Uncertainty and Global Sensitivity Analysis in the Evaluation of Investment Projects

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Abstract

This paper discusses the use of global Sensitivity Analysis (SA) techniques in investment decisions. Global SA is a branch of Statistics that complements and improves Uncertainty Analysis (UA) providing the analyst/decision-maker with information on how uncertainty is apportioned by the uncertain factors. In this work, we introduce global SA in the investment project evaluation realm. We then need to deal with two aspects: 1) the identification of the appropriate global SA method to be used and 2) the interpretation of their results from an investment uncertainty point of view. For task 1), we compare the performance of two family of techniques: non-parametric and variance decomposition based. For task 2), we explore the determination of the cash flow global importance (GI) for valuation criteria utilized in investment project evaluation. For the Net Present Value (NPV), we show that it is possible to derive an analytical expression of the cash flow GI, which is the same for all the techniques. This knowledge enables us to: 1) offer a direct way to compute cash flow GI; 2) illustrate the practical impact of global SA on the information collection process. For the Internal Rate of Return (IRR), we show that the same conclusions cannot be driven. In particular, a) one has to utilize a numerical approach for the computation of the cash flow influence, since an analytical expression cannot be found and b) different techniques can produce different ranking. These observations are illustrated by means of the application to a discounted cash flow model utilized in the energy sector for the evaluation of projects under survival risk . The quantitative comparison of cash flow ranking with respect to the NPV and IRR concludes the paper, illustrating that information gained on the NPV through global SA cannot be transferred to the IRR.

1 Introduction

This paper introduces the use of global Sensitivity Analysis (SA) techniques in investment valuation. When firms deal with investment projects, many factors are uncertain. Uncertainty Analysis (UA) is usually performed as part of the decision-making (DM) process, and dedicated subroutines are nowadays included in the most diffuse business software (Excel or Lotus) or in dedicated software packages as the one discussed in [31]. Through uncertainty propagation, the decision maker (DMr) is able to understand his/her degree of confidence in the decision ([1]), and to assess the project risk obtaining information about the likelihood of favorable and adverse scenarios ([31], [3], [12], [6], [13]). In the presence of limited time and resources, it could be of great help to the DM information on the factors on which to devote data collection resources so that to reduce uncertainty most effectively/rapidly. This information cannot be obtained by means of the standard UA methods available on most industrial DM software [18], but have to be accompanied by a global SA exercise. Several global SA methods have been recently developed in the literature, and they have not yet entered the investment project valuation realm ([2], [5], [8], [9], [10], [11], [12], [13], [14], [15], [17], [19], [18], [20], [24], [27], [28], [21], [22], [23]).

It is the purpose of this paper to illustrate the utilization and meaning of global SA in the uncertainty management of investment project evaluation. We undertake the analysis in two steps. The first step is the identification of the appropriate techniques to estimate cash flow GI in discounted cash flow (DCF) valuation models ([29], [3], [4]). The second step is the analysis of the application of cash flow GI and of its role in the DM process. For the first step, we examine the following global SA techniques: Sobol' global sensitivity indices $[S_r(x_i)$[1]$]$ ([28], [21], [22], [24]), the Pearson Correlation Coefficients ($PEAR$) and the Standardized Regression Coefficients (SRC) [20]. $S_r(x_i)$ belong to the family of variance decomposition based (VDB) techniques and estimate the GI of a parameter by means of the complete decomposition of the model variance. $PEAR$ and SRC are non-parametric (NP) global SA techniques and compute the GI by means of a regression of the output on the uncertain parameters.

We show that, if the DMr selects as a valuation criterion a Net Present Value (NPV) or one of its generalized forms, then the GI of cash flows: a) can be computed analytically; b) coincides with the fraction of the NPV variance associated with the cash flow; is equivalently estimated by all the techniques; c) has a straightforward interpretation

[1] Table 1 summarizes the acronyms used in this work

Acronyms	**Name**
UA	Uncertainty Analysis
SA	Sensitivity Analysis
VDB	Variance Decomposition Based
DM	Decision-Making
DMr	Decision-Maker
NP	Non Parametric
GI	Global Importance
$PEAR$	Pearson correlation coefficient
SRC	Standardized regression coefficient
$S_r(x_i)$	Sobol' Global Sensitivity Indices of order r
$S_T(x_i)$	Sobol' total indices
FAST	Fourier Amplitude Sensitivity Test

Table 1: Acronyms used in this work

in terms of the DMr uncertainty. It is then possible to study the relationship between GI, timing and uncertainty of a cash flow. We show that, if the DMr selects an Internal Rate of Return (IRR) to value the investment, an analytical approach is not feasible. As a consequence, not all the techniques can be equivalently used to estimate cash flow GI. In particular, NP techniques should not be utilized since their ability to correctly estimate cash flow global importance declines as the model becomes non-linear.

