The Importance of Assumptions in Investment Evaluation

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Abstract

This work illustrates a new method for estimating the importance of assumptions in investment evaluation. The most diffused sensitivity analysis schemes present limitations when used to assess the importance of individual parameters and cannot be employed to estimate the importance of groups of assumptions. However, such problems can be solved by making use of the Differential Importance Measure (DIM). We set forth the framework for the application of DIM at the parameter level of investment valuation models. We study the relationship between DIM and investment marginal behavior. We analyze the link between importance of a parameter and risk associated with it. We discuss general results for a sample valuation model. The numerical application to the valuation of an energy sector investment project follows. We rank the factors based on their importance and determine the project risk profile. We discuss the importance of groups of assumptions. Results will show that assumptions relating to revenues are the most influential ones, followed by discounting and operating cost assumptions. We discuss numerically the relationship between importance and risk, analyzing the effect of variable costs hedging through the comparison of the project risk profile in the presence and in the absence of such a hedging.

Keywords: Investment Analysis, Project Valuation, Sensitivity Analysis, Risk Analysis.

1 Introduction

This paper discusses the importance of assumptions in industrial investment evaluation. When firms are confronted with investment or business planning decisions, a number of factors influence the decision. The knowledge of which of the factors influences the investment the most is often sought by decision-makers for structuring and risk analysis purposes.

The decision-making (DM) process is a multidisciplinary effort bringing together economic, technical, financial and risk analysis [3]. The analysis is synthesized by the Financial Model (FM) or Business Plan (Figure 1) aimed at estimating the valuation criteria adopted by the decision-maker — a Net Present Value (V) or an Internal Rate of Return (IRR) for Shareholders, the Debt Service Coverage Ratio for Lenders ([1], [2], [3], [8], [12], [13], [14], [16], [17], [21]).

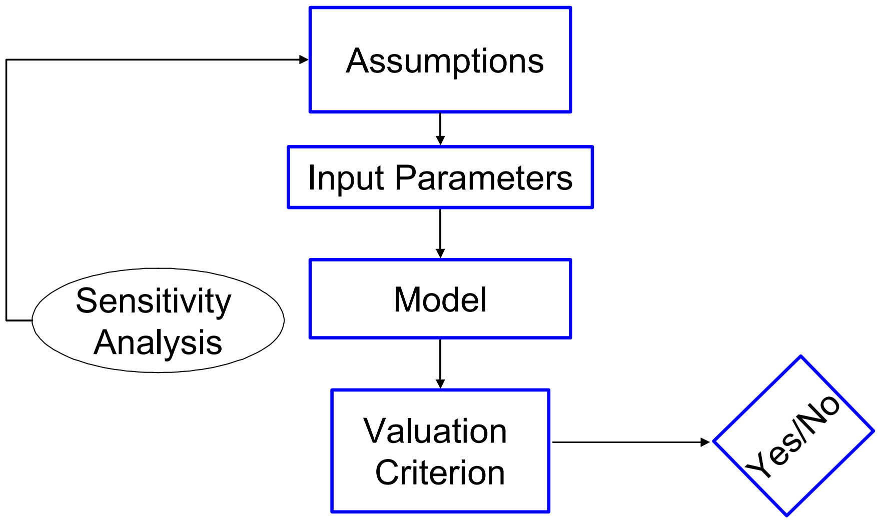

Figure 1: Investment decision making process

A series of factors influence the investment economics, such as investment costs, expected revenues, expected inflation, tax and accounting rules, etc.. We denote the set of all the input factors/assumptions by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_n)$. Letting V denote the valuation criterion, then:

$$V = v(\boldsymbol{\lambda}) \tag{1}$$

A critical role in the DM process is played by the reference value of the criterion, $V^0 = v(\boldsymbol{\lambda}^0)$, obtained fixing the input parameters at their reference value ($\boldsymbol{\lambda}^0$). $\boldsymbol{\lambda}^0$ represents the numerical assumptions reflecting the decision-maker view and knowledge of the investment factors and is usually referred to as base case.

Performing Sensitivity Analysis (SA) on V^0 is then an integral part of the DM process (Figure 1) ([9], [3]). SA can be used in a threefold mode ([11], [18], [19], [22]). The first mode is validation of model results, also called correctness test. The change in output that follows the change in input is utilized to check whether model reactions are consistent with the theory or with the analyst's expectations. In the second mode, the so-called stress test, SA supports risk analysis. Changes in the model output are used to test whether/how the investment can sustain changes in some of the assumptions without overcoming limit thresholds of the valuation criterion. Values of V below these thresholds could trigger investment rejection. In a third mode, SA can be used to assess the importance of the input parameters, *i.e.* which of the parameter influences the decision the most ([6], [4], [10], [18]). The most diffuse SA schemes are Tornado Diagrams and One-Way SA, that are usually implemented on decision-making software ([9], [23][1]). However, limitations are encountered in using such SA schemes to assess the importance of parameters ([4], [10], [11], [18], [19]). Especially in the presence of models with a high number of parameters, often SA is performed on a subset of the parameters, selected before assessing their importance. This leads to the risk of excluding relevant parameters from the analysis. In addition, the relative size of parameter changes is not taken into account, what can provoke errors in the classification ([5], [4]). Furthermore, these methods do not enable the assessment of the sensitivity of the model to changes in more than one parameter at a time: the importance of groups of assumptions cannot be estimated by these techniques ([5], [4]). Such information, instead, could reveal itself crucial in understanding the investment structure and its risk profile, as we are to discuss.

