Macroeconomics III

Spring 2016

A Cashless Economy with Imperfect Competition and Sticky Prices

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- Cashless Economy

- Firms have **market power** in setting power

- Goods prices: **flexible** vs. **sticky (**predetermined or staggered**)**

- Market structure

(i) Competitive producer of homogenous final good

(ii) Many monopolistic producers of differentiated intermediate goods

- Producers of homogenous final good Y: perfect competition

- Production function

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \varepsilon > 1 \tag{1}$$

- Problem: choose $Y_t(i)$, Y_t

$$\max \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

with P_t and $P_t(i)$ given

- Rewrite

$$P_t \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di$$

- FOC wrt to $Y_t(i)$:

$$\frac{\varepsilon}{\varepsilon - 1} P_t \frac{Y_t}{Y_t^{\frac{\varepsilon-1}{\varepsilon}}} \left(\frac{\varepsilon - 1}{\varepsilon} \right) Y_t(i)^{-\frac{1}{\varepsilon}} = P_t(i)$$

- Rearranging $\longrightarrow$ Demand function for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

- Derive aggregate **price level**

- Under **zero profits**:

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di$$

$$P_t Y_t = \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t di$$

$$P_t = P_t^{\varepsilon} \int_0^1 P_t(i)^{1-\varepsilon} di$$

- Obtain

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

- Households: Intertemporal Problem with Complete Markets

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \right\} \tag{2}$$

$$\underbrace{P_t C_t}_{\substack{\text{purchase} \\ \text{final good}}} + \mathbb{E}_t \left\{ Q_{t,t+1} B_{t+1} \right\} \leq W_t N_t + T_t + B_t + \underbrace{\int_0^1 \Gamma_t(i)}_{\text{profits of int.firms}} \tag{3}$$

$\longrightarrow$Usual FOCs

$$U_{c,t} = P_t \lambda_t \tag{4}$$

$$\lambda_t W_t = -U_{n,t} \tag{5}$$

$$Q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{6}$$

- Producer of **intermediate good i**

- Production function

$$Y_t(i) = A_t \; N_t(i) \tag{7}$$

- Price Setting under **Flexible** Prices

- Representative firm chooses $\{P_t(i),\ Y_t(i),\ N_t(i)\}$ to maximize:

$$P_t(i)Y_t(i) - W_t N_t(i) \tag{8}$$

subject to (7) and to demand function for good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

- Substituting for $Y_t(i)$ and $N_t(i)$
- Firm's problem becomes choosing $P_t(i)$ to max:

$$\left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} Y_t P_t - W_t \frac{Y_t}{A_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}$$

- FOC:

$$(1-\varepsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t + \varepsilon W_t \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1} \frac{Y_t}{A_t P_t} = 0 \tag{9}$$

Simplifies to

$$P_t(i) = \left(\frac{1}{1-\frac{1}{\varepsilon}}\right)\frac{W_t}{A_t} = \mu \ MC_t \tag{10}$$

MC_t is *nominal* marginal cost and $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ **desired** (constant) markup value.

Notice: flexible price allocation involves a **constant real** marginal cost:

$$MC_t^r \equiv \frac{\frac{W_t}{Pt}}{A_t} = \frac{\varepsilon - 1}{\varepsilon} \tag{11}$$

Staggered Prices: the Calvo Model

- **Staggered Prices: the Calvo Model**

- Assume now that firms adjust their price **infrequently** and that the opportunity to adjust follows an exogenous Poisson process.

- Each period there is a **constant** probability $(1-\alpha)$ that the firm will be able to adjust its price, independently of past history.

- The **expected** time between price adjustments is therefore $\frac{1}{1-\alpha}$.

- If the law of large numbers holds this implies that the fraction of firms **not** setting prices at t is α.

- The draw is independent of history, so that we do not need to keep track of firms changing prices over time.

- **Dynamics of the Aggregate Price Level**

$\longrightarrow$If the law of large number holds a fraction $(1-\alpha)$ of firms will reset the price at each point in time.