For the second step, we show that these results have a direct impact on the information collection process. To to so, we apply the previous results to the global SA of a sample model utilized in the energy sector for the evaluation of projects under survival risk ([4]). The model estimates three investment criteria: NPV, value at any time t (V_t), and IRR. For the project NPV and V_t the computation of the cash flow GI is direct thanks to the analytical results mentioned above. In particular, once the DMr has assessed a standard deviation of the cash flows — which is a direct output of a standard UA — then the computation of the GI is direct [eq.(12)]. From GI the DMr has immediate information on the influence of the cash flows on the DMr uncertainty. Reducing uncertainty in these cash flows associated to the highest values of GI would then provide the most effective way to reduce the uncertainty in NPV or V_t. We also show that knowing GI the DMr has immediate information on how much His/Her uncertainty would be reduced by gathering additional information. For the project IRR, we compute the cash flow GI numerically, comparing the results of $S_T(xi)$ computed via Extended FAST, $PEAR$ and SRC for the global SA of the investment IRR. We rank cash flows based on their GI w.r.t. the IRR. We then compare the cash flow ranking produced by the NPV global SA to the IRR global SA ranking, obtaining quantitative information through Savage Scores ([6], [8]). The comparison shows little agreement between the ranking: cash flows do not have the same influence on uncertainty in the investment NPV as they have on uncertainty in the investment IRR. As a consequence, global SA results for the NPV cannot be directly transferred to the IRR. More specifically, if one collects information on a cash flow which is influential on the IRR, one would not reduce uncertainty in the NPV effectively and vice-versa.

In Section 2, we present the principles of global SA and the techniques used in this paper. In Section 3, we discuss the global SA of equity valuation models. Section 4 presents the application of the results and techniques of Section 3 to a project evaluation model proposed by Beccacece, Gallo and Peccati [4] and in use in the energy sector. Conclusions are offered in Section 5.

2 Global Sensitivity Analysis

Let us denote the generic model output as:

$$Y = f(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = (x_1, x_2, ..., x_n)$ is the set of input parameters. For the purposes of this Section, $\mathbf{x}$ will be a uniformly distributed random vector in the unitary hypercube.

By global importance of x_i $[GI(x_i)]$ one means the relevance of x_i on Y when the entire distributions of $\mathbf{x}$ and Y are considered.

Techniques explored in the early literature to assess $GI(x_i)$ foresee the use of correlation and regression based methods that are classified under the name of Non-Parametric (NP) global SA techniques ([20]). Saltelli and Marivoet (1990) provide a thorough discussion of these techniques, their properties and applicability [20]. In this work, we compare the performance of two non-parametric techniques, PEAR and SRC [20], of which we report the definitions used in this work below, for the sake of notation clarity:

$$PEAR(x_i) = \frac{Cov(Y, x_i)}{\sigma_i \sigma_Y} \tag{2}$$

$$SRC(x_i) = \frac{b_i \sigma_i}{\sigma_Y} \tag{3}$$

where σ_i and σ_Y are the standard deviations of x_i and Y, respectively, and b_i is the coefficient of the multivariate linear regression of Y on $\mathbf{x}$.

NP techniques have the main advantage of simplicity from a computational point of view ([20], [24], [18], [28]). However, their ability in estimating correctly the global importance of an assumption is heavily linked to the linearity of the model, and it deteriorates as the model becomes non linear or interactions among parameters emerge. In the case of non-linear models, ranking parameters according to these techniques could become misleading.

The shortcomings of NP global SA techniques are overcome by the use of VDB techniques. Sobol' ([23]) proved that, under the assumption that the x_i are independent, the following decomposition of $f(\mathbf{x})$ is unique [23]:

$$f(\mathbf{x}) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i<j} f_{i,j}(x_i, x_j) + ... + f(x_i, x_j, ..., x_n) \tag{4}$$

with

$$\begin{array}{c} f_0 = \overline{Y} = \int \cdots \int f(\mathbf{x}) d\mathbf{x} \\ f_0 + f_i(x_i) = \int \cdots \int f(\mathbf{x}) \prod_{k \neq i} dx_k \\ f_0 + f_i(x_i) + f_{i,j}(x_i, x_j) = \int \cdots \int f(\mathbf{x}) \prod_{k \neq i,j} dx_k \\ \cdots \end{array} \tag{5}$$

The $f_{i,j,...k}(x_i, x_j, ..., x_k)$ terms in eq. (4) are orthogonal, and such that $\int f(x_i, , ...x_s) \prod_{k=i...s} dx_k = 0$. As a result of these properties, the variance of Y can be written as ([23], [28], [19]):

$$V_x[Y] = \sum_{i=1}^{n} V_i + \sum_{i<j} V_{i,j} + \sum_{i<j<m} V_{i,j,m} \cdots + V_{1,2,...n} \tag{6}$$

where

$$V_{i,j,...,m} = \int \cdots \int [f_{i,j...,m}(x_i, x_j, ..., x_m)]^2 \prod_{k=i,j,...,m} dx_k \tag{7}$$

$V_{i,j,...,m}$ are called partial or conditional variances ([2], [14], [19], [11], [10]). They are also deemed as interaction terms of order r, where r is the number of parameters they involve. $\sum_{i=1}^{n} V_i$ represents the portion of the model variance explained by the individual parameter uncertainty. Similarly, $\sum_{i<j} V_{i,j}$ is the portion of σ_Y^2 explained by terms containing parameter pairs and so on.

