What Drives Value Creation in Investment Projects? An Application of Sensitivity Analysis to Project Finance Transactions

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Abstract

Evaluating the economic attractiveness of large projects often requires the development of large and complex financial models. Model complexity can prevent management from obtaining crucial information, with the risk of a suboptimal exploitation of the modelling efforts. We propose a methodology based on the so-called "differential importance measure (D)" to enhance the managerial insights obtained from financial models. We illustrate our methodology by applying it to a project finance case study. We show that the additivity property of D grants analysts and managers full flexibility in combining parameters into any group and at the desired aggregation level. We analyze investment criteria related to both the investors's and lenders' perspectives. Results indicate that exogenous factors affect investors (sponsors and lenders) in different ways, whether exogenous variables are considered individually or by groups.

Keywords: *Risk Analysis*; *Finance: Investment Analysis*; *Sensitivity Analysis*.

This paper is currently under review at an international refereed journal.

1 Introduction

Investment planning is central to a corporation's growth and expansion. The valuation of new industrial opportunities is often accompanied by a sophisticated modeling exercise that includes the creation of financial models aimed at reproducing the potential investment's economics.

The use of financial models plays a crucial role in project finance transactions. According to Esty and Sesia (2007), project finance is a transaction that "[. . .] *involves the creation of a legally independent project company financed with nonrecourse debt (and equity from one or more corporations known as sponsoring firms) for the purpose of financing investment in a single purpose capital asset, usually with a limited life*". Project finance is usually associated with large capital-intensive ventures (for example, power plants, transportation infrastructure, telecom projects) with low redeployability values and limited recovery values in case of project defaults. Under these circumstances, lenders pay particular attention to project performance on a going concern basis because the possibility to repay principal and interest depends on the project's ability to generate sufficient cash flows. When the project is presented to potential lenders, the model becomes the shared platform for the negotiation between creditors and shareholders. To support negotiations

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effectively, the modeling effort requires considerable accuracy; this effort can become both costly and time consuming, as it involves the interaction and contribution of fiscal, technical, legal and financial consultants.

In this paper, we focus on such transactions for several reasons: *i)* project finance is a no-recourse form of financing, so lenders must dedicate time and resources to the careful estimation of the project's future performance via realistic models; *ii)* project finance is associated with the creation of a Special Purpose Vehicle (SPV) [Gatti (2007)], which enables us to concentrate our evaluation exercise on a single venture with limited economic life; i.e., no other case in standard corporate finance settings could enable us to separate the fate of the venture from any other existing project carried out at the same time by the sponsoring firm; *iii)* since financial models used for project finance evaluation are particularly complex, they challenge the Sensitivity Analysis (SA) exercise from both the methodological and numerical viewpoints.

This SA is crucial for our paper since the lack of an analytical expression and the dimensions of the model make it a black box [Diaconis (1988)]. Consequently, the model behavior as a function of the exogenous variables is unknown to the financial analyst.

The presence of many exogenous variables also means that SA is usually not performed on all exogenous variables. Conversely, to save time and expense, attention is restricted to a subset of inputs, usually pre-selected based on experience or qualitative statements. However, especially when the investment setting is new, such an approach may lead to the exclusion of relevant parameters from the analysis.

An additional issue is the selection of the SA method itself. Clemen (1997) (ch. 5) sets forth the central role of SA in the decision-making process and proposes a series of simple questions that can be answered through the use of one-parameter-at-a-time SA. In the literature, several authors include criticisms of the use of one-parameter-at-a-time SA in investment evaluation [Van Groenendaal and Kleijnen (1997), Van Groenendaal (1998), Van Groenendaal and Kleijnen (2002), Borgonovo and Peccati (2004) and Borgonovo and Peccati (2006)]. However, this type of SA has been decidedly improved in the recent literature. The recent development of new SA techniques provides analysts and decision makers with new tools that enable management to more fully exploit the information contained in the model [Frey (2002), Saltelli (2002), Borgonovo (2008)]. In particular, when applied to investment evaluation, the SA method should allow a decision maker:

Insight (1): to test the model's robustness and internal consistency;

Insight (2): to detect the model's response to changes in the parameters;

Insight (3): to determine the influence of each of the model's assumptions on the valuation criterion, i.e., to identify the Key Performance Drivers (KPD).

Insight (1) is necessary in the decision making process because, if model results do not comply with the underlying theory, they should not be used to make decisions. [For a discussion on modeling risk in financial applications, see Fabozzi (2000).] Insight (2) conforms to Samuelson's

classic statement of comparative statics [Borgonovo (2008)]. An analyst needs to determine how a change in an assumption affects valuation criteria. Insight (3) accomplishes the task of avoiding the screening out of relevant factors based simply on a-priori qualitative statements (that is, without quantitative support). We remark that obtaining these insights is particularly relevant for large models because an analyst has no other way to dissect the model results.[1] The absence of this information would prevent analysts and decision makers from fully exploiting the information contained in the (complex and costly) model. As a result, one would run the risk of undermining the modeling effort and the valuation process. This issue is particularly relevant in the case of complex transactions financed on a project-finance basis.

In this work, we aim to make the acquisition of these insights systematic. Our approach is based on the use of the differential importance measure (D) [Borgonovo and Peccati (2004), Borgonovo and Peccati (2006), Borgonovo and Apostolakis (2001)]. D extends comparative statics and elasticities, overcoming their limitations, especially regarding dimensionality issues [Borgonovo (2008)]. In this respect, we note that the presence of many parameters measured in different units raises two issues with traditional comparative statics methods. First, partial derivatives cannot be used as sensitivity measures to identify key project drivers [Borgonovo (2008)] — As an example, the partial derivative of a net present value (NPV) with respect to inflation carries the NPV units, since inflation is a pure number; however, the partial derivative of the NPV with respect to a cost is a pure number; therefore, one cannot compare the two partial derivatives to establish whether inflation is more important than costs.— Second, both analysts and decision makers feel the need to synthesize results by aggregating the long list of individual inputs into corresponding categories (e.g. revenue, fiscal, technical assumptions, etc.). One then faces a joint-SA problem. We show that exploiting the definition of D resolves the first issue, and exploiting its additivity resolves the second. Additivity, in fact, allows joint sensitivities to be obtained without additional model runs. Thus, analysts are free to set the level of detail in result communication.

Our first step is to allow the estimation of D in the context of large spreadsheet models, which imply the absence of a closed-form expression of the valuation criterion. We note that in previous literature applications of D, analytical expressions of the valuation criteria were available [see, for example, Borgonovo and Peccati (2004), Borgonovo and Peccati (2006)]. We then adapted and applied a numerical estimation algorithm whose mathematical aspects are set forth in Borgonovo and Apostolakis (2001).

We next discuss the financial and managerial interpretations of the results, by casting insights (1), (2) and (3) in the context of investment project valuation. Regarding insight (1), we find that it is possible to create an automated test of internal consistency by application of the algorithm. Regarding insight (2), we show that the sign of D completely reveals the dependence of the valuation criterion on the exogenous variables. Regarding insight (3), the method allows one to consider all exogenous variables and to rank them according to their influence. The case of project finance

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[1] Saltelli (2002) utilizes the following metaphor: "Sensitivity analysis for modelers? Would you go to an orthopedist who didn't use X-ray?"

is particularly interesting in this respect because the viability of the initiative must satisfy the valuation criteria of banks and sponsors simultaneously (Yescombe (2002); Gatti (2007), ch. 5). These criteria generally conflict with each other. Accordingly, when considering KPD, we cannot limit ourselves to the shareholder valuation perspective – based essentially on equity NPV – but must also take into account the lenders' viewpoint, which focuses on debt service coverage ratios (DSCRs) and loan life coverage ratios (LLCRs).

