base case values, i.e., it is the result of a deterministic calculation. The presence of epistemic uncertainty causes the decision-maker's view to be characterized by a (joint) subjective distribution on the parameters, so that the base case value of the parameters is just one of a (possibly infinite) set of values that parameters can assume. Quoting from Saltelli et al, 2004 [35], one runs the risk of drawing inappropriate conclusions generated by "*underestimation of predictive uncertainty"[35].* This issue is highlighted also in the work of Cheok et al (1998) [14]: "*the broad uncertainty distributions of these events can pose a challenge when ranking events based on risk importance ... (Cheok et al, 1998 [14])*" and "*consideration of uncertainty should be a factor of the integrated decision making process (Cheok et al, 1998 [14])*".

Monte Carlo analysis is utilized to determine the distributions of the importance measures. In His "Recommendations for Future Work", Lambert (1975a) [27] states that: "*It would also be useful to incorporate an option in the IMPORTANCE computer code to allow for an error analysis. This can be accomplished by placing prior distributions on the failure rate data and then use Monte Carlo simulation to determine the spread in the importance rankings*" (Lambert, 1975a; p. 229). It is not, however, till 20 years later that a first systematic approach to incorporate epistemic uncertainty in component rankings is introduced by Modarres and Agarwal [31], who propose "*methodologies for performing risk-based ranking under uncertainty*" (Modarres and Agarwal, 1996 [31]; p. 230).

This work has a twofold purpose. The first purpose is to build an approach that allows to rank and categorize PSA elements giving full credit to the decision-maker's state of belief. We recall that ranking PSA elements means to attribute them a relative importance, while categorizing PSA elements means to assign them to Safety/Risk Significance (S/RS) regions. S/RS regions are identified by threshold values set on (usually) two importance measures. As far as ranking is concerned, we complement the work of Modarres and Agarwal (1996) [31] by introducing a probability matrix ($\mathcal{P} = [p_{ij}]$) to be associated with the point estimate ranking. $\mathcal{P}$ contains the probabilities that SSC i is more important than SSC j. We discuss the properties of $\mathcal{P}$ (we show that $p_{ji} = 1 - p_{ij}$) and its use in establishing the decision-maker's degree of confidence in the importance measure results.

We then define the probabilistic inequalities that allow the creation of S/RS diagrams in the presence of epistemic uncertainty. By comparing the deterministic categorization with the results of the probability calculations, a decision-maker can assess Her/His confidence in the deterministic categorization. In addition, we propose a categorization method based on the expected value of the importance measures. Through a standard mathematical argument, we show that an expected value categorization is a synthesis bridge between the probabilistic and deterministic analyses.

The second purpose of this work is to deal with issues generated by the presence of epistemic uncertainty in the evaluation of changes. Cheok et al (1998) [14] evidence that "*the uncertainties in individual basic events may be correlated* (Cheok et al, 1998, p. 223)[14]". We show that, if a change affects identical components and epistemic uncertainty is taken into consideration, then the basic events become 100% correlated and the change must be modeled as affecting PSA model parameters. Thus, to evaluate the risk significance of such changes, one ought to determine the S/RS of the parameters impacted by the change. However, as RAW is not defined for parameters, such a categorization cannot be achieved through and S/RS plane formed using FV and RAW. A solution to such a problem is found by proposing a parameter categorization based on FV and DIM.

The application of the approach is discussed by means of the importance analysis of the large loss of coolant accident (LOCA) sequence of the Advanced Test Reactor (ATR) (Eide et al, 1991 [15]; Borgonovo et al, 2003 [7]). We obtain the deterministic, probabilistic and expected value categorizations of basic events with FV and RAW and parameters with FV and DIM. We utilize the results of the categorizations to assess the relevance of changes affecting parameters and basic events of components involved in the LOCA sequence.

On a broader perspective, we must remark the difference in the present work, and works in the global sensitivity analysis literature (see for example Frey and Patil (2002) [18], Helton and Davis (2002) [23], Saltelli (2002) [36], Patil and Frey (2004) [32], Borgonovo et al (2003) [7], Borgonovo (2006) [8], Helton et al (2006) [24]). The purpose of global sensitivity analysis is to identify the parameters that influences uncertainty in model results the most. In our analysis, we aim at characterizing and managing uncertainty concerning the results of a sensitivity analysis exercise which, given its own purpose, is local in nature.

The remainder of the paper is organized as follows. In Section 2, we present the definitions of the importance measures used in this work. In Section 3, we present a formal approach to PSA element categorization in the presence of epistemic uncertainty. Section 4 discusses the effect of epistemic dependence on the modeling of changes. In Section 5, we present the point estimate categorization, the corresponding probabilities and the results of expected value categorization for both basic events and probabilities of the Large LOCA sequence. In Section 6, we illustrate the application of the approach in the evaluation three changes affecting different basic events and parameters of systems involved in the ATR Large LOCA sequence. Conclusions are offered in Section 7.

2 Importance Measures Used in this Work

The PSA model is built to estimate the risk metric (R) of interest in the safety problem at hand. In the nuclear industry, R can be a core damage frequency (CDF) or Large Early Release Frequency (see [6]). The PSA model estimates R as a function of the basic event probabilities ($\mathbf{p}$) and initiating event frequencies $(\mathbf{f}_{IE})^2$:

$$R = h(\mathbf{f}_{IE}, \mathbf{p}) \tag{1}$$

If the decision-maker's state of belief were such that She/He can assign a precise value $\mathbf{f}_{IE}^0$, $\mathbf{p}^0$ to $\mathbf{f}_{IE}$ and $\mathbf{p}$, then R would take on the certain value:

$$R^0 = h(\mathbf{f}_{IE}^0, \mathbf{p}^0) \tag{2}$$

[2] $\mathbf{f}_{IE}$ represents the vector of initiating event frequencies: $\mathbf{f}_{IE} = \left(f_{IE}^1, f_{IE}^2, ..., f_{IE}^z\right)$, where z is the number of initiating events. Similarly, $\mathbf{p}$ is a vector $\mathbf{p} = (p_1, p_2, ..., p_m)$ where m is the number of basic events.

Importance measures convey information on the S/RS of a basic event or SSC by performing mathematical operations on R. The earliest literature on techniques for measuring the importance of events in technological starts with the works of authors as Birnbaum, Barlow, Proshan, Fussell, Vesely and Lambert the 1970's (see Lambert, 1975b [28] for a comprehensive introduction to importance measures).

The Fussell-Vesely (FV) importance of basic event j is defined as the fraction of R associated with basic event j (see eq. (9) in Lambert, 1975b, page 90; and also Cheok et al, 1998 [14]), i.e.,

$$FV_j(\mathbf{f}^0_{IE}, \mathbf{p}^0) = \left.\frac{fr(\cup MCS_j)}{R^0}\right|_{(\mathbf{f}^0_{IE}, \mathbf{p}^0)} \tag{3}$$

where $fr(\cup MCS_j)$ is the frequency of the union of the minimal cut sets (MCS) containing basic event j and R^0 is the base case value of the risk metric. Eq. (3) tells us that $FV_j(\mathbf{f}^0_{IE}, \mathbf{p}^0)$ is the fraction of the risk associated with basic event j, when the input variables, namely $\mathbf{f}_{IE}$ and $\mathbf{p}$, are at their nominal value, $(\mathbf{f}^0_{IE}, \mathbf{p}^0)$.