One defines the global sensitivity indices of order r $[S_r(x_i)]$ as the ratios of the individual terms in eq. (6) and $V_x[Y]$ ([28], [24], [21], [28]):

$$S_r(x_i) = \frac{\sum_{i<j...<m} V_{i,j,...,m}}{V_x[Y]} \tag{8}$$

Technique	Type	Computational Cost	Model Dependent	Complete σ^2 Decomposition
PEAR	NP	N	Yes	No
SPEAR	NP	N	Yes	No
Extended FAST	VDB	N	No	No
$S_r(x_i)$	VDB	$N \times 2^n$	No	Yes

Table 2: Advantages and Limitations of global SA techniques

The total order global sensitivity index $[S_T(x_i)]$ of parameter x_i is the sum of all the global sensitivity indices related to it:

$$S_T(x_i) = \frac{V_i + \sum_{j \neq i} V_{i,j} + ... + V_{1,2,...n}}{V_x[Y]} \tag{9}$$

i.e. the ratio of all individual and interaction terms involving x_i and $V_x[Y]$. Then, being $GI(x_i)$ the number that synthesizes the influence of the uncertainty in x_i on the uncertainty in Y, one defines the parameter global importance as ([28], [24], [6]):

$$S_T(x_i) = \frac{V_i + \sum_{j \neq i} V_{i,j} + ... + V_{1,2,...n}}{V_x[Y]} = GI(x_i) \tag{10}$$

In the remainder of the paper, $S_T(x_i)$ will denote the estimate of $GI(x_i)$ obtained by VBD techniques. Numerical estimation methods are the Fourier Amplitude Sensitivity Test (FAST) [24], the method of Sobol' ([27], [28]), and others ([14], [15]). We note that, the estimation of $GI(x_i)$ through VDB techniques becomes model-independent, but can be more costly from a numerical point of view than the estimation through NP techniques. In particular, if one wants to obtain the numerical assessment of $S_r(x_i)\ \forall r$, then one must utilize Sobol' numerical approach, whit a computational cost of $N \times 2^n$, where N is the sample size used in the Monte Carlo uncertainty analysis. If $GI(x_i)$ is estimated via Extended FAST, then the number of model runs required is N, but only $S_1(x_i)$ and $S_T(x_i)$ are estimated. The possible high computational cost can then make the use of Extended FAST or of NP techniques preferable, in spite of the non-estimation of the interaction terms. Table 2 summarizes these observations.

3 Global Sensitivity Analysis of Valuation Models

This Section discusses the global SA of equity valuation models. We assume that the analyst represents his/her uncertainty in the problem by assigning an epistemic distribution ([1]) to the cash flows [2]. We start with the global SA of valuation criteria expressed in an NPV form.

3.1 NPV Global SA

For NPV-like models, we write the valuation equations as follows ([7], [29]):

$$Y = f(\mathbf{x}) = \sum_{i=1}^{n} a_i x_i \tag{11}$$

where $\mathbf{x}$ is the vector of the cash flows, and $\mathbf{a}$ the appropriate discount factor.

The application of Sobol' variance decomposition [eq. (6)] to eq. (11) leads to the following expression of $GI(x_i)$:

$$GI(x_i) = \frac{a_i^2 \sigma_i^2}{\sigma_Y^2} = S_1[x_i] = S_T[x_i] \tag{12}$$

Applying the definition of $PEAR(x_i)$ and $SCR(x_i)$ to eq. (11), it is easy to see that:

$$PEAR(x_i) = \frac{a_i \sigma_i}{\sigma_Y} = SRC(x_i) \tag{13}$$

Thus, in the case of eq. (11), $SRC(x_i)$, $PEAR(x_i)$ and $S_T(x_i)$ are related as follows:

$$S_T(x_i) = S_1[x_i] = SRC(x_i)^2 = PEAR^2(x_i) = GI(x_i) \tag{14}$$

Eqs. (12), (13) and (14) suggest that:

- $GI(x_i)$ is the fraction of the NPV variance associated with x_i
- $GI(x_i)$ contains no interaction terms
- $SRC(x_i)$, $PEAR(x_i)$ and $S_T(x_i)$ techniques produce the same ranking [eq. (14)]. Thus, they can be equivalently used in identifying the most influential cash flows with respect to (w.r.t.) the investment NPV.
- $GI(x_i)$ is independent of the particular type of distribution chosen to characterize x_i, but dependent only on the cash flow variance.

[2] For the purposes of this work, we assume that cash flows are uncorrelated. The analysis of correlations and their effects are discussed in a separate work of the authors: Borgonovo and Peccati: "The Uncertainty and Global Sensitivity Analysis of Valuation Equations in the Presence of Cash Flow Correlations," IMQ, Bocconi University, Work in progress.

- eq. (14) provides the expression of *GI(x_i)* in portfolio models, where x_i are the (floating) stock prices and a_i are the (fixed) weights.