In this work, we show that the use of the Differential Importance Measure (DIM) ([5], [4]) enables one to overcome the above-mentioned limitations. In fact, DIM allows the assessment of the parameter importance taking into account the way parameters are varied automatically ([5], [4], [10]). DIM shares the additivity property and allows the computation of the importance of parameter groups straightforwardly ([4], [10]). Furthermore, utilizing the definition of risk proposed in Withe et al. [24], we show that parameter importance computed through DIM shares a direct interpretation in terms of risk significance of a parameter.

To apply DIM to industry FMs, we need to study the application of DIM at the level of the parameters that form the cash flows — previous works focuses on the cash flow level as in [5] and [7]. We decompose the cash flows in their input parameters, reproducing the main calculations of a FM while maintaining an analytical expression for $v(\boldsymbol{\lambda})$ (on cash

[1] Most of the standard Decision-Making Software (DATAPRO by the Treeage Corporatio, or Precision Tree by Palisade) are equipped with subroutines for Sensitivity Analysis based on parameter changes, delivering a Tornado Diagram or a one-way SA.

flow decomposition and analytical valuation see, for example, [12], [15]). A detailed discussion on the computation of DIM is then proposed, with the purpose of analyzing the relationship between the importance of a parameter and the marginal behavior of the valuation criterion, V, w.r.t. that parameter.

We apply the results to the evaluation of an energy sector investment. We compute the importance of individual assumptions, and rank the input factors. We then illustrate the determination of the importance of groups of assumptions. For the investment at hand parameters related to sales result as the most important ones, followed by assumptions on discounting, while fiscal assumptions play a minor role.

We discuss numerically the link between parameter importance and investment risk. In particular, we analyze the effect of hedging policies comparing the project risk profile in the presence and in the absence of hedging. We show that if variable cost pass-through is achieved, the risk associated with one of the most important parameters, namely fuel costs, is eliminated and study how the residual risk is redistributed among the remaining parameters. In particular, we shall note that the risk connected with the most influential assumption, namely the sale price, is reduced, since the tariff is no more responsible for remunerating variable costs.

Section 2 presents DIM definition, main properties, and its relationship to other SA techniques used in Economics and discusses the link between importance and risk. Section 3 illustrates the sample valuation model used in this work. In Section 4, we present the application of DIM to the model, highlighting the features of its computation at the parameter level. We study the relationship between the marginal behavior of V and the importance of the parameters. The numerical application to an energy sector investment project is presented in Section 5, with focus on the importance of assumptions when taken individually and in groups. We finally analyze how the project risk profile changes as a consequence of the introduction of hedging strategies. Conclusions are offered in Section 6.

2 Sensitivity Analysis

This section discusses definition and properties of the Differential Importance Measure (DIM) [4],[5] and its interpretation in terms of project risk. Let:

$$F = f(\mathbf{x}) \tag{2}$$

be a function differentiable at $\mathbf{x}^0 = (x_1^0, x_2^0, ..., x_n^0)$ and such that $\nabla f(\mathbf{x}^0)$ is not orthogonal to the increment vector $d\mathbf{x} = [dx_1, dx_2, \ldots, dx_n]^T$.

Then, the following importance measure (DIM) for the input x_s is defined [4]:

$$D_s(\mathbf{x}^0, d\mathbf{x}) = \frac{f_s(\mathbf{x}^0)dx_s}{\sum_{j=1}^{n} f_j(\mathbf{x}^0)dx_j} \tag{3}$$

where $f_s(\mathbf{x}^0)$ is the partial derivative of F w.r.t x_s at $\mathbf{x}^0$. $D_s(\mathbf{x}^0, d\mathbf{x})$ measures the parameter importance as the change in F provoked by a change in x_s, over the sum of the changes in F provoked by changes in all the input parameters.

We note that the definition in eq. (3) overcomes the limitations explained in the introduction and related to the use of traditional SA schemes, namely: non-consideration of the way parameters are varied, and impossibility of computing the importance of groups of assumptions. The way input parameters are varied is taken into account automatically by definition (3). Let us rewrite eq.(3) as:

$$D_s(\mathbf{x}^0, d\mathbf{x}) = \frac{f_s(\mathbf{x}^0)}{\sum_{j=1}^{n} f_j(\mathbf{x}^0)\frac{dx_j}{dx_s}} \tag{4}$$

Now, if one performs the sensitivity adopting the same change (H1) in all the parameters, then $\frac{dx_j}{dx_s} = 1 \; \forall j$ and [5]:

$$D1_s(\mathbf{x}^0) = \frac{f_s(\mathbf{x}^0)}{\sum_{j=1}^{n} f_j(\mathbf{x}^0)} \tag{5}$$

In this case, $D_s(\mathbf{x}^0, d\mathbf{x})$ is the ratio of the partial derivative of the model w.r.t. x_s at $\mathbf{x}^0$ divided by the sum of the derivatives at $\mathbf{x}^0$.

On the other hand, one could perform the sensitivity considering, for example, proportional changes (H2) in the parameters: $dx_j = \omega \cdot x_j^0 \; \forall j$. Then, $\frac{dx_j}{dx_s} = \frac{\omega \cdot x_j^0}{\omega \cdot x_s^0} = \frac{x_j^0}{x_s^0}$ and, hence [5]:

$$D2_s(\mathbf{x}^0) = \frac{f_s(\mathbf{x}^0)x_s^0}{\sum_{j=1}^{n} f_j(\mathbf{x}^0)x_j^0} \tag{6}$$

We note that in case H2 the importance of a parameter is directly linked to its elasticity $[E_s(\mathbf{x}^0) = f_s(\mathbf{x}^0)x_s^0/f(\mathbf{x}^0)]$ [5]:

$$D2_s(\mathbf{x}^0) = \frac{E_s(\mathbf{x}^0)}{\sum_{j=1}^{n} E_j(\mathbf{x}^0)} \tag{7}$$

In case H2, $D_s(\mathbf{x}^0, d\mathbf{x})$ is the ratio of the elasticity of the model w.r.t. x_s at $\mathbf{x}^0$ divided by the sum of the elasticities at $\mathbf{x}^0$ [eq. (7)]. Thus, elasticity can be interpreted as the importance of parameters for proportional changes in their values. We also note that if a parameter assumes the value $x_s^0 = 0$, then it has, by definition, zero elasticity and, as a consequence, $D2_s(\mathbf{x}^0) = 0$. This means that the model does not react to proportional changes in its value (a quantity proportional to 0 is again 0). However, it could react to different type of changes in the parameter values, as testified, for example, by eq.(5). We note that, different choices of parameter relative changes will always be automatically accommodated by the definition of DIM [eqs. (3) and (4)] (see also [4] and [5]).

