Macroeconomics III - Ph.D.

Incomplete contracts and collateral constraints: amplification vs persistence

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Financial frictions based on incomplete contracts

1. **Limited commitment:** based on the idea that borrower **cannot precommit** her human capital $\rightarrow$Lender cannot force borrower to repay debt

- Hence in case of default borrowers will never repay more than the value of their available assets

2. Alternative: limited **enforcement** $\rightarrow$ Lender can only recover a **fraction** of the value of collateral

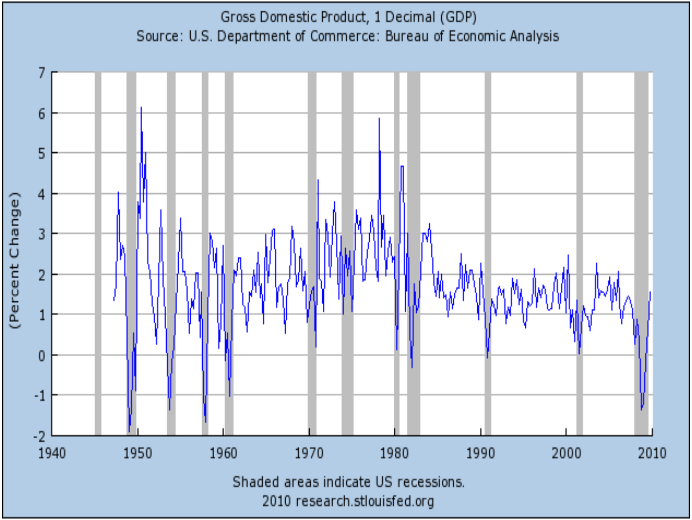
Baseline model with collateral constraint

- Kiyotaki and Moore (1997): entrepreneur use **durable asset** both as (i) **productive** input and (ii) **collateral** for borrowing
- Illustrate role of credit frictions in generating

1. **persistence** of shocks
2. **amplification** of shocks

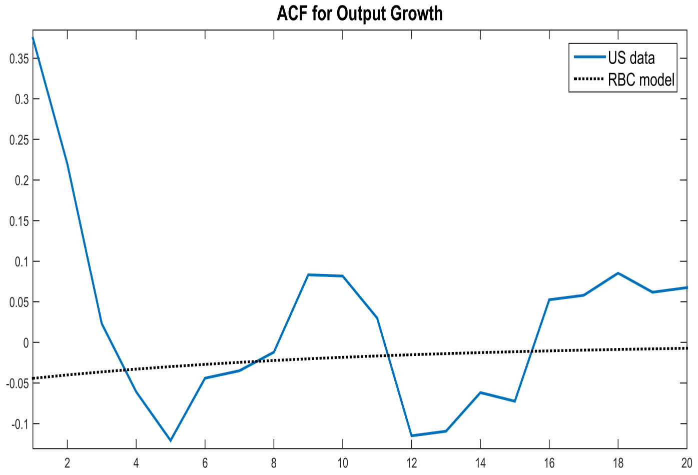
- Persistence and amplification reinforce each other

- In the data output growth is strongly serially correlated

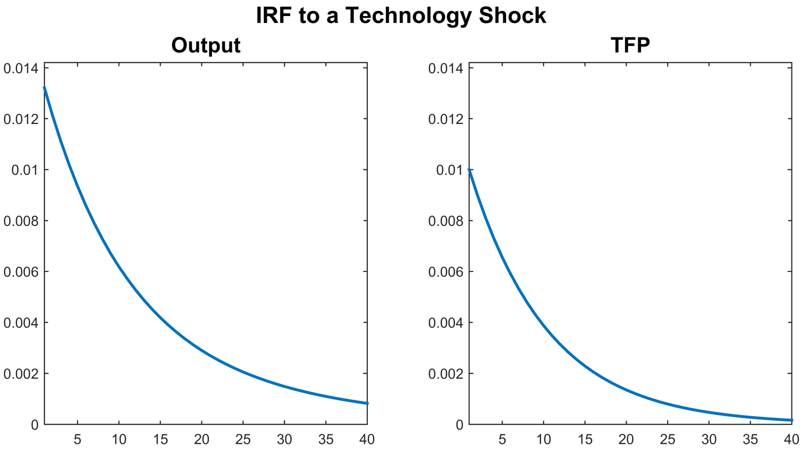


Gross Domestic Product, 1 Decimal (GDP)
Source: U.S. Department of Commerce: Bureau of Economic Analysis
(Percent Change)
7
6
5
4
3
2
1
0
-1
-2
1940
1950
1960
1970
1980
1990
2000
2010
Shaded areas indicate US recessions.
2010 research.stlouisfed.org