- eq. (14) enables one to study the relationship between cash flow importance and timing [*GI(x_i)*$\lesseqgtr$*GI(x_{i+1})*]. For example, in [7] and [4], a_i is written as:

$$a_i = c_i/(1+k)^i \tag{15}$$

  where k is the appropriate discount rate for x_i, and c_i denotes a coefficient related to the probability of receiving the cash flow, independently of its amount ([7], [4]). Then one finds that *GI(x_i)*>*GI(x_{i+1})*$\forall i$, if the following condition is satisfied:

$$1 > \frac{\sigma_{i+1}}{\sigma_i}\frac{c_{i+1}}{c_i}\frac{1}{(1+k)} \quad . \tag{16}$$

  In eq. (16), $\frac{\sigma_{i+1}}{\sigma_i}$ is the ratio of the uncertainty in the value of x_i and x_{i+1} ("epistemic uncertainty", [1]), $\frac{c_{i+1}}{c_i}$ is the ratio of the probability of receiving the cash flows, and $\frac{1}{(1+k)}$ is the discount factor for one period. $GI(x_i)$ will not necessarily follow the cash flow timing, if the effect of the individual cash flow epistemic uncertainty and/or probability distorts the discounting effect.

Let us discuss some implications of the above results from the DMr point of view:

- Once the DMr has assessed His/Her uncertainty in the cash flows then no additional model runs or Monte Carlo based approach are required. It is in fact enough that the DMr assigns/estimates a standard deviation of the cash flows and the computation of $GI(x_i)$ is directly performed through eq.(12).

- From eq.(12) the DMr has immediate information on which of the cash flow to gather informations in order to reduce uncertainty in the most effective way.

- Furthermore, from eq.(12) the DMr has immediate information on how much His/Her uncertainty would be reduced if gathering additional information reduced uncertainty in one or more of the cash flows.

Calculations in Section 4 illustrate these observations.

3.2 IRR Global SA

In the IRR case, using the notation of eq.(15), eq. (11) becomes ([7]):

$$\sum_{i=0}^{n} \frac{c_i x_i}{(1+IRR)^i} = 0 \tag{17}$$

([7], [4]). As opposite to the NPV case, an analytical expression of the IRR global importance cannot be obtained, in general. Thus, one has to resort to a numerical approach for the estimation of the cash flow global importance with respect to the project IRR [$GI^{IRR}(x_i)$]. The numerical approach involves the appropriate generation of random numbers following the x_i probability density functions and the computation of the project IRR corresponding to each sample generation ([27], [24]). Due to the lack of an analytical expression, comparisons of the estimates of $GI^{IRR}(x_i)$ and the cash flow ranking obtained using different global SA techniques (*i.e.* $S_T(x_i)$ vs $PEAR(x_i)$ or $SRC(x_i)$) will be possible only on a case by case basis. With this respect, we remark that the performance of NP techniques is expected to decrease rapidly if non-linear or interaction effects emerge in the model. In such cases, utilizing the results of these techniques to rank input parameters can be misleading ([6], [8], [14], [20]). Due to the non-linearity of IRR-like equations as eq. (17), therefore, VDB will be the most appropriate global SA techniques to be applied ([24], [21], [28], [12], [10], [11]). The numerical application in Section 4.3 will demonstrate these observations.

4 Application: Project Under Severe Survival Risk

In this Section, we present the application of the results of Section 3 to the global SA of a valuation model elaborated for the assessment of projects under serious survival risk and utilized in the energy sector ([4], [7]). The model was developed in a joint-venture, to be a tool agreed upon the partners to evaluate investment projects in regions where environmental and external risks could pose a threat to the life of the project at any time [4]. It is known that risks of an idiosyncratic nature cannot be factored directly into the cost of capital k [4]. Thus, the model utilizes an alternative approach through the introduction of extinction or survival probabilities (Q_j) [4]. Q_j is the probability of the project dying at year j. Since it is a certain event that the project life stops after period m, then $\sum_{s=1}^{m} Q_s = 1$. The investment is subdivided into a cost period and a cash-generating or operation period, with $m = n + k + 1$, where n is the number of cost periods and k the number of cash-generating periods. We denote the generic cash flow of period i with x_i. With particular reference to the cost period, the cash flows are denoted as γ_s, and with r_s in the income period. The model assumes that the s^{th} cash flow is received at the end of period s. The model estimates three valuation criteria, the project net present value (NPV), the value of the project at any time t (V_t) and the project internal rate of return (IRR) [7].

The cash flow discounting equations are generalized though the presence of extinction probabilities as follows ([4], [7]).

1. The project NPV ([4], [7]):

$$NPV = \sum_{j=0}^{m-1} \frac{x_j \sum_{s=j+1}^{m} Q_s}{(1+k)^j} \tag{18}$$

2. The project value at any time t, V_t ([4], [7]):

$$V_t = \frac{\sum_{h=t+1}^{m-1} x_h (1+k)^{t-h} \sum_{j=h+1}^{m} Q_j}{\sum_{s=t+1}^{m} Q_s} \tag{19}$$

3. The project IRR ([4], [7]):

$$0 = \sum_{j=0}^{m-1} \frac{x_j \sum_{s=j+1}^{m} Q_s}{(1+IRR)^j} \tag{20}$$

Rewriting the model based on the framework of eqs. (11) and (15), we have:

$$c_i = \sum_{s=i+1}^{m} Q_s \tag{21}$$

$$a_i^{NPV} = \frac{\sum_{s=i+1}^{m} Q_s}{(1+k)^i}, \quad i = 0, 1, .., n+k \tag{22}$$

and

$$a_i^{V_t} = \begin{cases} \frac{\sum_{j=i+1}^{m} Q_j / c_t}{(1+k)^{i-t}} & if\, i > t \\ 0 & if\, i \leq t \end{cases}, \quad i = 0, 1, .., n+k \tag{23}$$

Table 3 reports the expected value of the cash flows and the analyst uncertainty in the cash flow value (σ_i). k is set at 8%.