The RAW of basic event j is defined as (Cheok et al, 1998 [14]):

$$RAW_j(\mathbf{f}^0_{IE}, \mathbf{p}^0) = \left.\frac{R^j}{R^0}\right|_{(\mathbf{f}^0_{IE}, \mathbf{p}^0)} \tag{4}$$

where R^j is the value of the risk metric achieved when basic event j has happened. Eq. (4) shows that $RAW_j(\mathbf{f}^0_{IE}, \mathbf{p}^0)$ ranks basic events according to the risk that we achieve, if the event associated to basic event j happens.

The differential importance of basic event j is defined as (Borgonovo and Apostolakis, 2001 [6]):

$$DIM_j(\mathbf{f}^0_{IE}, \mathbf{p}^0) = \left.\frac{d_jR}{dR}\right|_{\mathbf{f}^0_{IE}, \mathbf{p}^0} \tag{5}$$

where d_jR is the change in risk provoked by a change in the probability of basic event j. The interpretation of $DIM_j(\mathbf{f}^0_{IE}, \mathbf{p}^0)$ [eq. (5)] is as follows: the greater the change in R provoked by a change in p_j, the more relevant is the corresponding basic event. Other two importance measures relevant in PSA are the Birnbaum importance measure (Birnbaum, 1969 [10]) and the criticality importance measure (for its definition, see Cheok et al, 1998 [14], Epstein and Rauzy, 2005 [16] or Borgonovo, 2007 [9]). It can be shown (see Borgonovo and Apostolakis, 2001 [6] and Borgonovo, 2007 [9]) that the Birnbaum importance measure produces the same ranking as DIM under the assumption of uniform changes in the basic event probabilities. DIM and the criticality importance measure produce the same ranking, under the assumption of proportional parameter changes. Finally, given the type of dependence of $h(\mathbf{f}_{IE}^0, \mathbf{p}^0)$ [eq. (1)] on the basic event probabilities, DIM, FV and the criticality importance measures give rise to the same basic event ranking under the assumption of proportional changes. We refer the reader to Borgonovo, 2007 [9] for a complete discussion on the relationship between DIM and other importance measures at the basic event level.

In many applications failure probabilities are estimated through the aid of submodels. For example, it is a typical reliability choice that the failure probability of equipment i is modeled via an exponential distribution [29]:

$$p(\lambda) = P(\text{equipment fails before } t; \lambda) = 1 - e^{-\lambda t} \tag{6}$$

with λ representing the failure rate. Note that it is not necessary that all probabilities are expressed through submodels and therefore are function of parameters. Thus, in general, R is a function of parameters, basic event probabilities and initiating event frequencies (see also Borgonovo and Apostolakis, 2001):

$$R = h(\mathbf{f}_{IE}, \mathbf{p}, \boldsymbol{\lambda}) = g(\mathbf{x}) \tag{7}$$

In the remainder, we shall use the symbol $\mathbf{x} = (x_1, x_2, ..., x_n)$ to denote a generic input ($\mathbf{f}_{IE}$, or $\mathbf{p}$ or $\boldsymbol{\lambda}$) to the PSA model.

In terms of importance measures, Borgonovo and Apostolakis (2001) [6] have shown that it is possible to extend both DIM and FV to the parameter level, while an extension of the definition of RAW is not feasible.[3] The FV of parameter λ_i is defined as (Borgonovo and Apostolakis, 2001 [6]; see also

[3]Indeed, the works of Vesely ([40]) have taught us that the true meaning of FV is for basic events. At the parameter level, the work of Borgonovo and Apostolakis (2001) showed that it is still possible to find what terms of the risk metric contain a parameter, and therefore to compute an equivalent FV at the parameter level. However, consider the reliability model:

$$R = f(\lambda_1, \lambda_2, \lambda_3) = e^{-\lambda_1 T} e^{-\lambda_2 T} e^{-\lambda_3 T} \tag{8}$$

with ■λ_1■λ_2■λ_3. We have $FV(\lambda_i) = 1$, $i = 1, 2, 3$. Thus, one would consider the three parameters as equally influential

Borgonovo et al, 2003[7]):

$$FV_i(\mathbf{x}^0) = \left. \frac{\sum T_R(x_i)}{R^0} \right|_{(\mathbf{x}^0)} \tag{9}$$

where $T_R(x_i)$ denotes any of the terms in the risk metric expression that contains parameter x_i. The sum in the numerator of eq. (9) is carried over all terms in R that contain x_i. The denominator is the point estimate value of R. Thus, according to eq. (9), $FV_i(\mathbf{x}^0)$ ranks parameters according to the portion of R^0 related to them. $FV_i(\mathbf{x}^0)$, then, could be interpreted as the fraction of the risk that is associated with x_i (Borgonovo and Apostolakis, 2001).

The definition of DIM for parameters is as follows (Borgonovo and Apostolakis, 2001):

$$DIM_i(\mathbf{x}^0) = \left. \frac{\frac{\partial R}{\partial x_i}\frac{x_i}{Y}}{\sum_{s=1}^{n} \frac{\partial R}{\partial x_s}\frac{x_s}{Y}} \right|_{\mathbf{x}^0} \tag{10}$$

According to eq. (10), $DIM_i(x^0)$ is the fraction of the change in risk that is associated with a change in x_i.

Finally, we remark that the risk metric dependence on individual parameters [$g(\mathbf{x})$, eq.(7)] is, in general, non-linear. Thus, as shown in Borgonovo and Apostolakis, 2001 [6] and FV and DIM give raise to different rankings (see also Borgonovo 2007 [9]).

3 PSA Element Categorization in the Presence of Epistemic Uncertainty

Categorizing PSA elements means assigning them to S/RS categories (Cheok et al, 1998 [14]). For components or basic events, this is usually done setting S/RS thresholds on the values of the FV and RAW (see Brewer and Canady (1999) [12])[4]. In general, let us denote with Y a PSA element, where Y can be an SSC, a basic event or a parameter, and with I_Y a generic importance measure of Y. Let also Th_I the corresponding threshold. To assign Y to a given region, one has to check whether the

using FV. However, the parameters have a different DIM. Thus, one can distinguish their relative importance through DIM. This example shows that FV and DIM do not provide the same information. As, due to the complexity of a PSA model, an analyst does not know the analytical expression of the risk metric as a function of the parameters, the joint use of FV and DIM also allows to avoid pitfalls of the type entailed in the previous example.

