Managerial Insights from Service Industry Models: A New Scenario Decomposition Method

by

E. Borgonovo and L. Peccati*

ELEUSI Research Center and Department of Decision Sciences (DECIS), Bocconi University, Viale Isonzo 25, 20135, Milano, Italy

Abstract

The service industry literature has recently assisted to the development of several new decision-support models. The new models have been often corroborated via scenario analysis. We introduce a new approach to obtain managerial insights in scenario analysis. The method is based on the decomposition of model results across sub-scenarios generated according to the high dimensional model representation theory. The new method allows analysts to quantify the effects of factors, their synergies and to identify the key drivers of scenario results. The method is applied to the scenario analysis of the workforce allocation model by Corominas et al (2004).

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*corresponding author: lorenzo.peccati@unibocconi.it

1 Introduction

The development of decision-support models dedicated to service industry problems has attracted renewed interest in O.R. (see Duder and Rosenwein (2001); Grossman and Brandeau (2002); Wu and Lin (2003); Kiely et al (2004); Corominas et al (2004); Corominas et al (2007a); Corominas et al (2007b); Hosanagar et al (2005); Ata and Shneorson (2006); Hong (2007); Debo et al (2008); Wahab et al (2008); Beraldi and Bruni (2009); Section 2). The literature review of Section 2 shows that profound efforts and great attention are concentrated into the model building phase, while the (equally important) phase of obtaining managerial insights is often left to qualitative reasoning. This exposes analysts to the risk of undermining the modelling effort. Two issues contribute to the problem. The first is model complexity. New models capture aspects of the managerial problems previously excluded from the analysis. Models, then, tend to become complex, with a closed-form expression of the decision-support criterion mostly unavailable. The model behavior becomes unknown to analysts generating the so called black-box effect (Saltelli et al (2000)). The second reason concerns parameter estimation: *"In most service systems, it is hard to estimate some system parameters reliably, and the parameters may also change over time. Therefore, it is important to understand how the optimal policy and its performance depend on various system parameters (*Ata and Shneorson (2006)*)."* These issues can be solved only by utilization of sensitivity analysis (SA) techniques that, by a thorough investigation of the model results, enable analysts to validate the output and obtain additional managerial insights (Saltelli et al (2000); Saltelli and Tarantola (2002); Borgonovo (2008); Borgonovo (2009); Borgonovo and Peccati (2008a)).

Scenario analysis (Kiely et al (2004), O'Brien (2004), Tietje (2005)) is the method that has been most widely utilized to perform SA of service industry models (Section 2). In a scenario analysis, the model is tested for alternative combinations of the exogenous variables and the corresponding model output values are recorded. These values provide decision-makers with the variability of the decision-support criterion. The explanation of the results, however, is usually left to "*qualitative*" (Tietje (2005)) reasoning, as a quantitative method for interpreting scenario results has not been developed yet.

This work introduces a new approach to the acquisition of managerial insights from scenario analysis. The tool integrates the qualitative aspects of scenario analysis with a quantitative methodology derived from the high dimensional model representation (HDMR) theory. In selecting the method, one needs to consider the following features of scenario analysis: a) absence of restrictions in parameter changes — exogenous variables undergo discrete changes when varying across scenarios (see Tietje (2005)); b) possible non-smooth model output behavior — the model does not need to be differentiable; and c) joint sensitivities — in a scenario analysis factors are, in general, formed by groups of exogenous variables; thus, factor variations coincide with the simultaneous change in several parameters (in the remainder, by factor we shall mean a group of $k \geq 1$ exogenous variables). The problem one is facing is, then, the SA of a potentially non-smooth model output in the presence of finite and simultaneous variations in groups of exogenous variables. Let us review how this problem has been addressed sofar. In Linear Programming, simultaneous variations in the objective function coefficients are the focus of the tolerance sensitivity approach (Wendell (2004)). Outside the Linear Programming realm, the works of Borgonovo and Apostolakis (2001) and Borgonovo (2008) address the joint sensitivity of generic models for small parameter changes. The work of Saltelli and Tarantola (2002) defines group sensitivity indices in the context of a global SA. As far as finite changes are concerned, Borgonovo (2009) introduces sensitivity measures for individual exogenous variables. To find sensitivity measures for factors, one then needs to extend the results of Borgonovo (2009) to group variations. Our first step is to prove that a change in model output is decomposed as a function of factors with the same structure of the parameter decomposition (Rabitz and Alis (1999)). This finding permits us to introduce factor finite change sensitivity indices (FCSI). The relationship between the factor FCSI and parameter FCSI's is investigated. We show that the first order FCSI of a factor formed by k parameters is the sum of all the FCSI's of order 1 to k of the parameters in the factor. Also, a closed-form expression for the relationship between higher order factor FCSI's and parameter FCSI's is derived.

These findings are then turned into a procedure for the estimation the factor FCSI's, based on the generation of appropriate sub-scenarios generated according to the HDMR theory. We discuss how to gain additional managerial insights by formulation of SA settings (Saltelli and Tarantola (2002)). Knowledge of the FCSI's enables analysts to quantify the contributions of factors, the relevance of their interactions and identify the key-drivers of scenario results.

We apply the methodology to the scenario analysis of the workforce allocation model by Corominas et al

(2004). The minimum (optimal) cost undergoes a 155% raise in passing from the first to the second scenario. The analysis reveals that the change cannot be explained by sole individual effects, but interactions play a relevant role: 40% of the change is generated by the cooperation between required capacity and extra-hour costs. Matching capacity and extra-hour costs profiles is, then, revealed as a key aspect on which "*to focus managerial attention during implementation* (Eschenbach (1992), p. 40-41)", in order to hedge losses.

The remainder of the paper is organized as follows. Section 2 offers a review of recent quantitative service industry literature. Section 3 introduces scenario analysis and three SA settings for obtaining additional managerial insights. Section 4 integrates scenario analysis and the HDMR theory and introduces sensitivity measure for factors. Section 5 presents the application of the method to the scenario analysis of Corominas et al (2004)'s workforce allocation model. Conclusions and further research perspectives are offered in Section 6.

2 Literature Review: Models and SA methods in recent works

This section presents the results of a literature review surveying the SA methods that have been employed to gain managerial insights from recently developed service industry decision-support models (Table 1). Due to space limitations, we cannot claim exhaustiveness. The indication obtained, however, is that scenario analysis is broadly utilized to corroborate model results and obtain additional insights (7 out of 12 works).

[Insert Table 1 about here]

Duder and Rosenwein (2001) address the problem of improving service quality in call-centers, with particular reference to the evaluation of "*zero abandonment policies* (Duder and Rosenwein (2001))". The model output (y) [see Table 2 for notation] is analytically expressed as a linear function of seven parameters. Model results are tested on a single scenario (no SA is performed).

Grossman and Brandeau (2002) propose a non-linear optimization model to "*address service systems where self-optimizing customers receive service from facilities with different service capabilities, seeking the best combination of ease of access to facilities with congestion at facilities* [Grossman and Brandeau (2002), p. 40]." Model results are tested on a single scenario and sensitivity bounds for toll flexibility are proposed in a 2-facility example.

In Wu and Lin (2003), a linear programming model is developed to solve a flow capturing problem. The model supports decision-makers in selecting "*the optimal locations*" of service facilities while maximizing "*the number of "captured" customers* [Wu and Lin (2003); p. 366]." The model is tested across scenarios generated by varying the number of facilities and the network spatial distribution.