Cash flows of the cost period are associated with less uncertainty than in the income period, since more information is available to the DMr on the former.

$\mathbf{x_i}$	**Base Case/Expected Value**	$\boldsymbol{\sigma_i}$
γ_0	0	0
γ_1	-10	2.5
γ_2	-30	2.5
γ_3	-50	5
γ_4	-70	5
r_5	600	50
r_6	750	25
r_7	900	50

Table 3: Cash flows numerical values

4.1 Global SA of the Project NPV

The NPV variance is given by:

$$\sigma_{NPV}^2 = \sum_{j=0}^{m-1} \left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^2 \tag{24}$$

Eq. (24) shows that σ_{NPV}^2 is given by the sum of the cash flow uncertainties (σ_j^2), discounted by $(1+k)^{-2j}$ and by the cash flow probabilities $(\sum_{s=j+1}^{m} Q_s)^2$. Using eqs. (24) and (10), the cash flow global importance with respect to the project NPV [$GI^{NPV}(x_i)$] is:

$$GI^{NPV}(x_i) = \frac{\left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^2}{\sigma_{NPV}^2} \tag{25}$$

As far as cash flow timing is concerned, for this model condition (16) becomes:

$$\frac{\sigma_i}{\sigma_{i+1}} > \frac{\sum_{s=i+2}^{m} Q_s}{\sum_{s=i+1}^{m} Q_s} \frac{1}{(1+k)} \quad \forall i, j \tag{26}$$

In eq. (26) the left hand side is always lower than 1, since $\sum_s Q_s$ decreases as s increases. Thus, deviations of the cash flow ranking from their timing will eventually be caused by σ_i.

The assumptions in Table 3, lead to the following numerical results for $GI^{NPV}(x_i)$ (Figure 1):

Table 4 below presents the cash flow ranking:

The most important cash flow is r_5, followed by r_6, γ_3, γ_1, r_7, γ_2, γ_4, γ_0 respectively. We recall that this ranking would be produced by any of the global SA techniques discussed in this paper, since eq. (14) holds (Section 3), if one utilized a numerical approach.

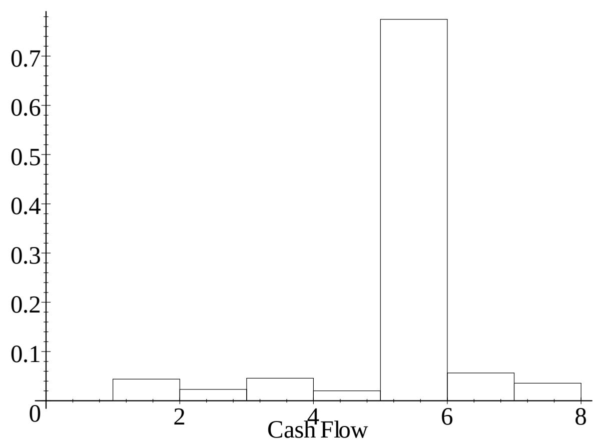


Figure 1: $GI^{NPV}(x_i)$

Cash Flow	γ_0	γ_1	γ_2	γ_3	γ_4	r_5	r_6	r_7
Ranking	8	4	6	3	7	1	2	5

Table 4: Cash Flow Ranking with PEAR, SRC and VDB for the global SA of the project NPV

As far as timing is concerned, table 4 shows that the ranking of the cash flows differs from their timing. Recalling eqs. (16) and (26), this result indicates that the individual cash flow epistemic uncertainty (σ_i) prevails on the discounting and probability effects.

In Section 3, we showed that $GI^{NPV}(x_i)$ coincides with the fraction of the NPV variance associated with x_i [eq.(12)]. Thus, the results of figure 1 share the following interpretation: around 96% of the model variance is originated by the cash flows of the revenue period, with 77% of the uncertainty associated with r_5. Based on the concluding remarks of Section 3, this result has a direct implication from an uncertainty management point of view. In fact, suppose that one would be able to know the value of r_5 exactly. At that moment $\sigma_5 = 0$. The NPV variance would follow from 80 to 17, with a 77% reduction. However, if one would come to know, say, γ_3 exactly, the variance would fall from 80 to 76 with only a 4.6% reduction.

Let us now discuss the global SA of the same model, but in the presence of higher DMr uncertainty. In particular, we consider a uniform increase in the cash flow uncertainties of 100%, *i.e.* $\sigma_i' = 2\sigma_i$. The new standard deviations are displayed in Table 5:

Cash Flow	γ_0	γ_1	γ_2	γ_3	γ_4	r_5	r_6	r_7
Standard Deviation	0	5	5	10	10	100	50	100

Table 5: The standard deviations in the case of increased uncertainty

The resulting NPV variance is now four times the previous one, in fact:

$$\sigma^{2\prime}_{NPV} = \sum_{j=0}^{m-1} \left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^{2\prime} = \sum_{j=0}^{m-1} \left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 4\sigma_j^2 = 4 \sum_{j=0}^{m-1} \left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^2 \tag{27}$$

The cash flows global importance is the same as before, since:

$$GI^{NPV}(x_i)' = \frac{\left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^{2\prime}}{\sigma^{2\prime}_{NPV}} = \frac{4 \left[\frac{\sum_{s=j+1}^{m} Q_s}{(1+k)^j} \right]^2 \sigma_j^2}{4\sigma^2_{NPV}} = GI^{NPV}(x_i) \tag{28}$$

In this case, the reduction of uncertainty associated with knowing cash flow r_5 exactly, would be still of 77% of the uncertainty, *i.e.* σ'_{NPV} would decrease from 318 to 72. We note that with the uncertainty of the previous case (Table 3) the NPV standard deviation was worth 79. This means that annulling uncertainty in the most influential cash flow (r_5) would reduce variance more effectively than halving the uncertainty in all the cash flows.