$\longrightarrow$Evolution of the **aggregate** price index:

$$P_t = \left[\alpha P_{t-1}^{1-\varepsilon} + (1-\alpha)(P_t^{new})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{12}$$

$\longrightarrow$In log-linear terms:

$$p_t = \alpha p_{t-1} + (1-\alpha) p_t^{new} \tag{13}$$

$\longrightarrow$Rate of inflation:

$$\pi_t = (1-\alpha)(p_t^{new} - p_{t-1})$$

Interpretation: positive inflation arises if and only if firms adjusting prices in any given period choose to charge prices that are **above the average price level** that prevailed in the economy in the previous period.

- **Optimal Price Setting**

$\longrightarrow$Problem of firm i able to reset its price

$\longrightarrow$Choose $P_t^{new}(i)$ to maximize

$$\sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} Y_{t+k}(i) \ \left[P_t^{new}(i) - MC_{t+k}\right]$$

subject to

$$Y_{t+k}(i) = \left(\frac{P_t^{new}(i)}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{14}$$

FOC

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left[Y_{t+k}(i) + [P_t^{new}(i) - MC_{t+k}] \frac{\partial Y_{t+k}(i)}{\partial P_t^{new}(i)} \right] \right\} = 0 \quad (15)$$

Notice

$$\frac{\partial Y_{t+k}(i)}{\partial P_t^{new}(i)} P_t^{new}(i) = -\varepsilon Y_{t+k} \left(\frac{P_t^{new}(i)}{P_{t+k}} \right)^{-\varepsilon} = -\varepsilon Y_{t+k}(i)$$

Rewrite:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(Y_{t+k}(i) - \varepsilon Y_{t+k}(i) \right) \right\}$$

$$= \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} MC_{t+k} \left(-\varepsilon \left(\frac{P_t^{new}(i)}{P_{t+k}} \right)^{-\varepsilon-1} \frac{1}{P_{t+k}} Y_{t+k} \right) \right\}$$

$\longrightarrow$Equivalently:

$$
\begin{aligned}
& \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} Y_{t+k}(i) \; (1 - \varepsilon) \right\} \\
= \;\; & -\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} MC_{t+k} \varepsilon Y_{t+k}(i) \frac{P_{t+k}}{P_t^{new}(i)} \frac{1}{P_{t+k}} \right\}
\end{aligned}
$$

$\longrightarrow$Rearranging :

$$P_t^{new}(i) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k \; Q_{t,t+k} \; MC_{t+k} \; Y_{t+k}\,(i) \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \alpha^k \; Q_{t,t+k} \; Y_{t+k}\,(i) \right\}} \tag{16}$$

Interpretation: **dynamic** markup equation.

$\longrightarrow$Notice

1. For $\alpha = 0$ equation (16) reduces to:

$$P_t(i) = \frac{\varepsilon}{\varepsilon - 1} MC_t$$

as in the flexible price model, i.e., firms set price as a simple (static) markup over the marginal cost.

2. Optimal price depends on a **forecast** of future values of aggregate demand conditions as well as on the future evolution of the marginal cost.

- **Equilibrium with Price Dispersion**

$$Y_t = C_t \tag{17}$$

We should now write:

$$\begin{aligned} N_t &= \int_0^1 \frac{Y_t(i)}{A_t} di \\ &= \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \\ &= \frac{Y_t}{A_t} D_t \end{aligned} \tag{18}$$

where $D_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ is a term that captures the **dispersion of relative prices** across producers.

- Possibility that D_t is time-varying hinges crucially on the assumed price setting structure.

- Under **Calvo pricing**, whereby firms adjust prices in a non-synchronized fashion, the dispersion of relative prices is potentially an important feature of the equilibrium.

