A Methodology for Determining Interactions in Probabilistic Safety Assessment Models by Varying One Parameter at a Time

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Abstract

In risk analysis problems, the decision-making process is supported by the utilization of quantitative models. Assessing the relevance of interactions is an essential information in the interpretation of model results. By such knowledge, analysts and decision-maker are able to understand whether risk is apportioned by individual factor contributions or by their joint action. However, models are oftentimes large, requiring a high number of input parameters, and complex, with individual model runs being time consuming. Computational complexity leads analysts to utilize one-parameter-at-a-time sensitivity methods, which prevent one from assessing interactions. In this work, we illustrate a methodology to quantify interactions in probabilistic safety assessment (PSA) models by varying one-parameter-at-a-time. The method is based on a property of the functional ANOVA decomposition of a finite change that allows to exactly determine the relevance of factors when considered individually or together with their interactions with all other factors. A set of test cases illustrates the technique. We apply the methodology to the analysis of the Core Damage Frequency of the Large Loss of Coolant Accident of a nuclear reactor. Numerical results reveal the non-additive model structure, allow to quantify the relevance of interactions and to identify the direction of change (increase or decrease in risk) implied by individual and joint factor variations.

Keywords: *Sensitivity Analysis*; *Importance Measures; Interaction Effects; Elementary Effects; Risk Analysis.*

1 Introduction

In the risk assessment of complex systems, decision-makers "*face a challenge when deciding how to allocate scarce resources to minimize the risks of failure. As resource constraints become tighter, balancing these failure risks is more critical, less intuitive, and can benefit from the power of quantitative analysis* [Dillon et al (2003); p. 354]." Indeed, the creation and utilization of quantitative models plays a central role in the decision-making process. Uncertainty quantification and model validation are then necessary to insure the correctness and consistency of the process [Saltelli (2002a), Saltelli et al (2000), Patil and Frey (2004), Borgonovo (2006), Saltelli et al (2009)]. The US Environmental Protection Agency "*recommends best practices to help determine when a model, despite its uncertainties, can be appropriately used to inform a decision. Specifically, it recommends that model developers and users ... perform sensitivity and uncertainty analyses. Sensitivity analysis evaluates the effect of changes in input values or assumptions on a model's results* [US EPA (2009), p. vii.]" The following quote from the White House's Office of Management and Budget is reported by Saltelli (2008): "*Sensitivity analysis is generally considered a minimum, necessary component of a quality risk assessment report.* [Saltelli (2008); p. 1]."

In this respect, assessing the relevance of interactions is an essential information towards the correct interpretation of model results. By such knowledge, analysts and decision-makers are able to understand whether risk is generated by factors individually or caused by their cooperation.

However, estimating interactions might become extremely demanding from a computational viewpoint. In fact, if a model contains k input factors, the number of possible interactions equals $2^k - 1$.[1] For $k > 30$, one has more than 1 billion interactions — see Table 1 for notation. — In real-life applications, probabilistic safety assessment (PSA) models contain hundreds of basic events and parameters. The large number of input factors and the model numerical complexity favour the use of one-at-a-time (OAT) sensitivity methods. Several PSA importance measures (Birnbaum, Risk Achievement Worth, Risk Reduction Worth [Cheok et al (1998)]) investigate the effect of OAT changes in components or parameters. However, OAT methods prevent analysts from detecting interactions (see Saltelli et al (2004), p. 52.).

In this work, we discuss a method to detect interactions in complex PSA models at the same computational cost of OAT methods. Core of the method is the evaluation of the model on two different scenarios. By utilizing properties of the functional ANOVA decomposition of a finite change, one can define sensitivity measures that exactly apportion the risk metric variation to the changes in the parameters [Borgonovo (2010)]. In particular, we shall make use of the following finite change sensitivity indices (FCSI) [Borgonovo (2010)]. The first order FCSI's (ϕ_i^1), that correspond to the portion of the finite change caused by a shift in factor x_i alone. The total order FCSI's [ϕ_i^T] that correspond to the fraction of the change in risk metric associated with a given factor. ϕ_i^T is a sensitivity measure that includes all contributions of x_i, individually and in interactions with all other factors. We then illustrate that, thanks to a symmetry effect entailed in the functional ANOVA expansion of a finite change, the total order FCSI's ($\boldsymbol{\phi}^T$) can be estimated

[1] In fact, there are $\binom{k}{1}$ parameters, $\binom{k}{2}$ pairs, $\binom{k}{3}$ triplets, ..., etc. Their sum equals $\sum_{i=1}^{k}\binom{k}{i} = 2^k - 1$.

Table 1: Notation and symbols used in this work

Symbol/Acronym	Meaning
y	Decision-support criterion/model output
OAT	One-at-a-time
SA	Sensitivity Analysis
FCSI	Finite change sensitivity index
PSA	Probabilistic Safety Assessment
ANOVA	Analysis of Variance
SSCC	Savage score correlation coefficient
LOCA	Loss of Coolant Accident
DOE	Design of Experiments
w.r.t.	with respect to
ATR	Advanced test reactor
CDF	Core damage Frequency
MCS	Minimal Cut Set
$x = (x_1, x_2, ..., x_k)$	Vector of the factors (parameters)
k	Number of factors
$X \subseteq R^k$	Input parameter (factors) space
M	Sample size in Montecarlo global SA
$f(x)$	Relationship between y and x
$f_{i_1,i_2,...,i_k}$	Generic term in the functional ANOVA expansion of f
X	Input parameter space
$f_i'(x_0)$	Partial derivative of f w.r.t. x_i
x^0, x^1	Any two points in X
x^-, x^+	Low and High levels in a Design of Experiment
$(x_i^0, x_{(-i)}^1)$	Point obtained by shifting x_i alone at x_i^1, with the other factors at x^0
$(x_i^0, x_{(-i)}^1)$	Point obtained by shifting x_i alone at x_i^0, with the other factors at x^1
ϕ_i^1	First order FCSI of factor x_i
ϕ_i^T	Total-order FCSI of factor x_i
$\phi_i^{\mathcal{I}}$	FCSI accounting for the interactions of factor x_i
$\phi^1, \phi^{\mathcal{I}}, \phi^T$	Vectors of all first, interaction and total order finite change sensitivity indices
$\overleftarrow{\phi}_i^1$	Reverse elementary FCSI of factor x_i
h	Gap in the model output change when x_i stays put
g	Sobol' g function
a	Parameters of Sobol' function

by varying factors OAT. By taking the difference between ϕ_i^T and ϕ_i^1 one quantifies, for each factor, the portion of the finite change associated with its interactions with all other factors. We denote such difference as $\phi_i^{\mathcal{I}} = \phi_i^T - \phi_i^1$, $i = 1, 2, ..., k$. The $\phi_i^{\mathcal{I}}$ then become synthetic indictors of the sign and relevance of interactions. Hence, by two OAT sensitivities (the first to compute $\boldsymbol{\phi}^1$, the second to compute $\boldsymbol{\phi}^T$] the method allows to quantify individual, total and interaction contributions of all factors.

We discuss the meaning of $\boldsymbol{\phi}^1, \boldsymbol{\phi}^{\mathcal{I}}$ and $\boldsymbol{\phi}^T$ in the context of the factor fixing setting. In particular, we split the setting into three insights. 1) Model structure. This insight answers the question: what portion of the model response (finite change) is attributable to interactions? We are going to see that knowledge of $\boldsymbol{\phi}^1, \boldsymbol{\phi}^{\mathcal{I}}$ and $\boldsymbol{\phi}^T$ provides an exact and quantitative answer to this question. 2) Direction of change. This insight provides indication on whether the decision-support criterion is increasing or decreasing when the parameters undergo the given changes. 3) Factor relevance. This insight concerns the identification of the most and least relevant factors in determining the model response.

Two well-known analytical test cases, the Ishigami function and Sobol' g functions, are used for a first illustration of the method and for discussing the derivation of insights through the above mentioned settings.

We then apply the method to a Probabilistic Safety Assessment (PSA) model. The case study is represented by the large Loss of Coolant Accident (LOCA) sequence of the Advanced Test Reactor (ATR). The model has been employed in previous SA studies [Borgonovo et al (2003)]. Model output is the core damage frequency (CDF) of the accident sequence. The CDF dependence on the basic event probabilities and parameters has a highly non-additive structure. Results indicate that, indeed, the model response is governed by interactions. Numerical findings reveal significant discrepancies between the ranking of obtained using sensitivity measures that include interactions and the ranking obtained using sensitivity measures that do not include interactions, both at the basic event and parameter levels.

The remainder of the paper is organized as follows. Section 2 illustrates the mathematical framework. Section 3 discusses the sensitivity analysis settings. Section 4 presents the analytical test cases. Section 5 presents the application to the PSA case study. Section 6 offers conclusions.

2 The Method

This section sets forth the mathematical framework of our work. We first review a relevant property of the functional ANOVA decomposition of a finite change. We then show how this property allows to find total order sensitivity measures at the same cost of elementary ones.

Let $y = f(x)$ denote the relationship between the model output (y) and the set of factors $\mathbf{x} = (x_1, x_2, ..., x_k)$ — see Table 1 for notation. — We next consider two generic points of the input parameter space, $\mathbf{x}^0$ and $\mathbf{x}^1$. The difference in the model output values attained when moving from $\mathbf{x}^0$ to $\mathbf{x}^1$ is denoted by $\Delta f = f(\mathbf{x}^1) - f(\mathbf{x}^0)$ (Figure 1).