[4] In Brewer and Canady (1999) [12], the value of the Safety thresholds are $Th_{RAW} = 2$, and $Th_{FV} = 0.01$, respectively.

importance measure of Y is greater than the threshold value:

$$I_Y(\mathbf{x}^0) > Th_I \tag{11}$$

In the deterministic case, inequalities of the type of eq. (11) allow to assign an SSC to a S/RS region with certainty. However, in most applications, information does not enable the decision-maker to determine x with certainty (Apostolakis, 1995 [4]). The PSA methodology foresees that the decision-maker is capable of assessing a prior/posterior distribution of the parameters[5]. PSA models are then kept alive, data are collected periodically and the distributions adjourned to reflect new evidence (see Martz and Waller, 1991 [29]; as an example of data collection, see Marshall et al, 1998 [30]). Accordingly, the decision-maker's state of belief of $\mathbf{x}$ is reflected by a joint distribution function. We denote as $\mathcal{X} \subseteq \mathbb{R}^n$ the set of all possible values that the inputs can assume (in other words, $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n$). We then denote as $\mathcal{A}$ the Borel algebra associated with $\mathcal{X}$ and with F the joint subjective probability measure characterizing the decision-maker's state of belief on $\mathbf{x}$. Then, $(\mathcal{X}, \mathcal{A}, F)$ denotes the probability space of interest in this work.

As a consequence of the introduction of epistemic uncertainty the inequality (11) becomes:

$$I_Y(\mathbf{x}) > Th_I \tag{12}$$

Being $\mathbf{x}$ an element of $(\mathcal{X}, \mathcal{A}, F)$, the above inequality becomes stochastic, i.e., one can only establish that the importance measure of the component is greater than the threshold value with a given probability. Now, let $\mathcal{X}^+_{I_Y} = \{\mathbf{x} : I_Y(\mathbf{x}) > Th_I\}$ and $\mathcal{X}^-_{I_Y} = \mathcal{X}_{I_Y} \backslash \mathcal{X}^+_{I_Y}$ or $\mathcal{X}^-_{I_Y} = \{\mathbf{x} : I_Y(\mathbf{x}) < Th_I\}$, then the probability that $I_Y(x) > Th_I$ is given by the probability that $x \in \mathcal{X}^+_{I_Y}$. This probability can be

[5] The foundations of PSA date back to the works of Lambert, Rasmussen in the 1970's, and find their operational formulation in the "risk triplets" of Kaplan and Garrick, 1981 [26] and in the work of Apostolakis (1990) in Science [3] It is a shared assumption of the PSA methodology that the decision maker is capable of specifying probabilities for all events. In the practice a Bayesian Decision maker is assumed {see Martz and Waller, 1991 [29]). Distributions are assigned and updated through extensive data anlysis and as soon as new evidence is available (in this respect, one says that PSA models are "kept alive").

estimated, via Monte Carlo propagation as follows:

$$P_F(I_Y(\mathbf{x}) > Th_I) = \lim_{M\to\infty} \frac{m}{M} \tag{13}$$

where

- m is the number of trials in which a value of x is sampled according to F such that $I_Y(x) > Th_I$
- M is the number of Monte Carlo trials.

We have mentioned that, when categorizing PSA elements, one introduces S/RS regions based on two thresholds $[Th_{I1}, Th_{I2}]$. Then, one obtains four regions[6]. Y belongs to a given S/RS region with the following probabilities:

$$\begin{array}{ccc} \text{Region} & \text{Probability} & \text{Name} \\ I & P_F\left\{(I1_Y(\mathbf{x}) > Th_{I1}) \cap (I2_Y(\mathbf{x}) > Th_{I2})\right\} & \text{Very high S/RS} \\ II & P_F\left\{(I1_Y(\mathbf{x}) > Th_{I1}) \cap (I2_Y(\mathbf{x}) < Th_{I2})\right\} & \text{High S/RS} \\ III & P_F\left\{(I1_Y(\mathbf{x}) < Th_{I1}) \cap (I2_Y(\mathbf{x}) > Th_{I2})\right\} & \text{High S/RS} \\ IV & P_F\left\{(I1_Y(\mathbf{x}) < Th_{I1}) \cap (I2_Y(\mathbf{x}) < Th_{I2})\right\} & \text{Low S/RS} \end{array} \tag{14}$$

The knowledge of the probabilities in (14) complements to the point estimate categorization. In fact, it provides the decision-maker's degree of confidence in the results of the point estimate categorization.

The probabilities in (14) can be computed in a similar way as the probabilities in eq. (13). Let $\mathcal{X}^1_{I_Y} = \{\mathbf{x} : (I1_Y(\mathbf{x}) > Th_{I1}) \cap (I2_Y(\mathbf{x}) > Th_{I2})\}$ the subset of $\mathcal{X}$ such that Y belongs to region 1, when $x \in \mathcal{X}^1_{I_Y}$. Then,

$$P_F\left\{(I1_Y(\mathbf{x}) > Th_{I1}) \cap (I2_Y(\mathbf{x}) > Th_{I2})\right\} = \lim_{M\to\infty} \frac{m_1}{M} \tag{15}$$

where

- m_1 is the number of trials in which a value of x is sampled according to F such that $(I1_Y > Th_{I1})$ and $(I2_Y > Th_{I2})$

[6] One usually deems as region of Very High S/RS region I, in which both importance measures overcome the thresholds. The region defined by values of both importance measures below the thresholds are called of Low S/RS, the other two regions of High S/RS.

- M is the number of Monte Carlo trials.

Operationally, eq. (15) can be implemented as follows. After a value of $\mathbf{x}$ (say $\mathbf{x}^1$) is sampled, one registers the two corresponding values $I1_Y(\mathbf{x}^1)$ and $I2_Y(\mathbf{x}^1)$. One then determines the region to which Y belongs, comparing these values to the threshold values. The procedure is repeated M times. We note that, in so doing, one also obtains the distributions of $I1_Y(x)$ and $I2_Y(x)$. Such distributions can then utilized to compute

$$\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] \text{ and } \mathbb{E}_F\left[I1_Y(\mathbf{x})\right] \tag{16}$$

Then, consider the inequality:

$$\mathbb{E}_F\left[I_Y(\mathbf{x})\right] > Th_I \tag{17}$$

Since $\mathbb{E}_F[Z]$, where Z is a random variable, is the quantity that expresses the decision-maker's expectation on Z, inequality (17) can be interpreted as answering the question of whether the decision-maker expects the importance measure to be greater than the threshold or not. One can then define the following four regions:

Region		Name
I	$\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] > Th_{I1}$ and $\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] > Th_{I2}$	Very high S/RS
II	$\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] > Th_{I1}$ and $\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] < Th_{I2}$	High S/RS
III	$\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] < Th_{I1}$ and $\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] > Th_{I2}$	High S/RS
IV	$\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] < Th_{I1}$ and $\mathbb{E}_F\left[I1_Y(\mathbf{x})\right] < Th_{I2}$	Low S/RS

(18)