- We **prove** that dispersion D_t is **bounded below** by 1

$$D_t \geq 1$$

→Define $v_{i,t} \equiv \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon}$

- We first have:

$$\begin{aligned} \left[\int_0^1 v_{i,t} di\right]^{\frac{\varepsilon}{\varepsilon-1}} &= \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= P_t^{\varepsilon} \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= 1 \end{aligned} \tag{19}$$

- Also:

$$\left[\int_0^1 v_{i,t}^{\frac{\varepsilon}{\varepsilon-1}} di\right] = \int_0^1 \left[\left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad (20)$$
$$= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$$
$$= D_t$$

- Combining (19) with (20) we have

$$\underbrace{\left[\int_0^1 v_{i,t}^{\frac{\varepsilon}{\varepsilon-1}} di\right]}_{D_t} \geq \left[\int_0^1 v_{i,t} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = 1$$

where the inequality follows from **Jensen's inequality**•

- **Monetary Policy Rule**

$$i_t = \gamma + \phi_\pi \pi_t + \varepsilon_t \tag{21}$$

- **Dispersion of Relative Prices and Inflation**

$$D_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di \tag{22}$$

$$= \int_{1-\alpha} \left(\frac{P_t^{new}}{P_t}\right)^{-\varepsilon} di + \left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} \int_{\alpha} \left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\varepsilon} di \tag{23}$$

$$= (1-\alpha)\widetilde{P}_t^{-\varepsilon} + \alpha \Pi_t^{\varepsilon} D_{t-1}$$

where $\widetilde{P}_t \equiv \frac{P_t^{new}}{P_t}$

- Rewrite price adjustment equation (12) (dividing through by $P_t^{1-\varepsilon}$):

$$1 = \alpha \Pi_t^{\varepsilon - 1} + (1 - \alpha) \left(\widetilde{P}_t \right)^{1-\varepsilon} \tag{24}$$

By combining (22) and (24) we can link relative price dispersion and inflation as follows:

$$D_t = (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\varepsilon - 1}}{1 - \alpha} \right)^{\frac{-\varepsilon}{1-\varepsilon}} + \alpha \Pi_t^{\varepsilon} D_{t-1} \tag{25}$$

- Log-linearize around a steady state with positive inflation $\pi > 0$

$$e^{\log(D_t)} = (1-\alpha)\left(\frac{1-\alpha\left(e_t^{\log(\Pi_t)}\right)^{\varepsilon-1}}{1-\alpha}\right)^{\frac{-\varepsilon}{1-\varepsilon}} + \alpha\left(\ e^{\log(\Pi_t)}\right)^{\varepsilon}\ e^{\log(D_{t-1})}$$

$\rightarrow$Obtain

$$d_t = \left\{\alpha\varepsilon\Pi^{\varepsilon}\left[1-\frac{1}{\Pi D}\left(\frac{1-\alpha\Pi^{\varepsilon-1}}{1-\alpha}\right)\right]^{\frac{1}{\varepsilon-1}}\right\}\pi_t + \alpha\Pi^{\varepsilon}d_{t-1} \qquad (26)$$

where $d_t \equiv \log\left(\frac{D_t}{D}\right)$.

In the particular case of **zero net steady state inflation** (i.e., $\Pi = 1$), we have (from 26) that $D = 1$. In this case we have:

$$A \equiv \left\{ \alpha \varepsilon \Pi^{\varepsilon} \left[1 - \frac{1}{\Pi D} \left(\frac{1 - \alpha \Pi^{\varepsilon - 1}}{1 - \alpha} \right) \right]^{\frac{1}{\varepsilon - 1}} \right\} = 0$$

and (26) reduces to:

$$d_t = \alpha d_{t-1}$$

- Even in the first-order approximation of the model the term d_t cannot be ignored if the point of approximation is a steady-state with $\Pi > 1$.

- If log linearize around **zero inflation** steady state

$$y_t = a_t + n_t \tag{27}$$

- Log-Linearization and the **New Keynesian Phillips Curve**

$$
\begin{aligned}
p_t^{new} &= (1-\alpha\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\alpha\beta)^k mc_{t+k}\right\} \qquad (28) \\
&= (1-\alpha\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\alpha\beta)^k \left(mc_{t+k}^r + p_{t+k}\right)\right\}
\end{aligned}
$$

where we used $mc_t = mc_t^r + p_t$.