Corominas et al (2004) develop a model for optimal workforce allocation in planning annualized hours. The model is formulated as a mixed integer linear program. Model results are tested on two scenarios involving different demand curves and working week type parameters. Generalizations of the model are proposed in Corominas et al (2007a) and Corominas et al (2007b). In these works, model results are tested on scenarios characterized by different demand curves and working week type parameters.

In Hosanagar et al (2005) the problem of caching service adoption in the internet service supply chain is addressed. Convex optimization problems are proposed and solved analytically "*to inform cache operators regarding service provisioning and optimal pricing policies* (Hosanagar et al (2005))". SA is performed by means of comparative statics. Monotonicity properties (i.e., the direction of change) of the solution for changes in the service parameters are derived.

Ata and Shneorson (2006) introduce an M/M/1 model to study a service facility in which the system manager dynamically controls arrival and service rates. Two decision-problems are analyzed: the rate-setting and the price-setting problems. Ata and Shneorson (2006) utilize comparative statics "*to study the dependence of the optimal policy on the system parameters*." In particular, Ata and Shneorson (2006) discuss monotonicity properties of the optimal policy as a function of the delay sensitivity parameter, in the value rate function and in the cost function.

Hong (2007) develops a nested logit model for determining the optimal "*location of foreign logistics services within the Chinese city"* of Shanghai. SA results are obtained by applying the model on sub-samples of the original data-set.

The model developed by Debo et al (2008) combines queuing literature and economic equilibrium models to take into account "*strategic interaction between the server and the customer* (Debo et al (2008))". The authors offer both analytical expressions of the model and a numerical experiment to validate the model results. In particular, model results are replicated for different choices of the parameters deemed relevant (scenario analysis).

Beraldi and Bruni (2009) develop a stochastic programming model for determining the optimal location of emergency vehicles in congested service systems. Model results are tested on different scenarios (see Beraldi and Bruni (2009), pages 328-329).

In Castillo et al (2009), the problem of workforce allocation is generalized to multicriteria optimization. Castillo et al (2009) report the following comment of the call center management concerning the model: "*The tool that you have created would allow scenarios to be run that would be very useful*".

Let us now summarize the findings of this (concise) literature review. A first indication is that scenario analysis is the most broadly applied method to corroborate model results (Table 1). A second is that the analysis is stopped after acquiring the values of the model output. The interpretation of scenario results is left to qualitative reasoning. In this way, many relevant managerial insights go overlooked, as we are to discuss in the next section.

3 Scenario Analysis and Managerial Insights

The origin of scenario analysis is often linked to the classical book by Khan and Wiener (1967). Since then, scenario analysis has became a major decision tool in economics and strategic management and has been extensively studied in O.R.. Several definitions of scenarios have been formulated: "*single deterministic realizations of all uncertainties over the planning horizon*" in Mulvey et al (1999), "*stories about how the future might turn out*" in O'Brien (2004), "*different states of the world in presence of uncertainty*" in Hinojosa et al (2005), and "*different possible future states of a system*" in Tietje (2005).

In the service industry, we recall the work of Kiely et al (2004), who propose a research-approach to scenario generation in the service sector. Scenarios are utilized "*to guide strategic planning*" by envisioning different possible realizations of the future (Kiely et al (2004), p. 131).

The aim of a scenario analysis is, indeed, a consistent elicitation of predictions. Cognitive aspects play a relevant role, as demonstrated by Jungermann and Thuring (1988). The study of the methodological aspects of a coherent scenario generation have been thoroughly discussed over time. The works of O'Brien (2004) and Tietje (2005) can be seen as summarizing the findings of previous literature. O'Brien (2004) proposes an eight-step scenario generation process that extends the 5-step approach proposed by Linneman and Kennell (1977). O'Brien (2004) discusses pitfalls in implementing scenarios and underlines consistency issues. Consistency is the focus of Tietje (2005), where it is considered "*a core part of a formative scenario analysis* (Tietje (2005); p. 419)." According to Tietje (2005), the *desirable* scenario generation method should result in scenarios that are *consistent* (to provide realistic descriptions of the future), *different* (to avoid redundant information), in a *small number* (to facilitate comparison), *reliable* (to allow replication if different scenario analysts were to start with the same assumptions) and *efficient*. Tietje (2005) introduces a quantitative approach to estimate consistency by the creation of a consistency matrix. Consistency analysis is followed by a scenario selection process, in which the number of scenarios is reduced by appropriate filtering so as to achieve a compact and efficient set (we refer to Tietje (2005), Section 2.2, *p.* 421, for further details).

A scenario analysis, if correctly implemented, allows an analyst/decision-maker to forecast future outcomes according to her/his state of knowledge (see also Jungermann and Thuring (1988)). By using scenario analysis to corroborate model results, a decision-maker assesses the values of the decision-support criterion given the forecasted realizations of the exogenous variables. The task of understanding "*what it was about the inputs that made the outputs come out as they did* (Little (1970); p. B469)" is usually addressed by qualitative reasoning, which does not necessarily lead to a rigorous interpretation of the results. The following questions, among others, remain unanswered (see also Borgonovo (2009)):

Setting 1: [1]How much does each factor impact the scenario results? What is the direction of change implied by the factor changes?

Setting 2: What portion of the results is due to interaction effects? Do interaction effects amplify or smoothen individual effects?

[1]Settings have been introduced in Saltelli and Tarantola (2002). A setting is a statement of the question a decision-maker/analyst wishes to answer through the SA exercise. A Setting then enables an analyst to identify the method that provides the consistent answer. Several works have underlined the risk of obtaining misleading insights SA. Koltay and Terlaky (2000) warn about the differences in mathematical and managerial interpretations of SA results when applied to linear programming models. Wallace (2000), Higle and Wallace, 2003 underline that the SA method employed to interpret model results must be consistent with the decision-maker's state of knowledge. Borgonovo and Peccati (2008a) provide a thorough discussion of the consistency between state-of-knowledge and SA methods.

Setting 3: What are the most important factors (key-drivers of the change)?

As we are to see, answering these questions is ripe of managerial insights. To obtain the answers, however, it is necessary to add to the quantitative aspects of scenario analysis. This is the task of the next section.

4 Generalized Scenario Analysis: Factor FCSI's and Scenario Decomposition

This section introduces the integration of scenario analysis with the HDMR theory to set forth a rigorous way to explain scenario analysis results.

The HDMR theory is the generalization of Functional ANOVA (Rabitz and Alis (1999); Efron and Stein (1981)). Functional ANOVA originates with the works of Hoeffding (see Hoeffding (1948)), and the problem of achieving the complete decomposition of the variance of a function of random variable. The most recent formulations are due to the works of Rabitz and Alis (1999), Sobol' (2001), Sobol' (2003) and Sobol' et al (2007). We summarize them in the next paragraphs. Let $(\Theta \subseteq \mathbb{R}^n, \mathcal{A}, \mu)$ be a measure space and consider a function f such that

$$y = f(\theta) \qquad f : \Theta \to \mathbb{R} \tag{1}$$

(see Table 2 for notation).