A categorization based on expected values would assume the meaning of assigning SSC's to S/RS regions based on the decision-maker's expectations, giving full credit to the view implied by Her/His state of belief. From a technical viewpoint, the expected value categorization represents a step in between the probabilistic approach and the deterministic one. In fact, the expected value of an importance measure ($\mathbb{E}_F\left[I_Y(\mathbf{x})\right]$) is equal to its point estimate $[I_Y(\mathbf{x}^0)]$ in the absence of epistemic uncertainty. This is readily obtained by Taylor expanding $I_Y(x)$ around $\mathbf{x}^0$.[7] The result suggests that the expected value

[7]

$$\mathbb{E}_F\left[I_Y(\mathbf{x})\right] = I_Y(\mathbf{x}^0) + \sum_{j=1}^{n}\sum_{i=1}^{n} \left.\frac{\partial^2 I_Y}{\partial x_j \partial x_i}\right|_{\mathbf{x}^0} Cov\left[x_i, x_j\right] + o(\sigma_X^2) \tag{19}$$

of an importance measure includes the point estimate value of the importance measure $[I_Y(\mathbf{x}^0)]$ plus terms that are generated by the decision-maker's uncertainty in the parameters. These terms are null when parameter are known with certainty.

We end this Section with a discussion concerning the effect of epistemic uncertainty on relative ranking (see also Modarres and Agarwal, 1996 [31]). We recall that ranking means to establish whether Y_i is more important than Y_j according to a given importance measure (see, for example Cheok et al, 1998 [14]). Thus, if $I_{Y_i} > I_{Y_j}$ one says that Y_i is more important than Y_j.[8] If the parameters are fixed at a certain value, then the inequality $I_{Y_i}(\mathbf{x}^0) > I_{Y_j}(\mathbf{x}^0)$ is deterministic, i.e., the statement "Y_i is more important than Y_j" is either true or false. Let $\mathcal{X}_{I_Y}^{Y_i>Y_j} = \{\mathbf{x} : I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\}$ and $\mathcal{X}_{I_Y}^{Y_i\leq Y_j} = \{\mathbf{x} : I_{Y_i}(\mathbf{x}) \leq I_{Y_j}(\mathbf{x})\}$. Clearly, $\mathcal{X} = \mathcal{X}_{I_Y}^{Y_i>Y_j} \cup \mathcal{X}_{I_Y}^{Y_i\leq Y_j}$, and $\mathcal{X}_{I_Y}^{Y_i>Y_j} \cap \mathcal{X}_{I_Y}^{Y_i\leq Y_j} = \varnothing$. The probability with which PSA element Y_i is more important than Y_j,

$$P_F(I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})) \tag{20}$$

is the probability with which $\mathbf{x} \in \mathcal{X}_{I_Y}^{Y_i>Y_j}$. The above framework allows to estimate $P_F(I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x}))$ through Monte Carlo simulation. Similarly to eqs. (13) and (15), $P_F(I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x}))$ is equal to:

$$P_F(I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})) = \lim_{M\to\infty} \frac{m_{ij}}{M} \tag{21}$$

where m_{ij} is equal to unity if Y_i is more important than Y_j in a given Monte Carlo run.

It is useful to arrange the information of eq. (20) in matrix form:

$$\mathcal{P} = [p_{i,j} = P_F(I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})) \quad \begin{matrix} i = 1, 2, ..., n \\ \\ j = 1, 2, ..., n \end{matrix}] \tag{22}$$

(by default, we let $p_{i,j} = 1$ when $i = j$). Note that the following holds for the elements of M:

$$p_{i,j} = 1 - p_{j,i}, \quad \text{when } i \neq j \tag{23}$$

[8] Ranking PSA elements means to attribute a relative importance. In other words, one is comparing PSA elements within themselves. In categorizing, one is introducing an external element (the threshold) and comparing the importance of PSA elements against the threshold values.

In fact,

$$P_F\left\{I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\right\} = 1 - P_F\left\{I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\right\} \tag{24}$$

Let us examine the insights that a decision-maker derives from $\mathcal{P}$. In the remainder, let Y_i be more important than Y_j in the point-estimate ranking (i.e., $I_{Y_i}(\mathbf{x}^0) > I_{Y_j}(\mathbf{x}^0)$). In the presence of epistemic uncertainty, the inequality $I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})$ is evaluated in several Monte Carlo runs. If $I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})$ is always true, then $p_{ij} = P_F\left\{I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\right\} = 1$(or 0) and correspondingly $p_{ji} = 1 - P_F\left\{I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\right\} = 0$ (or 1). This means that, if in $\mathcal{P}$ the non-diagonal elements are close to $p_{ij} = 1$ (or $p_{ji} = 0$), then the propagation of epistemic uncertainty does not alter the ranking induced by the point estimate values of the importance measure. One could say that the decision-maker is confident in the point-estimate ranking. Alternatively, it can happen that in some Monte Carlo runs $I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})$ and in some $I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})$. One can say that the decision-maker has a p_{ij} degree of confidence that Y_i is more important than Y_j. As the value of p_{ij} moves from 1 to 0.5 (correspondingly p_{ji} moves from 0 to 0.5), the decision-maker is progressively less confident that Y_i is more important than Y_j. A value of p_{ij} lower than 0.5 would signal that epistemic uncertainty leads to a ranking reversal. In fact, such a value indicates that $I_{Y_i}(\mathbf{x}) < I_{Y_j}(\mathbf{x})$ in most of the scenarios, while, based on the point-estimate ranking, one would expect $P_F\left\{I_{Y_i}(\mathbf{x}) > I_{Y_j}(\mathbf{x})\right\} > P_F\left\{I_{Y_i}(\mathbf{x}) < I_{Y_j}(\mathbf{x})\right\}$.

In the next Section, we discuss the implications generated by the presence of epistemic uncertainty in the evaluation of the S/RS of changes.

4 The Effects of Epistemic Uncertainty on Changes Evaluation

In this Section, we illustrate four different ways of modelling changes. As mentioned in the introduction, the need of PSA element categorization arises in applications such as the prioritization of maintenance activities, changes in quality assurance programs, changes in maintenance or testing policies, i.e., any activity that tends to alter the status quo of the plant (see Section 1). Before discussing the importance measure computation, a premise is necessary. The types of applications discussed in this work assume that changes are small. In general, however, changes can impact recovery actions or associated common-cause failures. One should consider whether eventual modifications to the PSA model are necessary to properly evaluate the impact of the change. In the case, the incorporation of these changes in the model is required before the importance measure calculations. We refer the reader to Smith, 1998 [37] for a detailed discussion.

Concerning the evaluation of changes affecting multiple basic events we can then proceed as follows. The four possible cases we consider are:

- Case a): changes affecting a single basic event.
- Case c): changes affecting individual parameters.
- Case b): changes affecting multiple basic events.
- Case d): changes affecting multiple parameters.

The need to distinguish between changes affecting individual or multiple basic events is related to the issues in the computation of joint importance measures mentioned in Section 1.

We now discuss the distinctive features in the use of importance measures in the evaluation of changes in the four cases.