We account for the valuation criteria utilized by shareholders and lenders by performing the SA on both types. This leads us to investigate whether KPD are the same for sponsors and lenders. Such an analysis is complicated by the presence of a large number of parameters. We then introduce a methodology to obtain a quantitative indication of the ranking agreement by synthesizing individual results. The methodology is based on the use of Savage scores, a statistical technique introduced in Iman and Conover (1987), which has found wide application in the SA of large models (Borgonovo (2006)).

We illustrate the methodology through its application to a full-fledged financial model developed for the evaluation of an infrastructure investment project — namely a parking lot — financed with no-recourse debt. The model was prepared and approved by the lead bank arranging credit, and the project sponsoring firms in order to determine the financial viability of the parking facility. The model has been implemented on a series of Excel spreadsheets requiring a set of 428 input parameters .

In the investment case study at hand, results show that on average KPD for individual contributions tend to be the same for sponsor and lender valuation criteria, with higher agreement on the most relevant factors. In a few but significant cases, however, parameters that are influential on sponsors' criteria are not influential on lenders' criteria. A notable example is the cost of capital (k_e), which is a significant contributor to the equity NPV but has null influence on the DSCR (see Section 2). In response to the decision-makers' need to obtain a comprehensive way of communicating the SA results, we discuss two levels of detail. First, we group the 428 parameters into the six main categories used in project finance practice. Revenue assumptions turn out to be the most relevant for both the NPV and the minimum DSCR (mDSCR), followed by construction costs; operational costs play only a minor role. We then further dissect the results by dividing the main categories into 17 groups based on subcategories selected by the analysts.

The remainder of the paper is organized as follows. In Section 2, we discuss the specific features of project finance valuation, focusing on sponsors' and lenders' criteria. In Section 3, we present the SA method we apply in this work, illustrating its mathematical properties and computational aspects. In Section 4, we present the application of the method to the SA of a full-fledged financial model used in a real-life application for measuring the financial viability of an investment in the construction and operation of a parking lot. We offer conclusions in Section 5.

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2 Financial Valuation: the Case of Project Finance Transactions

This section examines the characteristic features of project financing and their implications in the valuation of the economic attractiveness and the financial sustainability of this type of transaction. The analysis also aims to show the differences between the valuation perspectives of shareholders and lenders.

Project finance is an important part of the international syndicated loans market. Hainz and Kleimeier (2006) report that the value of project finance transactions closed in the period January 1980-March 2003 was about USD 960bn, and Esty and Sesia (2007) report that, in the US, the project finance loans market is larger than the initial public offering (IPO) market. Corielli et al. (2008) find that the average value of a project finance investment is about 512 million USD, with an average debt-to-equity ratio of 4.23.

Project finance originated in the energy generation sector and is now widely used to fund oil & gas, power and telecom projects [Gatti et al. (2007)]. It is the preferred way for firms to limit their balance sheet exposure when entering risky foreign markets. In addition, project finance is used more intensively in developing countries as an efficient way to help bridge infrastructure gaps quickly (Hammami et al. (2006)). More recently, project finance schemes have been sought to fund Internet and e-commerce projects.

At the heart of a project finance scheme is a nexus of contracts (Jensen and Meckling (1976)) revolving around a specially incorporated entity, the SPV, which becomes the counterpart for all the operating and financial contracts (Vinter (1998)). A group of sponsoring firms (the SPV's shareholders), and to a larger extent a bank syndicate headed by a Mandated Lead Arranger, provide the money needed to design, build and operate a new project. Loans are fully guaranteed by all the company's assets, supplemented by a large set of covenants that aims to restrict the SPV's use of the funds (Smith and Warner (1979)). Very often, the loans are granted on a no-recourse or limited recourse basis; in this way, sponsors limit their responsibility for the project performance to their original equity injection. In other words, project finance allows sponsors to fund the venture "off balance sheet".

The success of a project finance transaction depends on the project's capacity to generate sufficient cash during its operating phase so that it matches the cash needed for debt service (interest and principal repayment) and dividends paid to the project sponsors. The operating phase usually lasts for a long but finite period; this implies that – contrary to what happens in standard corporate finance settings – the SPV will not reinvest cash flows for further development of the initiative, but instead will distribute all available cash to the participating counterparts. Note that the SPV has no reinvestment mandate (or capability); project finance analysts are allowed to assume a null SPV retention ratio. This marks a departure from traditional investments dividend policies. As an example, stability of future cash flows is found as a key of dividend payment decisions in Brav et al (2008). Amromin et al (2008) examine the relationship between dividend taxation and dividend distribution policies. Li and Zhao (2008) study *"how informational asymmetries affect firms' dividend policies."*

Given the importance of cash flow generation in project finance transactions, it is not surprising that extensive negotiations surround the estimation of SPV cash flows. This estimation is accomplished by means of a financial model that tries to forecast the financial statements of the SPV in order to accurately predict the SPV's economic and financial performance through a spreadsheet model (Benninga (2000)). The model tries to adhere as closely as possible to what would be the actual reported statements of the project company; thus, full consideration is given to accounting and fiscal rules in the country or region where the SPV operates.

The core of the model is the cash flow statement, from which lenders' and investors' cash flows are estimated. The first cash flow of interest is the project free cash flow (FCF), defined as revenues less operating expenses, less correction for changes in working capital and taxes. Tax outflows are estimated via an income statement built in compliance with the fiscal and accounting rules of the country where the SPV operates. The income statement is also necessary to estimate profits. Because the SPV retention ratio equals zero, profits will be paid as dividends after satisfying lenders' requirements for a minimum level of cash reserves (debt service reserve account). The project FCF represents the cash available before debt service and cash remittance to shareholders. FCF is then allocated to interest payment, principal repayment and debt-related reserves. After all debt-holders' cash flows are subtracted, the remaining cash constitutes the free cash flow to equity (FCE). Once identified and estimated, the debt and equity cash flows feed into the valuation criteria.

In deciding whether to move forward with a project, sponsoring firms and banks apply different criteria. Operating from the perspective of an SPV's shareholders, sponsors base their decisions on standard equity NPV, which becomes an adjusted present value when third-party financing is present [Myers (1974)]. Since project finance is characterized by a closed lifecycle without the possibility of scope changes or reinvestment for expansion or abandonment [Zettl (2002), Bernardo et al (2007), Joos and Zhdanov (2008)], real options are not a concern because flexibility is practically absent [Dixit and Pyndyk (1994)][2]. The financial model applied for the SA of this work is based on the assumption that there is no possibility of delays in the investment decision and no abandonment or expansion options. Under these conditions [Dixit and Pyndyk (1994)], an investor should apply the NPV rule; that is, undertake the project if NPV is positive.

The perspective of lenders is different [Gatti (2007)]. In particular, due to the peculiar investment structure, lenders focus on the project's debt repayment capability [Nevitt and Fabozzi (1995), Gatti (2007)]. Hence, criteria utilized by financial institutions to investigate lending decisions for industrial projects look at debt service. In the practice of project finance, the two most often applied criteria are the DSCR and the LLCR. The DSCR is a period-on-period (typically year-on-year) measure that quantifies the capacity of the operating cash flows to service the debt.