Let $\mu = \prod_{i=1}^{k} \mu_i$ any product measure. Under the sole requirement that f is measurable, it

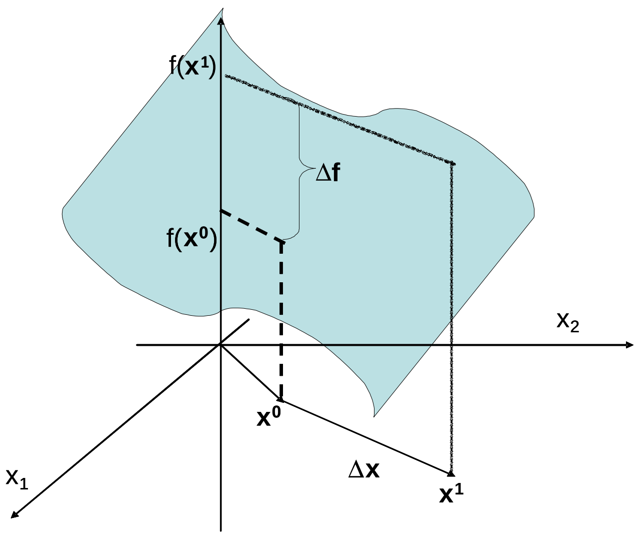

Figure 1: Finite change in a 2-factor model output when the parameters shift from $\mathbf{x}^0$ to $\mathbf{x}^1$.

has been shown that Δf can be decomposed in a finite number of terms as follows [Rabitz and Alis (1999), Sobol' (2003), Borgonovo (2010)]:

$$\Delta f = \sum_{s=1}^{k} \sum_{i_1<i_2<...<i_s}^{k} \Delta f_{i_1,i_2,...,i_s} \tag{1}$$

where $f_{i_1,i_2,...,i_s}(\mathbf{x}^0)$ is a generic summand of the functional ANOVA expansion of f, $\Delta f_{i_1,i_2,...,i_s} = f_{i_1,i_2,...,i_s}(\mathbf{x}^1) - f_{i_1,i_2,...,i_s}(\mathbf{x}^0)$ and $\mathbf{x}^1$ and $\mathbf{x}^0$ are any two points in the input parameter space. We note that Δf is any change in f (not necessarily infinitesimal), and the summands $f_{i_1,i_2,...,i_s}$ are obtained by projection according to a generic product measure. When μ_i is set equal to the Dirac-δ measure, as stated by Sobol' (2003), the decomposition of a finite change is obtained by nested applications of the finite-difference operator. The summands in eq. (1) become

$$\begin{cases} \Delta_i f = f(x_i^1, \mathbf{x}^0_{(-i)}) - f(\mathbf{x}^0) \\ \Delta_{i,j} f = f(x_i^1, x_j^1, \mathbf{x}^0_{(-i,j)}) - \Delta_i f - \Delta_j f - f(x^0) \\ ... \end{cases} \tag{2}$$

In eq. (2), $(x_i^1, \mathbf{x}^0_{(-i)}) = (x_1^0, x_2^0, ..., x_{i-1}^0, x_i^1, x_{i+1}^0, ..., x_k^0)$ denotes the point obtained by shifting factor x_i at value x_i^1, while the remaining $k-1$ parameters $(\mathbf{x}^0_{(-i)})$ are fixed at $\mathbf{x}^0$. Similarly, $(x_i^1, x_j^1, \mathbf{x}^0_{(-i,j)})$ indicates that factors x_i and x_j are shifted, with the others remaining fixed.

Consider the first equalities in eq. (2) and denote them as ϕ_i^1 [Borgonovo (2010)]:

$$\phi_i^1 = \Delta_i f = f(x_i^1, \mathbf{x}^0_{(-i)}) - f(\mathbf{x}^0) \tag{3}$$

If taken as a sensitivity measure, ϕ_i^1 represents the individual contribution of x_i to the finite change in f. We refer to ϕ_i^1 as to the FCSI of x_i. One can also consider the normalized version of this index:

$$\Phi_i^1 = \frac{\phi_i^1}{\Delta f} \tag{4}$$

Note that, as changes become small, under the assumption that f is differentiable, one has that

$$\phi_i^1 \simeq f_i'(x^0)\mathrm{d}x_i = \mathrm{d}_i f \text{ and } \Phi_i^1 \simeq DIM_i(x^0) \tag{5}$$

i.e., ϕ_i^1 and Φ_i^1 tend to the partial differential of f and the differential importance (DIM) of x_i, respectively [see Appendix A in Borgonovo (2010).]

Consider then the second order terms in eqs. (2). One can set:

$$\phi_{i,j} = f(x_i^1, x_j^1, \mathbf{x}^0_{(-i,j)}) - \Delta_i f - \Delta_j f - f(x^0) \tag{6}$$

$\phi_{i,j}$ [eq. (6)] represents the portion of Δf that is due to the interaction between x_i and x_j. In particular, when changes are small, it can be proven that $\phi_{i,j}(x_i, x_j)$ is related to the second order partial derivative of f w.r.t. x_i and x_j ($f''_{i,j}(x^0)$) as follows [Borgonovo (2010); Appendix A]:

$$\phi_{i,j}(x_i, x_j) \simeq f''_{i,j}(x^0)\mathrm{d}x_i\mathrm{d}x_j \tag{7}$$

Thus, for small changes, $\phi_{i,j}(x_i, x_j)$ equals the rate of change of f w.r.t. x_i and x_j multiplied by the corresponding small deviations. This means that $\phi_{i,j}(x_i, x_j)$ conveys information on second order local interactions.

By utilizing eqs. (3) and (6), one can rewrite eq. (1) as follows:

$$\Delta f = \sum_{s=1}^{k} \sum_{i_1 < i_2 ... < i_s} \phi_{i_1 i_2 ... i_s} \tag{8}$$

Eq. (8) states that the sum of all the FCSI's ($\phi_{i_1 i_2 ... i_s}$, $s = 1, 2, ..., k$) equals the finite change in f. Since eq. (8) follows from a functional ANOVA expansion, all the FCSI's [$\phi_{i_1 i_2 ... i_s}$] are orthogonal [Borgonovo (2010)]. Let us now consider the sum of all terms in eq. (8) that involve x_i. Let us write

$$\phi_i^T := \sum_{s=1}^{k} \sum_{\substack{i_1 < i_2 ... < i_s \\ i \in i_1 < i_2 ... < i_s}} \phi_{i_1 i_2 ... i_s} \tag{9}$$

ϕ_i^T represents the fraction of the change in f associated with the change in x_i and equals the sum of its individual and interaction contributions to the finite change. It can be shown that, if f is n times differentiable and parameter changes are small, then

$$\phi_i^T \simeq f_i'(x^0)\mathrm{d}x_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} f''_{j,i}(x^0)\mathrm{d}x_j\mathrm{d}x_i + ... + f^n_{1,2,..,n}(x^0)\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_n \tag{10}$$

Eq. (10) shows that φ_l^T includes all partial derivatives of f related to x_i. Thus, it includes the local interaction effects of all orders associated with x_i. For this reason, ϕ_i^T is named the total order FCSI of x_i [Borgonovo (2010)].

Consider next the differences $\phi_i^{\mathcal{I}} = \phi_i^T - \phi_i^1$, $i = 1, 2, ..., k$. One has

$$\phi_i^{\mathcal{I}} = \phi_i^T - \phi_i^1 = \sum_{s=2}^{k} \sum_{\substack{i_1<i_2...<i_s \\ i\in i_1<i_2...<i_s}} \phi_{i_1 i_2 ... i_s} \tag{11}$$

$\phi_i^{\mathcal{I}}$ is the sum of $2^k - 2$ terms and is extended to all the interactions of parameter x_i. Note that, by construction, $\phi_i^{\mathcal{I}}$ equals the portion of the change in f provoked by the change in factor i in interactions with all the other factors, with exclusion of its individual contribution.

By eq. (9), estimation of all ϕ_i^T's requires $2^k - 1$ model evaluations. This clearly would make the computation prohibitive in the case of complex models. In this respect, one notes that a similar problem arises in the estimation of variance-based global sensitivity indices. More precisely, in global SA, the cost would be of $M \cdot (2^k - 1)$, where M is the Montecarlo sample size. However, Saltelli (2002a)'s Theorem 1 shows that a cost of $M \cdot (k + 2)$ model runs is necessary to obtain both first and total order variance-based sensitivity indices. The result stems from properties of global sensitivity indices proven in Sobol' (1993), Homma and Saltelli (1996). In other words, variance-based total order sensitivity indices are obtained at the same cost of the first order ones. Since the mathematical framework in eqs. (1) and (2) allows to transfer the properties of variance decomposition to the decomposition of a finite change, one would expect that the total order FCSI's can be computed at the same cost of first order FCSI's.

The next paragraphs confirm such expectation. We start with a result illustrated in Borgonovo (2010).