- Case a): changes affecting one basic event at a time. After identifying the basic event in the PSA model affected by the change, using the S/RS plane of FV and RAW the safety significance of individual basic events is readily established. The decision-maker can then accept or reject the change based on the S/RS of the affected basic event.
- Case b): changes affecting individual parameters. At the parameter level, one can build the S/RS plane using FV [eq. (9)] and DIM [eq. (10)]. Comparing the $FV_i(\mathbf{x}^0)$ and $DIM_i(\mathbf{x}^0)$ to their thresholds, the decision-maker has information regarding the S/RS of the change[9].

[9] Similarly to what discussed in the previous footnote, to decide on the S/RS of the change, an S/RS threshold for parameter DIM (Th_{DIM}) must be established. One could adopt a uniform contribution criterion, setting $Th_{DIM} = 1/N_p$, with N_p the number of parameters.

- Case c): changes affecting multiple basic events. There are two possible situations. In the first case, one or more of the individual basic events affected by the change is/are risk significant. We note that, as the importance of a single PSA element is lower than or equal to the importance of a set of elements, a change affecting multiple PSA elements will be more safety/risk significant than a change affecting only individual PSA elements. Thus, one can consider a change safety/risk significant, if the change involves at least one safety/risk significant basic event. The second situation foresees that none of the involved basic events is individually safety/risk significant. In this case, the analyst has still no information on whether the combined change is safety/risk significant, as the S/RS plane provides information on the S/RS of basic events individually. One needs then to compute of the joint importance of the basic events. As underlined in Cheok et al (1998) [14] (see also our introduction), this is not feasible with RAW and not straightforward with FV. However, a direct way to obtain an importance measure of joint basic event changes is to make use of DIM. We recall that, under the assumption of proportional changes, DIM produces the same basic event ranking as FV (see also Borgonovo and Apostolakis, 2001 [6] and Borgonovo, 2007 [9]). Furthermore: i) as shown in Borgonovo (2007), one can infer the individual basic event DIM from other importance measures, without running additional calculations; ii) DIM shares the additivity property, and, therefore, the importance of the change is the sum of the individual importance measures (DIM's) of the basic events affected by the change (Borgonovo and Apostolakis, 2001). Thus, the importance of the joint change is readily appreciated using DIM[10]. The use of DIM for the evaluation of joint changes at the basic event level also shares the following interpretation: a) DIM ranks basic events according to their impact on the change in the risk metric; b) the effect of the change is supposed to be a small one, in accordance with

[10] To decide on the S/RS of the change, an S/RS threshold for DIM (Th_{DIM}) must be established. One could adopt a uniform contribution criterion, setting $Th_{DIM} = 1/N_{BE}$, with N_{BE} the number of basic events. The rationale is that, since DIM is the fraction of the change associated with each basic event, if all the basic events contributed in the same way, their contribution would be equal to $1/N_{BE}$. The decision-maker can then compare the sum of the DIM's of the basic events involved in the change against this threshold value and decide on the S/RS of the change.

the underlying philosophy.

- Case d): changes affecting multiple parameters. In this case the evaluation is done using $FV_{j,l,..,k}$ and $DIM_{j,l,..,k}$. We recall that the calculation of $FV_{j,l,..,k}$ requires the identification of the terms in the risk metric that involve any of the parameters in the group, and therefore one has to re-run the FV algorithm. $DIM_{j,l,..,k}$ is the sum of the the DIM's in the group and therefore requires no additional runs. Looking at the S/RS region to which $FV_{j,l,..,k}$ and $DIM_{j,l,..,k}$ belong, the decision-maker has information regarding the S/RS of the change.

The above observations have been stated with reference to the deterministic case. They also hold in the presence of epistemic uncertainty, but with a further feature, which we are to discuss.

In Cases b and d, the following issue is generated by the presence of epistemic uncertainty in expected value assignment [Section 3, eq. (18).] Let $Y_i, Y_j, ..., Y_k$ denote a group of PSA elements. Then, one would need to estimate:

$$\mathbb{E}_F\left[I_{Y_i,Y_j,...,Y_k}\right] \tag{25}$$

In general, importance measure of the group is not the sum of individual importance measures of the elements in the group, i.e.,

$$I_{Y_i,Y_j,...,Y_k} \neq I_{Y_i} + I_{Y_j} + ... + I_{Y_k} \tag{26}$$

Inequality (26) implies that, in general:

$$\mathbb{E}_F\left[I_{Y_i,Y_j,...,Y_k}\right] \neq \mathbb{E}_F\left[I_{Y_i}\right] + \mathbb{E}_F\left[I_{Y_j}\right] + ... + \mathbb{E}_F\left[I_{Y_k}\right] \tag{27}$$

Thus, to compute the expected value of the importance of groups when importance measures are not additive, one would have to repeat the calculations as many times as many the groups of interest are. On the other hand, if $I = DIM$, one has[11]:

$$\begin{aligned}\mathbb{E}_F\left[DIM_{Y_i,Y_j,...,Y_k}\right] &= \mathbb{E}_F\left[DIM_{Y_i} + DIM_{Y_j} + ... + DIM_{Y_k}\right] = \\ &= \mathbb{E}_F\left[DIM_{Y_i}\right] + \mathbb{E}_F\left[DIM_{Y_j}\right] + ... + \mathbb{E}_F\left[DIM_{Y_k}\right]\end{aligned} \tag{28}$$

[11] We recall that the expecation operator is linear, i.e., $\mathbb{E}_F[aX + bY] = a\mathbb{E}_F[X] + b\mathbb{E}_F[Y]$.

Eq. (28) implies that the expected value of the DIM of a group of PSA elements can be found from the individual expected DIM's of the elements in the group, thanks to the linearity of the expectation operator and the additivity of DIM. Thus, the information on group importance under uncertainty can be found through DIM without further model runs.

In the following Sections, we discuss the results of the application of this approach to a reference PSA model.

5 Application: the Effect of Epistemic Uncertainty on Ranking and on Categorization

In this Section, we discuss the numerical findings of propagation of epistemic uncertainty in both the ranking and categorization of the elements of a PSA Model.

The reference model is the large loss of coolant accident (LOCA) sequence of the Advanced Test Reactor, a research reactor located in the National Engineering Environmental Laboratory, Idaho Falls, Idaho (Eide et al, 1991 [15]). The Event Tree used in this paper is the same as in Borgonovo et al, 2003. Two major safety systems are involved in the large LOCA accident, namely the SCRAM system and the Firewater Injection system. Failure of the SCRAM system leads directly to core damage. If the SCRAM system is successful, the Firewater Injection System must also be successful to prevent core damage (CD.)

The model contains 44 basic events, for a total of 289 MCSs (for a detailed list of the parameters and basic events, Borgonovo et al, 2003, p. 180). The number of parameters is 31. The number of parameters is lower then the number of basic events, as the same parameters are used for basic events of identical components to account for complete epistemic dependence (Apostolakis and Kaplan, 1981 [2]). As from the original model, an error factor[12] equal to 10 has been associated with all the basic

[12] As often happens in risk analysis, the lognormal distribution:

$$f_X(x) = \frac{1}{\xi X\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(X)-\eta}{\xi}\right]^2}$$

is expressed through its mean and error factor. The error factor is given by the ratio between the median and the 5^{th} percentile of the distribution, which, in the lognormal case, also equal the ratio of the 95^{th} percentile to the median. From events and no correlation has been assumed among the parameters.