DIM shares the additivity property w.r.t. the various inputs, *i.e.*, the DIM of some set of parameters $(s_1, ..., s_k,\ k \leq n)$ coincides with the sum of the individual DIMs of the parameters in that set ([4], [5]):

$$D_{s_1,...,s_k}(\mathbf{x}^0, d\mathbf{x}) = \frac{f_{s_1}(\mathbf{x}^0)dx_{s_1} + f_{s_2}(\mathbf{x}^0)dx_{s_2} + ... + f_{s_k}(\mathbf{x}^0)dx_{s_k}}{\sum_{j=1}^{n} f_j(\mathbf{x}^0)dx_j} \tag{8}$$

Thus, the joint importance of assumptions can be directly computed using DIM: the sensitivity of the output to a set of assumptions is found by simply by adding the importance of individual the assumptions in the group. We illustrate this computation in Section 5.

Note that $\sum_{j=1}^{n} D_s(\mathbf{x}^0, d\mathbf{x}) = 1$.

Let us now turn to the relationship between importance and risk. In the application to financial models, $F = f(\mathbf{x})$ becomes $V = v(\boldsymbol{\lambda})$. One can re-write eq. (3) as:

$$D_s(\boldsymbol{\lambda}^0, d\boldsymbol{\lambda}) = \frac{d_s V}{dV} \tag{9}$$

Thus, $D_s(\boldsymbol{\lambda}^0, d\boldsymbol{\lambda})$ is the fraction of the change in V associated with a change in parameter λ_s ([5], [4]). We recall that investment risk can be defined as "*... the potential variability of financial outcomes...*" [24]. Thus, as the parameter with the highest DIM causes the biggest variation in the investment value, such parameter is also the most relevant risk driver of the project. Following this interpretation, we shall use the term "investment risk profile" to denote the set of the parameter importances.

3 An analytical expression for V

Financial models based on the discounted cash flow methodology estimate the investment value (V) as ([21], [17], [12]):

$$V = f(\boldsymbol{\Phi}, k) = \boldsymbol{\Phi}\mathbf{K}_0^{\mathrm{T}} \tag{10}$$

where $\boldsymbol{\Phi} = \{\Phi_j,\ j = 0...N\}$, is the cash flow vector, N is the number of periods used to model the investment, $\mathbf{K}_0 = \{(1+k)^{-j},\ j = 0...N\}$ is the discount factor vector, and k is the discount rate.

As far as timing is concerned, one usually distinguishes between a construction period (CP) and an operation period (OP). If the model does not consider capital expenditures during the operation period, one can state $\Phi_0 \leq 0$ and $\Phi_j \geq 0$ for $j = 1...N$, with Φ_0 representing the net present value of the outflows associated with the investment costs. Φ_j ($j = 1...N$) are then estimated using the pro-forma financial statements of the investment vehicle projected over the investment life [2]. The financial statements will take into account fiscal and corporate rules of the country in which the investment vehicle operates [24].

Under the following assumptions:

- The investment is performed through a special purpose vehicle with optimized financial structure. *I.e.*, shareholders are capable of remitting all the available cash at any j.
- The investment is carried out in a full equity mode.[2]
- 0 days payable and receivables and negligible financial income [3],

one gets:

$$V = h(-a, t, \mathbf{r}, \mathbf{o}, \mathbf{d}, k) = -a + [(1-t)(\mathbf{r} - \mathbf{o}) + t\mathbf{d}]\,\mathbf{K}_1^{\mathrm{T}} \tag{11}$$

where $a = \Phi_0$ represents the investment costs and coincides with the cash outflows, t represents the income tax rate, $\mathbf{r}, \mathbf{o}, \mathbf{d}$ represent revenues, operating expenses and depreciation charges, respectively, and $\mathbf{K}_1 = \{(1+k)^{-j}, j = 1...N\}$.

In order to understand how numerical assumptions influence the investment decision, one needs to express V in terms of more fundamental parameters.

1. $\mathbf{r}$. Revenues. Two are the typical situations: sales to the market or contractually regulated sales to specified customers. An expression that captures the cash revenues of the project with contractual sale and price escalation, is the following:

$$r_j = (1-\tau)p_{j-1}(1 + i_j^p)X_{j-1}(1 + g_j) \tag{12}$$

   where τ is the applicable revenue tax rate, p_{j-1} is the price of sales at period $j-1$, i_j^p is the price escalation index between period $j-1$

[2] This assumption is commonly utilized in the earliest phases of project evaluation. In fact, when the project is in its first stages, information on the leverage and on the cost of debt that lenders are willing to offer to the project is often unavailable.

[3] Ideally the cash is remitted "istantaneously" to shareholdelrs and does not sit in the SPC accounts a sufficient enough time to generate relevant interests

and period j, X_{j-1} is the quantity of goods sold in period $j-1$, g_j is the growth/decrease in sales from period $j-1$ to period j.