Autocorrelation function of GDP growth



Theoretical impulse responses: RBC model



- Criticism : RBC model has **weak propagation mechanism** (Cogley and Nason, 1995)

Kiyotaki and Moore (1997): Amplification + Persistence

Key ingredients

1. Credit constraints + balance sheet effect
2. Forward looking asset prices
3. Two goods: consumption + capital (land) in **fixed supply** (no depreciation) $\longrightarrow$ Asset price is the *relative* price of capital good

Basic intuition: static and intertemporal multiplier

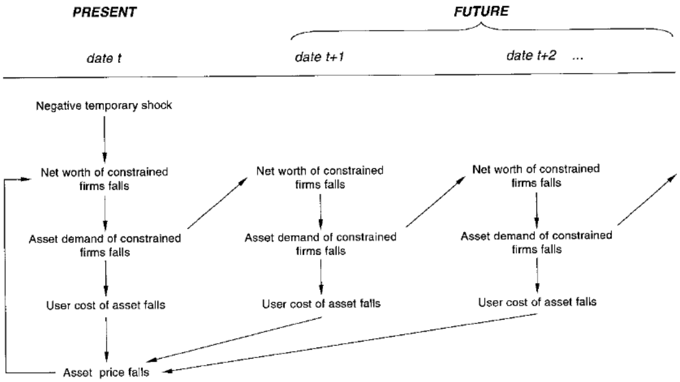


FIG. 1

Basic intuition: static and intertemporal multiplier

- Constrained firms

- (*time t*) $\downarrow$ Productivity $\longrightarrow \downarrow$ NW_t $\longrightarrow \downarrow$ borrowing (**binding** constraint) $\longrightarrow \downarrow$ land demand (t)
- (*time t+1*) Land is used in t+1 production $\longrightarrow \downarrow$ NW_{t+1} $\longrightarrow$ borrowing (t+1)$\longrightarrow \downarrow$ land demand (t+1)...

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- Unconstrained firms

↑ Demand of land (since total supply fixed) → ↓ user cost **in each period** (anticipated effect)

$$\underbrace{z_t}_{\text{user cost}} = q_t - \frac{q_{t+1}}{R}$$

→Integrating forward:

$$\underbrace{q_t}_{\substack{\text{asset}\\\text{price}}} = \underbrace{\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j z_{t+j}}_{\substack{\text{PDV of}\\\text{user costs}}}$$

Basic intuition: static and intertemporal multiplier (con't)

- **Large fall** in q_t due anticipated fall in user cost in **future** periods $\longrightarrow$ Large fall in constrained firms' net worth and capital demand
- Notice: there is amplification because of persistence

Entrepreneurs ("Farmers")

- Produce **tradable** $(ak_{e,t})$ and **non-tradable** $(ck_{e,t})$ output

$$y_{e,t} = \underbrace{(a + c)k_{e,t-1}}_{\substack{\text{linear} \\ \text{prod. function}}}$$

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- Flow of funds

$$c_{e,t} + \underbrace{q_t(k_{e,t} - k_{e,t-1})}_{\substack{\text{purchase} \\ \text{new land}}} \leq \underbrace{(a+c)k_{e,t-1}}_{\text{output}} - \underbrace{Rb_{t-1}}_{\substack{\text{payment} \\ \text{old debt}}} + \underbrace{b_t}_{\text{new debt}}$$

- Consumption

$$c_{e,t} \geq ck_{e,t-1}$$

- *If* E. decide to consume only non-tradable output $\rightarrow$ use proceeds of tradable output (+ new loans - repayment old loans) to purchase more K (land)