As a result of this analysis, global SA results point out how to direct resources in information collection so as to reduce uncertainty in the most effective way. Such an information can be particularly useful to the DMr in the presence of limited time and resources.

4.2 Global SA of V_t

Let us now consider the global SA of the project value at time t, V_t. $\sigma^2_{V_t}$ is given by:

$$\sigma^2_{V_t} = \sum_{h=t+1}^{m} \sigma_h^2 \left[\frac{(1+k)^{t-h} \sum_{j=h+1}^{m} Q_j}{\sum_{s=t+1}^{m} Q_s} \right]^2 \tag{29}$$

$\sigma^2_{V_t}$ is plotted in Figure 2 for the input data of Table 3.

Applying Sobol' variance decomposition theorem to eqs. (19), $GI^{V_t}(x_i)$ turns out to be:

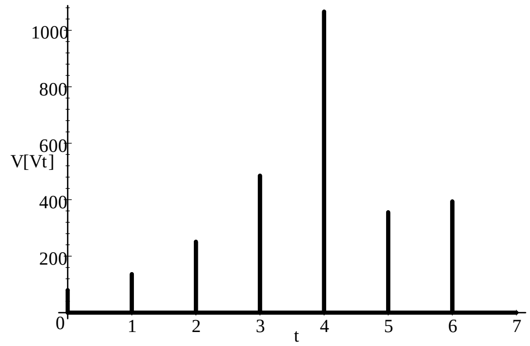


Figure 2: $\sigma^2_{V_t}$ as a function of t

t	1	2	3	4	5	6	7
γ_1	4						
γ_2	6	5					
γ_3	3	3	3				
γ_4	7	6	5	4			
r_5	1	1	1	1	1		
r_6	2	2	2	2	2	1	
r_7	5	4	4	3	3	2	1

Table 6: Cash Flow ranking as a function of time

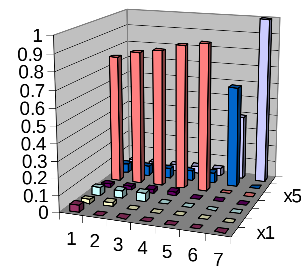
$$GI^{V_t}(x_i) = \begin{cases} \dfrac{\sigma_i^2\left[\frac{(1+k)^{t-i}\sum_{j=i+1}^{m}Q_j}{\sum_{s=t+1}^{m}Q_s}\right]^2}{\sum_{h=t+1}^{m}\sigma_h^2\left[\frac{(1+k)^{t-h}\sum_{j=h+1}^{m}Q_j}{\sum_{s=t+1}^{m}Q_s}\right]^2} & if\, i > t \\ 0 & if\, i \leq t \end{cases}, \; i = 0, 1, .., n+k \tag{30}$$

$GI^{V_t}(x_i)$ is the fraction of the project value variance associated with x_i at any time t.

Figure 3 shows $GI^{V_t}(x_i)$ as a function of t.

The ranking of the cash flows as a function of time is reported in Table 6.

Table 6 shows that r_5 is the most influential parameter till it is received ($t = 5$). r_6 ranks second until r_5 is received. γ_4 is always the



	1	2	3	4	5	6	7
x1	0.044	0	0	0	0	0	0
x2	0.023	0.024	0	0	0	0	0
x3	0.046	0.047	0.049	0	0	0	0
x4	0.02	0.021	0.022	0.023	0	0	0
x5	0.77	0.81	0.83	0.87	0.89	0	0
x6	0.056	0.059	0.06	0.063	0.065	0.61	0
x7	0.041	0.039	0.039	0.044	0.045	0.39	1

Figure 3: $GI^{V_t}(x_i)$ as a function of time

least influential cash flow. The ranking in the cash flows is mainly driven by uncertainty effects. We note that, in the presence of increased uncertainty, the same results as in the discussion of Section 4.1 would be obtained.

4.3 Global SA of the project IRR

For the global SA of the project IRR [eq. (20)], we have utilized a numerical approach, since an analytical expression for σ^2_{IRR} is not achievable. A random sample of size $N = 5000$ has been generated.

The project IRR distributions in the base case (Table 3) and in the case of increased uncertainty are shown in Figure 4.

Figure 4 shows that increased uncertainty causes the IRR distribution to become less symmetric. The estimates of $E[IRR]$ and σ_{IRR} are 0.53 and 5% respectively, with the data of Table 3.

The subroutine for the calculation of $GI^{IRR}(x_i)$ is based on the extended FAST ([24], [26]). The numerical estimates are shown in Figure 5.