Proposition 1 *[(see Proposition* 1 *in Borgonovo (2010) for a formal proof)]Consider points* $\mathbf{x}^1$ *and* $(x_i^0, \mathbf{x}_{(-i)}^1)$. *Then*

$$\phi_i^T = f(\mathbf{x}^1) - f(x_i^0, \mathbf{x}_{(-i)}^1) \qquad i = 1, 2, ..., k \tag{12}$$

In Proposition 1, $(x_i^0, \mathbf{x}_{(-i)}^1)$ is the point obtained by shifting all factors from $\mathbf{x}^0$ to $\mathbf{x}^1$, but x_i. $f(\mathbf{x}^1) - f(x_i^0, \mathbf{x}_{(-i)}^1)$ is the difference in the corresponding model output values. The equality states that such difference equals ϕ_i^T. An intuitive explanation is as follows. By adding and subtracting $f(\mathbf{x}^0)$ from $f(\mathbf{x}^1) - f(\mathbf{x}_{(-i)}^1)$, one obtains $f(\mathbf{x}^1) - f(x_i^0, \mathbf{x}_{(-i)}^1) = \Delta f - \Delta f_{(-i)}$, where Δf is the change in f when all factors jump from $\mathbf{x}^0$ to $\mathbf{x}^1$, and $\Delta f_{(-i)}$ is the change in f when all factors shift but x_i. Thus, $f(\mathbf{x}^1) - f(x_i^0, \mathbf{x}_{(-i)}^1)$ equals the gap in Δf that is created when all factors move from $\mathbf{x}^0$ to $\mathbf{x}^1$ but x_i. It is then possible to prove that this gap is exactly equal to the portion of the change in f related to x_i, namely, ϕ_i^T (see Proposition 1 in Borgonovo (2010) for a formal proof).

We then observe that point $(x_i^0, \mathbf{x}_{(-i)}^1)$ is obtained by placing oneself at $\mathbf{x}^1$ and shifting only x_i to x_i^0. In other words, point $(x_i^0, \mathbf{x}_{(-i)}^1)$ is obtained by shifting one parameter in the direction $\mathbf{x}^1 \to \mathbf{x}^0$ — we shall refer to it as "reverse direction" — The corresponding change in f is equal

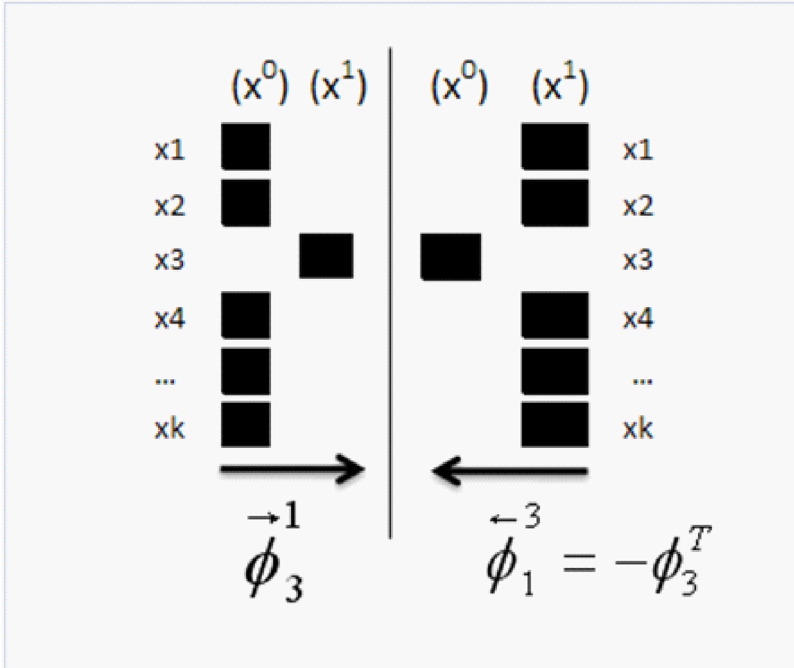

Figure 2: A visual representation of the symmetry effect that allows to estimate ϕ^T by one-parameter-at-a-time variations.

to $f(x_i^0, \mathbf{x}_{(-i)}^1) - f(\mathbf{x}^1)$. This quantity, however, equals the first order FCSI of x_i obtained when parameters shift in the reverse direction. Denoting the reverse direction FCSI by $\overleftarrow{\phi}_i^1$, one has:

$$\overleftarrow{\phi}_i^1 = f(x_i^0; \mathbf{x}_{(i)}^1) - f(\mathbf{x}^1) \tag{13}$$

By comparing eqs. (13) and (12) one obtains

$$\phi_i^T = -\overleftarrow{\phi}_i^1 \tag{14}$$

The above finding can be summarized in the following.

Proposition 2 *The total order FCSI's when factors shift in the direction* $\mathbf{x}^0 \to \mathbf{x}^1$ *equal the opposite of the first order FCSI* ($\overleftarrow{\phi}_i^1$) *when factors shift in the reverse direction,* $\mathbf{x}^1 \to \mathbf{x}^0$.

Figure 2 visualizes Proposition 2. The left part of Figure 2 shows the shift in one parameter (x_3) in the direction $\mathbf{x}^0 \to \mathbf{x}^1$. One obtains the corresponding first order FCSI's (ϕ_3^1). Conversely (right part of Figure 2), by starting at ($\mathbf{x}^1$) and shifting x_3 in the direction $\mathbf{x}^1 \to \mathbf{x}^0$, one obtains the reverse-direction first order FCSI, $\overleftarrow{\phi}_3^1$. By changing sign to $\overleftarrow{\phi}_3^1$, one obtains ϕ_3^T. By subtracting ϕ_3^1 from ϕ_3^T, one obtains the interaction FCSI of x_3, $\phi_3^{\mathcal{I}}$. By repeating this steps k times, one estimates $\boldsymbol{\phi}^1, \boldsymbol{\phi}^{\mathcal{I}}$ and $\boldsymbol{\phi}^T$ for all factors.

From the computational viewpoint, this leads to the following result: $k+1$ model runs allow to

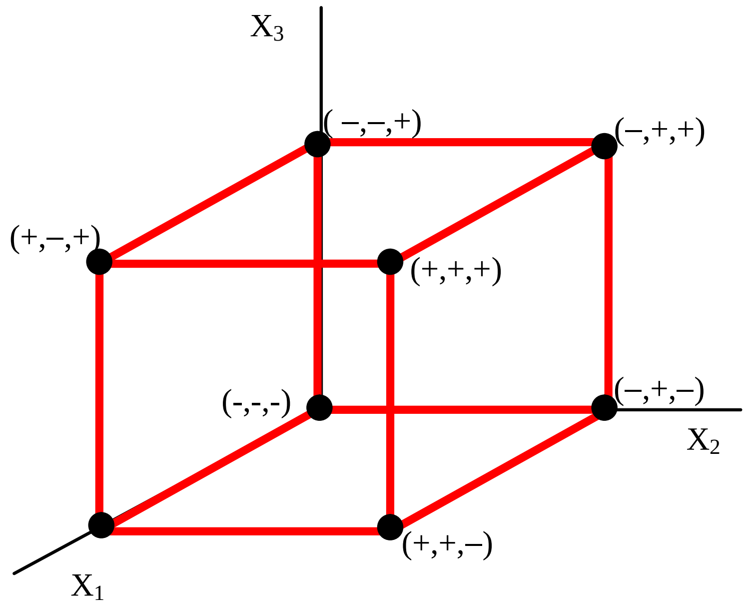

Figure 3: A 2^3 design (see Montgomery and Myers (1995); p. 92).

obtain the first order contributions $[\boldsymbol{\phi}^1]$. By additional $k+1$ model runs in the reverse direction one determines the k total order sensitivity measures $[\boldsymbol{\phi}^T]$. In summary, by two OAT sensitivities one estimates the total, individual and interaction contributions of all factors.

In the remainder of this section, we investigate how the above concepts relate to the screening exercise. We consider a design of experiment as described by Myers and Montgomery (1995). In particular, let us focus on the full-factorial 2^k design framework (Figure 3 reports a 2^3 example; see Myers and Montgomery (1995) p. 92).

In a 2^k design, each factor has two levels, $(-)$ and $(+)$. We shall refer to low and high level, respectively [see Figure 3]. Let us denote by $\mathbf{x}^+ = (+,+,....,+)$ and $\mathbf{x}^- = (-,-,....,-)$ the points at which all factors are at the high level and low levels respectively, and by $f^+ = f(\mathbf{x}^+)$, $f^- = f(\mathbf{x}^-)$ the corresponding model output values. The link between DOE and the ANOVA decomposition of a finite change is then obtained by applying eq. (8) to decompose the change $f^+ - f^-$.

Let us now further examine the meaning of eq. (2). Morris (1991) defines one estimate of an individual effect via the following quantity:

$$d_i = \frac{f(x_i^- + \Delta_i, \mathbf{x}^+_{(-i)}) - f(\mathbf{x}^+)}{\Delta_i} \tag{15}$$

In Campolongo et al (2007), the absolute values of elementary effects are taken as sensitivity measures in order to avoid type two errors in randomization:

$$|d_i| = \left| \frac{f(x_i^- + \Delta_i, \mathbf{x}_{(-i)}^+) - f(\mathbf{x}^+)}{\Delta_i} \right| \tag{16}$$

One notes that the first of the equalities in eq. (2) is the numerator of eq. (15). Therefore, one has

$$\phi_i^1 = \Delta_i \cdot d_i \tag{17}$$

Eq. (17) shows that ϕ_i^1 and d_i differ only by a normalization factor. In particular, by appropriate scaling, one can always choose $\Delta_i = 1$ and ϕ_i^1 and d_i coincide. Thus, the first order FCSI's coincide with one estimate of the elementary effect of factor i in a screening exercise, given that x^0 and x^1 are chosen as $(-)$ and $(+)$ levels.

