**Macroeconomics III- Ph.D.**
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# Problem Set 2: Financial Shocks

*due April 20, 2016*

Note: you should also hand in the Dynare code used to answer question 7

Consider a small open endowment economy inhabited by a representative agent

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (\log c_t + \psi \log k_t), \quad \psi > 0, \quad \beta \in [0,1] \tag{1}$$

s.t. the budget constraint:

$$c_t + q_t k_t + \underbrace{R_{t-1} b_{t-1}}_{\substack{\text{service} \\ \text{cost of debt}}} \leq \underbrace{y_t}_{\text{endowment}} + q_t k_{t-1} + b_t$$

and borrowing constraint:

$$b_t \leq \chi_t q_t k_t \tag{2}$$

where $k_t$ is end-of-period t holdings of a durable asset (e.g., housing) which yields direct utility, $b_t$ is borrowing at time t at the world *gross* real interest rate $R_t$ (which is exogenous), and $\chi_t$ is *loan-to-value exogenous* process (our measure of a "financial shock"). The durable asset is in fixed supply $\overline{k} = 1$. (Note: it is without loss of generality that $q_t$, rather than $q_{t+1}$, is featured on the RHS of (2)).

1. Derive FOCs of this problem. Comment especially on the efficiency condition for the durable asset

2. Derive the equilibrium when the borrowing constraint is not binding (hint: it should feature 4 equations in 4 endogenous variables). What is the effect of a negative financial shock (i..e, a reduction in $\chi_t$) in this case?

3. Derive the equilibrium with a *binding* borrowing constraint ((hint: it should feature 5 equations in 5 endogenous variables)

4. Describe the deterministic steady state. Derive the conditions that insure that the borrowing constraint is binding. What is the steady state effect on consumption, borrowing and asset price of a fall in $\chi$? Derive analytically.

5. Characterize the equilibrium dynamics under the assumption that shocks are sufficiently small so that in the neighborhood of the steady state the borrowing constraint can be assumed to be binding. For simplicity, begin by assuming that utility is *linear* in consumption. Take a first order Taylor expansion around the steady state.

6. Assume that the LTV parameter follows the stochastic process (in percent deviations from steady state):

$$\widehat{\chi}_t = (1-\rho)\chi + \rho\widehat{\chi}_{t-1} + \varepsilon_t$$

Characterize the effects on consumption and the price of the asset of a contraction in $\widehat{\chi}_t$.

7. Assume general concave preferences of the form given in (1). Assume that $y = 1$ in the steady state, and choose the following values for the parameters:

| $\chi$ | $R$ | $\beta$ | $\rho$ |
|---|---|---|---|
| 0.8 | 0.01 | 0.96 | 0.98 |

Important: choose the value of $\psi$ such that $q = 1$ holds in the steady state.

- Write a Dynare code to solve the model and generate impulse responses of consumption, housing, asset price and debt to a negative financial shock (under the assumption that the borrowing constraint is binding). Compare the case of endogenous borrowing limit:

$$b_t = q_t \overline{\chi}_t$$

with the one in which the borrowing limit does not depend on the asset price:

$$b_t = \overline{\chi}_t$$

- Which version of the model generates amplification? Explain.