In Figure 5, the top of each bar corresponds to the value of $S_T(x_i)$. One notes that r_1 is now the most influential cash flow, followed by r_5, γ_2, γ_3, γ_4, r_6,r_7. The shadowed portion of each bar in Figure 5 shows the portion of the cash flow importance explained by first order terms ($S_1^{IRR}(x_i)$). Thus, the influence of r_7, and r_6 with respect to the IRR is

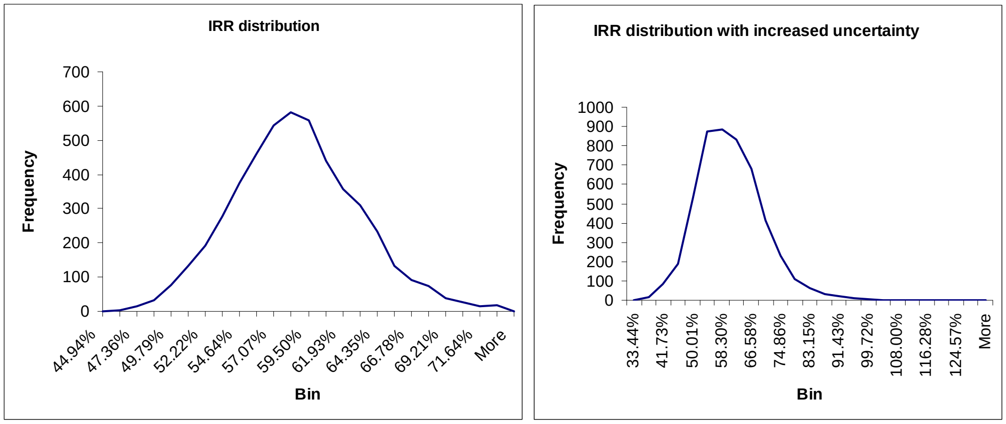


Figure 4: IRR distribution generated by uncertainty in the cash flows

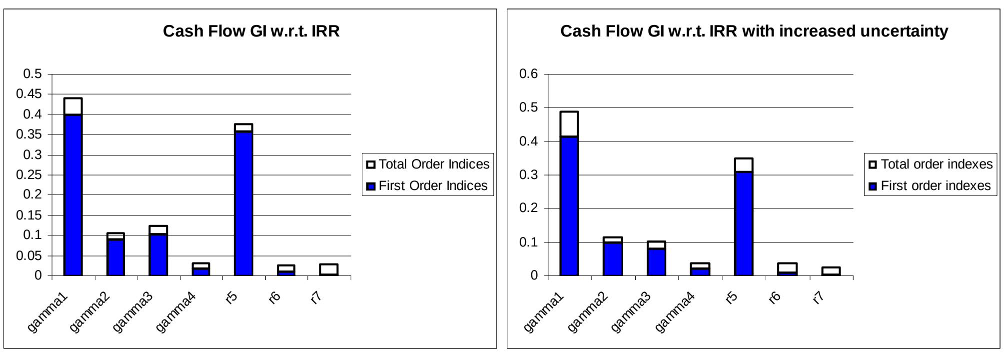


Figure 5: $GI^{IRR}(x_i)$ in the case of Table 2, on the right, and of increased uncertainty, on the left

mainly due to interaction effects, while the importance of the cost cash flows ($\gamma_i, i = 1...4$) and of r_5 is mainly determined by individual terms.

Let us now compare the ranking obtained through $S_T(x_i)$ to the one obtained making use of $PEAR(x_i)$ and $SRC(x_i)$ for the values in Table 3. The estimates of $GI(x_i)$ obtained using $PEAR(x_i)$, $SRC(x_i)$ with a sample of size $N = 5000$ are shown in Table 7.

Table 7 shows that SRC and $PEAR$ produce the same ranking. However, such a raking is different from the ranking obtained using $S_T(x_i)$. In fact, as a consequence of the nonlinearity of the IRR w.r.t. the cash flows, $PEAR$ and SRC cannot estimate the importance of interaction terms.

Cash Flow	γ_0	γ_1	γ_2	γ_3	γ_4	r_5	r_6	r_7
Rank using $S_T(x_i)$	0	1	4	3	5	2	7	6
Rank using $PEAR$ and SRC	0	1	3	4	5	2	6	7

Table 7: Estimating GI(xi) using PEAR and SRC would lead to misleading conclusions

$x_{\mathbf{i}}$	**NPV Rank**	**IRR Rank**
γ_1	4	1
γ_2	6	4
γ_3	3	3
γ_4	7	5
r_5	1	2
r_6	5	7
r_7	2	6

Table 8: Ranking of the cash flows with respect to IRR and NPV

4.4 Comparison of NPV and IRR global SA results

Table 8 reports the cash flow ranking with respect to the investment NPV and IRR obtained ordering the cash flows according to $GI(x_i)$.