Eq.(12) can be expressed as a function of the initial sale price, π, and sale quantity χ as follows:

$$r_j = (1-\tau)\chi\pi \prod_{n=1}^{j}(1+i_n^p)(1+g_n^p) \tag{13}$$

2. **o**. Operating expenses can be modeled as fixed and variable costs:

$$o_j = C_j + c_j X_j \tag{14}$$

where C_j denotes the fixed costs in period j, c_j the unit costs for period j and X_j the quantity of goods sold in period j.

The modeling choice usually foresees the escalation of the initial costs by means of appropriate indices. Letting Γ and γ denote the starting operational fixed and variable costs, eq. (14) becomes:

$$o_j = \Gamma \prod_{n=1}^{j}(1+i_n^C) + \chi\gamma \prod_{n=1}^{j}(1+i_n^c)(1+g_n) \tag{15}$$

with i_j^C, i_j^c being the cost escalators of period $(j-1) \to j$.

3. **d**. The depreciation amount at period j (d_j) is a function of the depreciation method chosen by the firm and of the tax and accounting rules of the country or region where the investment vehicle operates [24]. If a straight line depreciation method is chosen, d_j assumes the following form:

$$d_j = \alpha_j FA_0 \tag{16}$$

where α is the appropriate depreciation rate and FA_0 is the asset book value at the end of construction. Often accounting rules require capitalization of all the costs sustained by the investor during the construction period. Under the assumption of full equity financing:

$$FA_0 = a \tag{17}$$

and considering a constant depreciation rate throughout the life of the project ($\alpha_j = \alpha$ and consistently $\alpha \leq \frac{1}{N}$), one is allowed to write:

$$d_j = \alpha a \quad j = 1...N \tag{18}$$

Substituting eqs. (13, 15 and 18) into eq. (11), one gets:

$$V = -a + \sum_{j=1}^{N} \Phi_j (1+k)^{-j} \tag{19}$$

with

$$\Phi_j = \left\{ \begin{matrix} [(1-\tau)\pi \prod_{n=1}^{j}(1+i_n^p) - \gamma \prod_{n=1}^{j}(1+i_n^c)]\chi \prod_{n=1}^{j}(1+g_n^p) - \\ -C_0 \prod_{n=1}^{j}(1+i_n^C) \end{matrix} \right\} (1-t) + \alpha t a \tag{20}$$

If the same escalator (i) is used for revenues and costs, and its value is assumed constant over time eq. (20) becomes:

$$V = -a + \sum_{j=1}^{N} \frac{(1-t)(1+i)^j \{\chi[(1-\tau)\pi - \gamma](1+g)^j - \Gamma]\} + t \cdot \alpha a}{(1+k)^j} \tag{21}$$

In the framework of eq. (1), the investment value V is a function of the following 11 parameters:

$$V = v(a, t, \tau, \chi, \pi, \Gamma, \gamma, i, g, \alpha, k) \tag{22}$$

Rational investors will accept only investments with $V > 0$. Setting $V > 0$, rearranging eq. (21) considering that $\sum_{j=1}^{N} \frac{+t \cdot \alpha a}{(1+k)^j} = t\alpha a\theta$, with $\theta = \frac{1}{k}\left(1 - \frac{1}{(k+1)^N}\right)$, one gets the following condition for an investment to have a positive V:

$$\sum_{j=1}^{N} \frac{(1-t)(1+i)^j \left\{\chi[(1-\tau)\pi - \gamma](1+g)^j - \Gamma]\right\}}{(1+k)^j} > a(1 - t\alpha\theta) \tag{23}$$

Letting

$$EBIDTA_j = \left\{\chi[(1-\tau)\pi - \gamma](1+i)^j(1+g)^j - \Gamma(1+i)^j]\right\}, \tag{24}$$

one gets:

$$V > 0 \Longleftrightarrow \sum_{j=1}^{N} \frac{EBITDA_j}{(1+k)^j} > \frac{a}{(1-t)}(1 - t\alpha\theta) \tag{25}$$

In the following analysis, eq. (25) will prove useful in understanding the relationship between the importance of the parameters and the investment value marginal behavior.

4 Importance and Marginal Behavior

In this section, we study the application of DIM at the parameter level of the valuation model [eq. (21)] and its relationship to the marginal behavior of the model.

An observation first. In computing the importance of cash flows [5], parameters (the cash flows themselves) have the same dimension. Therefore both H1 and H2 hold. However, parameters that compose cash flows have, in general, different dimensions. For instance, costs have monetary units while escalation indices are pure numbers. Therefore case H1 is not applicable. Case H2, instead, is still applicable. For proportional changes eq. (9) becomes:

$$D2_s(\boldsymbol{\lambda}^0) = \frac{v_s(\boldsymbol{\lambda}^0)\lambda_s^0}{\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)\lambda_j^0} \tag{26}$$

Let us discuss briefly eq.(26). The sign of $D2_s(\lambda)$ depends both on the partial derivatives of V^0 w.r.t. the parameters and on the parameter sign and magnitude. The following cases are given:

- If $\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)\lambda_j^0 > 0$,
  - then if λ_s^0 and $v_s(\boldsymbol{\lambda}^0)$ have the same sign, then $D2_s(\boldsymbol{\lambda}^0) > 0$,
  - else if λ_s^0 and $v_s(\boldsymbol{\lambda}^0)$ have opposite signs, then $D2_s(\boldsymbol{\lambda}^0) < 0$.
- If $\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)\lambda_j^0 < 0$ the two previous conclusions are reversed.

Table 1 summarizes the analytical expressions of the partial derivatives ($v_s(\boldsymbol{\lambda}^0)$) and the parameter importances ($D2_s(\boldsymbol{\lambda}^0)$).