Problem of Entrepreneurs

$$\max \sum_{t=0}^{\infty} \left(\beta_e^t\right) c_{e,t}$$

$$c_{e,t} + q_t(k_{e,t} - k_{e,t-1}) \leq (a + c)k_{e,t-1} - Rb_{t-1} + b_t \tag{1}$$

$$Rb_t \leq q_{t+1}k_{e,t} \tag{2}$$

$$c_{e,t} \geq ck_{e,t-1} \tag{3}$$

- Need to show that both (2) and (3) **hold with equality**

Proving that E. will borrow up to maximum leverage

- Guess and verify
- Suppose both (2) and (3) hold with **equality**. Substituting into (1)

$$q_t\left(k_{e,t}-k_{e,t-1}\right)=ak_{e,t-1}+\underbrace{\frac{q_{t+1}k_{e,t}}{R}}_{b_t}-Rb_{t-1}$$

- Rearranging:

$$k_{e,t}=\frac{\overbrace{(a+q_t)k_{e,t-1}-Rb_{t-1}}^{\text{net worth}}}{\underbrace{q_t}_{\substack{\text{cost of}\\ \text{land}}}-\underbrace{(q_{t+1}/R)}_{\substack{\text{collateralized}\\ \text{value 1 unit}\\ \text{of land}}}}\equiv\frac{nw_t}{z_t}$$

Remarks 1

- Equation

$$k_{e,t} = \frac{(a + q_t)k_{e,t-1} - Rb_{t-1}}{z_t} \equiv \frac{nw_t}{z_t}$$

$\longrightarrow$Shows that demand of capital depends on net worth.

- Notice that this holds conditional on the **borrowing constraint** being **binding**

Remarks 2

- Rewrite previous equation

$$\underbrace{q_t k_{e,t}}_{\substack{\text{land}\\ \text{demand}}} = nw_t + \underbrace{\frac{q_{t+1}}{R} k_{e,t}}_{\substack{\text{amount borrowed}\\ \text{against value}\\ \text{of land}}}$$

- Notice z_t is usually defined as the **user cost** of land. Or alternatively the difference between the current price of land and the amount that can be borrowed against it
- If (2) binds also at $t-1 \longrightarrow Rb_{t-1} = q_t k_{e,t-1} \longrightarrow$

$$k_{e,t} = \frac{a k_{e,t-1}}{z_t} \tag{4}$$

- In **steady state** $\longrightarrow k_{e,t} = k_{e,t-1} = k_e \longrightarrow$

$$a = z \tag{5}$$

Equilibrium with binding constraint

- We need to show that, after repaying debt, the entrepreneur will use all units of **tradable** output to purchase new land (i.e., to **"invest"**)
- E. borrows up to the **maximum** borrowing limit

Two alternative uses of 1 unit of tradable output

1. Consume $\rightarrow$ Path of consumption $\{1, 0, 0, 0, ..\}$.

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- Will generate non-tradable output $c \cdot (1/z_t)$ and tradable output $a \cdot (1/z_t)$ in t+1
- In turn, (a/z_t) units of tradable output can be used to purchase new land in t+2 (and borrow) $\rightarrow$ Additional non-tradable output (c/z_{t+1}), etc.
- The path of consumption will be:

$$\left\{ \underbrace{0}_{t} \quad \underbrace{\frac{c}{z_t}}_{t+1}, \quad \underbrace{\frac{a}{z_t} \cdot \frac{c}{z_{t+1}}}_{t+2}, \quad \frac{a}{z_t}\frac{a}{z_{t+1}}\frac{c}{z_{t+2}}.. \right\}$$

- Using (5), the NPV reads:

$$0 + \beta_e \frac{c}{a} + \beta_e^2 \frac{c}{z} + \beta_e^3 \frac{c}{z} + .. = \frac{\beta_e}{1 - \beta_e} \frac{c}{a}$$

- Hence for both (2) and (3) to hold with equality in equilibrium it must hold:

$$\underbrace{\frac{\beta_e}{1-\beta_e}\frac{c}{a}}_{\substack{\text{NPV of}\\ \text{investing 1 unit}\\ \text{of tradable output}}} > \underbrace{1}_{\substack{\text{NPV of}\\ \text{consuming 1 unit}\\ \text{of tradable}}} \tag{6}$$

- K-M assume this condition holds to insure that borrowing constraint **always binding** + E. will devote all tradable output to investment in land (and consume non-tradable output only)
- Rewrite equivalently

$$\frac{c}{a} > \frac{1-\beta_e}{\beta_e} \tag{7}$$

$$\rightarrow \frac{a+c}{a} > \frac{1}{\beta_e} \tag{8}$$

Lenders (savers)