In the DOE terminology, eq. (3), implies that the first order FCSI's (ϕ_i^1) are obtained by shifting parameters OAT from the low to the high level $[(+) \rightarrow (-)]$. Similarly, eq. (6) shows that second order FCSI's are proportional to one estimate of second order interaction effects. They are obtained by shifting all factor pairs in the reverse direction $[(-) \rightarrow (+)]$ and orthogonalizing.

To cast Proposition 1 in the DOE context, one needs to set $\mathbf{x}^1 = \mathbf{x}^+$ and $\mathbf{x}^0 = \mathbf{x}^-$ and apply eq. (12). One has:

$$\phi_i^T = f^+ - f(x_i^-, \mathbf{x}_{(-i)}^+) \tag{18}$$

where $(x_i^-, \mathbf{x}_{(-i)}^+)$ is the point obtained by shifting all factors at the high level but x_i. Then, $f(x_i^-, \mathbf{x}_{(-i)}^+) - f^+$ is one estimate of an elementary effect in the reverse direction $(+) \rightarrow (-)$**,** and coincides with the corresponding reversed first order FCSI.

In terms of DOE schemes, we note that the proposed approach makes use of a finite change in model output across two points in the input parameter space, namely $\mathbf{x}^0$ and $\mathbf{x}^1$. The assumption under which the method insures detection of interactions is model monotonicity.

In the case of non-monotonic input-output relations, a result indicating that interactions do not matter might not testify the absence of interactions, as the consequence of compensation effects (type II error). The choice of the points at which to evaluate the model, then, becomes crucial to evidence interactions. In particular, the DOE and screening literature suggests to utilize a larger sample size $\mathbf{x}^0$, $\mathbf{x}^1, ..., \mathbf{x}^s$ $(s > 2)$, possibly with suitable randomization. We refer to Morris (1991), Campolongo et al (2007), Saltelli et al (2009) for sampling schemes in the screening exercise, and to Myers and Montgomery (1995) for a comprehensive description of several design of experiment (DOE) schemes. One notes that the method proposed here would then be applicable to the decomposition of each of the jumps in model output across any pair of these points.

In the next Section, we discuss the SA settings utilized in this work.

3 Settings for Finite Change Sensitivity Indices

An SA setting is defined as "*a way of framing the sensitivity analysis quest in such a way that the answer can be confidently entrusted to a well-identified measure* [Saltelli et al (2008); p. 24.]

The concept of SA setting is originated by Saltelli and Tarantola (2002), and further elaborated in Saltelli et al (2004), Saltelli et al (2006). In Saltelli et al (2004), the following settings are introduced: factor prioritization, factor fixing, variance cutting and factor mapping. The factor prioritization setting concerns the identification of the most relevant factors. The variance cutting setting is related to the identification of the smallest groups of factors that, when fixed, allow to achieve a pre-determined variance-reduction level. The factor mapping setting is developed in the context of Montecarlo filtering and answers the question of identifying "*what factor is most responsible for producing realizations of Y in the region of interest* (Saltelli et al (2004); p. 55)". The purpose of the factor-fixing setting is to identify the input factors that, if fixed, do not cause any "*significant loss of information* (Saltelli et al (2004); p. 54)" to the decision-maker. Screening methods are the most appropriate tools in the context of the "*factor fixing*" setting (Saltelli et al (2004); p. 54). The reason is as follows. SA methods can be grouped in the categories of local, screening, and global methods. Examples of local SA techniques are the Birnbaum, the Criticality, the Fussell-Vesely, the Joint and the Differential importance measures [Birnbaum (1969), Vesely (1998), Armstrong (1995), Borgonovo and Apostolakis (2001)]. By construction, local methods provide insights on model behavior around one point in the input parameter space [Saltelli (1999), Borgonovo et al (2003), Patil and Frey (2004)]. When the decision-maker's confidence in the parameters is not complete, uncertainty propagation is necessary to assess the decision-maker's degree of confidence in the importance measure results [a study on the effect of uncertainty in local importance measures can be found in Borgonovo (2008).]. Conversely, global SA methods take the decision-maker's uncertainty into account directly in the sensitivity measures. Thus, the sensitivity measures reflect the decision-maker's state of belief [Saltelli (2002b), Borgonovo (2006)]. However, when models are computationally intensive, the application of global SA techniques might be restrained. In fact, in spite of the considerable improvements in computation (see Theorems 1 and 2 in Saltelli (2002a)), global SA requires a high number of model runs for accurate estimation of the sensitivity measures. In particular, as proven in Saltelli (2002a), $M(k+2)$ model runs are necessary to estimate the first and total order global sensitivity indices. M is the appropriate sample size for numerical estimation — M can be of order 10^3 or more. — If model runs are time consuming, the computational cost might prevent the direct application of a global SA method. In these circumstances, however, it is recommended to adopt a two-step approach, by letting the global SA exercise be preceded by a screening exercise [see Saltelli et al (2004)]. In particular, "*In these cases, one of the aims in modelling is to come up with a* short list *of important factors* [Saltelli et al (2004); p. 91]." Fixing non-relevant input parameters, in fact, can lead to notable reductions in computational cost.

In the remainder of this section, we elaborate further the factor fixing setting, in the light of the information on model structure, direction of change and factor relevance that can be derived by knowledge of ϕ_i^1, ϕ_i^T and $\phi_i^{\mathcal{I}}$.

Model structure. The SA question is states as follows: are model results driven by individual factors or by interactions? This question is answered by comparing the magnitude of ϕ_i^1, $\phi_i^{\mathcal{I}}$ and

ϕ_i^T. In fact, if $\left|\phi_i^1\right| >> \left|\phi_i^{\mathcal{I}}\right|$ for all i, then one can conclude that the model response is additive, i.e., the model responds separately to parameter changes. Conversely, if $\left|\phi_i^1\right|$ is comparable to $\left|\phi_i^{\mathcal{I}}\right|$, or $\left|\phi_i^1\right| < \left|\phi_i^{\mathcal{I}}\right|$, interactions cannot be neglected.

Direction of change: The SA question becomes: do parameter changes cause the model output to increase or decrease? The origin of this setting is linked to the SA question stated in Economics by Samuelson (1947): "*it is hoped to formulate qualitative restrictions on slopes, curvatures etc. of our equilibrium,*" so as to be able to understand whether the parameter changes impact the decision-support criterion positively or negatively (Samuelson (1947) p. 20.). As a reference, the decision-support criterion (model output) can be a risk metric, an expected utility, a net present value, etc.. It is then natural to ask the "what-if" question of whether, when factors are changed, risk, expected utility or net present value (etc.) increase or decrease, respectively. To answer this question, one needs to retain the sign of the sensitivity measures. The sign of ϕ_i^1 indicates whether the individual factor variations impact the decision-support criterion positively or negatively. The sign of $\phi_i^{\mathcal{I}}$ indicates whether interactions overlap in a constructive or disruptive way with individual actions. The sign of ϕ_i^T summarizes whether the overall consequence of a change in x_i (alone and together with its interactions) is an increase or decrease in model output.

Factor relevance. The SA question is: how influential is a factor in driving model results? The answer to this question is provided for by the absolute value of the sensitivity measures. By utilizing $\left|\phi_i^1\right|$ one ranks the factors based on their individual actions. By utilizing $\left|\phi_i^T\right|$ one ranks parameters with inclusion of all their interactions with the other factors. In this respect, we note that the insights on model structure should be gained before choosing the sensitivity measures. In fact, if interactions prevail over individual actions, a ranking based on first order sensitivity measures might not be completely reflective of the actual model behavior.

4 Analytical Test Cases

In this section, by application to analytical examples, we illustrate the determination of the FCSI's $(\phi_i^1, \phi_i^{\mathcal{I}}, \phi_i^T)$ by the OAT scheme presented in Section 2, and the interpretation of results in the light of the Settings of Section 3.

4.1 Test Case I: the Ishigami Function

The Ishigami function has been extensively used in the SA literature as a test case, especially after the work by Homma and Saltelli (1996). We write it in the following form

$$f = \sin(x_1) + 7\sin(x_2)^2 + 0.1x_3^4\sin(x_1) \tag{19}$$

We let $\mathbf{x}^0 = (1, 1, 1)$ and $\mathbf{x}^1 = (2, 2, 2)$. The two corresponding model output values are $f(\mathbf{x}^0) = 5.8821$ and $f(\mathbf{x}^1) = 8.1519$. Hence, the model output undergoes the finite change $\Delta f = 2.2698$. Since the model is smooth ($Y \in C^\infty(\mathbb{R}^3)$), the decomposition in eq. (1) applies.