- Hence firms that are allowed to reset the price choose to do so as a weighted **average** over the **expected future nominal marginal cost**. Equation (28) above points clearly to the two factors that drive the decision of a firm to deviate from the average price level prevailing in the previous period:

- The presence of the aggregate price level denotes the willingness to maintain (in expectations) the *relative* price unchanged.

- The term involving mc_t^r denotes the desire to *change* the expected relative price in order to avoid any gap that may emerge between expected and desired markup.

- Rewrite equation (28) as a first order difference equation in p_t^{new}

$$p_t^{new} = (1 - \beta\alpha)(mc_t^r + p_t) + \beta\alpha p_{t+1}^{new} \tag{29}$$

- By combining equation (29) with (13) we can obtain a forward looking equation for inflation :

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \left[\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right] mc_t^r \tag{30}$$

- The longer prices are fixed (i.e., for higher α, since prices are kept fixed for an average length of $1/(1-\alpha)$ periods), the less firms are sensitive to changes in the real marginal cost, as current demand conditions matter less.

- **Canonical Representation**

$$U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$$

- Log-linear approximation of the **real marginal cost**:

$$\begin{aligned} mc_t^r &= (w_t - p_t) - a_t \\ &= \varphi n_t + \sigma c_t - a_t \\ &= (\varphi + \sigma) y_t - (1+\varphi) a_t \end{aligned} \tag{31}$$

where the last expression follows from (27).

- **Fully flexible prices** $\rightarrow mc_t^r = 0 \rightarrow$ **natural** level of output

$$y_t^n = \left(\frac{1+\varphi}{\sigma+\varphi}\right) a_t \tag{32}$$

- **Real Marginal Cost and Output Gap**

$$x_t \equiv y_t - y_t^n \tag{33}$$

From equation (31) we can write:

$$\begin{aligned} mc_t^r &= (\varphi + \sigma)\left(y_t - \left(\frac{1+\varphi}{\varphi+\sigma}\right)a_t\right) \\ &= (\varphi+\sigma)x_t \end{aligned}$$

- **The New Keynesian Phillips Curve**

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t \tag{34}$$

where

$$\kappa \equiv \frac{(\varphi + \sigma)(1-\alpha)(1-\beta\alpha)}{\alpha}$$

$\rightarrow$Notice:

$$\frac{\partial \kappa}{\partial \alpha} < 0$$

for any given value of φ, σ, β. Hence a **higher degree of price stickiness** translates into a **flatter** aggregate supply curve.

$\longrightarrow$Notice:

1. Inflation rises as output deviates from its **natural** level. Hence it is not a rise in output per se that produces inflation.

2. By iterating (34) forward we obtain:

$$\pi_t = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \kappa \, x_{t+j} \right\} \tag{35}$$

$\longrightarrow$Inflation is a forward-looking variable, i.e., it depends on current and expected future deviations of output from its natural level.

- **Dynamic IS Equation**

$\rightarrow$From Euler

$$c_t = \mathbb{E}_t \{c_{t+1}\} - \sigma^{-1} (r_t - \gamma) \tag{36}$$

where $r_t \simeq \log(1 + r_t)$.

- Substituting $c_t = y_t$ yields:

$$x_t = \mathbb{E}_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) \tag{37}$$

where

$$r_t^n \equiv \gamma + \sigma E_t\{y_{t+1}^n - y_t^n\} = \gamma + \frac{\sigma(1+\varphi)}{\sigma+\varphi}\mathbb{E}_t\{\Delta a_{t+1}\} \tag{38}$$

→N**atural real rate of interest**.

- Notice the the natural real rate of interest is determined by **real** factors outside the control of monetary policy.

- Integrating *dynamic IS equation* forward:

$$x_t = -\frac{1}{\sigma}\mathbb{E}_t\left\{\sum_{j=0}^{\infty}\left(r_{t+j} - r_{t+j}^{n}\right)\right\} \tag{39}$$

- **Canonical Model**

For any given process for $\{r_t^n\}$ a for a given policy process $\{i_t\}$:

$$x_t = \mathbb{E}_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) \tag{40}$$

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa x_t \tag{41}$$

- **Monetary Policy Trade-Offs**

- To control inflation the CB does not need to generate a recession.