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[2]Bernardo et al (2007) modify the capital asset pricing model to include the effect of growth options. Joos and Zhdanov (2008) utilize real options to *"to capture investment and abandonment options in the research-intensive biotechnology industry."* Conversely, the conditions of the car park project under examination were such that no flexibility could be included in the financial evaluation by the professional analysts investigating the project's viability.

It is defined as follows [Gatti (2007)]:

$$DSCR_t = \frac{FCF_t}{P_t + I_t} \quad t = 1, 2, ..., T_L \tag{1}$$

where FCF_t is the free cash flow generated by the project at time t, P_t is the principal repayment for period t, I_t is the interest repayment for period t and T_L is the loan tenor, that is, the length of the repayment period. Loan contract default clauses require the SPV to maintain the minimum value of the DSCR over time greater than a predetermined threshold. Denoting this value as $DSCR_{Th}$, we can write:

$$\min_t DSCR_t > DSCR_{Th} \tag{2}$$

where $DSCR_{Th}$ is a number greater than one whose magnitude depends on the Bank's risk perception of the project finance transaction. In practice, $DSCR_{Th}$ ranges from 1.2 to 1.9. The SPV's failure to maintain such $DSCR_t$ at any period in which the loan is present, may trigger default.

The LLCR is a project-life measure of debt repayment capability and is defined as [Gatti (2007)]:

$$LLCR_t = \frac{\sum_{s=t}^{T_{Debt}} \frac{FCF_t}{(1 + k_d)^{s-t}}}{D_t} \tag{3}$$

where t is the time of interest, T_{Debt} the debt tenor, k_d the discount factor, D_t the debt outstanding at time t. The numerator in eq. (3) represents the present value at time t of the FCFs generated by the project from t to T_{Debt} discounted at k_d.

The cash flows (both FCF and cash flow to equity) depend on the several factors influencing the investment performance, such as macroeconomic parameters (future inflation), market driven parameters (demand, price of goods sold, raw material costs), financial aspects (leverage, spreads, currency), technical aspects (plant efficiency) and construction costs. Correspondingly, the valuation criteria depend on the same exogenous factors. An analytical example of the dependence of valuation criteria on the exogenous variables can be found in eq. (21) of Borgonovo and Peccati (2006), which represents the present value of an investment in the energy sector. However, such an analytical expression is possible only under the assumptions of a perfectly efficient financial structure and a simplified timing of the cash inflows and outflows, with the cash outflows concentrated at $t = 0$.

For complex projects, however, such assumptions are usually not realistic in an advanced phase of the valuation process. Financing institutions need to accurately compute the total amount of credit to be disbursed to the project. Since the amount of debt at the end of construction includes capitalized interest that is computed on a monthly or daily basis, applying a yearly approximation could lead to misleading estimates. In addition, project costs are often an itemized list that can include more than 100 items, and the model elaborates numerous intermediate calculations to account for escalation, compute appropriate depreciation amounts, estimate debt outflows reflecting the loan agreement's repayment schedule and interest calculation rules, include tax and accounting

rules, etc. (Therefore, the calculations are implemented on large spreadsheets.) It is obvious that analytical approaches are not practical.

As a result, models are complicated, large, and usually not analytically known (Yescombe (2002), Finnerty (2007)). Consequently, the decision maker is often denied the substantial aid provided by a closed-form expression for the NPV or the mDSCR as a function of the exogenous variables. Decision-makers then run the risk of making important decisions based on a "black box" that processes a list of inputs and estimates valuation criteria.

In addition, building accurate financial models consumes both time and resources, because the costs of technical consultants and financial advisors hired to provide inputs or financial modeling as well as model auditing can be significant.

Gaining insights into the model's behavior and understanding the influence of exogenous variables on the investment performance is, then, crucial in adding value to the modeling exercise and in better informing the decision-makers. The next section examines the technical aspects of how to accomplish these tasks.

3 Sensitivity Analysis of Financial Models: Estimating the Differential Importance Measure

In Section 1, we discussed three types of the insights that a decision maker derives from SA. In this section, we explain how these insights can be obtained for large models by using an SA approach based on the differential importance measure.

Let

$$V = v(\boldsymbol{\lambda}) \quad v : \Lambda \subseteq \mathbb{R}^n \to \mathbb{R} \tag{4}$$

denote the relationship that links the valuation criterion (V) to the exogenous variables $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_n)$. (We recall that V differs for lenders and sponsors; for example, it is an equity NPV or Internal Rate of Return for a shareholder and an mDSCR for a lender.) We let $\boldsymbol{\lambda}^0$ denote the reference (base case) value of the exogenous variables on which the valuation criterion depends. The numerical values of the exogenous variables reflect the current assumptions and state of knowledge of the decision maker concerning $\boldsymbol{\lambda}$. We assume that $v(\boldsymbol{\lambda})$ is differentiable at $\boldsymbol{\lambda}^0$ and that ∇v is not orthogonal to $\mathrm{d}\boldsymbol{\lambda} = [\mathrm{d}\lambda_1, \mathrm{d}\lambda_2, \ldots, \mathrm{d}\lambda_n]^T$ at $\boldsymbol{\lambda}^0$.

The sensitivity of V on exogenous variable λ_s at λ_0 can be defined as [Borgonovo and Apostolakis (2001), Borgonovo and Peccati (2004), Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = \frac{\mathrm{d}_s V}{\mathrm{d}V} = \frac{v_s(\boldsymbol{\lambda}^0)\mathrm{d}x_s}{\sum_{i=1}^n v_i(\boldsymbol{\lambda}^0)\mathrm{d}x_i} \tag{5}$$

where $v_s(\boldsymbol{\lambda}^0)$ is the partial derivative of F with regard to λ_s at $\boldsymbol{\lambda}^0$. So, $D_s(\boldsymbol{\lambda}^0, d\boldsymbol{\lambda})$ measures the parameter importance as the change in V produced by a change in λ_s, over the sum of the changes in F produced by changes in all the input parameters. $D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda})$ is the fractional change in V that follows a (small) change in λ_s. In fact, the numerator in eq. (5), is the change produced by a variation in λ_s, while the denominator is $\mathrm{d}V$, that is, the differential of V, which equals the change

in V produced by a simultaneous change in all the parameters.

It can be shown that D has the following properties [Borgonovo and Apostolakis (2001), Borgonovo and Peccati (2004)]:

- D generalizes partial derivatives if one assumes a uniform change in the parameters. In fact, if one assumes

$$\mathrm{d}\lambda_i = \mathrm{d}\lambda_s \ \forall s, i = 1, 2, ..., n, \tag{6}$$

then it holds that [Borgonovo and Peccati (2004), Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = D1_s^0 = \frac{v_s(\boldsymbol{\lambda}^0)}{\sum_{i=1}^{n} v_i(\boldsymbol{\lambda}^0)} \quad s = 1, 2, .., n. \tag{7}$$

Eq. (7) implies that $D1_s^0 \propto v_s(\boldsymbol{\lambda}^0)$; that is, under the assumption of equal variations in the exogenous variables, the differential importance of a parameter is proportional to the corresponding partial derivative. This implies that measuring sensitivity based on partial derivatives is equivalent to state an assumption of uniform changes in the parameters. Borgonovo and Peccati (2004) show that, if V is an NPV and $\boldsymbol{\lambda}$ the (vector of) expected cash flows, then the (vector of) $D1_s^0$ coincides with the cash flow profile. Borgonovo and Peccati (2006), however, note that if one moves at the level of the exogenous variables that determine the cash flows, then this conclusion no longer holds. In fact, when the exogenous variables are measured in different units, the uniform change assumption cannot be adopted as eq. (6) cannot hold.