- Use land to produce output $\longrightarrow$ Concave production function

$$y_{s,t} = G\underbrace{\left(\overline{k} - k_{e,t-1}\right)}_{\substack{\text{land employed}\\ \text{by savers}}} \qquad G^{'} \geq 0;\ G^{''} \leq 0$$

Efficient allocation (Social Planner)

- Planner allocates land across Entrepreneurs and Savers in order to equalize the **marginal product** across the two uses

$$G'(\overline{k} - \underbrace{k^*_{e,t}}_{\substack{\text{socially} \\ \text{efficient level} \\ \text{of land allocated} \\ \text{to E.}}}) = a + c \quad \text{for all } t$$

Market equilibrium

- Savers solve

$$\max \sum_{t=0}^{\infty} \left(\beta_s^t\right) c_{s,t} \qquad \underbrace{\beta_s > \beta_e}_{\text{Savers are more patient}}$$

$$c_{s,t} + q_t\left(k_{s,t} - k_{s,t-1}\right) \leq G\left(k_{s,t-1}\right) - Rb_{t-1} + b_t$$

- Lagrangian

$$\sum_{t=0}^{\infty} \left(\beta_s^t\right) c_{s,t}$$
$$-\beta_s^t \lambda_{s,t} \left\{c_{s,t} + q_t(k_{s,t} - k_{s,t-1}) - G(k_{s,t-1}) + Rb_{t-1} - b_t\right\}$$

- First order conditions

$$\beta_s^t - \beta_s^t \lambda_{s,t} = 0 \ \rightarrow \lambda_{s,t} = 1$$

$$-\beta_s^t \lambda_{s,t} q_t + \beta_s^{t+1} \lambda_{s,t+1} (q_{t+1} + G^{'}(k_{s,t})) = 0$$

$$-\beta_s^{t+1} \lambda_{s,t+1} R + \beta_s^t \lambda_{s,t} = 0$$

- Combining

$$\beta_s = R^{-1} \tag{9}$$

$\rightarrow$Notice: Savers' discount factor pins down real interest rate

$$\underbrace{q_t}_{\substack{\text{marg cost} \\ \text{1 unit of land}}} = \underbrace{\beta_s (q_{t+1} + G^{'}(k_{s,t}))}_{\substack{\text{marginal benefit} \\ \text{1 unit of land}}}$$

- Rewrite

$$\underbrace{q_t - \frac{q_{t+1}}{R}}_{z_t} = \underbrace{\left(\frac{G'(k_{s,t})}{R}\right)}_{\substack{\text{discounted} \\ \text{marg. product} \\ \text{of land}}} \tag{10}$$

→Savers equate user cost of land to discounted marginal product

- Let's go back to the Entrepreneurs
- Assuming $\frac{\beta_e}{1-\beta_e}\frac{c}{a} > 1$ holds (borrowing constraint binding)
- Recall that we have

$$k_{e,t} = \frac{(a+q_t)k_{e,t-1} - Rb_{t-1}}{q_t - (q_{t+1}/R)}$$

- Can write

$$\begin{aligned} z_t k_{e,t} &= (a+q_t)k_{e,t-1} - \underbrace{Rb_{t-1}}_{\substack{=q_t k_{e,t-1} \\ \text{since borr.constr.} \\ \text{binding}}} \\ &= (a+q_t)k_{e,t-1} - q_t k_{e,t-1} \end{aligned}$$

- Using $z_t = G^{'}(k_{s,t})/R$, obtain 1^{st} order difference equation in $k_{e,t}$

$$\frac{G^{'}(\bar{k} - k_{e,t})}{R} k_{e,t} = a k_{e,t-1} \tag{11}$$

- Following assumptions insure unique and stable solution to (11)

1. $G^{'}(k - k_{e,t})$ is monotonically increasing in k
2. $G^{'}(\bar{k}) < a$ and $G^{'}(0) > a$

Steady state and (in)efficiency

- Evaluating (11) at the s.s and simplifying

$$G^{'}(\overline{k} - k_e) = Ra \tag{12}$$

- Compare to social planner efficiency condition (evaluated at ss)

$$G^{'}(\overline{k} - k_e^*) = a + c$$

- Rewrite (8), using $\beta_s = 1/R > \beta_e$

$$\frac{a + c}{a} > \frac{1}{\beta_e} > R \tag{13}$$

- Hence

$$a + c > Ra$$

which implies

$$G^{'}(\overline{k} - k_e^*) > G^{'}(\overline{k} - k_e)$$

$$k_e^* > k_e$$

- Hence the market equilibrium is characterized by a **suboptimal** amount of land allocated to Entrepreneurs $\longrightarrow$ Output is too low $\longrightarrow$ Key implication of **financial frictions**