In Section 3, we have seen that, in the presence of epistemic uncertainty, both the ranking and the categorization exercises becomes probabilistic. One then faces the problem of obtaining the estimates of the probabilities in eqs. (14) and (20). To do so, we have extracted the MCS equations produced by the SAPHIRE code and implemented them in an ad-hoc software code. The Monte Carlo sample has been generated through the LpTau quasi-random sequence generator proposed by Sobol' 1967 ([38]) with a sample of size $M = 4096$.

In the remainder of this Section, subsections 5.1 and 5.2 discuss the numerical findings concerning the effects of epistemic uncertainty in the ranking of PSA elements and in their categorization, respectively.

5.1 Ranking

We start with the results for the relative ranking of basic events. Table 1 reports the ranking induced by point estimate values of $FV_j(\mathbf{f}_{IE}^0, \mathbf{p}^0)$ and $RAW_j(\mathbf{f}_{IE}^0, \mathbf{p}^0)$ [eqs. (3) and (4)] as outputted from a standard PSA software code (SAPHIRE, [39]).

[Insert Table 1 about here]

Figure 1 reports the elements of matrix $\mathcal{P}$ [eq. (22)] obtained through uncertainty propagation (eq. 21). Due to space reasons, Figure 1 shows the first 36 columns, with the complete matrix being 44×44.

[Insert Figure 1 about here]

In Figure 1, a grey background indicates an element of $\mathcal{P}$ whose value is around 0.5, (more precisely between 0.40 and 0.60), a lighter grey background indicates an element of $\mathcal{P}$ with value in $(0.6 - 0.75]$

knowledge of the mean and error factor, one can then derive the two parameters (η, ξ) of the lognormal distribution from:

$$\begin{cases} \xi = \frac{\ln(ErrorFactor)}{1.645} \\ \eta = \ln(Mean) - \frac{\xi^2}{2} \end{cases}$$

or in $[0.25 - 0.40]$. A black background would be applied to cells whose value denotes the presence of a ranking reversal (See Section 3 for a description of the potential effects of epistemic uncertainty).

One notes that grey cells appears in groups in Figure 1. In order to understand the results, note that in Table 1 basic events are ranked in groups: basic events $3, 4, 5, 11, 12, 22, 37, 36, 39, 41, 42$ rank 1^{nd}, basic events $6, 7, 8, 9, 10, 12, 13, 14$ rank 14, etc.. The reason is that these basic events are logically related by an "or" gate. In fact, basic events related by an "or" relation have the same RAW, as proven in Borgonovo and Apostolakis, 2001 ([6]; Appendix $C.2$, p. 210). Since a logical relationship is a structural feature, it is unaffected by epistemic uncertainty. Thus, the basic events related by an "or" gate maintain the same RAW in all Monte Carlo trials. Indeed, the appearance of blocks of grey cells in Figure 1 confirms this behavior.

Figure 2 shows the effect of epistemic uncertainty on FV rankings.

[Insert Figure 2 about here]

One notes that now that epistemic uncertainty affects a higher number of elements of P and in a sparse way (rather than in groups). To further understand these results, we have computed the uncertainty ranges of the importance measures and the variability in ranking, reported in Figure 3.

[Insert Figure 3 about here]

In Figure 3, graphs $3a$ and $3c$ report the 10% confidence intervals on the importance measure values and on the ranking for RAW. Graph $3a$ show that there is little overlapping among the uncertainty ranges in RAW. Graph $3c$ shows that rank overlapping appears across groups of basic events. Graphs $3b$ and $3d$ refer to FV. Graph $3b$ shows a high overlapping of the FV values. Correspondingly, graph $3d$ shows a much more diffuse variability in the ranking.

Let us now examing the use of the results of Figures 1 and 2 to answer the question of the degree of confidence in the point estimate categorization. Figures 1 and 2 allow to determine how likely it is that a basic event which is high-ranked in a point estimate analysis is low-ranked when full credit is given to epistemic uncertainty. Let us start with RAW and consider basic events $3, 4, 5, 11, 12, 22, 37, 36, 39, 41, 42$. These basic events rank $1nd$ in the point estimate analysis. Following the rows corresponding to these basic events in Figure 1, one notes that these basic events rank first with probability 1. This result confirms the point-estimate ranking: these basic events are the most risk-significant also in the presence of epistemic uncertainty. Let us then follow what happens for basic events ranked 42 in Table 2. Following the corresponding lines in Figure 2, one notes that the probability with which they become more important than any other basic event is null. Again, one obtains a 100% confirmation of the point estimate calculation. Figure 2 shows that epistemic uncertainty makes the decision-maker less confident in the point-estimate ranking for basic events with "intermediate" importance. For example, there is a 30% chance that basic events $16, 17, 18, 19, 20$ become more important than basic events $6, 7, 8$ and 9. Thus, Figure 1 shows that, a decision-maker using RAW is 100% confident about the most and least relevant basic events, but can have some uncertainty in the intermediate ranking.

Let us now turn to FV rankings (Figure 2 shows the corresponding matrix $\mathcal{P}$; the use of dark cells is the same as in Figure 1). When epistemic uncertainty is propagated, no basic event ranking is confirmed with probability 1. Let us start with the first ranked basic events. Basic event 22 (who ranks first according to point-estimate) has 28%, 33%, 46%, 44%, 29% and 30% chances of ranking lower than basic events 4 (ranked $10), 5$ (ranked 6), 11 (ranked $3), 12$ (ranked $5), 30$ (ranked 4) and 39 (ranked 7) respectively. For basic event 11 (second according to the point-estimate ranking) we note that: i) there are non-negligible ranking exchange probabilities with the top 10 ranked basic events; ii) the value $p_{11,23} = 0.25$ means that there is a 25% chance that basic event 11 ranks lower than basic event 23, which ranks 14. Basic event 30 has 43% and 29% chances to become less important than basic event 37 (which ranks 14) and basic event 38 (which ranks 17) respectively. Basic event 12 has 31% and 29% chances of ranking lower than basic events 35 and 37, which rank 12^{th} and 14^{th} respectively. Basic event 5 has a 30% chance of ranking higher than basic event 22, all other significant p_{ij}'s are with basic events ranking among the first ten. For basic event 39, one notes that the number of p_{ij}'s cells that are highlighted in dark-grey in the corresponding row increases, signalling a lower confidence in the ranking.