- D generalizes Elasticity if one assumes a proportional change in the parameters. In fact, if

$$\frac{\mathrm{d}\lambda_i}{\lambda_i^0} = \omega = \frac{\mathrm{d}\lambda_s}{\lambda_s^0} \ \forall s, i = 1, 2, ..., n, \tag{8}$$

it turns out that [Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = D2_s^0 = \frac{v_s(\boldsymbol{\lambda}^0)\lambda_s^0/V^0}{\sum_{i=1}^{n} v_i(\boldsymbol{\lambda}^0)\lambda_i^0/V^0} = \frac{E_s^0}{\sum_{i=1}^{n} E_i^0} \quad s = 1, 2, ..., n, \tag{9}$$

where E_s^0 is the elasticity of V with respect to λ_s at $\boldsymbol{\lambda}^0$. Eq. (9) implies that measuring sensitivity based on elasticity is equivalent to stating an assumption of proportional changes in the parameters [Borgonovo and Peccati (2004), Borgonovo and Peccati (2006).] In fact, eq. (9) implies that D and Elasticity differ only for a normalization factor if one assumes proportional parameter variations. Borgonovo and Peccati (2004) show that if V is an NPV and $\boldsymbol{\lambda}$ the vector of expected cash flow, then $D2_s$ is the fraction of the NPV associated with λ_s. Borgonovo and Peccati (2006) show that, when $\boldsymbol{\lambda}$ represents the parameters that determine the cash flows (and not the cash flows themselves), then $D2$ represents the fraction of the change in NPV related to a change in λ_s.

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- D has the additivity property. Let $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_k}$ be a set of k input factors ($k < n$). The sensitivity of V on $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_k}$ is related to the individual sensitivities as [Borgonovo and Apostolakis (2001)]:

$$D_{i_1, i_2, \ldots, i_k}(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = \sum_{i=1}^{k} D_{i_k}(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}); \tag{10}$$

that is, the differential importance of a group of parameters is equal to the sum of the differential importance of each parameter in the group. The additivity property [eq. (10)] allows one to obtain directly from the individual sensitivities the sensitivity of V on any parameter set. In the remainder of this paper, we shall see that this property is the key to synthesizing results and to reducing the computational effort in the SA of complex financial models.

- The immediate consequence of the additivity property is that the sum of the D_s of all parameters equals unity [Borgonovo and Apostolakis (2001)]:

$$\sum_{i=1}^{n} D_i(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = 1. \tag{11}$$

In the remainder of this section, we discuss the technical aspects of the application of D to the SA of large project valuation models. The first key feature is that in real-life applications financial models are characterized by a large number of exogenous variables. The complexity of the transformation implied by the spreadsheet model makes it impossible to utilize an analytical approach for the computation of D. One must consequently resort to numerical estimation through a dedicated algorithm. This feature represents a first contribution of the present work. In fact, in previous applications of D in the industrial investment realm, analytical expressions of the valuation criteria were available to the analysts. We make use of the estimation algorithm for D developed by Borgonovo and Apostolakis (2001), after having adapted it to spreadsheet modeling. The algorithm is based on the steps presented in Table 1 (see Borgonovo and Apostolakis (2001)).

[Insert Table 1 about here]

The algorithm is composed of an external and internal loop. The external loop consists of four steps that define the sequences of model evaluations and test convergence. The internal loop consists of two steps for the estimation of the sensitivity measures. The loop is repeated until the level of accuracy set by the analyst is achieved.

In Table 1, m is the number of times the algorithm is repeated, n the number of parameters. $\Delta\lambda_s^j$ is, then, the discrete change in parameter λ_s at step j. The basis for the algorithm in Table 1 is the operational definition of D following from eq. (5) (Borgonovo and Apostolakis (2001)):

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = \lim_{\Delta\boldsymbol{\lambda} \to \mathbf{0}} \frac{\Delta V_s}{\sum_{i=1}^{n} \Delta V_i} = \lim_{\Delta\boldsymbol{\lambda} \to \mathbf{0}} \frac{V(\lambda_s^0 + \Delta\lambda_s; \boldsymbol{\lambda}^0_{(-s)}) - V(\boldsymbol{\lambda}^0)}{\sum_{i=1}^{n} V(\lambda_i^0 + \Delta\lambda_i; \boldsymbol{\lambda}^0) - V(\boldsymbol{\lambda}^0)} \tag{12}$$

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where $\Delta\boldsymbol{\lambda}$ is the vector of all parameter changes, ΔV_s is the change in V due to the change in exogenous variable λ_s while the other exogenous variables are kept at $\boldsymbol{\lambda}^0$ (denoted in eq. (12) as $\boldsymbol{\lambda}^0_{(-s)}$). To turn eq. (12) into an algorithm, one considers $\Delta\boldsymbol{\lambda}$ as independent variables and introduces the functions:

$$r_s(\Delta\boldsymbol{\lambda}) = \frac{V(\lambda_s^0 + \Delta\lambda_s; \boldsymbol{\lambda}^0_{(-s)}) - V(\boldsymbol{\lambda}^0)}{\sum_{i=1}^n V(\lambda_i^0 + \Delta\lambda_i; \boldsymbol{\lambda}^0_{(-i)}) - V(\boldsymbol{\lambda}^0)} \quad s = 1, 2, ..., n \tag{13}$$

Each function in [eq. (13)] is continuous in $\Delta\boldsymbol{\lambda}$, if V is differentiable. This fact, combined with a well-known multivariate calculus result (see Burkill and Burkill (1970), pp. 33-34) guarantees that $r_s(\Delta\boldsymbol{\lambda})$ converges to $D_s(\boldsymbol{\lambda}_0)$ with continuity. Then, it follows that, if $r_s(\Delta\boldsymbol{\lambda})$ tends to $D_s(\boldsymbol{\lambda}_0)$ for $\Delta\boldsymbol{\lambda} \to \mathbf{0}$ on a continuous basis, then it will converge to the same limit for every discrete sequence $\Delta\boldsymbol{\lambda}^j$, $j = 1, 2, ...$, such that $\Delta\boldsymbol{\lambda}^j \to \mathbf{0}$. A way of defining the discrete sequence of $\Delta\boldsymbol{\lambda}^j$ (Step 1 in Table 1) is to set:

$$\Delta\lambda_s^j = \frac{\lambda_s}{\omega_s^j} \quad s = 1, 2, ..., n \tag{14}$$

where ω_s^j is a diverging and increasing sequence of integers, such that $\lim_{j\to+\infty} \omega_s^j = +\infty$. With these definitions, $\lim_{j\to+\infty} \Delta\lambda_s^j = 0$, $\forall s = 1, 2, ..., n$.

One can then test the convergence of the algorithm as j progresses (Step 4 in Table 1). The convergence test utilized in our implementation is based on Cauchy's convergence criterion, as in Borgonovo and Apostolakis (2001). Since any convergent sequence is a Cauchy's sequence, for every small number ε there will exist an index $j_s^*(\varepsilon)$, such that for all m and k greater than $j_s^*(\varepsilon)$:

$$\left| r_s^m(\Delta\boldsymbol{\lambda}) - r_s^k(\Delta\boldsymbol{\lambda}) \right| < \epsilon \quad \forall m, k > j_s^*(\varepsilon) \tag{15}$$

From the numerical viewpoint then, when j is greater than m or k, D_s is estimated with an error smaller than ϵ. In particular, we use a percentage test of the form:

$$\max_s \left| \frac{r_s(\Delta\boldsymbol{\lambda}^j) - r_s(\Delta\boldsymbol{\lambda}^{j+1})}{r_s(\Delta\boldsymbol{\lambda})} \right| < \varepsilon; \tag{16}$$

that is, the algorithm stops when the percentage discrepancy in the estimation of D in two consecutive steps is lower than a small predetermined positive number. Note that if eq. (16) is satisfied at $j = j^*$, then Cauchy's convergence criterion insures that this difference will remain lower than ε for $j > j^*$.