Table 8 shows that, there is little agreement for the relevance of cash flows w.r.t. uncertainty in the IRR and in the NPV. Quantitative information on ranking agreement can be found by making use of the Savage Score correlation coefficient. We refer the reader to [6] and [8] for the mathematical definition. For the purposes of this work, we need only to recall that a high Savage Score correlation coefficient means good agreement on the most and least influential model cash flows. The Savage Score correlation coefficient of the ranking in Table 8 is 27%. Such value indicates that high influence and low relevance cash flows tend not to coincide. In Table 8 it is evident the change in ranking of γ_1: from almost non influential on the NPV, to most influential cash flow on the IRR. This result shows that, in general, cash flow influence will depend upon the valuation criterion chosen by the DMr[3]. As a practical implication, then, results obtained for the SA of the NPV cannot be transferred to the SA of IRR and vice-versa. This means that, if the DMr would collect information on r_5, then He/Her would reduce His/Her uncertainty on the NPV in the most effective way (see Section 4.1.), but uncertainty on the IRR would not be reduced in

[3]The authors obtained similar results for the local SA of the same model [7]

the most effective way. To do so, information on γ_1 should be gathered.

5 Conclusions

We have discussed the use of global SA techniques in the evaluation of business decisions. Global SA is a set of techniques that have been recently developed in the literature [18] to complement Uncertainty Analysis (UA), providing information on how uncertainty in the model output is generated by uncertainty in the input factors. Nowadays, most of the standard software used in industrial decision-making is equipped with Monte Carlo subroutines that enable the DMr to perform uncertainty or scenario analysis. By means of UA, the DMr can quantify His/Her uncertainty in the project and assess the likelihood of favorable and adverse scenarios. If in addition to UA a global SA is performed, then the DMr is able to derive quantitative information on what are the assumptions influencing uncertainty the most. These parameters are the ones deserving better attention in the information gathering and data collection processes, in order to reduce uncertainty in the fastest way.

It has been the purpose of this work to introduce global SA in the investment project evaluation realm. To do so, we have examined the global SA of the most used valuation criteria. We have then discussed the investment NPV — or one of the generalized forms — and IRR global SA to uncertainty in the cash flows. For the NPV valuation criterion we have seen that:

- The global importance of the cash flows $[GI^{NPV}(x_i)]$ can be obtained analytically.
- $GI^{NPV}(x_i)$ can be seen as the product of three effects: the cash flow uncertainty, the probability of receiving the cash flow and the discount factor.
- The ranking of the cash flows does not necessarily follow their timing, but uncertainty and probability effects can drive the results.
- $GI^{NPV}(x_i)$ is independent of the type of cash flow epistemic distribution, but dependent only on σ_i^2.
- $GI^{NPV}(x_i)$ can be equivalently estimated using variance decomposition based techniques, and non-parametric techniques (SRC and $PEAR$).

These results imply that, once the DMr has estimated His/Her uncertainty in the investment cash flows, then their global importance can be found directly, without further calculations, since an analytical expression is available. More in detail, it is enough that the DMr assesses a standard deviation of the cash flows, and their global importance is directly found from eq. (12). From a computational point of view, the advantage of the above results is that they rule out any problem related to the numerical computation of cash flow global importance.

Different results, however, have been obtained for the IRR valuation criterion. We have seen that:

- $GI^{IRR}(x_i)$ must be computed numerically, since an analytical expression for $GI^{IRR}(x_i)$ is not achievable, in general.
- Due to the non-linearity of the model, PEAR, SRC and VDB techniques produce different cash flow ranking
- VDB techniques produce the most reliable estimates since PEAR and SRC fail in assessing the importance of interaction terms in the case of non-linear models.

The fact that the global SA of the IRR can be performed only numerically, brings into the picture the limitations connected with the computational aspects illustrated in Section 2 (Table 2).

We have illustrated the previous results numerically, through the global SA of a model developed for the evaluation of projects under serious survival risk proposed by Beccacece, Gallo and Peccati (2000) and in use in the energy sector [4]. We have discussed the cash flow global importance in two cases, a base case and a case of uniform increase in uncertainty in the cash flows. The application of the general NPV results listed above has enabled us to: - derive the cash flow global importance analytically. - quantify the reduction in uncertainty in the NPV associated with the reduction of the uncertainty in cash flows. We have discussed how the global SA results can be utilized in the information collection process in order to manage uncertainty effectively. More precisely, we have seen that reducing uncertainty in the first cash flow of the revenue period (r_5) would lead to the fastest reduction in the uncertainty on the NPV. The analytical approach has been utilized for the global SA of the value of the project as a function of time. Similar results w.r.t. those of the NPV has been obtained.

We have then performed the global SA of the project IRR. We have resorted to a numerical approach utilizing the extended FAST method to obtain quantitative estimates for $GI^{IRR}(x_i)$ $[S_T(x_i)]$ and we have also compared the results to the estimates of $GI^{IRR}(x_i)$ obtained by utilizing the $PEAR$ and SRC methods. Results confirmed the foreseen poor performance of $PEAR$ and SRC, due to the non-linear dependence of the IRR on the cash flows. We have then compared the cash flow ranking w.r.t. the NPV with the IRR ranking. Results have shown that little agreement is obtained. In particular, the parameter influencing uncertainty in the IRR the most (γ_1) is not the one influencing the NPV the most (r_5). Thus, if a DMr collects information on γ_1, then He/Her would reduce uncertainty only on the IRR and not on the NPV. This result shows that information obtained from the SA of the NPV cannot be transferred to the SA of the IRR. Thus, also global SA results state the non-equivalence of the IRR and NPV valuation criteria.

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