Let us now consider basic event 29, which would be the 7th most important basic event according to the point estimate ranking. One notes that there are 17 p_{ij}'s which are now in grey, indicating a higher variability in the raking. In particular, note the following values: $p_{29,17} = 0.49$, $p_{29,31} = 0.28 = p_{29,32}$. The value of $p_{29,17} = 0.49$ shows that basic event 29 ranks ahead than basic event 17 in 49% of the cases. This represents a reversal of the point estimate ranking, which saw basic event 17 as 11^{th} and basic event 29 as 8^{th} (the corresponding cells are black in Figure 2). The values of $p_{29,31}$ and $p_{29,32}$ show that there is a non-negligible probability of basic event 29 becoming less important than basic events 31 and 31, which rank 21^{st} in the nominal ranking. Continuing with the remaining basic events, one notes an increasing effect of epistemic uncertainty. The presence of epistemic uncertainty makes the decision-maker progressively less confident in the point-estimate ranking, and its effects becomes particularly relevant starting from the seventh most important basic event.

The same analysis has been carried out at the parameter level, with FV and DIM as importance measures. Results show the following: i) the response of the two importance measures is similar; ii) the impact of epistemic uncertainty is progressively felt. In particular, a decision-maker is confident about the relative ranking of the first 7 parameters, with an increasing number of elements of $\mathcal{P}$ being close to 0.5, from the 8^{th} ranked parameter onwards.

The analysis carried out in this Section has concerned the impact of epistemic uncertainty on relative rankings. In the next Section, we analyze the impact of epistemic uncertainty on PSA elements categorization .

5.2 Categorization

In this Section, we discuss the effects of epistemic uncertainty in the categorization of PSA elements.

We start with basic events. The following threshold values have been utilized in this exercise: $Th_{RAW} = 10$ and $Th_{FV} = 0.05$. The S/RS categories are displayed in Table 2 (column 5).

[Insert Table 2 about here]

Eight basic events are in the very high S/RS category, ten are in the high S/RS category (six in region II and four in region III) and 27 are in the low S/RS region.

The probabilities [eq. (14)] with which a basic event is assigned to a given category are reported in Table 2, columns 2 to 5. Basic events 41, 42, 43 and 44 are assigned to the corresponding point-estimate S/RS region with probability 1. Of the remaining, 33 basic events are assigned with highest probability to the same S/RS region of the point-estimate categorization, 8 basic events to a different S/RS region. For example, basic event 1 is assigned by point-estimate to S/SRS region IV (i.e., non risk-significant). When epistemic uncertainty is considered, it is assigned to this region with probability 0.89 and, with probability 0.11, it is assigned to region III. Basic event 6 is assigned to region IV (the same as with the point-estimate assignment) with probability 0.72, to region II with probability 0.26 and to region I with probability 0.02. Basic events 4 and 5 have 65% and 54% chances to belong to region II, respectively. However, they were assigned by point estimate categorization to region I. Basic event 23 has a 0.73 probability of belonging to region II, while it is assigned to region I by point estimate assignment. A similar behavior is registered for basic events 17, 19, 29, 30 and 35. For all these basic events, the region to which they belong with the highest probability is different from the point-estimate assignment region. We note that, however, all the probabilistic assignment regions are of lower safety significance than the point-estimate ones. It is possible to explain this last finding, by:

- obtaining the distribution of the importance measures (FV and RAW in this case) for each basic event;
- comparing the distributions to the threshold value;
- computing the probabilities in eq. (13) that the importance measures are below/above the threshold values.

As an example, let us apply the previous steps to basic event 23. The distributions of $FV_{BE_{23}}(\mathbf{x})$ and $RAW_{BE_{23}}(\mathbf{x})$ are reported in Figure 4.

[Insert Figure 4 about here]

The vertical lines in Figure 4 evidence the thresholds. We have that $P(RAW_{BE_{23}}(\mathbf{x}) < Th_{FV}) = 0.005$. Thus, the RAW of basic event 23 lies above the threshold in almost all of the MC trials. Similarly, we have that $P(FV_{BE_{23}}(\mathbf{x}) < Th_{FV}) = 0.73$, i.e., the FV of basic event 23 is below the threshold in 73% of the Monte Carlo trials. On the other side, the point estimate value of the FV for this basic event is 0.062, which is above the threshold. This explains why the basic event is categorized by point-estimate in S/RS region I, but has a high probability of being in S/RS region II. Indeed, 0.73 in Table 2 is the value of the probability that the basic event lies in S/RS region II.

In Table 2, consider basic event 12. Basic event 12 has 51% and 49% chances of belonging to regions I and II, respectively. Thus, the degree of confidence of the decision-maker on the point estimate categorization is low for this basic event. Finally, consider basic events 16 and 17. Let us refer to column 8 in Table 2. This column reports the categorization obtained utilizing the expected value of the importance measures. This type of categorization is defined according to eq. (18). Table 2 shows that the two types of assignment give raise to very similar categorizations. Indeed, the only shift is registered for BE_{16} and BE_{17}, whose S/RS regions are reversed by the two methods. This signals that, for the reference example, the uncertainty terms in eq. (19) do not play a major role, and, thus, $\mathbb{E}_F\left[FV_{BE_i}(\mathbf{x})\right]$ and $\mathbb{E}_F\left[RAW_{BE_i}(\mathbf{x})\right]$ are mainly driven by the their point estimates.

Let us now turn to the categorization of parameters. The same procedure has been followed as for the basic events. The point-estimate categorization at the parameter level is reported in Table 3.

[Insert Table 3 about here]

Let us first discuss the point-estimate results for the model parameters. We recall that the model is non-linear at the parameter level, as one or more parameters are shared by basic events that appear in the same MCS's, due to the complete epistemic dependence for the failure rates of identical components. Figure 6 displays the parameter S/RS plane, with $Th_{DIM} = Th_{FV} = 0.008$.

[Insert Figure 6 about here]

We note that 17 parameters fall in the very high S/RS category, 3 in the high S/RS (zero in region II and seven in region III), and 11 in the low S/RS category.

To analyze the effect of epistemic uncertainty, let us refer to Table 3. We note that the region of highest probability for parameters 3, 6, 9, 15, 20, 23, 28 is region I, while they are assigned to regions of lower S/RS by the point estimate calculations. The explanation of these result is found by applying the three steps discussed above for the analysis of the distributions. In particular, one finds values of $P(DIM_i(\mathbf{x}) > Th_{DIM}) > 0.7$ for all these parameters.

Figure 6 also reports the S/RS regions obtained with expected value assignment. We note that: *i*) twenty-one parameters are ranked in S/RS region I, four more than with the point estimate analysis; *ii*) one parameter is in S/RS region II, while three parameters were in this region with the point estimate categorization (see Table 3); *iii*) nine parameters are assigned to S/RS region IV, while eleven were assigned with the point estimate rankings. Thus, expected value categorization differs more than point estimate categorization at the parameter level than at the basic event level, for this case study.

Overall, the analysis confirms Cheok et al (1998)'s statement reported in the introduction that uncertainty is relevant in the integrated decision making process (see also Modarres and Agarwal, 1996 [31]). The previous findings have shown that, different assignment methods can bring to different basic event and parameter categorizations, due to the different information that each of the methods conveys. Ideally, one would like to retain the information of all the methods, as in Tables 2 and 3. These tables allow decision makers to compare the different categorization, providing their degree of confidence in the deterministic categorization results.