[Insert Table 2 about here]

From the SA viewpoint, y [eq. (1)] is the model output and $f(\theta)$ the functional relationship that binds y and the exogenous variables (parameters, $\theta \in \Theta$). A closed-form for $f(\theta)$ does not need to be known. The central result of the HDMR theory states that under the assumption of $\mu = \prod_{i=1}^{n} \mu_i$, any $f \in \mathcal{L}(\Theta, \mathcal{A}, \mu)$ can be expanded as (Sobol' (1993), Rabitz and Alis (1999)):

$$f(\theta) = f_0 + \sum_{i=1}^{n} f_i(\theta_i) + \sum_{i<j} f_{i,j}(\theta_i, \theta_j) + ... + f_{1,2,...n}(\theta_1, \theta_2, ..., \theta_n) \tag{2}$$

where

$$
\begin{cases}
f_0 = \mathbb{E}_\mu[Y] = \int \cdots \int f(x) d\mu \\
f_i(x_i) = \mathbb{E}_\mu[Y|x_i] - f_0 = \int \cdots \int f(x) \prod_{k \neq i} d\mu_k - f_0 \\
f_{i,j}(x_i, x_j) = \mathbb{E}_\mu[Y|x_i, x_j] - f_i(x_i) - f_j(x_j) - f_0 \\
...
\end{cases}
\tag{3}
$$

Eq. (2) has been widely employed in statistics and global sensitivity analysis (Saltelli and Tarantola (2002)). In fact, by adding the assumption that $f \in \mathcal{L}^2(\Theta, \mathcal{A}, \mu)$ and squaring the difference $f - f_0$, one obtains the complete decomposition of the variance of f (functional ANOVA). The variance decomposition is proven for the first time in Efron and Stein (1981).

Borgonovo (2009) proves that any finite change in f can be decomposed in $2^n - 1$ terms via eq. (2). In this case, let θ^0,$\theta^1 \in \Theta$, $y^0 = f(\theta^0)$, $y^1 = f(\theta^1)$ and $\Delta f = y^1 - y^0 = f(\theta^1) - f(\theta^0)$. Then, under the same assumptions of eq. (2), one obtains (Borgonovo (2009), Theorem 2),

$$
\Delta f = f(\theta^1) - f(\theta^0) = \sum_{i=1}^{n} \Delta f_i + \sum_{i<j} \Delta f_{i,j} + ... + \Delta f_{1,2,...n} \tag{4}
$$

One refers to the terms in the RHS of eq.(4) as to "effects". The first order terms Δf_i [eq.(4)] are the contributions of individual parameter changes. The second order terms ($\Delta f_{i,j}$) account for the interactions of all parameters pairs, excluding individual effects (see also Borgonovo (2009)). The third order terms reveal the strength of the interaction of all triplets, with exclusion of second and first order effects. The last term reflects the portion of the change in f that can be explained only as a residual interaction of order n.

We now address how to merge scenario analysis and HDMR theory. As mentioned in Section 3, in a scenario analysis factors are usually represented by groups of exogenous variables. Thus, analysts are interested in the effect of a group of parameters rather than in individual parameter effects. Let us refer to the model by Corominas et al (2004). The capacity curve, which is one of the factors, is a group of T variables ($C = \{c_t\}$, $t = 1, 2, ..., T$). When passing from scenario 0 to scenario 1, the whole curve is shifted and an analyst is interested in the overall effect of C, and not in the separate effects of each c_t. Since eq. (4) deal with individual parameters, it needs to be generalized to the case of parameter groups. We proceed as follows. First, we partition θ in $Q < n$ groups, each called a factor and denoted by γ_l, $l = 1, 2, ..., Q$.

Definition 1 *We call*

$$\underbrace{\theta_1 \quad \theta_2 \quad ... \quad \theta_{s_1}}_{\gamma_1} \quad \underbrace{\theta_{s_1+1} \quad \theta_{s_1+2} \quad ... \quad \theta_{s_2}}_{\gamma_2} \quad ... \quad \underbrace{\theta_{s_{kQ-1}+1} \quad \theta_{s_{Q-1}+2} \quad ... \quad \theta_n}_{\gamma_Q} \tag{5}$$

factor partition of the parameters, and

$$\gamma = \left\{ \gamma_1, \gamma_2, ..., \gamma_Q \right\} \tag{6}$$

the vector of factors.

We then utilize the formalization of Tietje (2005), according to whom a scenario is "*a set of system variables (impact factors) each of which is allowed to take only a small number of different levels (two to five)*". For each of the Q factors (γ_l, $l = 1, 2, ..., Q$), s_l different levels $\gamma_l^1, \ldots, \gamma_l^{s_l}$ are defined. In Tietje (2005), it is suggested to keep s_l between two to five. Scenario γ^k is the vector $\gamma^k = (\gamma_1^{m_1}, \ldots, \gamma_Q^{m_Q})$ with each factor set at a given level. For simplicity, in the remainder, let us refer to two scenarios, namely scenario 0 and 1: $\gamma^0 = \left\{ \gamma_1^0, \gamma_2^0, ..., \gamma_Q^0 \right\}$ and $\gamma^1 = \left\{ \gamma_1^1, \gamma_2^1, ..., \gamma_Q^1 \right\}$ denote the values of the factors and, similarly, $y^0 = f(\gamma^0)$ and $y^1 = f(\gamma^1)$ denote the corresponding model output values.

We have the following result (we refer to Table 2 for notation).

Theorem 1 *Let f be a measurable function, and let γ be the vector of factors [eq. (6)]. Then,*

$$\Delta f = f(\gamma^1) - f(\gamma^0) = \sum_{l=1}^{Q} \sum_{i_1 < i_2 \ldots < i_l} \Delta_{i_1, i_2, \ldots, i_l} f \tag{7}$$

where

$$\begin{cases} \Delta_i f = f(\gamma_i^1; \gamma_{(-)}^0) - f(\gamma^0) \\ \Delta_{i_1, i_2} f = f(\gamma_{i_1}, \gamma_{i_2}^1; \gamma_{(-)}^0) - \Delta_{i_1} f - \Delta_{i_2} f - f(\gamma^0) \\ \ldots \end{cases} \tag{8}$$

Proof. Let Γ denote the space of parameter groups, with measure space $(\Gamma, \mathcal{F}, \mu_\gamma)$. Consider then a Dirac δ-density of the type $\delta(\gamma) = \prod_{l=1}^{Q} \delta(\gamma_l - \gamma_l^0)$. Eq. (8) then follows by inserting $\delta(\gamma)$ into eqs. (3). Eq. (4) then becomes eq. (7). ■

Theorem 1 states that the decomposition of a change in model output with respect to factors preserves the structure of the decomposition with respect to individual variables. It is then possible to define finite change sensitivity indices (FCSI) for factors.

Definition 2 *We call*

$$\xi_l^1 := \Delta_l f \quad (l = 1, 2, ..., Q) \tag{9}$$

the FCSI of factor γ_l,

$$\xi_{l_1,l_2,...,l_k}^k := \Delta_{l_1,l_2,...,l_k} f \quad (k = 1, 2, ..., Q) \tag{10}$$

the FCSI of order k *of the group of factors* $\gamma_{l_1}, \gamma_{l_2}, ..., \gamma_{l_k}$,

and

$$\xi_l^T := \Delta_l f + \sum_{l \neq j} \Delta_{l,j} f + ... + \Delta_{1,2,...,Q} f \quad (l = 1, 2, ..., Q) \tag{11}$$

the total order FCSI of factor γ_l.