Asset prices and demand for capital

- Demand for land (with binding borrowing contstraint)

$$
\begin{aligned}
k_{e,t} &= \frac{q_t k_{e,t-1}}{z_t} + \frac{a k_{e,t-1} - R b_{t-1}}{z_t} \\
&= \left(\frac{k_{e,t-1}}{1 - \beta_s \frac{q_{t+1}}{q_t}} \right) + \frac{a k_{e,t-1} - R b_{t-1}}{q_t - \beta_s q_{t+1}}
\end{aligned}
$$

- Consider a permanent fall in asset prices ($\downarrow \widehat{q}_t \downarrow \widehat{q}_{t+1}$). First term is unaltered
- Log-linearizing around ss (considering only second term)

$$
\widehat{k}_{e,t} = \frac{q \overbrace{(Rb - a k_e)}^{>0 \text{ at ss}}}{z} \left(\widehat{q}_t - \beta_s \widehat{q}_{t+1} \right)
$$

Demand for capital falls as asset prices fall permanently ($\beta_s < 1 \longrightarrow$ Effect of current fall in q_t prevails)

Summary of key points in K-M

1. Output and capital are **inefficiently low** because of borrowing frictions. Notice that this holds despite the fact that E. have access to a more productive technology than Savers. **Financial frictions** do not allow resources to be channeled to the most productive segment of the economy

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2. Ability of E. to obtain credit is limited by **collateral**. A binding borrowing constraint makes the demand of capital (land) dependent on **net worth**
3. Land purchases by E. depend on asset prices. A permanent fall in asset prices **reduces** the demand for land (due to the negative effect on net worth $\rightarrow$ reduces ability to borrow)

Dynamics

Effects of productivity shocks: amplification and persistence

- Aggregate unexpected rise in productivity of both E. and S
- Start from equilibrium conditions

$$k_{e,t} = \frac{(a + q_t)k_{e,t-1} - Rb_{t-1}}{z_t} \tag{14}$$

$$z_t = \frac{G'(\bar{k} - k_{e,t})}{R} = z(k_{e,t}) \tag{15}$$

- Assume productivity time varying
- Combining can rewrite:

$$z(k_{e,t})k_{e,t} = (\underbrace{a_t}_{\substack{\text{time}\\\text{varying}}} + q_t)k_{e,t-1} - Rb_{t-1}$$

- Two channels of rise in productivity

1. Tradable output increases (=rise in income) $\rightarrow$Increase demand for land
2. Asset price rises $\rightarrow$Net worth rises $\rightarrow$ Further increase in demand for land

- Rewrite

$$z(e^{\log k_{e,t}})e^{\log k_{e,t}} = (a+q_t)e^{\log k_{e,t-1}} - Re^{\log b_{t-1}}$$

- Log-linearizing LHS (using $z = a$ in ss)

$$\begin{aligned} z(e^{\log k_{e,t}})e^{\log k_{e,t}} &\simeq z(k_e)k_e + \left[z^{'}(k_e)k_e^2 + ak_e\right]\widehat{k}_{e,t} \\ &= ak_e + ak_e\left(1 + \underbrace{\frac{z^{'}(k_e)k_e}{a}}_{\substack{\text{elasticity of} \\ \text{user cost} \\ \text{wrt to K demand}}}\right)\widehat{k}_{e,t} \end{aligned}$$

- Now turn to RHS

$$(a_t + q_t)k_{e,t-1} - Rb_{t-1} \simeq ak_e\widehat{a}_t + \left[k_e\left(a + \frac{Ra}{R-1}\right)\right]\underbrace{\widehat{k}_{e,t-1}}_{=0} + \left[\frac{Ra}{R-1}k_e\right]\widehat{q}_t$$

where we have used $q = Ra/(R-1)$ in steady state

- Equating

$$(1+\zeta)\widehat{k}_{e,t} = \underbrace{\widehat{a}_t}_{\substack{\text{direct}\\ \text{impact of}\\ \text{shock at}\\ \text{time t}}} + \underbrace{\frac{R}{R-1}}_{\substack{\text{amplification}\\ >>1}}\widehat{q}_t$$