In the next Section, we show that the information gained through the basic event and parameter categorizations can be utilized in the evaluation of changes that alter the status quo of the plant.

6 An Application: the Evaluation of Changes with Epistemic Uncertainty

In this Section, we apply the categorization results of the previous Section to the evaluation of the following changes:

- ChA-1: change in the testing interval on the SCRAM system.
- ChA-2: change in QA procurements on deep-well pump 1.
- ChA-3: change in QA procurement on the three pumps.

We start with the modeling of ChA-1. ChA-1 affects the testing of the SCRAM system. The testing scheme has the purpose to check that the sensors transmit the proper signal to the electronic sub-logic. This is represented in the model by basic event 42. If no epistemic dependence is considered and a unique basic event is involved in the change, one can model the change as affecting an individual basic event. The evaluation is then straightforward using the categorization of basic events obtained in Section 5. We make reference to Table 6. We note that BE_{42} is ranked in region II by the point estimate results. Therefore, one would conclude that ChA-1 affects a PSA element that is in the high S/RS category. Let us then investigate whether the consideration of epistemic uncertainty can change this conclusion. Table 6 still shows that expected value results would place the basic event in a region of high S/RS, while it is in region IV with probabilistic assignment. The decision-maker can use this information to accept or reject ChA-1.

We now focus on ChA-2. The change affects pump 1 random properties, namely its failure to start and failure while running. Since pumps 2 and 3 are not affected by ChA-2, ChA-2 must be modeled as affecting pump 1 individually. Therefore we can evaluate the change as altering basic events BE_{18} and BE_{19} that represent these two failure modes in the PSA model. Using Table 2, we note that BE_{18} and BE_{19} are not individually S/RS, since they are assigned to S/RS regions IV by all methods. To have information on their combined effect, we follow the approach proposed in Section 4, evaluating the change through the joint DIM, $DIM_{BE_{18},BE_{19}}$. The individual DIMs are computed through simple manipulations by exploiting the availability of FV results (see Borgonovo, 2007). We get: $DIM_{BE_{18}}(\mathbf{x}^0) = DIM_{BE_{19}}(\mathbf{x}^0) = 0.0042$. Thus, $DIM_{BE_{19},BE_{19}}(\mathbf{x}^0) = DIM_{BE_{18}}(\mathbf{x}^0) + DIM_{BE_{19}}(\mathbf{x}^0) = 0.0084$. Comparing this number with $Th_{DIM_{BE}}$, the analyst has information on whether the change exceeds or not the threshold. This information can than be used to accept or reject the change.

We are now left with the evaluation of ChA-3. ChA-3 has to be modeled at the parameter level, since it affects a set of redundant components (see Section 4). In particular, the change affects parameters 14 and 15 that are shared among the three pumps and represent their failure-to-start probability and failure-to-run rate, respectively.

Referring to Table 8, we note that both parameters are assigned to S/RS region I, by both point estimate and expected value assignments. Their combined effect, can be evaluated through the use of their joint FV and DIM. Let us start with the point estimate assignment. In this case, $FV_{14,15}(\mathbf{x}^0) = 0.070$ and their joint $DIM_{14,15}(\mathbf{x}^0) = DIM_{14}(\mathbf{x}^0) + DIM_{15}(\mathbf{x}^0) = 0.03077$. Then, one has all the quantitative information necessary to evaluate the change, by comparing these values to the threshold values. This information is then used by the decision-maker to accept or reject (most likely) the change.

Finally, suppose that one wants to compare this result with the one She/He would obtain using the expected values assignment. Then, to find $\mathbb{E}_F[FV_{14,15}(\mathbf{x})]$ one has to repeat Monte Carlo trials computing $FV_{14,15}(\mathbf{x})$ in each of the trials. With DIM, however, the expected importance of a group is straightforwardly obtained as sum of the expected DIM's in the group [eq. (28).] Thus, in the above example, using Table 7 one obtains: $\mathbb{E}_F[DIM_{14,15}(\mathbf{x})] = \mathbb{E}_F[DIM_{15}(\mathbf{x})] + \mathbb{E}_F[DIM_{15}(\mathbf{x})] = 0.03073$.

We also remark that we did not formulate the final decision-maker conclusions while commenting the three changes evaluation. This has had the purpose of maintaining this application an exercise and not to give general conclusions for two reasons: 1) we are dealing with just one sequence of the PSA model. The basic events of this example also appear in other sequences; 2) the decision-maker final resolution on the changes does not only involve quantitative information derived by importance measures, but undergoes a peer review through a panel of experts (Apostolakis, 2005 [5]) which integrates quantitative and qualitative information.

7 Conclusions

We have presented a methodical approach that allows a risk informed decision-maker to derive information on the categorization of PSA elements in the presence of epistemic uncertainty.

In the presence of epistemic uncertainty, a decision-maker cannot rank SSC's with certainty, nor assign them SSC to a safety/risk significance (S/RS) regions with certainty, but only with a given probability. We have then structured the framework and proposed a method for the estimation of such probability. For the ranking exercise, we have formalized the notion of probabilistic ranking through probability matrix $\mathcal{P}$, whose entries represent the probabilities that basic event (or another PSA element) i is more important than element j. We have discussed its properties and its use in the determination of the decision-maker's degree of confidence in the deterministic ranking.

For the categorization, we have discussed two procedures to assign SSC's to S/RS regions in the presence of epistemic uncertainty. The first, called probabilistic assignment, consists in estimating the probability with which a certain PSA element belongs to a given S/RS region. We have provided a formal definition of these probabilities and the relevant Monte Carlo estimation equations. We have also formalized a categorization procedure based on the expected value of the importance measures. We have discussed that such an assignment method would be a bridge between a deterministic and a probabilistic categorization.

We have then investigated the impact of epistemic uncertainty on the evaluation of changes. We have seen that changes that affect PSA model elements related by epistemic dependence (that is the case of redundant identical components) are properly modeled at the parameter level. On the other hand, changes that affect individual or multiple failure modes, where no epistemic dependence is involved, are properly modeled at the basic event level.

We have presented the application of the proposed framework to the PSA model of the Large LOCA sequence of the ATR reactor. We have discussed results for the probability matrix P, and for the categorization of basic event and parameters. We have compared the deterministic categorization with the categorization obtained assigning basic events and parameters to safety regions based on the expected value of the importance measures. We have discussed how the information obtained through these procedures allows decision makers to appreciate "*the sensitivity of component categorizations to uncertainties in the parameter values* [Regulatory Guide 1.174]".

We have then illustrated, through the same case study, the evaluation of three changes, which require modeling both at the basic event and parameter levels. We have seen that the S/RS of changes affecting individual basic events or parameters can be directly established from the S/RS plane. For changes affecting multiple basic events, a two step evaluation has been performed, with the second step relying on DIM to compute the importance of groups.

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