Eqs. (9), (10) and (11) define the factor FCSI's. We are shortly to discuss how they generalize the parameter FCSI's introduced in Borgonovo (2009). Before, let us summarize their meaining. ξ_l^1 [eq. (9)] is the effect of factor γ_l. It equals the change in f provoked by moving factor γ_l from γ_l^0 to γ_l^1, while the other factors are kept at γ_l^0. $\xi_{l_1,l_2,...,l_k}^k$ [eq. (10)] equals the interaction effect of factors $\gamma_{l_1}, \gamma_{l_2}, ...$ and γ_{l_k}. For instance, let us consider the group FCSI of order 2. $\xi_{s,t}^2$ is the residual change in f provoked by the simultaneous shift of factors γ_s and γ_t from γ_s^0,γ_t^0 to γ_s^1,γ_t^1 respectively $[f_{s,t}(\gamma_1^0, ..., \gamma_t^1, ..., \gamma_s^1, ..., \gamma_Q^0) - f(\gamma^0)]$, after subtraction of their individual effects $(-\Delta_t f - \Delta_s f)$. ξ_l^T [eq. (11)] is the total order FCSI of group γ_l. The sum in eq. (11) includes all terms in eq. (7) involving group γ_l. ξ_l^T is called total effect of factor γ_l and coincides with the portion of Δf associated with group γ_l.

We now investigate how factors FCSI's of factors are related to FCSI's of the parameters in the group (see Definition 1). Let θ^0 and θ^1 denote the values assumed by the exogenous variables (parameters, see eq. (5)) in scenarios 0 and 1, respectively. One observes that $y_0 = f(\theta^0) = f(\gamma^0)$ and $y_1 = f(\theta^1) = f(\gamma^1)$. Then, the following holds.

Theorem 2 ***Part 1)*** *Relationship between first order factor FCSI* (ξ_i^1) *and parameter FCSI's* ($\varphi_{t_1,t_2,...,t_k}^k$)**.** *Let* $\gamma_i = \{\theta_t, \ t = s_i + 1, s_i + 2, ..., s_i + n_i\}$ *the* i^{th} *factor in Definition 1 and let* $t_1, t_2, ...t_k$ *any subset of* k *indices such that* $\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_k} \in \gamma_i$. *Then, one has:*

$$\xi_i^1 = \sum_{k=1}^{n_i} \sum_{t_1 < t_2 ... < t_k} \varphi_{t_1,t_2,...,t_k}^k \tag{12}$$

where $\varphi^k_{t_1,t_2,...,t_k}$ is the FCSI of exogenous variables θ_{t_1}, θ_{t_2},..., θ_{t_k}.

Part 2) *Relationship between second order factor FCSI and parameter FCSI's ($\varphi^k_{t_1,t_2,...,t_k}$).* *Let γ_i and $t_1, t_2, ...t_k$ as in Part 1. In addition, let $\gamma_j = \{\theta_r,\ r = s_j+1, s_j+2, ..., s_j+n_j\}$ be the j^{th} factor in Definition 1 and let $r_1, r_2, ...r_m$ any subset of m indices such that $\gamma_{r_1}, \gamma_{r_2}, ..., \gamma_{r_m} \in \gamma_j$. One has:*

$$\xi^2_{i,j} = \Delta_{\gamma_i,\gamma_j} f = \sum_{u=\max(n_i,n_j)}^{n_i+n_j} \sum_{s_1<s_2...<s_u} \varphi^u_{t_1,t_2,...,t_u} \tag{13}$$

Proof. Part 1. We use the subscripts $|_{\gamma_i}$ and $|_{\theta_t}$when referring to the effect of a factor or of an exogenous variables, respectively. By expressing eq. (8) as a function of the parameters, one has:

$$f(\gamma^1_i; \gamma^0_{(-)}) - f(\gamma^0) = f(\theta^1_{t_1}, \theta^1_{t_2}, ..., \theta^1_{t_{n_i}}; \theta^0_{(-)}) - f(\theta^0) \tag{14}$$

We recall that in Borgonovo (2009) (Corollary 1) the following equalities for the parameter effects hold:

$$\Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_{n_i}}} f = f(\theta^1_{t_1}, \theta^1_{t_2}, ..., \theta^1_{t_{n_i}}; \theta^0_{(-)}) - \sum_{k=1}^{n_i-1} \sum_{t_1<t_2...<t_k} \Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_{n_k}}} f - f(\theta^0) \tag{15}$$

Hence, there follows that:

$$f(\theta^1_{t_1}, \theta^1_{t_2}, ..., \theta^1_{t_{n_i}}; \theta^0_{(-)}) - f(\theta^0) = \Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_{n_i}}} f + \sum_{k=1}^{n_i-1} \sum_{t_1<t_2...<t_k} \Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_k}} f \tag{16}$$

i.e.,

$$\Delta_{\gamma_i} f = \sum_{k=1}^{n_i} \sum_{t_1<t_2...<t_k} \Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_{n_i}}} f \tag{17}$$

By definition of $\varphi^k_{t_1,t_2,...,t_k}$,

$$\Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_{n_i}}} f = \varphi^k_{t_1,t_2,...,t_k} \tag{18}$$

which concludes the proof of Part 1.

Part 2. We note that:

$$\begin{gathered} \Delta_{\gamma_i,\gamma_j} f = f(\gamma^1_i, \gamma^1_j; \gamma^0_{(-)}) - \Delta\gamma_{i_1} f - \Delta\gamma_{i_2} f - f(\gamma^0) \\ f(\theta^1_{t_1}, \theta^1_{t_2}, ..., \theta^1_{t_{n_i}}, \theta^1_{r_1}, \theta^1_{r_2}, ..., \theta^1_{r_{n_j}}; \theta^0_{(-)}) - \Delta\gamma_i f - \Delta\gamma_j f - f(\theta^0) \end{gathered} \tag{19}$$

From Corollary 1 in Borgonovo (2009), there follows that:

$$f(\theta^1_{t_1}, \theta^1_{t_2}, ..., \theta^1_{t_{n_i}}, \theta^1_{r_1}, \theta^1_{r_2}, ..., \theta^1_{r_{n_j}}; \theta^0_{(-)}) - f(\theta^0) = \sum_{k=1}^{n_i+n_j} \sum_{t_1<t_2...<t_k} \Delta_{\theta_{t_1},\theta_{t_2},...,\theta_{t_k}} f \tag{20}$$

Hence, substituting in eq. (19), one obtains:

$$\Delta_{\gamma_i\gamma_j} f = \sum_{k=1}^{n_i+n_j} \sum_{t_1<t_2\ldots<t_k} \Delta_{\theta_{t_1},\theta_{t_2},\ldots,\theta_{t_k}} f - \Delta_{\gamma_i} f - \Delta_{\gamma_j} f \tag{21}$$