We first focus on $v_s(\boldsymbol{\lambda}^0)$, $s = 1...11$ (Table 1). Noting that under the assumptions of Section 3 $t\alpha\theta < 1$, with some algebraic manipulations one finds:

- $v_\tau(\boldsymbol{\lambda}^0)$, $v_\Gamma(\boldsymbol{\lambda}^0)$, $v_\gamma(\boldsymbol{\lambda}^0)$, $v_t(\boldsymbol{\lambda}^0)$, $v_a(\boldsymbol{\lambda}^0) < 0$ $\forall \boldsymbol{\lambda}^0$. This means that increases in taxes, investment costs, operational fixed and variable costs will always decrease V.

  In particular, in many investment situations, an economic hedge against variable costs is sought [20]. This is achieved by splitting the sale price into two or more portions, with one such portion directly replicating the variable costs. If the sale contract is negotiated so that the sale price reflects the splitting, variable costs

s	λ_s	$v_s(\boldsymbol{\lambda}^0)$	$D2_s(\boldsymbol{\lambda}^0)$
1	a	$-1+t\alpha\theta$	$\frac{a(-1+\alpha t\theta)}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
2	t	$\sum_{j=1}^{N} \frac{-EBITDA_j+\alpha a}{(1+k)^j}$	$\frac{t\sum_{j=1}^{N} \frac{-EBITDA_j+\alpha a}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
3	τ	$\sum_{j=1}^{N} \frac{-(1-t)\chi\pi(1+i)^j(1+g)^j}{(1+k)^j}$	$\frac{\tau\sum_{j=1}^{N} \frac{-(1-t)\chi\pi(1+i)^j(1+g)^j}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
4	χ	$\sum_{j=1}^{N} \frac{-(1-t)(1+i)^j}{(1+k)^j}$	$\frac{\chi\sum_{j=1}^{N} \frac{(1-t)[(1-\tau)\pi-\gamma](1+i)^j(1+g)^j}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
5	π	$\sum_{j=1}^{N} \frac{(1-t)[(1-\tau)\chi(1+i)^j(1+g)^j]}{(1+k)^j}$	$\frac{\pi\sum_{j=1}^{N} \frac{(1-t)[(1-\tau)\chi(1+i)^j(1+g)^j]}{(1+k)^j}}{\sum_{i=1}^{n} \frac{\partial V}{\partial\lambda_i}\lambda_i^0}$
6	Γ	$\sum_{j=1}^{N} \frac{-(1-t)(1+i)^j}{(1+k)^j}$	$\frac{\Gamma\sum_{j=1}^{N} \frac{-(1-t)(1+i)^j}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
7	γ	$\sum_{j=1}^{N} \frac{-(1-t)(1+i)^j(1+g)^j}{(1+k)^j}$	$\frac{\sum_{j=1}^{N} \gamma\frac{-(1-t)(1+i)^j(1+g)^j}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
8	i	$\sum_{j=1}^{N} \frac{j(1+i)^{j-1}(1-t)\left\{\chi[(1-\tau)\pi-\gamma](1+g)^j-\Gamma\right\}}{(1+k)^j}$	$\frac{\sum_{j=1}^{N} i\frac{j(1+i)^{j-1}(1-t)\left\{\chi[(1-\tau)\pi-\gamma](1+g)^j-\Gamma\right\}}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
9	g	$\sum_{j=1}^{N} \frac{(1-t)\left\{\chi[(1-\tau)\pi-\gamma](1+i)^j j(1+g)^{j-1}\right\}}{(1+k)^j}$	$\frac{\sum_{j=1}^{N} g\frac{(1-t)\left\{\chi[(1-\tau)\pi-\gamma](1+i)^j j(1+g)^{j-1}\right\}}{(1+k)^j}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
10	α	$\sum_{j=1}^{N} \frac{+ta}{(1+k)^j}$	$\frac{\alpha ta\theta}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$
11	k	$\sum_{j=1}^{N} -j\frac{\Phi_j}{(1+k)^{j+1}}$	$\frac{k\sum_{j=1}^{N} -j\frac{\Phi_j}{(1+k)^{j+1}}}{\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0}$

Table 1: Partial Derivatives and DIM for the sample model

are then “passed-through” to the customer. In this case, eq. (21) becomes:

$$V = -a + \sum_{j=1}^{N} \frac{(1-t)\left\{\chi[(1-\tau)\pi^{pt}](1+i)^j(1+g)^j - \Gamma(1+i)^j]\right\} + t\alpha a}{(1+k)^j} \tag{27}$$

where π^{pt} is the sale price, net of the variable cost portion. It is easy to see that π^{pt} is related to π by $\pi^{pt} = \pi - \frac{\gamma}{(1-\tau)}$. As a result, V does not depend on γ any more, and correspondingly the variable cost risk is hedged against. π^{pt}, in the energy sector, is called “capacity charge.” It is the portion of the sale charge that covers fixed costs, remuneration of the investment costs, and grants investors the required return.

- $v_\chi(\boldsymbol{\lambda}^0) > 0$ and $v_g(\boldsymbol{\lambda}^0) > 0$ if $\pi > \gamma$. This means that, increasing production benefits V only if the company is able to sell at a unit price higher than its unit variable costs. This result reflects the marginal cost law for bidding. *I.e.*, no rational investor would bid at $\pi < \gamma$ in a free market [20].

- $v_\pi(\boldsymbol{\lambda}^0) > 0\ \forall \boldsymbol{\lambda}^0$. As expected, increases in sale price always benefit the investment.