Being $k = 3$ in this example, only 8 model evaluations are necessary for the complete decomposition of Δf. The decomposition is achieved by recursive application of the finite difference operator

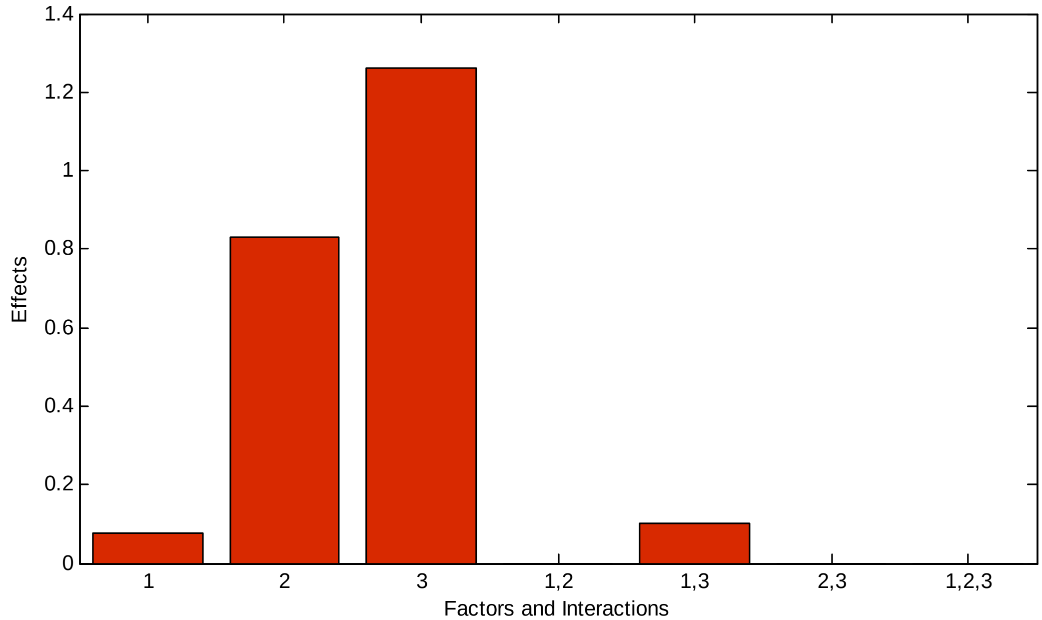

Figure 4: Complete decomposition of $\Delta f = f^+ - f^-$ for the Ishigami test function. The FCSI's of all orders, ϕ_1^1, $\phi_2^1, \phi_3^1, \phi_{1,2}^2, \phi_{1,3}^2, \phi_{2,3}^2, \phi_{1,2,3}^3$ are displayed. One notes that only a weak interaction effect between x_1 and x_3 is registered.

[eq. (2); see also Borgonovo (2010).] The FCSI's of all orders (ϕ_1^1, $\phi_2^1, \phi_3^1, \phi_{1,2}^2, \phi_{1,3}^2, \phi_{2,3}^2, \phi_{1,2,3}^3$) are plotted in Figure 4.

From Figure 4, one notes that the sum of the sensitivity measures equals the finite change: $\phi_1^1 + \phi_2^1 + \phi_3^1 + \phi_{1,2}^2 + \phi_{1,3}^2 + \phi + \phi_{1,2,3}^3 = 2.2698 = \Delta f$, as per eq. (8). Figure 4 also shows that the only interaction is the one between x_1 and x_3, in agreement with the Ishigami function structure.

The total order FCSI's of the parameters can then be found as sum of the lower order FCSI's, by a direct application of the definition [eq. (9)]:

$$\begin{aligned}
\phi_1^T &= \phi_1^1 + \phi_{1,2}^2 + \phi_{1,3}^2 + \phi_{1,2,3}^3 = 0.1763 \\
\phi_2^T &= \phi_2^1 + \phi_{1,2}^2 + \phi_{2,3}^2 + \phi_{1,2,3}^3 = 0.8312 \\
\phi_3^T &= \phi_3^1 + \phi_{1,3}^2 + \phi_{2,3}^2 + \phi_{1,2,3}^3 = 1.3639
\end{aligned}$$

However, by Proposition 2, the ϕ_i^T's can be estimated directly by 3 OAT sensitivities. One needs to utilize $\mathbf{x}^1$ as the reference point and to re-evaluate the Ishigami function in the direction $\mathbf{x}^1 \to \mathbf{x}^0$. The three new model output values, which we denote by $f(x_i^-; \mathbf{x}_{(-i)}^1)$ are: $f(x_i^-; \mathbf{x}_{(-i)}^1) = [7.9756, 7.3207, 6.7880]$. The differences $f(\mathbf{x}_{(-i)}^1) - f(x_i^0; \mathbf{x}_{(-i)}^1)$ $(i = 1, 2, 3)$ then equal the reverse first order FCSI's, $\overleftarrow{\phi}_i^1$, i.e., the opposite of ϕ_i^T by eq. (14) (see also Proposition 2, Section 2). One obtains:

$$\boldsymbol{\phi}^T = [0.1763, 0.8312, 1.3639]$$

The first order, interaction and total order FCSI's are displayed in Figure 5.

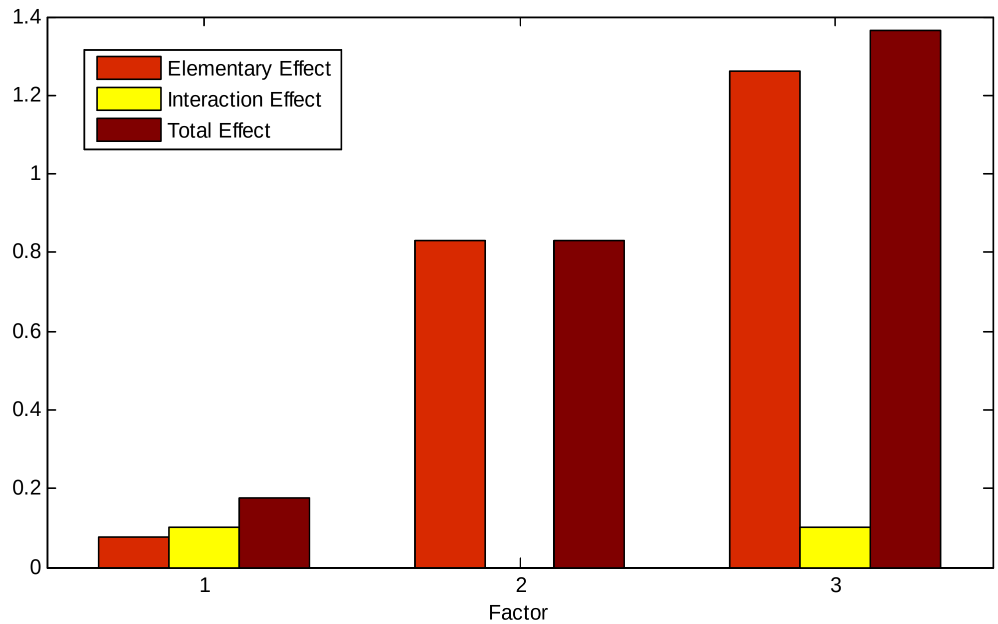

Figure 5: ϕ_i^1 (left), $\phi_i^{\mathcal{I}}$ (middle), ϕ_i^T (right) for the 3 factors of the Ishigami model. One notes that individual contributions are predominant, with a weak presence of interactions. All sensitivity measures are positive.

By the settings of Section 3, one gathers the following insights from Figure 5.

Model structure. The only interaction is the one between x_1 and x_3. Individual actions prevail for factors x_1 and x_3. These results are in agreement with past studies on the Ishigami function.

Direction of change. All individual and interaction sensitivity measures are positive. This means that when jumping from $\mathbf{x}^0$ to $\mathbf{x}^1$, all factors have a positive impact on the model output. Interactions reinforce individual effects.

Factor relevance. From Figure 5, one observes that x_3 is the most relevant factor followed by x_2 and x_1.

In the next section, we discuss a test case in which model behavior is dominated by interactions.

4.2 Sobol' g functions

Sobol' g functions have been utilized as a test case in several important works related to functional ANOVA: we recall Sobol' (1993), Sobol' (2001), Sobol' (2003), Sobol' et al (2007). A Sobol' g function is represented by the following expression

$$g = \prod_{i=1}^{k} \frac{|4x_i - 2| + a_i}{1 + a_i} \tag{20}$$

where $\mathbf{a}$ is a vector of parameters. In our test case, we utilize the same vector $\mathbf{a}$ as in Sobol' et al (2007): $\mathbf{a} = \left[0\ 1\ 4.5\ 9\ 99\ 99\ 99\ 99\right]$. We let $\mathbf{x}^0 = [-1, -1, -1, -1, -1, -1, -1, -1]$ and $\mathbf{x}^1 = [1, 1, 1, 1, 1, 1, 1, 1]$. Since $g \in C(\mathbb{R}^k)$, eq. (1) applies.