One needs to observe that

$$\sum\nolimits_{k=1}^{n_i+n_j} \sum_{t_1<t_2\ldots<t_k} \Delta_{\theta_{t_1},\theta_{t_2},\ldots,\theta_{t_k}} f = \sum\nolimits_{k=1}^{n_i} \sum_{t_1<t_2\ldots<t_k} \Delta_{\theta_{t_1},\theta_{t_2},\ldots,\theta_{t_k}} f + \sum\nolimits_{p=1}^{n_j} \sum_{r_1<r_2\ldots<r_p} \Delta_{\theta_{r_1},\theta_{r_2},\ldots,\theta_{r_p}} f + \\ + \sum\nolimits_{u=\max(n_i,n_j)}^{n_i+n_j} \sum_{s_1<s_2\ldots<s_u} \Delta_{\theta_{s_1},\theta_{s_2},\ldots,\theta_{s_u}} f \tag{22}$$

Then, by utilizing Part 1 [eq. (12)] to expand $\Delta_{\gamma_i} f$ and $\Delta_{\gamma_j} f$, and substituting into eq. (21), one obtains:

$$\Delta_{\gamma_i,\gamma_j} f = \sum\nolimits_{k=1}^{n_i} \sum_{t_1<t_2\ldots<t_k} \Delta_{\theta_{t_1},\theta_{t_2},\ldots,\theta_{t_k}} f + \sum\nolimits_{p=1}^{n_j} \sum_{r_1<r_2\ldots<r_p} \Delta_{\theta_{r_1},\theta_{r_2},\ldots,\theta_{r_k}} f + \\ + \sum\nolimits_{u=n_i}^{n_i+n_j} \sum_{s_1<s_2\ldots<s_u} \Delta_{\theta_{s_1},\theta_{s_2},\ldots,\theta_{s_u}} f - \sum\nolimits_{k=1}^{n_i} \sum_{t_1<t_2\ldots<t_k} \Delta_{\theta_{t_1},\theta_{t_2},\ldots,\theta_{t_k}} f - \sum\nolimits_{p=1}^{n_j} \sum_{r_1<r_2\ldots<r_p} \Delta_{\theta_{r_1},\theta_{r_2},\ldots,\theta_{r_k}} f \tag{23}$$

Hence,

$$\Delta_{\gamma_i,\gamma_j} f = \sum_{u=n_i+1}^{n_i+n_j} \sum_{s_1<s_2\ldots<s_u} \Delta_{\theta_{s_1},\theta_{s_2},\ldots,\theta_{s_u}} f = \sum_{u=\max(n_i,n_j)}^{n_i+n_j} \sum_{s_1<s_2\ldots<s_u} \varphi^u_{t_1,t_2,\ldots,t_u} \tag{24}$$

■

Part 1 of Theorem 2 states that the first order FCSI of a factor is the sum of all the FCSI's of the parameters contained in the factor. Hence, ξ^1_i encompasses all individual effects, the effects of all pairs, triplets (and so on so forth) that can be formed with the parameters in the group. With reference to Corominas et al (2004)'s model, Proposition 2 states that the effect of translating the whole capacity curve from one scenario to the other ($C^0 = \{c^0_t\} \to C^1 = \{c^1_t\}$) is the sum of the effects of all the shifts in the individual c_t's, $t = 1, 2, ..., T$, when considered individually, in pairs, triplets etc..

Part 2 of Theorem 2 states that the interaction effect between two factors includes the interaction effects of all (and sole) the groups of order $\max(n_i, n_j) + 1$ to $n_i + n_j$ that can be formed by adding parameters of factor γ_j to the parameters of factor γ_i. In Part 2 of Theorem 2, we have used the second-order FCSI's for notation simplicity, but the result can be readily extended to higher order interactions.

We now turn these findings into a procedure for the estimation of $\xi^k_{\iota_1,\iota_2,\ldots,\iota_k}$ while conducting scenario analysis (Table 3).

[Insert Table 3 about here]

The first step consists in developing the scenarios and evaluating the model as discussed in Section 3 (i.e., one performs scenario analysis of model output as foreseen by the scenario analysis methodology). The second step consists in determining the partition in Definition 1. This is equivalent to establishing what factors are varied across scenarios. The third step (Table 3) consists in running the model on sub-scenarios. The sub-scenarios are selected according to eq. (7). The model is first run on sub-scenarios involving one-factor-at-a-time variations ($\gamma_l^0 \to \gamma_l^1$, $l = 1, ..., Q$). By computing the difference between the model output in scenario 0 ($f(\gamma^0)$) and the values obtained in each sub-scenario, one estimates the first order indices, ξ_l^1. To quantify two-factor interactions (Table 3), the model needs to be run on sub-scenarios formed by shifting two-factors-at-a-time. By taking the difference between the values attained by the model in each sub-scenario and $f(\gamma^0)$, and subtracting the first order indices of the pair, one obtains the second order FCSI's. This step is repeated $Q-1$ times, since the term of order n can be computed as a residual difference. The last step of the procedure consists in summing the sensitivity indices of orders 1 to Q of each factor so as to obtain the total effect ξ_l^T.

In the next section, we apply the methodology to gain managerial insights in the scenario analysis of the service industry model by Corominas et al (2004).

5 Application: Explaining the AHDV-1 Model Results on Two Scenarios

In this section, we apply the method introduced in Section 4 to gain additional managerial insights from the scenario analysis of the model proposed by Corominas et al (2004) [eq. (25)].

Flexibility is a key feature in both the manufacturing and the service industries. For a thorough discussion of flexibility and the problem of its measurement, we refer to Wahab et al (2008) and to the references therein contained. To achieve flexibility, managers in the manufacturing sector can rely both on machines and inventories. In the service sector, the problem has a different nature "*because of the absence of any equivalent to inventory* (Corominas et al (2004), p. 217)". Thus, to match "*capacity to demand over time*" (Corominas et al (2004), p. 218), the focus is on workforce. In workforce allocation problems, constraints arise in view of regulations and collective enterprise (union) agreements. The concept of annualized hours plays a central role (Corominas et al (2004), Corominas et al (2007a) Corominas et al (2007b)).

Corominas et al (2004) develop a mixed integer linear programming model to determine optimal workforce allocation. The model equations are as follows:

$$
\begin{array}{ll}
\multicolumn{2}{c}{\min_{x,d} \sum_{t=1}^{T} \alpha_t d_t} \\
\multicolumn{2}{c}{s.t.} \\
\sum_{k=1}^{K} h_k x_{kt} + d_t \geq c_t & (\forall t) \\
\sum_{k=1}^{K} x_{kt} = N & (\forall t) \\
\sum_{t=1}^{T} x_{kt} = N \cdot r_k & (\forall k) \\
x_{kt} \geq 0 \text{ and integer} & (\forall k, \forall t) \\
d_t \geq 0 & (\forall t)
\end{array}
\tag{25}
$$

The objective function in eq. (25) represents the temporary worker cost. $A = \{\alpha_t, \, t = 1, 2, ..., T\}$ and d_t denote the hourly cost and the number of external workforce hours, respectively. N denotes the number of permanent staff-members. The planning horizon is denoted by T. It is assumed that the firm is capable of forecasting demand at times $t = 1, 2, ..., T$ and then to estimate the corresponding required capacity ($C = \{c_t, \, t = 1, 2, ..., T\}$) — see Table 2. — If demand cannot be satisfied by permanent staff capacity, temporary workers are hired. Working weeks differ based on the number of working hours (e.g., 35, 38, 40). K denotes the number of working weeks types (Table 2). h_k denotes the number of hours in each type of week. A fixed number of working weeks of each type can be performed by each staff member in the planning horizon T (such number is denoted by r_k). x_{kt} denotes the number of permanent workers assigned to a working week of type k at time t (see Table 2). The first constraint states that the firm's (offered) capacity must exceed (or be equal) to the required capacity at any t. The second concerns the number of workers to be assigned to each type of working week at any period t. The third constraint binds the number of weeks of a given type that can be performed by each worker during the planning horizon. The fourth and fifth constraints are technical.