- $v_i(\boldsymbol{\lambda}^0)$. The sign of $v_i(\boldsymbol{\lambda}^0)$ depends on the sign of the terms

$$\left\{\chi[(1-\tau)\pi - \gamma](1+g)^j - \Gamma\right\} \tag{28}$$

If $g > 0$, then a necessary and sufficient condition for $v_i(\boldsymbol{\lambda}^0) > 0$ is that

$$EBITDA_0 > 0 \tag{29}$$

If $g < 0$, a sufficient condition for $v_i(\boldsymbol{\lambda}^0) > 0$ is given by:

$$\chi[(1-\tau)\pi - \gamma] > \frac{\Gamma}{(1+g)^N} \tag{30}$$

- $v_\alpha(\boldsymbol{\lambda}^0) > 0\ \forall \boldsymbol{\lambda}^0$. This result is a consequence of the optimal financing structure assumption. An increase in depreciation rates increases the tax burden and increases the cash available to shareholders [24]. In the presence of a perfectly efficient financial structure, all the cash is remitted to shareholders. Thus, increasing depreciation rates will increase V, since more cash can be redirected towards shareholders.

- $v_k(\boldsymbol{\lambda}^0) < 0 \; \forall \boldsymbol{\lambda}^0$. In fact:

$$v_k(\boldsymbol{\lambda}^0) = \sum_{j=1}^{N} -j \frac{(1-t)\left\{\chi[(1-\tau)\pi - \gamma](1+i)^j(1+g)^j - \Gamma(1+i)^j]\right\} + t \cdot \alpha a}{(1+k)^{j+1}} \tag{31}$$

  Re-writing eq. (31) in terms of Φ_j as:

$$v_k(\boldsymbol{\lambda}^0) = \sum_{j=1}^{N} -j \frac{\Phi_j}{(1+k)^{j+1}} \tag{32}$$

  and recalling that $\Phi_j > 0$ for $j = 1...N$, then $v_k(\boldsymbol{\lambda}^0) < 0 \; \forall \boldsymbol{\lambda}^0$.

Combining Table 1 with the partial derivative analysis above, one gets to the following conclusion on the sign of the parameter importance for the model at hand:

- If $\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0 > 0$ ($n = 11$ in our case)
  - $D2_\pi(\boldsymbol{\lambda}^0), D2_\alpha(\boldsymbol{\lambda}^0) > 0 \; \forall \boldsymbol{\lambda}^0$
  - $D2_\chi(\boldsymbol{\lambda}^0)$, if $\pi > \lambda$, $D2_g(\boldsymbol{\lambda}^0) > 0$ if $\pi > \lambda$ and $g > 0$, $D2_i(\boldsymbol{\lambda}^0) > 0$ if $g > 0$ and $EBITDA_0 > 0$
  - $D2_a(\boldsymbol{\lambda}^0)$, $D2_\tau(\boldsymbol{\lambda}^0)$, $D2_\Gamma(\boldsymbol{\lambda}^0)$, $D2_\gamma(\boldsymbol{\lambda}^0)$, $D2_t(\boldsymbol{\lambda}^0)$ and $D2_k(\boldsymbol{\lambda}^0) < 0 \; \forall \boldsymbol{\lambda}^0$
- If $\sum_{i=1}^{n} v_i(\boldsymbol{\lambda}^0)\lambda_i^0 < 0$ the signs are reversed.

5 Numerical Application: Energy Sector Investment Analysis

We consider the valuation of a "green-field"[4] investment project in the energy sector. The sale of energy is regulated by a 28 year concession contract. The project is characterized by a total investment cost of $a = 1000$ $[m]$, where m stands for "monetary unit". The investment is performed through a special purpose vehicle subject to the investment country regulation and fiscal law. Fees and taxes levied on revenues (τ) amount at 5%. The income tax rate is $t = 33\%$. The energy is produced at quantity $\chi = 5.64$ $[q]$, which already incorporates plant availability and load factor. The sale price is $\pi = 48$ $[m/q]$. In order to operate

[4]By greenfield investment it is meant an investment where construction of the plant or transmission facility starts at $t = 0$. Generally, revenues start only at the end of construction

s	$\lambda_{\mathbf{s}}$	λ_s^0	$D2_s(\boldsymbol{\lambda}^0)$	$Rank$
1	a	$1000[m]$	-37	4
2	t	33%	-17	7
3	τ	5%	-4	8
4	χ	$5.64[q/m]$	45	3
5	π	$48[m/q]$	80	1
6	Γ	$13[m]$	-4	9
7	γ	$20[m]$	-35	5
8	i	7%	27	6
9	g	0	0	11
10	α	$1/28$	3	10
11	k	15%	-56	2

Table 2: Investment Base Case Assumptions and Importance Analysis results

the facility, fixed costs of $\Gamma = 13[m]$ per year are sustained. A variable cost of $\gamma = 20$ $[m/q]$ is sustained per year. The price escalation rate is set equal to the forecasted inflation at $i = 7\%$. No growth is expected for this investment ($g = 0$). This assumption is "forced" by the context of this investment, since production is fixed at the moment of the initial plant design, as capacity[5] is determined over the investment horizon. The average depreciation rate is $\alpha = 1/28$, and the return on equity is set at $k = 15\%$.

Under these assumptions, the base case investment value is $V^0 = 98$ $[m]$.

Columns $1 - 3$ of Table 2 summarize the base case assumptions.

As far as the investment marginal behavior is concerned, a series of one parameter at a time SA's on V (Figure 2) confirms the theoretical expectations.

From Figure 2, we note that the investment value is linearly decreasing in a, t, τ, and linearly increasing in χ, as expected from the discussion in Section 4. V is non-linearly increasing in i (Figure 2). In fact, $v_i(\boldsymbol{\lambda}^0) > 0$, since $EBITDA_0 = 131 > 0$ (see eqs. (28) and (29)). The one way sensitivity on k shows that V decreases non-linearly in k and that the project IRR is around 16%.