To compute the total order FCSI's, one needs to perform the following steps. The first is to evaluate $g(\mathbf{x}^1)$. In our case, one obtains $g(\mathbf{x}^1) = 4.0584$. Then, by starting at $\mathbf{x}^1$ and shifting one parameter at a time from x_i^1 to x_i^0, one computes the differences $g(\mathbf{x}^1) - g(x_i^0; \mathbf{x}_{(-i)}^1)$. By eq. (14) (see Proposition 2), the ϕ_i^T's are then obtained by reversing the signs of these differences. One obtains

$$\boldsymbol{\phi}^T = [-8.12, -5.41, -2.50, -1.48, -0.16, -0.16, -0.16, -0.16] \tag{21}$$

A second series of OAT factor variations provides ϕ_i^1, and, by difference, $\phi_i^{\mathcal{I}}$. By starting at $\mathbf{x}^0$, one evaluates $g(\mathbf{x}^0) = 73.10$. Shifting one-parameter at a time from x_i^0 to x_i^1, with the others at $\mathbf{x}^0$, one obtains the first order FCSI's

$$\boldsymbol{\phi}^1 = [-48.73, -41.76, -27.85, -19.50, -2.78, -2.78, -2.78, -2.78] \tag{22}$$

By computing the differences $\phi_i^T - \phi_i^1$, one derives the interaction FCSI's:

$$\boldsymbol{\phi}^{\mathcal{I}} = [40.61, 36.36, 25.35, 18.02, 2.62, 2.62, 2.62, 2.62] \tag{23}$$

Figure 6 allows a visual comparison of ϕ_i^1, $\phi_i^{\mathcal{I}}$ and ϕ_i^T.

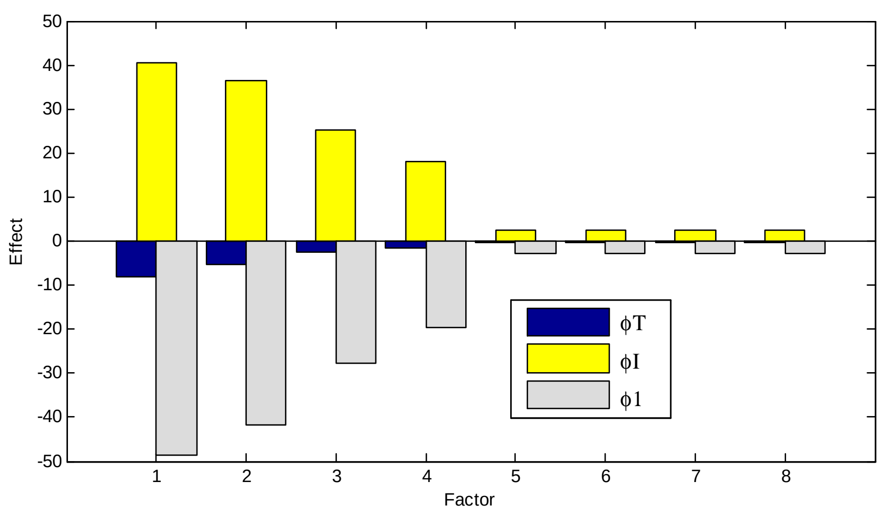

Figure 6: ϕ_i^T (negative dark, first in each triplet), $\phi_i^{\mathcal{I}}$ (positive light, second in each triplet) and ϕ_i^1 (negative light, third of each triplet) for the 8 factors. One notes that interactions act in the opposite direction in respect of the individual factor impacts.

Let us then interpret the results in Figure 6 in view of the settings of Section 3.

Model structure. The results in Figure 6 show that interactions play an important role in the model response, as the magnitude of the interaction FCSI's is comparable to the magnitude of the first order indices.

Direction of change. Figure 6 shows that the shift from $\mathbf{x}^0$ to $\mathbf{x}^1$ of each factor has a negative impact on the model output (the ϕ_i^1 are all negative). Interaction effects $[\phi_i^{\mathcal{I}}]$, instead, are positive, thus contrasting the individual parameter actions. The total order FCSI's $[\boldsymbol{\phi}^T]$, are negative, but their magnitude is lower than the corresponding first order FCSI's. Thus, interactions smoothen the impact of the individual factor changes.

Factor relevance. By utilizing the absolute values of the sensitivity indices, one notes that x_5, x_6, x_7 and x_8 have a low relevance on the model output. On the other hand, x_1 is the most relevant factor, followed by x_2, x_3 and x_4. These results hold both when first order and total order FCSI's are considered [Figure 6].

In the next Section, we apply the method to a PSA model. We note that the class of PSA models for coherent systems satisfy the monotonicity assumptions stated above.

5 An Application: the Advanced Test Reactor Large Loss of Coolant Accident Sequence

In this section, we apply the method to a PSA case study. We make reference to the large LOCA sequence of the ATR reactor PSA model utilized in Borgonovo et al (2003).

The sequence contains one initiating event, 44 basic events, 239 minimal cut sets (MCS) and 31 parameters. The list of factors is reported in Table 2. The numerical values are the same as in Table 1 of Borgonovo et al (2003).

The dependence of the CDF on basic event probabilities is multilinear, i.e., of the form [Borgonovo and Apostolakis (2001), Borgonovo et al (2003)]:

$$CDF = f_{LLOCA} \cdot (\sum_{i=1}^{n_{MCS}} \prod_{u=1}^{n_{BE_i}} p_{s_u}) \tag{24}$$

where f_{LLOCA} is the initiating event frequency, $n_{MCS} = 239$ is the number of minimal cut sets, n_{BE_i} is the number of basic events in minimal cut set i, and p_{s_u} is the conditional probability of the u^{th} basic event in the i^{th} MCS. The 44 basic event probabilities are given in column 3 of Table 2.

At the parameter level, some of the failure probabilities are expressed through an exponential model

$$p_{s_k}(T) = 1 - e^{-\lambda_{s_k} T} \tag{25}$$

The relationship between the CDF and the factors becomes non-linear

$$CDF = f_{LLOCA} \cdot [\sum_{i=1}^{n_{MCS}} \prod_{k=1}^{n_{BE_i}} p_{s_k}(\lambda)] \tag{26}$$

as some of the $p_{s_k}(\lambda)$ are of the exponential form of eq. (25). Since the same λ is shared across

Table 2: Basic events (first column), meaning (second column), values of the Probabilities (third column) and corresponding parameters for the ATR large LOCA sequence, as per Table 1 in Borgonovo et al (2003).

Nr.	Event	Probability	Parameter
1	Operator failure to isolate after excavation error	$8.00 \cdot 10^{-2}$	x_1
2	Firewater injection system (FWIS) disabled by excavation error	$1.25 \cdot 10^{-4}$	x_2
3	Insufficient flow through bottom head injection	$1.50 \cdot 10^{-6}$	x_3
4	Lower FIS manual valve GT-T-84 failure to restore after TM	$2.70 \cdot 10^{-5}$	x_4
5	No flow from firewater injection system	$3.48 \cdot 10^{-5}$	x_5
6	Failure to actuate valve lcv-7b	$5.00 \cdot 10^{-4}$	x_6
7	Failure to actuate valve lcv-7a	$5.00 \cdot 10^{-4}$	x_6
8	Lower FWIS injection valve LCV-7B spuriously closes	$3.00 \cdot 10^{-4}$	$x_7 = \lambda_v = 3 \cdot 10^{-6}$
9	Valve LCV-7B ICC fails to operate	$1.00 \cdot 10^{-3}$	x_8
10	Lower FIS injection valve LCV-7B fails to open	$7.00 \cdot 10^{-4}$	x_9
11	Common cause failure of valve paths to open	$7.00 \cdot 10^{-5}$	x_{10}
12	Common cause loss of both FIS paths due to AOVs failure	$4.30 \cdot 10^{-5}$	x_{11}
13	Lower FIS injection valve LCV-7A fails to open	$7.00 \cdot 10^{-4}$	x_9
14	Valve LCV-7A ICC fails to operate	$1.00 \cdot 10^{-3}$	$x_8 = \lambda_v = 3 \cdot 10^{-6}$
15	Lower FWIS injection valve LCV-7A spuriously closes	$3.00 \cdot 10^{-4}$	x_7
16	Deepwell pump 1 heating and ventilation fails	$1.40 \cdot 10^{-2}$	x_{12}
17	Deepwell pump 1 is in TM (plant-specific)	$1.94 \cdot 10^{-2}$	x_{13}
18	Deepwell pump 1 fails to start	$3.00 \cdot 10^{-3}$	x_{14}
19	Deepwell pump 1 fails to run	$2.99 \cdot 10^{-3}$	$x_{15} = \lambda_p = 3 \cdot 10^{-5}$
20	Deepwell Pump 1 instrumentation and control (ICC) fails	$1.00 \cdot 10^{-3}$	x_{16}
21	Deepwell Pump #1 Breaker Spuriously Opens	$3.00 \cdot 10^{-5}$	$x_{17} = \lambda_b = 3 \cdot 10^{-7}$
22	Level control faults	$8.38 \cdot 10^{-5}$	x_{19}
23	Power failure at 4160 vac commercial bus ‘d’	$5.60 \cdot 10^{-4}$	x_{20}
24	Deepwell pump 3 is in TM (plant-specific)	$7.05 \cdot 10^{-3}$	x_{18}
25	Deepwell pump 3 fails to start	$3.00 \cdot 10^{-3}$	x_{14}
26	Deepwell pump 3 fails to run	$2.99 \cdot 10^{-3}$	$x_{15} = \lambda_p = 3 \cdot 10^{-5}$
27	Deepwell Pump 3 instrumentation and control (ICC) fails	$1.00 \cdot 10^{-3}$	x_{16}
28	Deepwell Pump #3 Breaker Spuriously Opens	$3.00 \cdot 10^{-5}$	$x_{17} = \lambda_b = 3 \cdot 10^{-7}$
29	Deepwell pump 3 heating and ventilation fails	$1.40 \cdot 10^{-2}$	x_{12}
30	Deepwell pump 4 TM (plant-specific)	$2.62 \cdot 10^{-2}$	x_{21}
31	Deepwell pump 4 fails to start	$3.00 \cdot 10^{-3}$	x_{14}
32	Deepwell pump 4 fails to run	$2.99 \cdot 10^{-3}$	$x_{15} = \lambda_p = 3 \cdot 10^{-5}$
33	Deepwell Pump 4 instrumentation and control (ICC) fails	$1.00 \cdot 10^{-3}$	x_{16}
34	Deepwell pump #4 breaker spuriously opens	$3.00 \cdot 10^{-5}$	$x_{17} = \lambda_b = 3 \cdot 10^{-7}$
35	Heating and ventilation fails for pump 4	$1.40 \cdot 10^{-2}$	x_{12}
36	Power failure at 4160 V atr bus 670-e-1	$1.08 \cdot 10^{-3}$	x_{22}
37	Common cause loss of scram system	$1.50 \cdot 10^{-5}$	x_{23}
38	Common cause failure of low outlet pressure sensor trains (C)	$7.20 \cdot 10^{-6}$	x_{24}
39	Common cause failures of low outlet pressure 2:3 logics	$3.00 \cdot 10^{-5}$	x_{25}
40	Failure of rod clutch coil controllers (rcccs)	$2.60 \cdot 10^{-6}$	x_{26}
41	2/3 Sensor trains fail to signal lop sublogic u	$4.77 \cdot 10^{-7}$	x_{27}
42	Failure to insert at least three safety rods in	$6.50 \cdot 10^{-7}$	x_{28}
43	Common cause failure of RCCCs to release	$5.00 \cdot 10^{-4}$	x_{29}
44	Failure of sufficient rcccs to release	$5.00 \cdot 10^{-4}$	x_{30}
LL0CA	Initiating event frequency	$4.56 \cdot 10^{-6} (1/y)$	flloca