Corominas et al (2004) evaluate the model on two scenarios. The output of interest in our analysis is the (minimum) cost generated by the optimal choice, here denoted by $f_* = f(\gamma)$. In the first scenario, f_* is equal to $f_*^0 = f(\gamma^0) = 896$. In the second scenario, we have $f_*^1 = f(\gamma^1) = 2282$. Hence, in the second scenario the firm would incur an extra cost of 1386.

A natural question that a decision-maker would like to answer, is, then: how can we explain this 155% jump? To provide a structured answer to the question, we apply the procedure of Section 4, together with the three Settings of Section 3. As suggested by step 2 in Table 3, we determine the factors (Definition 1). The scenarios are characterized by different required capacity (C), and extra-hour cost (A) profiles (Figure 1, upper chart) and annualized hours constrains (H) (Table 4). Thus, three factors are shifted across the scenarios, namely C, H and A. The parameter partition is, then given by

$$\underbrace{c_1 \quad c_2 \quad ... \quad c_T}_{C} \quad \underbrace{h_1 \quad h_2 \quad ... \quad h_k \quad r_1 \quad r_2 \quad ... \quad r_k}_{H} \quad \underbrace{\alpha_1 \quad \alpha_2 \quad ... \quad \alpha_T}_{A} \tag{26}$$

with $T = 46$ in both scenarios, and $k = 3$ and 4 in scenarios 0 and 1 respectively. The input data are displayed in Figure 1 and in Table 4. The numerical values for C and H are taken from Corominas et al (2004).

[Insert Figure 1 about here]

[Insert Table 4 about here]

In scenario 0, the required capacity curve (C, $C = \{c_t,\ t = 1, 2, ..., 46\}$) (see Figure 1 in Corominas et al (2004) from which is taken) is seasonal with a smooth behavior and one peak. In scenario 1, C follows a two-peak pattern (see Figure 2 in Corominas et al (2004)). The hourly cost curve ($A = \{\alpha_t,\ t = 1, 2, ..., 46\}$) assumes values between a minimum of 5/hour to a maximum of 10/hour with the two behaviors shown in Figure 1, bottom graphs. The H parameters vary from the three week-type frame of scenario 0 to the four week-type frame of scenario 1 (Table 4).

The second step is the estimation of the scenario FCSI's. Applying eq. (7), Δf_* is decomposed as follows:

$$\Delta f_* = \Delta_C f + \Delta_H f + \Delta_A f + \Delta_{C,H} f + \Delta_{C,A} f + \Delta_{A,H} f + \Delta_{C,H,A} f \tag{27}$$

In eq. (27), the first three terms represent the effects of the changes in C, H and A, when they vary individually. To estimate $\xi_C^1 = \Delta_C f$, $\xi_H^1 = \Delta_H f$ and $\xi_A^1 = \Delta_A f$ the model is run in three sub-scenarios, in which C, A and H are shifted one-at-a-time to the respective values of scenario 1 , while the other factors are kept at scenario 0 (Table 3, step 3). The second order terms $\Delta_{C,A}f$, $\Delta_{C,H}f$, and $\Delta_{A,H}f$ represent the marginal effects of the interactions (C - A), (C - H) and (A - H), respectively. They are estimated by re-running the model in three sub-scenarios, with the pairs of factors shifted to scenario 1, with the residual factor kept at the value of scenario 0 (Table 3, step 3). Finally, $\Delta_{C,A,H}f$ represents the residual effect of the interaction of the three factors. A total of 6 model runs is necessary for the scenario decomposition.

The numerical estimates of the FCSI's are reported in Figure 2.

[Insert Figure 2 about here]

Let us start with interpreting the results in the light of Setting 1 (Section 3, page 6). The first three columns of Figure 2 display ξ^1_C, ξ^1_H and ξ^1_A. As ξ^1_C is positive, the change in the required capacity curve from seasonal to peaked causes an increase in the total cost for the firm. On the other hand, the allocation of working hours on a 4-work-week-type basis rather than on a 3-work-week-type basis leads to a reduction in the total cost to the firm (in fact, $\xi^1_H = \Delta_H f$ is negative). ξ^1_A is negative, but it is much smaller than ξ^1_C, ξ^1_H. This has the following managerial interpretation. The shift in hourly cost profile from scenario 0 to scenario 1 reduces the total cost, but with a much lower individual effect than the shift in H. Let us then consider interaction effects (Setting 2). Interaction effects are reported in bars 4 to 6 of Figure 2. The most significant interaction is the one between C and A, with $\xi^2_{C,A} \simeq 500$. The magnitude of $\xi^2_{C,A}$ indicates that this interaction plays a relevant role, as it accounts for around 40% of the change. The positive sign of $\xi^2_{C,A}$ indicates that the interaction between C and A is, indeed, a cooperation, and amplifies their individual effects. In particular, it is worth noting that$A$ has no individual effect, and becomes relevant only in interaction with C. $\xi^2_{C,H}$ and $\xi^2_{A,H}$ are much smaller than $\xi^2_{C,A}$, i.e., the interactions between (C and H) and (A and H) have a low impact on Δf_*. Finally, $\xi^3_{C,H,A}$, the residual effect three-factor interaction, is negative and non-negligible.

Overall, Figure 2 shows that the model response cannot be interpreted as the superimposition of individual effects, but interactions play a relevant role. We recall that if an analyst were to use differential techniques to explain the change, she/he would not be able to estimate interaction effects (see also Borgonovo (2009)).

We turn to the identification of the key-drivers (Setting 3). Through step 4 in Table 3, one obtains the total order FCSI's reported in Figure 3.

[Insert Figure 3 about here]

Figure 3 shows that C is the key-driver of the scenario results, followed by H and A. The above discussion, however, points out that the relevance of A is not determined by its individual effect, but is due to its interaction with C; similarly, part of the relevance of C is due to its cooperation with A.

The above results deliver the following additional insights on "*the factors on which to focus managerial attention during implementation* (Eschenbach (1992))". The positive sign of ξ_C^T, and the negative sign of ξ_H^T indicate that a change in demand from seasonal to peaked is softened by a simultaneous change of work-hours allocation from the three-working-week type to the four–working-week type scheme. However, the capacity curve shift is amplified by the simultaneous change in extra-hous costs. The cooperation effect of these two factors causes an additional 40% increase in costs. We note that interaction effects between two factors γ_1 and γ_1 disappears as soon as one of them stays put — it is a consequence of eq. (8). — In this respect, management is probably more likely to have partial control over A (through negotiations) rather than over C (which is driven by external forces). Then, in the case of a seasonal-to-peaked demand shift, being able to hold extra-hour costs still would reduce losses considerably (40% in the present case). In summary, the analysis also reveals two keys in the hands of management to hedge potential losses provoked by a change in demand from seasonal to peaked. We finally note that these insights could not have been obtained by a purely qualitative observation of the scenario results.