The values of $D2_s(\boldsymbol{\lambda}^0)$ are listed in Table 2. We note that $\sum_{j=1}^{n} v_j(\boldsymbol{\lambda}^0)\lambda_j^0 > 0$ and since all the parameters are greater than 0, the sign of $D2_j(\boldsymbol{\lambda}^0)$ will reflect the sign of the partial derivatives. The last column of Table 2 shows the parameter ranking. The most influential parameter is π,

[5]Capacity is the term used to denote the power (MW) available to a power plant to generate electricity.

Group	Importance
R	124
O	-39
I	-37
F	-18
M	26
D	-56

Table 3: Importance of groups of assumptions

with $D2_\pi(\boldsymbol{\lambda}^0) = 79$. k follows with $D2_k(\boldsymbol{\lambda}^0) = -56$. The values of the importance of χ, a, and γ are 44, -36 and -34 respectively (Figure 3). These last three parameters show a similar influence on V. i follows with $\left|D2_i(\boldsymbol{\lambda}^0)\right| \cong 26$. t ranks 7^{th} with $\left|D2_k(\boldsymbol{\lambda}^0)\right| = 17$. τ, Γ α are the less relevant parameters, with their $\left|D2_s(\boldsymbol{\lambda}^0)\right|$ around $2-4$. Being there no growth for the investment over the investment horizon, g is non-influential, and $D2_g(\boldsymbol{\lambda}^0) = 0$.

Recalling the interpretation of DIM in terms of risk, π is the parameter that bears the greatest fraction of the project risk, followed by k, χ, a, γ, i, τ, Γ, α and g.

Let us now discuss the importance of groups of assumptions. As an example, let us introduce the following categories: revenue assumptions ($R = \{\pi, \chi\}$), operating cost assumptions ($O = \{\gamma, \Gamma\}$), investment cost assumptions, $I = \{a\}$, fiscal assumptions ($F = \{t, \tau, \alpha\}$), macroeconomic assumptions ($M = \{i, g\}$) and discounting assumptions ($D = \{k\}$). The importance of these groups can be straightforwardly determined by adding the importance of the parameters in the group, thanks to the additivity property of DIM [eq.(8)]. The results are given in Table 3.

From Table 3, it is immediate to note that revenue assumptions are the most influential ones, followed by discounting assumptions, while fiscal assumptions have the lowest influence. Assumptions related to investment and operating costs have a similar importance, and play an intermediate role.

We now turn to the numerical illustration of the relationship between parameter importance and risk. Let us first discuss the project risk profile in the case of case variable cost pass-through [Section 4, eq. (27)]. In this case, $\gamma = 0$, and $\pi^{pt} = 27$ is the revised sale price that leaves the base case value unchanged [eq. (27)]. Applying DIM one obtains the ranking of the parameters listed in Table 4.

W.r.t. the non-hedged case, not only the dependence on variable costs has been removed, but the overall project risk profile has changed

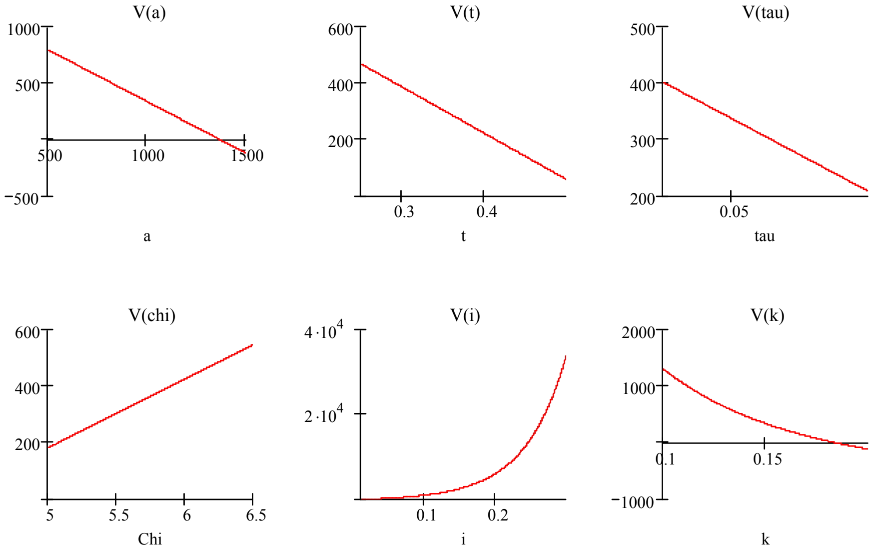

Figure 2: One way sensitivity analyses on 6 factors tests model correctness

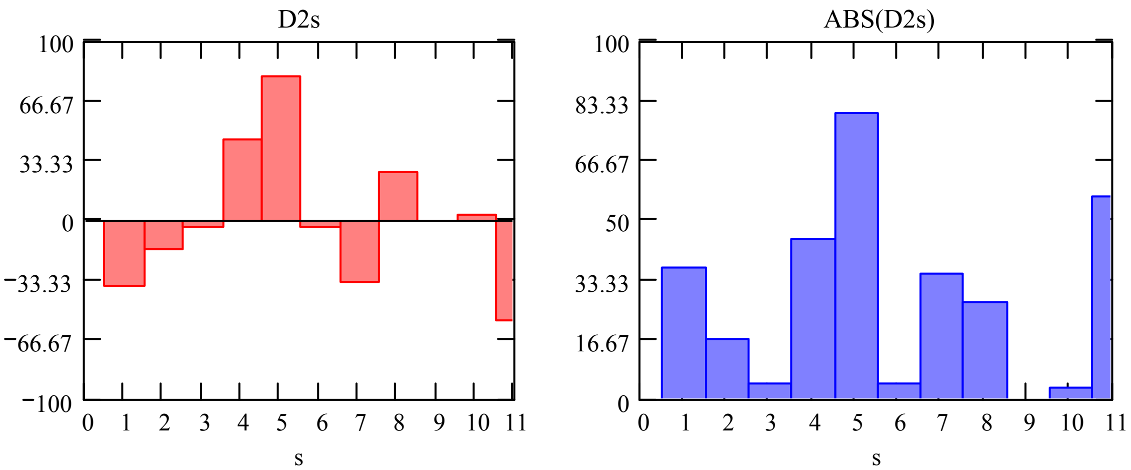

Figure 3: $D2_s(\boldsymbol{\lambda}^0)$ and $\left|D2_s(\boldsymbol{\lambda}^0)\right|$ in the base case.