some of the basic event probabilities, the CDF is a function of 31 distinct parameters, the 31^{st} being the initiating event frequency (see also Borgonovo et al (2003)).

The models in eqs. (24) and (26) are non-additive. In particular, at the basic event level the CDF [eq. (24)] has the same functional form as the multilinear model in eq. (11) of Saltelli et al (2009). At the parameter level, linearity is lost and the model becomes non-linear and non-additive. We now perform the SA of the model by utilizing the method of Section 2 both when the CDF is expressed as a function of the basic events [eq. (24)] and of the parameters [eq. (26)].

We select the 5^{th} and 95^{th} percentiles of the basic events and parameters distributions as points $\mathbf{x}^0$ and $\mathbf{x}^1$, respectively. As in Borgonovo et al (2003), the distributions are lognormal, with an error factor of 10. The corresponding CDF values are $CDF(\mathbf{x}^0) = 7.9580 \cdot 10^{-8}$ and $CDF(\mathbf{x}^1) = 1.9994 \cdot 10^{-12}$. By applying the results of Section 2, one estimates ϕ_i^1, $\phi_i^{\mathcal{I}}$ and ϕ_i^T with 92 model runs at the basic event level and 64 model runs at the parameter level[2].

Table 3 displays the results at the basic event level.

Let us discuss the results of Table 3 with the help of the settings of Section 3.

Model structure. Table 3 shows that $\phi_i^T \simeq \phi_i^{\mathcal{I}} >> \phi_i^1$ for all factors. This result reveals that interactions prevail over individual actions in determining the model response.

Direction of change. All first order FCSI's are positive. This means that each shift in basic event probability from the 5^{th} to the 95^{th} percentile increases the CDF. This result follows from the coherent structure of the system. The presence of a negative sign in first order FCSI's would, in fact, reveal the presence of some non-coherent feature of the system (see Lu and Jiang (2007)). As far as interactions are concerned, the positive sign means that interactions reinforce individual actions.

Factor relevance. The most relevant factor is the initiating event frequency (factor nr 45), which is ranked first by $\left|\phi_i^1\right|$, $\left|\phi_i^{\mathcal{I}}\right|$ and $\left|\phi_i^T\right|$ (Table 3). Among the 44 remaining basic events, results are as follows. If one utilizes as sensitivity measure $\left|\phi_i^1\right|$, basic event 22 ("Level control faults") ranks second, followed by basic event 11 ("Common cause failure of valve paths LCV-7A and LCV-7B to open") and basic event 12 ("Common cause loss of both FIS paths due to failure of AOVs"). We recall that ranking basic events with $\left|\phi_i^1\right|$ is equivalent to considering individual actions only. However, the model structure insights suggest that interactions play a primary role in the model response. If one includes interactions in the analysis, and ranks the factors according to $\left|\phi_i^T\right|$, one obtains the following results. Basic event 22 ranks 9^{th}, while basic events 11 and 12 rank 12^{th} and 17^{th}, respectively. The most important basic event becomes nr 30 ("Deepwell pump 4 is in TM"), followed by basic event 29 ("Deepwell pump 3 heating and ventilation fails") and basic event 17 ("Deepwell pump 1 is in TM").

In order to further analyze the ranking agreement with inclusion and exclusion of interactions, we make use of the Savage Score correlation coefficients (SSCC) (Iman and Conover (1987)). Let us denote by $\rho_{\phi_i^1,\phi_i^T}$, $\rho_{\phi_i^1,\phi_i^{\mathcal{I}}}$ and $\rho_{\phi_i^{\mathcal{I}},\phi_i^T}$ the SSCC's between the ranking obtained with $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^T$,

[2] These figures ought to be contrasted to the total number of interactions, namely $2^{45}+2$ and $2^{31}+2$ at the basic event and parameter levels, respectively.

Table 3: Sensitrivity measures and ranking for the 45 basic events. One notes that interactions prevail over individual impacts.

Basic Event	ϕ^1	$\phi^{\mathcal{I}}$	ϕ^T	Rank ϕ^1	Rank $\phi^{\mathcal{I}}$	Rank ϕ^T
BE1	2.36E-13	2.36E-09	2.36E-09	15	18	18
BE2	2.36E-13	2.36E-09	2.36E-09	15	18	18
BE3	9.44E-13	9.34E-11	9.44E-11	10	37	37
BE4	1.70E-11	1.68E-09	1.70E-09	7	23	23
BE5	2.19E-11	2.17E-09	2.19E-09	5	20	20
BE6	2.95E-14	2.95E-10	2.95E-10	24	33	33
BE7	2.95E-14	2.95E-10	2.95E-10	24	33	33
BE8	1.77E-14	1.77E-10	1.77E-10	28	35	35
BE9	5.89E-14	5.89E-10	5.89E-10	18	28	28
BE10	4.13E-14	4.13E-10	4.13E-10	22	31	31
BE11	4.40E-11	4.35E-09	4.40E-09	3	12	12
BE12	2.70E-11	2.67E-09	2.70E-09	4	17	17
BE13	4.13E-14	4.13E-10	4.13E-10	22	31	31
BE14	5.89E-14	5.89E-10	5.89E-10	18	28	28
BE15	1.77E-14	1.77E-10	1.77E-10	28	35	35
BE16	1.70E-14	1.70E-08	1.70E-08	30	5	5
BE17	2.34E-14	2.34E-08	2.34E-08	27	4	4
BE18	3.62E-15	3.62E-09	3.62E-09	36	13	13
BE19	3.61E-15	3.61E-09	3.61E-09	37	14	14
BE20	1.21E-15	1.21E-09	1.21E-09	39	25	25
BE21	3.36E-17	3.36E-11	3.36E-11	43	40	40
BE22	5.26E-11	5.21E-09	5.26E-09	2	9	9
BE23	6.39E-13	6.39E-09	6.39E-09	11	8	8
BE24	1.22E-14	1.22E-08	1.22E-08	33	7	7
BE25	5.21E-15	5.21E-09	5.21E-09	34	10	10
BE26	5.20E-15	5.20E-09	5.20E-09	35	11	11
BE27	1.73E-15	1.73E-09	1.73E-09	38	22	22
BE28	4.87E-17	4.87E-11	4.87E-11	42	38	38
BE29	2.45E-14	2.45E-08	2.45E-08	26	3	3
BE30	3.72E-13	2.99E-08	2.99E-08	13	2	2
BE31	4.26E-14	3.43E-09	3.43E-09	20	15	15
BE32	4.26E-14	3.42E-09	3.42E-09	21	16	16
BE33	1.42E-14	1.14E-09	1.14E-09	32	26	26
BE34	4.24E-16	3.19E-11	3.19E-11	40	41	41
BE35	2.00E-13	1.61E-08	1.61E-08	17	6	6
BE36	1.53E-14	1.23E-09	1.23E-09	31	24	24
BE37	9.42E-12	9.33E-10	9.42E-10	8	27	27
BE38	4.52E-12	4.48E-10	4.52E-10	9	30	30
BE39	1.88E-11	1.87E-09	1.88E-09	6	21	21
BE40	6.25E-17	6.25E-13	6.25E-13	41	43	43
BE41	3.00E-13	2.97E-11	3.00E-11	14	42	42
BE42	4.08E-13	4.04E-11	4.08E-11	12	39	39
BE43	3.13E-17	3.13E-13	3.13E-13	44	44	44
BE44	3.13E-17	3.13E-13	3.13E-13	44	44	44
fLL0CA	1.98E-10	7.86E-08	7.87E-08	1	1	1

Table 4: Savage score correlation coefficients on the ranking of basic events obtained with inclusion and exclusion of interactions.