6 Conclusions

Several new quantitative decision-support models have been developed to assist decision-making in the service industry sector. The models capture an increasing number of aspects of the managerial problems under analysis. However, the lack of a proper methodical approach to gaining managerial insights exposes analysts to the risk of partially exploiting the modelling efforts. A literature review has indicated scenario analysis as the most widely utilized method for corroborating and understanding service industry model results.

This work has proposed a new method to support the acquisition of managerial insights from scenario analysis. Central to the method is the integration of scenario analysis and the HDMR theory. The HDMR theory, in fact, allows to preserve the main features of scenario analysis: simultaneous and finite parameter variations and non-smooth model response. We have proven that the HDMR decomposition of model output in terms of factors preserves the structure of the HDMR decomposition with respect to exogenous variables. It has then been possible to introduce factor finite change sensitivity indices (FCSI). We have shown that the first order FCSI of a factor equals the sum of all FCSI's of the parameters contained in the factor. A closed-form expression for the relationship between higher order factor FCSI's and parameter FCSI's has also been derived. Knowledge of the factor FCSI's enables a scenario analyst to: *i*) apportion the change to its sources in an exact fashion; *ii*) quantify the effect of interactions; *iii*) identify the key drivers of the decision-making problem. Thus, scenario analysts are provided with a quantitative explanation of scenario analysis results, that reduces the reliance on purely qualitative statements. A 4-step estimation procedure has been introduced. Once the main scenarios have been developed, the procedure foresees to run the model on sub-scenarios selected according to the HDMR theory and enables the estimation of the factor FCSI's.

To demonstrate the method, a numerical case study has been discussed, namely, the scenario analysis of Corominas et al (2004)'s workforce allocation model. In Corominas et al (2004), the model is run on two scenarios. A 155% increase in minimum (optimal) cost is registered. By application of the procedure (Table 3), we have been able to identify the individual and cooperation effects of the factors. The required capacity curve has been identified as the most important factor, followed by the change in working week parameters and by extra hour costs. As far as interactions are concerned, the cooperation between required capacity and extra-hour costs leads to a 40% additional fall in total costs. Thus, hedging capacity changes with a proper extra-hour cost profile (i.e., being able to revert the sign of the interaction term) would be an important tool in the hands of management to smoothen potential losses.

A final remark. The proposed scenario decomposition method nests naturally into scenario analysis. Thus, it enables one to gain additional insights while preserving the rigor in scenario development and the methodological advances achieved by scenario literature. As this is the first work in this direction, the present paper also paves the way to further research, with special reference to applications in different industry sectors (Manufacturing, Finance, Inventory — see Borgonovo and Peccati (2008b)) and to problems involving the implementation of non-small sets of scenarios.

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Table 1: A synthetic overview on recently developed service industry decision-support models

Paper	**Model**	**Application**	**Sensitivity Analyses**
Duder and Rosenwein (2001)	Linear equation	Zero abandonments	None
Grossman and Brandeau (2002)	Non-linear programming	Pricing decisions	Sensitivity bounds on toll flexibility
Wu and Lin (2003)	Linear programming	Facility location	Scenario analysis
Corominas et al (2004)	Mixed Integer Programming	Workforce allocation	Scenario analysis
Corominas et al (2007a)	Mixed Integer Programming	Workforce allocation	Scenario analysis
Corominas et al (2007b)	Mixed Integer Programming	Workforce allocation	Scenario analysis
Hosanagar et al (2005)	Convex Programming	Resource allocation in caching services	Comparative statics
Ata and Shneorson (2006)	M/M/1	Dynamic control of a service system	Comparative statics
Hong (2007)	Nested Logit	Logistic service facility location	Robustness test
Debo et al (2008)	Queuing Equilibrium	Services with "credence" characteristics	Scenario analysis
Castillo et al (2009)	Free disposable Hull analysis	Workforce allocation with multiple-objectives	Scenario analysis
Beraldi and Bruni (2009)	Stochastic programming	Emergency service vehicle location	Scenario analysis

Table 2: Notation and Symbols used in this work

Symbol	**Meaning**
N	number of staff members
T	planning horizon (non-holidays weeks)
$C = \{c_t\}$	capacity required at t to satisfy demand; $t = 1, 2, ..., T$
K	number of types of working weeks
h_k	number of hours corresponding to week of type k
r_k	number of working weeks of type k to be performed by each staff member
α_t	hourly cost of external workforce at t
d_t	number of hours provided by temporary workers in period t
y	model output (decision support criterion)
θ_l	Parameter
γ_l	A factor (parameter group) [eq. (5)]
$f(\cdot)$	Model output relationship to the parameters or parameter groups
$(\gamma^1_{i_1}, ..., \gamma^1_{i_k}; \gamma^0_{(-)})$	factors $\gamma_{i_1}, ..., \gamma_{i_k}$ are shifted at scenario 1, the remainder are kept at scenario 0.
$\Delta_{i_1,i_2,...,i_k} f$	Orthogonalized change in f due to the changes in $\gamma_{i_1}, \gamma_{i_2}, ..., \gamma_{i_k}$ [eq. ()]
$\xi^k_{i_1,i_2,...,i_k}$	Finite change sensitivity index (FCSI) of order k for factors [eq.(10)]
ξ^1_l	First order FCSI for factor γ_l [eq. (9)]
ξ^T_l	Total order FCSI for factor γ_l [eq. (11)]
$\varphi^k_{i_1,i_2,...,i_k}$	FCSI of order k for parameters
C, A, H	Groups of parameters in the scenario analysis of the model of Corominas et al (2004) [eq. ()]

Table 3: Steps for scenario decomposition and estimation of the FCSI

Step	**Action**	**Quantity Estimated**
1	Develop scenarios and run the model accordingly	$f(\gamma^0), f(\gamma^1), ...$
2	Determine the factors	$\gamma = (\gamma_1, \gamma_2, ..., \gamma_Q)$ [eqs. (5) and (6)]
3	For $k = 1, 2, ...Q$ 2.a For $l_1 < l_2 < ... < l_k$ compute $f(\gamma^1_{l_1}, \gamma^1_{l_2}, ..., \gamma^1_{l_k}; \gamma^0_{(-)}) - f(\gamma^0)$ 2.b If $k > 1$, subtract all effects of order $s = 1, 2, ..., k-1$	$\xi_{l_1, l_2, ..., l_k}$
4	For $l = 1, 2, ..., Q$ sum all effects related to factor γ_l	ξ^T_l

Table 4: Working-week types (K), hours (h) and allocation (r) in Scenario 1

	k	$\mathbf{h_k}$	$\mathbf{r_k}$		**k**	$\mathbf{h_k}$	$\mathbf{r_k}$
	1	30	11		1	30	8
Scenario 1	2	37.5	24	**Scenario 2**	2	35	15
	3	45	11		3	40	15
					4	45	8