s	$\lambda_{\mathbf{s}}$	$D2_s(\boldsymbol{\lambda}^0)$	**Rank**	**Non pass-through Rank**
1	a	-12	4	4
2	t	-6	6	7
3	τ	-1	9	8
4	χ	15	2	3
5	π^{pt}	15	2	1
6	Γ	-1	7	9
7	γ	0	11	5
8	i	9	5	6
9	g	0	11	11
10	α	1	8	10
11	k	-19	1	2

Table 4: Parameter DIM and ranking in the pass-through and non pass-through case

(Figure 4).

In particular (Table 4):

- γ is non influential. It ranked 5^{th} in the non-pass through case.
- k is the most important factor, as opposed to π in the non-hedged case.
- π^{pt} is as important as χ and ranks 2^{nd}.

  This change is due to the fact that less risk is now associated with the sale price. In fact, π^{pt} is responsible for remunerating fixed operational costs, investment costs and assuring the investor the required return, while π, in addition to these, carried the burden of variable cost remuneration.
- τ, Γ and α are still the less relevant factors, but their ranking is reversed.

6 Conclusions

This work has discussed the assessment of assumption importance in investment valuation, by means of the link between sensitivity and risk analysis . Traditional SA schemes based on parameter changes, while well-suited for stress and correctness test purposes, lack of a systematic approach that prevents one from using them in finding parameter importance. Furthermore, they do not enable the sensitivity analysis of the model on more than one parameter at a time. We have seen that these

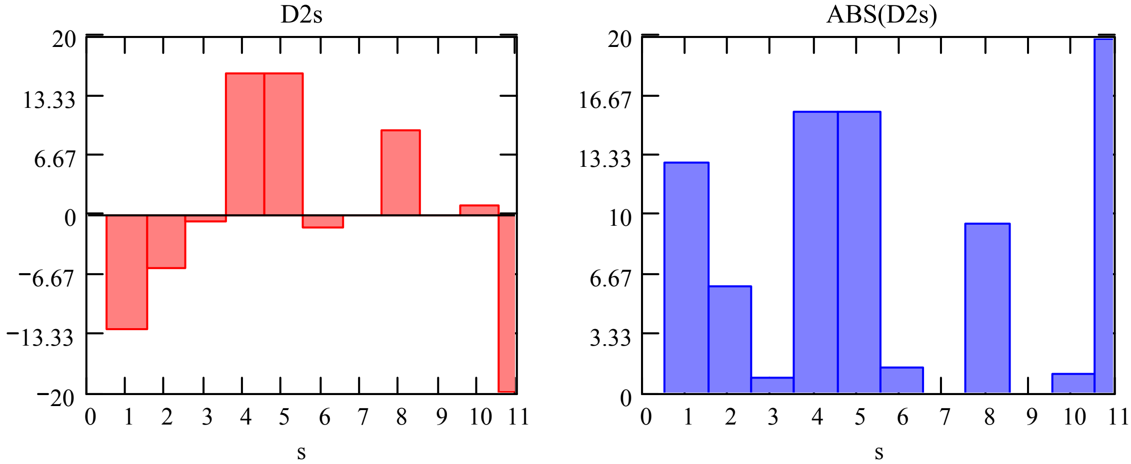

Figure 4: $D2_s(\boldsymbol{\lambda}^0)$ and $\left|D2_s(\boldsymbol{\lambda}^0)\right|$ in the presence of hedging (variable cost pass-through)

problems can be overcome by making use of the Differential Importance Measure (DIM).

We have stated the framework for the application of DIM at the parameter level of valuation models. We have noted that, while at the cash flow level both a uniform (H1) and a proportional (H2) sensitivity analyses are possible (Section 4), in the application to the parameter SA, H1 is not allowed.

We have seen that parameter importance estimated through DIM shares a direct interpretation in terms of project risk. Namely, the parameters with the highest DIM are also the main risk drivers.

We have then applied DIM to a sample valuation model providing an analytical expression for the cash flow decomposition in terms of the input factors. We have studied how the model marginal behavior w.r.t. the parameters is related to the parameter DIMs.

The numerical application to a model for the valuation of a green field investment in the energy sector has followed. We have ranked the parameters based on their importance and determined the investment risk profile. We have then determined the joint importance of assumptions, exploiting DIM additivity property. We have seen that assumptions related to revenues are more relevant than cost assumptions; macroeconomic assumptions play an intermediate role, while a less relevant role is played by fiscal assumptions.

Finally, we have analyzed the effect of a hedging policy on the project risk profile. The comparison of the project risk profiles in the presence and in the absence of hedging, has shown that a hedged assumption, i.e. an assumption that does not bear risk anymore, has null importance. More in detail, in the absence of a variable cost hedging, the sale price resulted the most important parameter, being responsible for the remuneration of fixed costs, variable costs and return on the investment. In the presence of a variable cost hedging, instead, the discount rate resulted as the most relevant factor, followed by the sale price with no role played by variable costs (γ). This result shows that variable cost hedging not only completely eliminates the risk associated with a relevant factor (γ), but also alleviates a portion of the risk associated with the sale price.

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