- By stabilizing output at its natural level the CB is also stabilizing inflation.

- Consider a hybrid version of equation (34) (for $\beta \simeq 1$) featuring a backward-looking component:

$$\pi_t = \delta E_t\{\pi_{t+1}\} + (1 - \delta)\pi_{t-1} + \kappa x_t \tag{42}$$

For $\delta = 0$:

$$\pi_t = \pi_{t-1} + \kappa x_t \tag{43}$$

If π_{t-1} rises above average it is clear that the CB needs to generate a recession to stabilize *current* inflation. This persistence feature of inflation emerges clearly from the data.

- **Uniqueness and Stability of the Equilibrium**

Compact form:

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \mathbf{M}\ \mathbb{E}_t \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} + \frac{1}{\sigma + \kappa\phi_\pi} \begin{pmatrix} \kappa \\ 1 \end{pmatrix} r_t^n \tag{44}$$

where

$$\mathbf{M} \equiv \frac{1}{\sigma + \kappa\phi_\pi} \begin{pmatrix} \sigma\beta + \kappa & \sigma\kappa \\ 1 - \beta\phi_\pi & \sigma \end{pmatrix}$$

- Blanchard-Khan 1980

A necessary and sufficient condition for the system (44) to exhibit a **unique** bounded solution is that the number of **non-predetermined** endogenous variables (i.e., jumpy variables) equal the **number of roots** of $\mathbf{M}$ that lie **inside the unit circle**

- **Solving the Model**

- Assume that the monetary shock in (21) and the technology shock follow respectively:

$$\varepsilon_t = \rho^{\varepsilon}\varepsilon_{t-1} + u_t^{\varepsilon} \tag{45}$$

$$a_t = \rho^{a} a_{t-1} + u_t^{a} \tag{46}$$

where u_t^{ε} and u_t^{a} are iid processes with mean zero and variance σ_{ε}^2 and σ_a^2 respectively.

- **Monetary Shock**

- Method of **undetermined coefficients**.

- Conjecture the solution:

$$x_t = a_x \varepsilon_t \tag{47}$$

$$\pi_t = a_\pi \varepsilon_t \tag{48}$$

Notice that (45), (47) and (48) jointly imply:

$$\mathbb{E}_t \{x_{t+1}\} = a_x \rho^{\varepsilon} \varepsilon_t$$

$$\mathbb{E}_t \{\pi_{t+1}\} = a_\pi \rho^{\varepsilon} \varepsilon_t$$

Substituting (21) and the above conjectured solutions in (37) we obtain

$$x_t = \varepsilon_t \left[a_\pi \left(\frac{\rho^\varepsilon - \phi_\pi}{\sigma} \right) + \rho^\varepsilon a_x - \frac{1}{\sigma} \right] \tag{49}$$

Equating the coefficient on ε_t in (49) to the one in (47) we obtain

$$a_x \left(1 - \rho^\varepsilon\right) = a_\pi \left(\frac{\rho^\varepsilon - \phi_\pi}{\sigma} \right) - \frac{1}{\sigma} \tag{50}$$

Substituting the conjectured solutions in (34) we obtain

$$\pi_t = \varepsilon_t \left[\beta a_\pi \rho^\varepsilon + \kappa a_x\right] \tag{51}$$

Equating the coefficient on ε_t to the one in (48) yields

$$a_\pi = \left(\frac{\kappa}{1 - \beta\rho^\varepsilon}\right) a_x \tag{52}$$

The system of equations (50), (52) can be solved for the two unknows a_π and a_x, yielding the solutions:

$$x_t = -\,\Gamma_x \varepsilon_t \tag{53}$$

$$\pi_t = -\,\Gamma_\pi \varepsilon_t \tag{54}$$

where

$$\Gamma_x \equiv \frac{(1-\beta\rho^{\varepsilon})}{\sigma(1-\rho^{e})\,(1-\beta\rho^{\varepsilon}) + \kappa(\phi_\pi - \rho^{\varepsilon})} > 0$$

and

$$\Gamma_\pi \equiv \frac{\kappa}{\sigma(1-\rho^{e})\,(1-\beta\rho^{\varepsilon}) + \kappa(\phi_\pi - \rho^{\varepsilon})} > 0$$

- Notice

1. Both coefficients Γ_x and Γ_π are positive. Hence a contractionary (expansionary) monetary policy shock lowers (raises) *both* inflation and the output gap. Since the natural level of output is unaffected by monetary shocks, the same effect translates into **actual output** also.