We need two additional observations about implementation. Regarding Step 1 of Table 1, we note that, if arbitrary relative parameters changes are allowed, then one needs to define a distinct sequence ω_s^j for each parameter. In the case of proportional changes, however, one needs

to introduce only one sequence because eq. (8) implies

$$\omega_s^j = \omega_l^j \qquad \forall s, l = 1, 2, ..., n \qquad (17)$$

Also in the case of uniform changes, it is sufficient to define a single sequence of ω_s^j because eq. (6) implies that once $\Delta\lambda_1^j$ is determined, the other parameter variations are equal. However, in the case of financial models, and more generally in the case of economic models, parameters have different dimensions [Borgonovo and Peccati (2004), (2006).] For example, inflation indices are "pure numbers" being the ratios of homogeneous quantities, while costs are denominated in the corresponding currency. Consequently, one cannot compare a change of one inflation unit (a pure number) with a unit change in construction cost (denominated, for example, in EUR, USD or GBP). Hence, an assumption of uniform parameter changes does not hold for most financial models. Instead, one ought to compare proportional changes in the factors [eq. (8)]. We recall that, in this case, the ranking obtained with D is the same as the ranking obtained utilizing Elasticity [eq. (9)]. The natural sensitivity measure to answer this question is $D2$, eq. (9) [Borgonovo and Peccati (2006).] Thus, the subroutine utilized in this work foresees to perform the steps in Table 1, with the sequence generated by eq. (17).

The second observation concerns the differentiability assumption. By the above discussion, it is clear that the presence of discontinuities would impair convergence of the algorithm. In particular, several operational research models are piecewise defined due to the presence of the max() or $|\cdot|$ functions or of caps and rounding operators. The input parameter space, Λ, is then partitioned in K subsets ("regions") Λ_j such that $\bigcup_{j=1}^{K} \Lambda_j = \Lambda$ and $\Lambda_j \cap \Lambda_i = \varnothing$ ($i \neq j$, $i, j = 1, 2, ..., K$) over which the model output can take on a different functional form (we refer to Borgonovo and Peccati (2009) for further details). Denoting the functional form on region Λ_j by $v_{\Lambda_j}(\boldsymbol{\lambda})$, then, if $\boldsymbol{\lambda}^0$ is on the border of two elements of the partition, V might not be differentiable on Λ, even if each of the $v_{\Lambda_j}(\boldsymbol{\lambda})$'s is a smooth function of the parameters (however, in that case, it will be differentiable at any internal point). On the other hand, the approach presented in this work is a local one, and D provides the importance of exogenous variables at $\boldsymbol{\lambda}^0$. Thus, differentiability has not to be insured in the entire Λ, but at $\boldsymbol{\lambda}^0$. Hence, in the presence of a potentially non-smooth model behavior over Λ, care has to be taken to insure that $\boldsymbol{\lambda}^0$ is a regular point for V. In this respect, we note that the sophistication of financial models utilized for project finance transactions stems from the financial side (e.g., exact reproduction of accounting and fiscal rules, the need for a high level of detail). On the mathematical side, the transformations involve a set of smooth operations, which lead to a regular behavior of V.

Finally, we discuss a methodology based on the Savage score correlation coefficients that we are going to use. As we shall discuss, one of the insights that can be gained from the analysis is the investigation of whether parameters have the same influence with respect to different valuation criteria. Let C_1 and C_2 denote two generic criteria and let $R_i^{C_1}$ and $R_i^{C_2}$ denote the respective rank of factor λ_i^0 for criteria C_1 and C_2. If there are n factors, then $\mathbf{R}^{C_1}$ and $\mathbf{R}^{C_2}$ are two vectors with n

components. Iman and Conover (1987) provide a synthetic way to express the agreement between $\mathbf{R}^{C_1}$ and $\mathbf{R}^{C_2}$. The technique consists of calculating the correlation coefficient on the corresponding Savage scores. Savage scores have been introduced to emphasize the agreement among the top-ranked factors [Iman and Conover (1987), Campolongo and Saltelli (1997), Kleijnen and Helton (1999).] Denoting the rank of factor λ_i by R_i, then its Savage score is defined as:

$$SS_i = \sum_{h=R_i}^{n} \frac{1}{h} \tag{18}$$

For example, if $n = 428$ and $R_i = 1$, the Savage score of this factor is 6.64. Let $\rho_{\mathbf{SS}^{C_1}, \mathbf{SS}^{C_2}}$ the corresponding correlation coefficient on Savage Scores. $\rho_{\mathbf{SS}^{C_1}, \mathbf{SS}^{C_2}}$ conveys information on whether the key-drivers are the same for the two criteria. Let then $\rho_{\mathbf{R}^{C_1}, \mathbf{R}^{C_2}}$ denote the correlation coefficient on pure ranks. $\rho_{\mathbf{R}^{C_1}, \mathbf{R}^{C_2}}$ provides information on the overall ranking agreement, without the emphasis on the top ranked factors. As the works of Iman and Conover (1987), Campolongo and Saltelli (1997), Kleijnen and Helton (1999), Borgonovo (2006) underline, by comparing $\rho_{\mathbf{R}^{C_1}, \mathbf{R}^{C_2}}$ against $\rho_{\mathbf{SS}^{C_1}, \mathbf{SS}^{C_2}}$ one obtains information on whether the agreement (or discrepancy) among the ranking is at the level of the most important factors. As an example, suppose that $\rho_{\mathbf{SS}^{C_1}, \mathbf{SS}^{C_2}} > \rho_{\mathbf{R}^{C_1}, \mathbf{R}^{C_2}}$. This result would signal that the agreement on the key-drivers is higher than the agreement on the low-ranked factors. Similarly, the converse indication is generated by $\rho_{\mathbf{SS}^{C_1}, \mathbf{SS}^{C_2}} < \rho_{\mathbf{R}^{C_1}, \mathbf{R}^{C_2}}$.

The next section presents the application of the method in obtaining financial decision-making insights into the valuation of a large project in an advanced phase of negotiation and modeling.

4   Application: Valuing a Project Finance Investment in a Parking Lot

The purpose of this section is to illustrate information and insights gained by the application of the SA method proposed in Section 3 for planning and evaluating an infrastructure initiative. The project consists of the construction and operation of a parking lot through a project finance scheme. There is a single sponsor and the sale is by definition a merchant one; that is, the project cannot count on a single buyer — an off-taker — for all the production available during the operating phase. —

The financial model has been developed by the sponsor in the initial due-diligence phase. Upon the sponsor's request for financing from the Mandated Lead Arranging Bank, the Bank took over the financial modeling exercise. The resulting financial model parallels the investment timing. It foresees a two-year construction period, in which cash outflows are modeled monthly. The operation period is modeled annually over a $20-$year time horizon. The total investment cost (construction plus interests) is estimated at around 40 million euros. The financial structure of the SPV foresees 70% debt financing and 30% equity, which is split into equal portions of ordinary shares and shareholder-subordinated loans. The spreadsheet model contains 40 calculation worksheets; to estimate the valuation criteria, it requires a set of $n = 428$ inputs to be supplied by the analysts. The model provides a detailed estimation of the project cash flows, which can generate all the necessary valuation criteria. We focus on the equity NPV and the mDSCR as representative of the

sponsor's and lender's respective viewpoints. The base case assumptions lead to a positive NPV and to an mDSCR value of about 1.3.