Amplification at time t

- The **time-t** effect on capital demand goes beyond the direct increase in productivity
- Capital gain effect on Entrepren. Land is **scaled up** by a factor $R/(R-1)$ due to the possibility of leveraging up their net worth
- Amplification can be large $\rightarrow$ If $R = 1.05 \rightarrow R/(R-1) = 21$

Persistence beyond time t

- Recall that we have $k_{e,t} = \frac{(a+q_t)k_{e,t-1} - Rb_{t-1}}{z_t}$
- If borrowing constraint binds between any two periods (t, t+1) $\longrightarrow$ (using also (15))

$$z(k_{e,t+1})\ k_{e,t+1} = ak_{e,t}$$

- Log-linearizing

$$z(e^{\log k_{e,t+1}})e^{\log k_{e,t+1}} = ae^{\log k_{e,t}}$$

$$\left[z^{'}(k_e)k_e^2 + ak_e\right]\widehat{k}_{e,t+1} = ak_e\widehat{k}_{e,t}$$

$$\underbrace{\left[1 + \frac{z^{'}(k_e)k_e}{a}\right]}_{1+\zeta}\widehat{k}_{e,t+1} = \widehat{k}_{e,t}$$

$\longrightarrow \zeta > 0 \longrightarrow$ Effect **persistent** beyond time t

Persistence implies amplification

- Recall:

$$q_t = z(k_{e,t}) + \frac{1}{R} q_{t+1}$$

$\longrightarrow$

$$q_t = \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j z(k_{e,t+j})$$

$\longrightarrow$ Response of current asset price depends on **current and future** land purchases

- Log-linearizing

$$\widehat{q}_t = \zeta \frac{R-1}{R} \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j \underbrace{\widehat{k}_{e,t+j}}_{(1+\zeta)^{-1}\widehat{k}_{e,t}}$$

$$= \left[\zeta \frac{R-1}{R} \frac{1}{1-\frac{1}{R(1+\zeta)}}\right] \widehat{k}_{e,t}$$

- To be combined with:

$$(1+\zeta)\widehat{k}_{e,t} = \widehat{a}_t + \frac{R}{R-1}\widehat{q}_t$$

Reduced form solution

$$\widehat{q}_t = \zeta \widehat{a}_t$$

$$\widehat{k}_{e,t} = \underbrace{\frac{1}{1+\zeta}\left(1+\zeta\frac{R}{R-1}\right)}_{>>1}\widehat{a}_t \tag{16}$$

→Effect on land purchase at time t **can far exceed** the initial impulse in productivity

Static vs dynamic multiplier

- Recall $\widehat{q}_t = \zeta \frac{R-1}{R} \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j \widehat{k}_{e,t+j}$
- Suppose **shut off persistence**: asset price $\widehat{q}_t$ depends only on land purchase **at time t**

$$\widehat{q}_t = \zeta \frac{R-1}{R} \widehat{k}_{e,t} + \text{ const}$$

- Recall that at time t: $(1+\zeta)\widehat{k}_{e,t} = \widehat{a}_t + \frac{R}{R-1}\widehat{q}_t \rightarrow$ Obtain

$$\widehat{k}_{e,t} = \widehat{a}_t$$

$\rightarrow$

$$\widehat{q}_t = \underbrace{\zeta \frac{R-1}{R}}_{\substack{\text{static} \\ \text{multiplier of q}}} \widehat{a}_t$$

Static vs dynamic mulitpliers

$$\underbrace{\zeta\frac{R-1}{R}}_{\substack{\text{static}\\ \text{multiplier of } q}} < \underbrace{\zeta}_{\substack{\text{dynamic}\\ \text{multiplier of q}}}$$

$$\underbrace{1}_{\substack{\text{static}\\ \text{multiplier of } \mathrm{k}_e}} < \underbrace{\frac{1}{1+\zeta}\left(1+\zeta\frac{R}{R-1}\right)}_{\substack{\text{dynamic}\\ \text{multiplier of } k_e}}$$

Issues

1. Shocks unexpected: why don't Entrepreneurs **insure**?
2. Is amplification a **general** result? Cordoba and Ripoll (2004): amplification small for standard preferences.
3. Yet what about **other shocks** (different from productivity)?
4. No uncertainty + constraint always binding. No role for precautionary motive→Dampen motive for accumulation of debt