$\boldsymbol{\rho}_{\cdot,\cdot}$	$\boldsymbol{\phi}^1$	$\boldsymbol{\phi}^{\mathcal{I}}$	$\boldsymbol{\phi}^T$
$\boldsymbol{\phi}^1$	1	−0.03	−0.03
$\boldsymbol{\phi}^{\mathcal{I}}$	−0.03	1	1
$\boldsymbol{\phi}^T$	−0.03	1	1

$\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^{\mathcal{I}}$, and $\boldsymbol{\phi}^{\mathcal{I}}$ and $\boldsymbol{\phi}^T$, respectively. Table 4 displays the results.

Table 4 reveals a low agreement between the ranking of basic events obtained with first order and total order indices ($\rho_{\phi_i^1,\phi_i^T} = -0.03$; $\rho_{\phi_i^1,\phi_i^{\mathcal{I}}} = -0.03$). Conversely, the value $\rho_{\phi_i^{\mathcal{I}},\phi_i^T} = 1$ shows perfect agreement between the ranking induced by $\phi_i^{\mathcal{I}}$ and ϕ_i^T. This result indicates that the ranking of the basic events obtained when including interactions in the sensitivity measures differs substantially from the ranking obtained when interactions are excluded. This is a consequence of the non-additivity of the model [eq. (24)].

We now report the results at the parameter level. The input-output dependence is represented by eq. (26). Table 5 displays the numerical results.

Let us discuss the results in Table 5 with the aid of the settings (Section 3).

Model structure. Table 5 shows that interactions prevail over individual effects also at the parameter level.

Direction of change. Table 5 shows that all individual, interaction and total sensitivity measures are positive. Similarly to the finding at the basic event level, this is a reflection of the coherent structure of the system.

Factor relevance. When individual effects are considered, the most relevant parameter is f_{LLOCA}, followed by parameters x_{19}, "Level control faults", x_{10}, "Common cause failure of valve paths LCV-7A and LCV-7B to open" and x_{11}, "Common cause loss of both FIS paths due to failure of AOVs." However, when interaction effects are included in the analysis, x_{19} ranks 9^{th}, x_{10} and x_{11} rank 10^{th} and 12^{th}, respectively. The second most important parameter becomes x_{12} ("Deepwell pump 1 heating and ventilation fails"), ranked 5^{th} by individual effects, followed by x_{21} ("Deepwell pump 4 in TM"), ranked 19^{th} by individual effects, and x_{13} ("Deepwell pump 1 is in TM"), ranked 13 by individual effects.

We then analyze the ranking agreement by computing $\rho_{\phi_i^1,\phi_i^T}$, $\rho_{\phi_i^1,\phi_i^{\mathcal{I}}}$ and $\rho_{\phi_i^{\mathcal{I}},\phi_i^T}$ on the parameters, with exclusion of f_{LLOCA}. Table 6 shows the results.

Table 6 shows a very low value of the correlation coefficient between $\left|\phi_i^1\right|$ and $\left|\phi_i^{\mathcal{I}}\right|$ and between $\left|\phi_i^1\right|$ and $\left|\phi_i^T\right|$. Conversely, the ranking agreement between $\left|\phi_i^{\mathcal{I}}\right|$ and $\left|\phi_i^T\right|$ is complete. This signals that interactions, if included in the analysis, change the results obtained by individual sensitivity measures, in agreement with the result obtained at the basic event level.

Table 5: Sensitrivity measures and raking for the 31 parameters. Also at the parameter level interaction effects prevail over the individual actions

Parameter	ϕ_i^1	$\phi_i^{\mathcal{I}}$	ϕ_i^T	**Rank** ϕ_i^1	**Rank** $\phi_i^{\mathcal{I}}$	**Rank** ϕ_i^T
x_1	2.36E-13	2.36E-09	2.36E-09	22	13	13
x_2	2.36E-13	2.36E-09	2.36E-09	22	13	13
x_3	9.44E-13	9.34E-11	9.44E-11	13	26	26
x_4	1.70E-11	1.68E-09	1.70E-09	8	17	17
x_5	2.19E-11	2.17E-09	2.19E-09	6	15	15
x_6	6.42E-13	5.30E-10	5.31E-10	14	22	22
x_7	2.45E-13	3.32E-10	3.32E-10	21	24	24
x_8	2.45E-12	9.43E-10	9.45E-10	11	19	19
x_9	1.23E-12	7.10E-10	7.11E-10	12	21	21
x_{10}	4.40E-11	4.35E-09	4.40E-09	3	10	10
x_{11}	2.70E-11	2.67E-09	2.70E-09	4	12	12
x_{12}	2.65E-11	3.95E-08	3.96E-08	5	2	2
x_{13}	2.34E-14	2.34E-08	2.34E-08	25	4	4
x_{14}	3.78E-13	1.13E-08	1.13E-08	17	6	6
x_{15}	3.73E-13	1.13E-08	1.13E-08	18	7	7
x_{16}	3.61E-14	3.98E-09	3.98E-09	24	11	11
x_{17}	5.16E-16	1.14E-10	1.14E-10	28	25	25
x_{18}	1.22E-14	1.22E-08	1.22E-08	27	5	5
x_{19}	5.26E-11	5.21E-09	5.26E-09	2	9	9
x_{20}	6.39E-13	6.39E-09	6.39E-09	15	8	8
x_{21}	3.72E-13	2.99E-08	2.99E-08	19	3	3
x_{22}	1.53E-14	1.23E-09	1.23E-09	26	18	18
x_{23}	9.42E-12	9.33E-10	9.42E-10	9	20	20
x_{24}	4.52E-12	4.48E-10	4.52E-10	10	23	23
x_{25}	1.88E-11	1.87E-09	1.88E-09	7	16	16
x_{26}	6.25E-17	6.25E-13	6.25E-13	29	29	29
x_{27}	3.00E-13	2.97E-11	3.00E-11	20	28	28
x_{28}	4.08E-13	4.04E-11	4.08E-11	16	27	27
x_{29}	3.13E-17	3.13E-13	3.13E-13	30	30	30
x_{30}	3.13E-17	3.13E-13	3.13E-13	30	30	30
x_{31}	1.98E-10	7.85E-08	7.87E-08	1	1	1

Table 6: Savage score correlation coefficients on the ranking of parameters obtained with inclusion and exclusion of interactions.

$\boldsymbol{\rho}_{\cdot,\cdot}$	$\boldsymbol{\phi}^1$	$\boldsymbol{\phi}^{\mathcal{I}}$	$\boldsymbol{\phi}^T$
$\boldsymbol{\phi}^1$	1	0.06	0.06
$\boldsymbol{\phi}^{\mathcal{I}}$	0.06	1	1
$\boldsymbol{\phi}^T$	0.06	1	1

6 Conclusions

In the analysis of complex systems, decision-makers benefit from the utilization of quantitative models. Knowledge of the relevance of interactions is essential in correctly interpreting and explaining model results. However, the computational complexity of the problems often leads decision-makers to rely on OAT techniques. In this work, we have shown a method to determine the relevance of interactions in complex PSA models at the same computational cost of OAT sensitivity methods.

The method rests on the functional ANOVA decomposition of a finite change discussed in previous literature [Rabitz and Alis (1999), Sobol' (2003), Borgonovo (2010)]. From this decomposition, one can then define sensitivity measures (the finite change sensitivity indices (ϕ^1,$\phi^{\mathcal{I}}$ and ϕ^T)) that identify the portion of the finite change in model output caused by interactions. Furthermore, a symmetry property entailed in the decomposition allows one to estimate the sensitivity measures at the same cost as OAT methods.

We have cast the interpretation of the results in the context of the factor fixing setting showing that knowledge of ϕ^1,$\phi^{\mathcal{I}}$ and ϕ^T allows analysts to infer insights on model structure, direction of change and factor relevance.

A first series of numerical experiments has been performed by means of two widely utilized SA case studies, the Ishigami and Sobol' g functions.

We have then applied the method to the PSA model of the ATR large LOCA sequence. We have carried out the analysis at two levels, the basic event and parameter levels. The model contains 45 basic events and 31 parameters. The CDF is multilinear and non-additive as a function of the basic events, non-linear and non-additive as a function of the parameters. We have chosen the 5^{th} and 95^{th} percentiles of the factors distributions as sample points. Numerical findings show that: 1) interactions prevail over individual effects; 2) all sensitivity measures are positive, revealing the coherent structure of the system; 3) the initiating event frequency is the most relevant factor both at the parameter and basic event levels, and both when individual and total effects are considered; however, notable discrepancies arise in the basic events and parameter ranking when interactions are quantified in the sensitivity measures.

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