The SA has been performed by implementing the computational algorithm proposed in Section 2 through a Visual Basic subroutine. We have set $\varepsilon = 10^{-3}$. Convergence was obtained after 10 iterations (that is, $j^* = 10$ using the notation of Section 3) and with a total computational time of around 5 minutes.[3] The importance of each of the 428 factors, D_s ($s = 1, 2, \ldots, 428$), has been estimated with an accuracy of $\varepsilon \leq 10^{-3}$ (see Section 3).

In addition, at each iteration an internal consistency test has been implemented as follows. Given their complexity, financial models are usually equipped with warning or error messages to help analysts correct possible faults. The most diffuse one is an error message signaling imbalance between total assets and total liabilities for any year. In each iteration of the proposed algorithm all inputs are varied; if some changes produce erroneous model responses, one can register the corresponding warning signal, thus detecting which input causes the fault. Possible inconsistencies can then be corrected. Thus, an automated model consistency test is a first benefit of the proposed SA method.

In all iterations, 68 inputs registered a value $D_s = 0$. This result implies that this subset of inputs plays no role in the financial calculations. Further examination of the model structure enabled analysts to realize that these inputs were indeed disconnected from the financial calculations in the evolution of the model. These inputs were excluded from subsequent analysis.

The above two results can be ascribed as insights of type (1), in so far as they contributed to corroborate the model and to test its correctness. They thereby increased the degree of confidence in the model's results.

The identification of the direction of change in the valuation criteria and the influence of each parameter [insights (2) and (3) of Section 1] can be deducted simultaneously from the sign and magnitude of D_s. In fact, from eqs. (5) or (9), it is easy to see that the numerator of D_s is the change in V produced by a change in λ_s. Because the denominator is positive, the sign of D_s indicates whether a change in the assumption impacts the NPV or the mDSCR positively or negatively. Table 2 reports the ranking and direction of change of the 10 most influential factors. The table's last two rows report the least influential inputs on the NPV and the mDSCR, respectively. These last two parameters correspond to very detailed assumptions and reflect the thorough effort put by the analysts in the modeling exercise, where they tried to carefully reproduce the reality of the investment setting.

[Insert Table 2 about here]

Regarding the direction of impact (insight (2)), results reported in columns 1 and 4 in Table 2 are consistent with expectations. For example, increases in k_d and k_e lead to decreases in the mDSCR and the NPV; increases in tariffs lead to increases in the NPV and the mDSCR.

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[3]The short computational time is related to the nature of the spreadsheet model and is typical of financial applications; in more general computer aided engineering applications, however, a longer computational time might be expected.

As Table 2 indicates, the same five parameters exercise the most influence over both the NPV and the mDSCR; all of them concern revenue assumptions. The parameter ranking sixth for the NPV is k_e, the cost of capital; this indicates that the assumption related to the discount factor is very relevant for investors, who adopt the NPV as a valuation criterion. By contrast, this same k_e is not influential on the mDSCR because it does not play any role in eq. (1). The parameter that ranks 6th for the mDSCR, the cost of debt (k_d), is also relevant for the NPV, for which it ranks 10th.

We note that leverage ranks 8th for the mDSCR, while it is the 30th most important parameter for the NPV. Leverage determines the total amount of funds that lenders disburse and so it is closely linked to the project debt service capability. Accordingly, it has a strong direct impact on $I_t + P_t$, the denominator of the DSCR in eq. (1). Meanwhile, its impact is weaker on the NPV.

Among fiscal assumptions, the income tax rate ranks 45th for the NPV, but 358th (that is, non influential) for the mDSCR. This result is related to the choice of the lenders of utilizing a "before tax" project FCF in the numerator of eq. (1). As a consequence, taxes do not impact the mDSCR. We finally observe that the lowest ranked parameters concern very detailed assumptions; for example, the cost of onsite geological inspection or the number of days required before the investment vehicle paid connection costs to the local electricity provider.

We now investigate the level of agreement between the ranking of exogenous variables for NPV and DSCR, respectively. To do so, we make use of the methodology discussed in Section 3. In the following lines, $\mathbf{R}^{NPV}$ and $\mathbf{R}^{mDSCR}$ denote the vector of factor ranking with respect to the NPV and mDSCR respectively. Similarly, $\mathbf{SS}^{NPV}$ and $\mathbf{SS}^{mDSCR}$ denote the corresponding vector of factor Savage scores. We start with studying the set of the ranking shifts for the parameters (Figure 1).

[Insert Figure 1 about here]

Figure 1 reports the distribution of the ranking shifts. Excluding the 68 non-influential factors, 30 parameters rank the same, while a total of 338 factors rank differently when considering their respective influences on the NPV and the mDSCR. On average, factors shift 9 positions. k_e has the largest shift, (-362) positions; it ranks 6th for the NPV and 368 for the mDSCR.

To obtain a synthetic view of the level of agreement, we then estimate $\rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}}$ and $\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}}$ as suggested in Section 3. One obtains $\rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}} = 0.88$ and $\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}} = 0.93$. The value of $\rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}} = 0.88$ indicates an overall ranking agreement. The fact that $\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}} = 0.93 > \rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}} = 0.88$ indicates that discrepancies are concentrated in the ranking of the least influential factors.

In practice, investment parameters are usually grouped into the categories of revenue, operating expenses, construction costs, financial, fiscal and macroeconomic assumptions (Table 3).

[Insert Table 3 about here]

Column 2 of Table 3 displays the number of parameters in each category. The construction costs category includes the highest number of inputs, 219. This large quantity reflects the high level of detail the Bank utilized in estimating construction costs. Such detail is justified because construction costs determine the amount of credit to be disbursed to the project. The revenue assumption category includes 135 parameters. This high number is a consequence of the sophisticated revenue calculation method; the calculation is based on the number of parking slots, rotations, tariffs and occupation times, and allows for intra-day variations. The category with the lowest number of parameters, namely macroeconomic assumptions, has only one input because a unique inflation index has been used for escalation purposes at all instances.

Management finds it desirable to obtain the importance of the investment categories, in addition to ranking individual inputs/items. The results will indicate what assumptions drive the valuation results. The additivity property of D [eq. (10)] allows this process to be streamlined; a category's importance is the sum of the importance ranks of its constituent parameters. No further model runs are necessary to estimate the importance of groups; this represents notable savings in computational cost.

The third and fourth columns in Table 3 report the category ranking for the NPV and the mDSCR, respectively. Revenue assumptions, followed by construction cost assumptions are the most important groups for both criteria. Financial assumptions rank 3rd for the NPV and 5th for the mDSCR. Fiscal assumptions rank 4th for both criteria. Operating expenses rank 5th for the NPV and 3rd for the mDSCR. Macroeconomic assumptions are the least important for both criteria.

[Insert Figure 2 about here]

Figure 2 shows the magnitude and the direction of impact of each category. While there is one discrepancy in ranking — construction costs rank second for shareholders while they rank third for lenders and the converse happen to financing assumptions — the direction of each group's impact is the same for the NPV and the mDSCR; that is, a group whose positive change causes an increase in the NPV also increases the mDSCR. In particular, increases in revenue and macroeconomic assumptions improve the project's economic performance from the perspectives of both the sponsor and the lender.

