Macroeconomics III- Ph.D.
Prof. Tommaso Monacelli
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Problem Set 1

due April 13, 2016 in class

Consider an economy with two agents, households (who are unproductive) and entrepreneurs. The *households* solve:

$$\max \sum_{t=0}^{\infty} \gamma^t (C_{h,t} + \eta K_{h,t})$$

$$\underbrace{C_{h,t}}_{\text{consumption}} + \underbrace{q_t \left(K_{h,t} - K_{h,t-1}\right)}_{\text{durable investment}} + \underbrace{R_{t-1} B_{h,t-1}}_{\text{interest on loan contract signed in t-1}} \leq \underbrace{B_{h,t}}_{\text{new debt}}$$

where q_t is the price of a durable good (e.g, "land") in units of consumption good. The durable good is in fixed supply (normalized to 1).

1. Derive FOCs of the household's problem. Show that they imply that the real interest rate R_t is constant and that the price of the durable asset K is also constant.

The second set of agents, the *entrepreneurs*, have a technology for the production of a homogeneous final good which employs land:

$$Y_t = A_t K_{e,t-1}^{\alpha}$$

They face a constraint on borrowing:

$$R_t B_{e,t} \leq (1 - \chi) q_{t+1} K_{e,t} \tag{1}$$

At any time t, the amount $R_t B_{e,t}$ that the entrepreneur agrees to repay in t+1 is tied to the future value of the durable asset $q_{t+1} K_{e,t}$.

The entrepreneurs solve:

$$\max \sum_{t=0}^{\infty} \beta^t \log C_{e,t} \qquad \beta < \gamma$$

subject to the budget constraint:

$$C_{e,t} + q_t \left(K_{e,t} - K_{e,t-1}\right) + R_{t-1} B_{e,t-1} \leq B_{e,t} + Y_t$$

2. Let ψ_t be the multiplier on (1). Derive the FOCs. Explain in detail the intuition for the FOC on K_e. Compare the two cases: $\psi_t = 0$ vs $\psi_t > 0$.

3. Write down the equilibrium conditions in the consumption and durable good market, and the one in the credit market

4. Use the steady state version of the entrepreneurs' Euler condition to derive the condition under which the borrowing constraint is binding in the steady state. Provide an intuition.

5. Assume that the borrowing constraint is binding. Show that in this case the equilibrium can be described in terms of two equations in the two endogenous variables $\{C_{e,t}, K_{e,t}\}$, with $\{A_t\}$ exogenous. What are the endogenous predetermined and the endogenous predetermined variables? What is the state space of the model?

6. Assume, for simplicity, that $\chi = 0$. Guess a solution of the form:

$$C_{e,t} = (1-d) Y_t = (1-d) A_t K_{e,t-1}^{\alpha}$$

where d is a constant to be determined. Show that the equilibrium process for $log(Y_t)$ can be written:

$$\log\left(Y_{t+1}\right) = \phi_1 + \phi_2 \log\left(Y_t\right) + \log A_{t+1}$$

with ϕ_1 and ϕ_2 are constant that need to be determined. Provide an intuition for this result.

7. What does this result imply for the persistence properties of the model? Compare with the case in which the borrowing constraint is *not* binding.