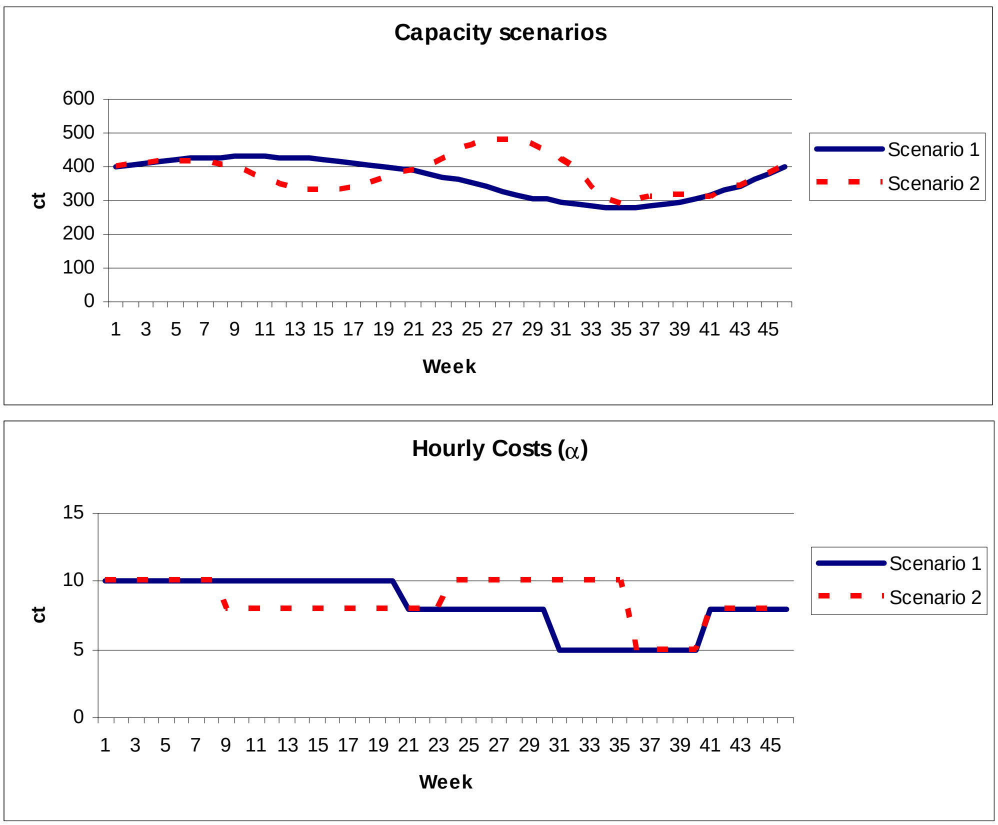

Figure 1: Capacity ($C = \{c_t\}$) and external workers hourly costs ($A = \{\alpha_t\}$) in Scenarios 1 and 2.

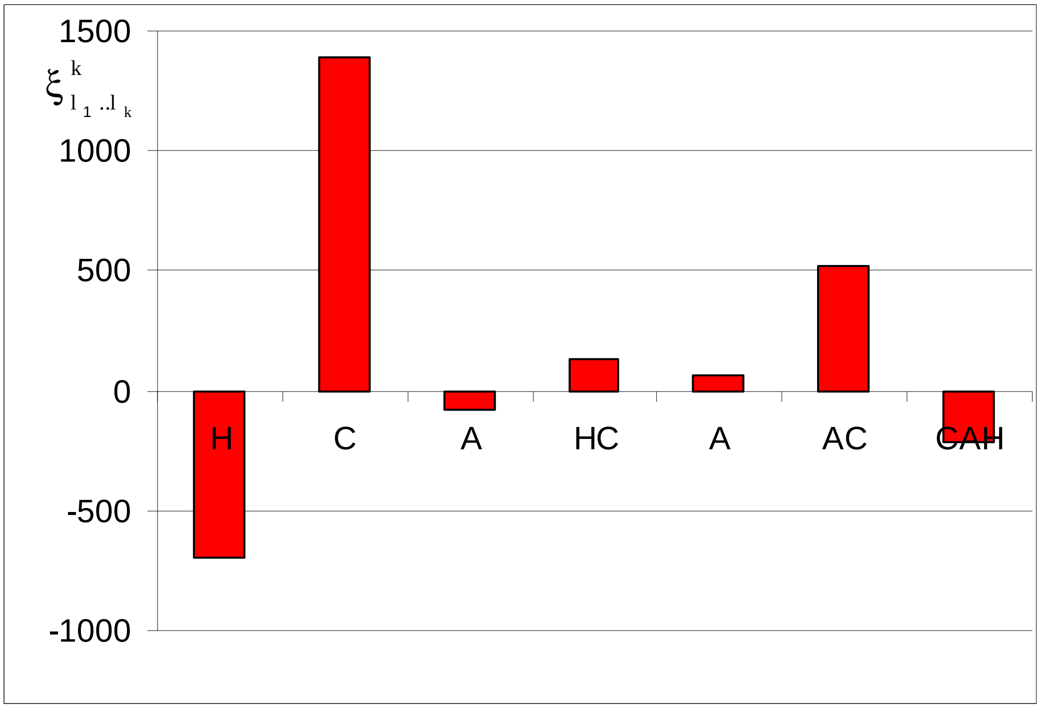

Figure 2: FCSI of first order ($\xi^1_C, \xi^1_H, \xi^1_A$; bars 1 to 3), second order ($\xi^2_{C,H}, \xi^2_{C,A}, \xi^2_{A,H}$, bars 4 to 6) and third order ($\xi^3_{C,H,A,}$) in the scenario analysis of Corominas et al (2004)'s model.

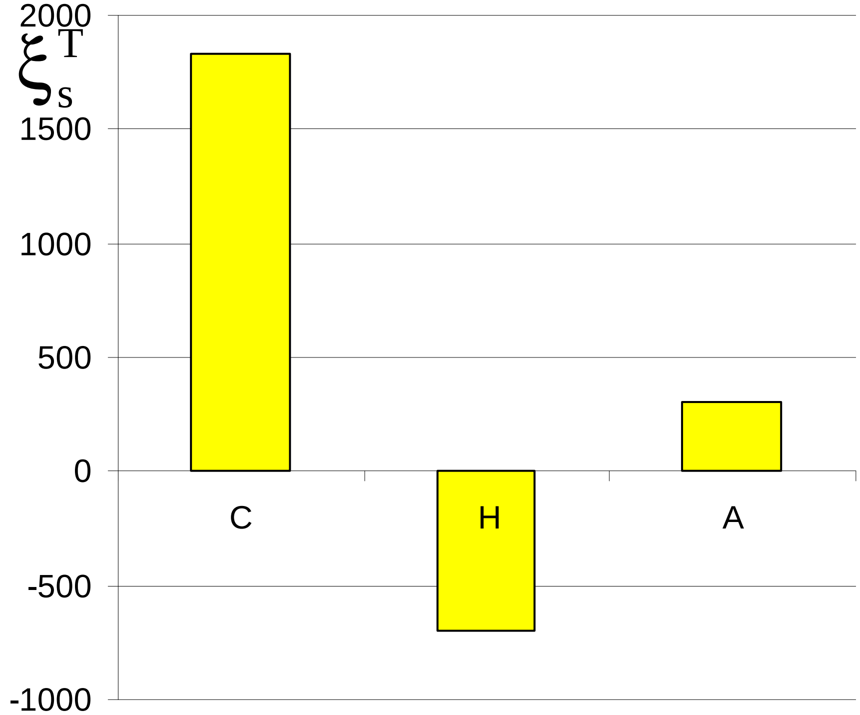

Figure 3: Total effects of factors C, H and A (ξ_C^T, ξ_H^T, ξ_A^T) in the scenario analysis of Corominas et al (2004)'s model.