The negative sign associated with fiscal assumptions reflects the fact that increases in taxes would lead to decreases in the project's economic performance. An increase in construction costs would affect project performance negatively from the perspectives of both the sponsor and the lender. Lenders, however, are more exposed than sponsors because (as mentioned above) lenders use construction costs as the basis for estimating the amount of credit to disburse. Consequently, this factor directly affects lenders' exposure.

The negative sign associated with financial assumptions reflects the fact that an increase in the cost of money (debt/capital) has a negative effect on both the NPV and the mDSCR. Conversely, an increase in leverage negatively affects the DSCR but positively affects the NPV. The negative sign

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accordingly suggests that the impact of k_e (ranking 6th) is not offset by a proportional increase in leverage (ranking 45th). Sponsors are more exposed to changes in financial assumptions than lenders are, because for sponsors this category includes such parameters as Shareholder Loan Percentage and Interest Rate, which are not relevant to creditors.

During the presentation of the results, management found it informative to further analyze the results by splitting each main category into subcategories. Revenue assumptions were subdivided into Tariffs (45 inputs), Occupation Days (20), Number of Rotations (20), Percentage of Occupation (20), Available Car Parking Slots (10), Occupation Time (10) and Number of Motorbike Slots (10). Within the financial assumption category, the Cost of Capital is separated from the other assumptions to isolate its influence. Construction Costs are also split into 4 subcategories corresponding to the actual sub-categorization in the model (Number of Rooms to be built, Number of Parking Slots, Green Spaces, Automation). Figure 3 reports the results of the analysis.

[Insert Figure 3 about here]

Figure 3 shows that assumptions about Tariffs are the most important ones for determining both the project NPV and mDSCR. Assumptions concerning Slots Occupation (Occupation Days Per Year, Occupation Percentage, Number of Rotations) follow, and are relevant from the perspective of both debt and equity criteria. In accord with Figure 2, construction costs are more relevant for lenders than for sponsors. Also, financial assumptions are more relevant from the perspective of equity than from debt. The cost of capital (k_e) highly impacts investors' NPV. Overall, Figures 2 and 3 show that fiscal assumptions and financing assumptions are less relevant than are assumptions about revenues or construction costs. Also, operating expenses play a minor role in the project's economic performance.

We note that the flexibility in choosing the level of detail (Figures 2 and 3 and Table 3) is crucial for the purpose of communicating results, especially when the number of parameters is large. For example, we have seen that calculations of construction costs and revenues were broken down to high levels of detail, but a decision maker might want to understand their influences as aggregates.

5   Conclusions

In this work, we have examined a method to better exploit the information generated by financial models for evaluating industrial investments. The high level of detail and intricacy of models used in such large and often complex initiatives makes SA essential for increasing the understanding of model results. We formalized a systematic approach based on the Differential Importance Measure (D) which allows analysts to gain systematic information on: 1) model correctness and robustness, 2) model response to changes in the exogenous variables, and 3) the influence of each assumption concerning these exogenous variables (inputs, factors) on the valuation criteria.

In the SA of complex models, two main problems arise: the management of the great number of inputs and the need to assess the sensitivity of the model output to groups of inputs. The first has been solved by applying an algorithm based on Cauchy's convergence criterion, which allows an accurate estimation of D.

The second problem has been solved by applying the additivity property of D. By additivity an analyst can, in fact, assess the joint relevance of parameters without additional model runs. This allows analysts full flexibility in the choice of groups of inputs as well as aggregation levels. The approach also eliminates the risk of excluding important factors from the analysis ex-ante.

We have applied the algorithm to a financial model for the evaluation of a project-financed parking lot. With 428 input parameters, the model realistically reproduces the investment settings. This approach has allowed us to obtain the sensitivity measures of all the exogenous variables. We then identified the key-drivers and screened out the irrelevant parameters in a rigorous way, without a-priori selecting the relevant parameters. The utilization of a warning signal has enabled us to test model correctness automatically.

In analyzing results, we considered the equity or shareholder's perspective (synthesized in an equity NPV) as well as the lender's perspective on debt performance (synthesized in the mDSCR). We discussed individual parameter ranking and notable discrepancies. For example, we have seen that while the cost of capital is relevant for determining the NPV, it does not affect the mDSCR. Similarly, the income tax rate impacts the NPV more than it does the mDSCR. Conversely, leverage is one of the most significant parameters for lenders, but not for sponsors. The introduction of a comparison method based on Savage scores enabled us to obtain quantitative measures of the ranking agreement. Results show that differences are concentrated among the least relevant factors.

We further explored the ranking agreement by analyzing the importance of parameter categories. Parameters were first grouped into the six standard categories in investment project financial analysis. Results show that Revenue Assumptions, followed by Construction Costs, are the most important drivers of economic performance from the perspectives of both sponsors and lenders. Dividing Revenue Assumptions into subcategories, we found that Tariffs are the project's KPD. Regarding the direction of change (insight (2) in Section 1), results show that the NPV and the mDSCR responded in the same direction to changes in the groups; that is, a group that influenced the NPV positively influenced the mDSCR in the same way. By contrast, individual factors can affect the NPV and the mDSCR in different ways.

We finally note that the flexibility in assessing the combined effect of factors that the presented method allows, helps meet the need of decision makers to understand with different levels of detail the influence of factors aggregated into categories.

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Table 1: Steps for the numerical estimation of D (from Borgonovo and Apostolakis (2001)).

Loop	nr.	Step
External	1	Define the $\Delta\lambda_s^j$, $j = 1, 2, ..., m$, $s = 1, 2, ..., n$ sequences
	2	Perform the steps of the internal loop (steps 2.1 and 2.2 below)
	3	Set a discrepancy
	4	Test convergence and eventually stop iterations
Internal	2.1	Compute V with all the parameters at $\boldsymbol{\lambda}^0$
	2.2	For a given j and for $s = 1, 2, ..., n$ compute $r_s^j = r_s(\Delta\boldsymbol{\lambda})$ [eq. (13)]

Table 2: Parameter ranking for sponsors (NPV) and lenders (mDSCR). Columns 1 to 3 display the most and least relevant parameters results from the sponsors' perspective. Columns 4 to 6 display corresponding information from the lenders' perspective.

Sign	Rank NPV	Parameter	Sign	Rank $mDSCR$	Parameter
+	1	Nr of parking slots	+	1	Nr. of parking slots
+	1	Daily occupation	+	1	Daily occupation
+	3	First 2 hs tariff	+	3	First 2 hs tariff
+	3	First 2 hs rotation number	+	3	First 2 hs rotation number
+	5	First 2 hs occupation %	+	3	First 2 hs occupation %
-	6	k_e (cost of equity)	-	6	k_d (cost of debt)
+	7	Tariff after the first 2 hs	-	7	Lots construction costs
+	7	Rotation number after first 2 hs	-	8	Leverage (debt/total assets)
-	9	VAT on revenues	+	9	Tariff after first 2 hs
-	10	k_d (cost of debt)	+	9	Rotation number after first 2 hs
+	11	Night tariff	+	9	Occupation % after 2hs
+	11	Number of rotations per hr after first 2 hs	-	12	VAT on revenues
+	11	Nightly occupation %	+	13	Night tariff
-	14	Rooms construction costs	+	13	Number of night rotation
-	367	Geological inspection cost	+	13	Night occupation %
-	368	Days payables for electricity connection costs	-	368	Construction site set up cost

Table 3: Ranking of the six assumption categories for the NPV and the mDSCR.

Category	Parameters	$\mathbf{R}^{NPV}$	$\mathbf{R}^{mDSCR}$
Revenue Assumptions	135	1	1
Construction Cost Assumptions	219	2	2
Financial Assumptions	29	3	5
Fiscal Assumptions	19	4	4
Operating Expenses	25	5	3
Macroeconomic Assumptions (Inflation)	1	6	6

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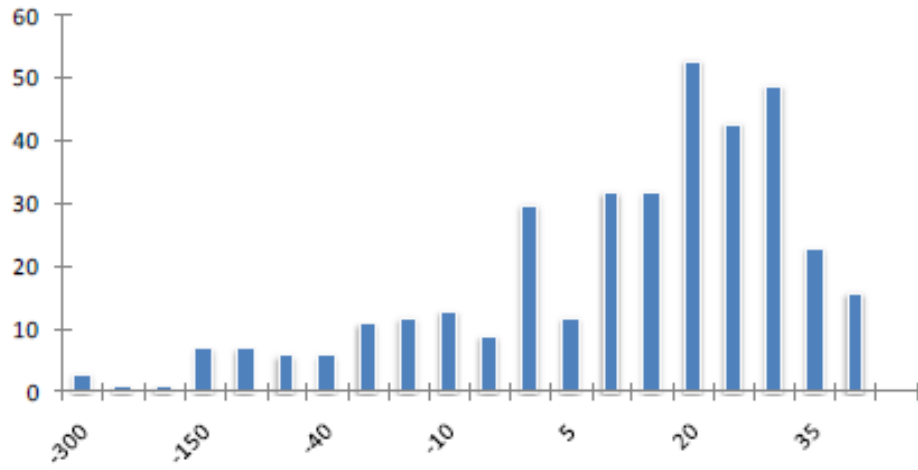
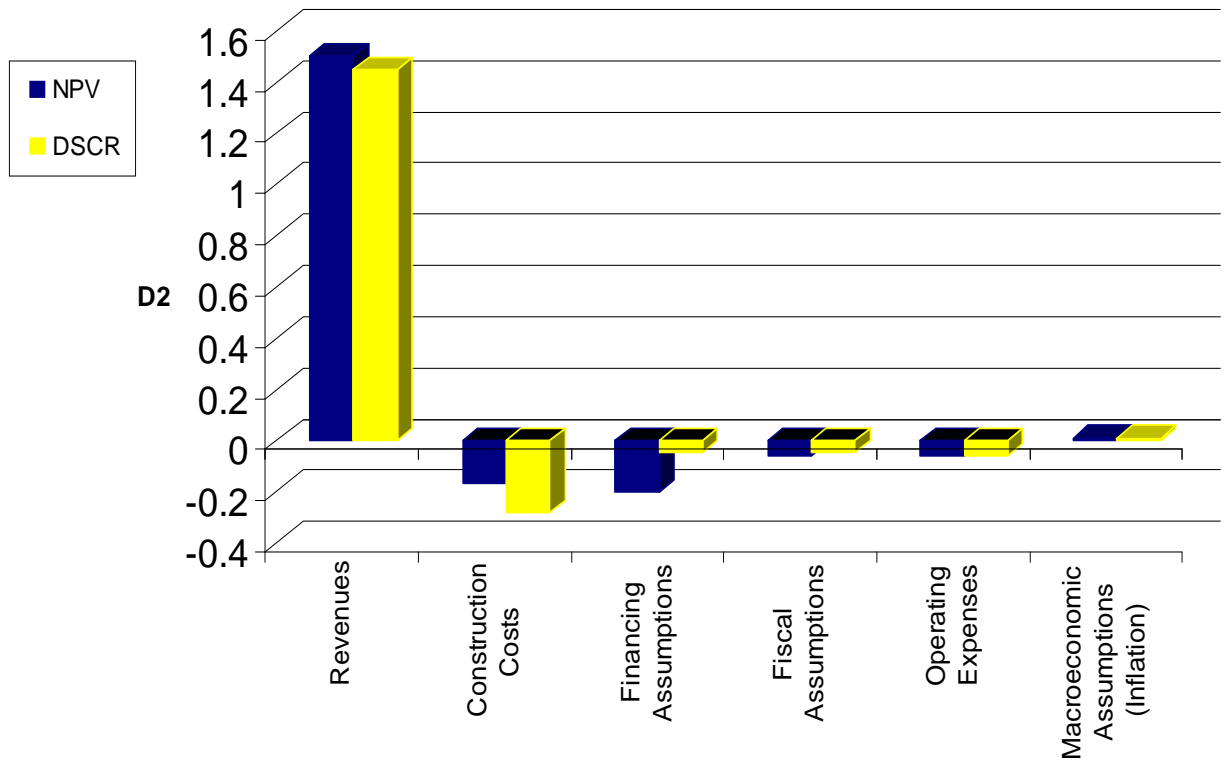**Figure 1:** Histogram of ranking shifts.

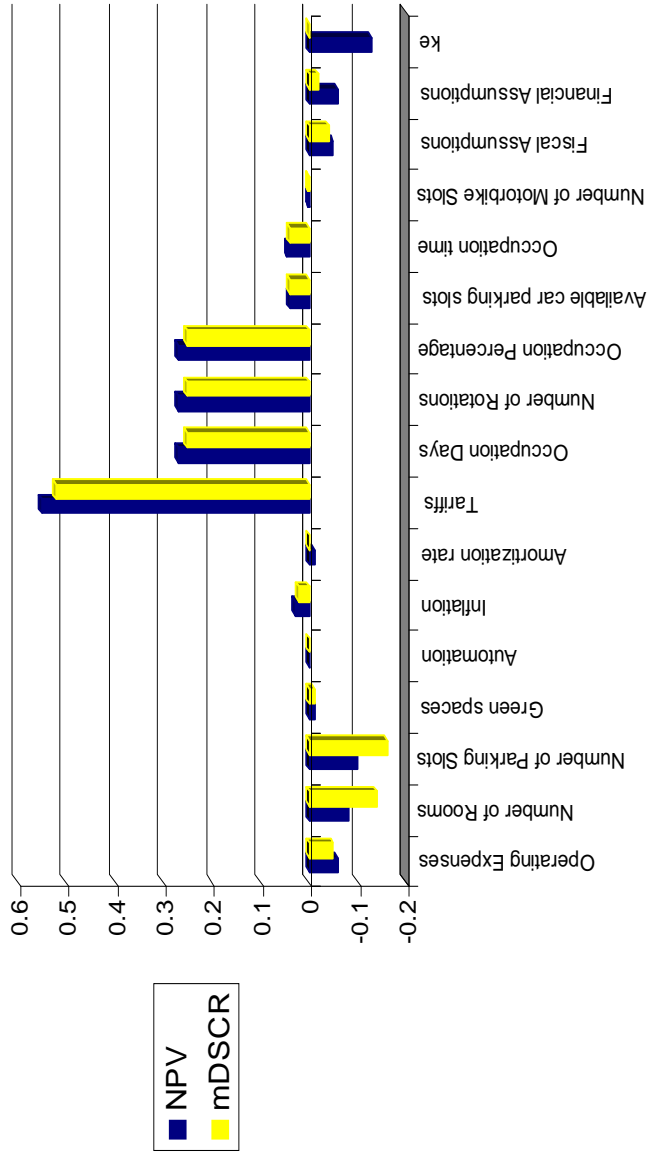**Figure 2:** $D2$ for each of the assumption categories.

Figure 3: Influence of parameters grouped into 17 categories.