2. The role of the degree of **price stickiness**, via its effect on κ, the slope of the NKPC.

$$\frac{\partial \Gamma_x}{\partial \kappa} < 0$$

- As $\alpha \longrightarrow 0$ (flexible prices), $\kappa \longrightarrow \infty$, which implies $\Gamma_x \longrightarrow 0$. In this case the effect of a monetary policy shock on the output gap is nil (monetary policy neutrality).

- Conversely, the effect of a monetary shock on the output gap (or output) is maximized as $\alpha \longrightarrow 1$ (full price rigidity) and $\kappa \longrightarrow 0$.

- Effects of a monetary shock on **inflation**.

$$\frac{\partial \Gamma_\pi}{\partial \kappa} > 0$$

.

$\rightarrow$A monetary policy shock produces a **smaller** effect on inflation the **larger** the degree of price stickiness.

- The higher the degree of price stickiness (ie, low κ), the weaker each firm's tendency to match any given variation in demand (induced by the monetary policy action) with a variation in prices (as opposed to output)

- **Technology Shock**

-Using (46) we can write the natural real interest rate as:

$$r_t^n = \gamma - \left[\frac{\sigma(1+\varphi)(1-\rho^a)}{(\sigma+\varphi)} \right] a_t$$

- We conjecture the solution:

$$x_t = b_x a_t \tag{55}$$

$$\pi_t = b_\pi a_t \tag{56}$$

Substituting (21) and the above conjectured solutions in (37) we obtain

$$x_t = a_t \left[b_x \rho^a + b_\pi \left(\frac{\rho^a - \phi_\pi}{\sigma} \right) - \frac{(1+\varphi)(1-\rho^a)}{(\sigma+\varphi)} \right] \tag{57}$$

Equating the coefficient on a_t in (57) and (55) yields

$$b_x (1-\rho^a) = b_\pi \left(\frac{\rho^a - \phi_\pi}{\sigma} \right) - \frac{(1+\varphi)(1-\rho^a)}{(\sigma+\varphi)} \tag{58}$$

Similarly, by substituting the conjectured solutions in (34) we obtain

$$b_\pi = \left(\frac{\kappa}{1-\beta\rho^a}\right) b_x \tag{59}$$

Substituting (59) in (58), and solving for b_x we can write

$$x_t = -\,\Theta_x a_t \tag{60}$$

$$\pi_t = -\,\Theta_\pi a_t \tag{61}$$

where

$$\Theta_x \equiv \frac{\frac{(1+\varphi)}{(\sigma+\varphi)}\sigma(1-\beta\rho^a)(1-\rho^a)}{\sigma(1-\beta\rho^a)(1-\rho^a)+\kappa(\phi_\pi-\rho^a)} > 0$$

$$\Theta_\pi \equiv \frac{\frac{(1+\varphi)}{(\sigma+\varphi)}(1-\rho^a)\sigma\kappa}{\sigma(1-\beta\rho^a)(1-\rho^a)+\kappa(\phi_\pi-\rho^a)} > 0$$

Notice

1. A positive technology shock produces a contraction in both the *output gap* and *inflation*.

2. For $\kappa \to \infty$ (flexible prices) we have $\Theta_x \to 0$. In other words, under flexible prices, the output gap is always zero, since output will constantly replicate its flexible-price counterpart.

3. Effects of a technology shock on *output*:

$$
\begin{aligned}
y_t &= x_t + y_t^n \\
&= \left(\frac{1+\varphi}{\sigma+\varphi} - \Theta_x \right) a_t \\
&= \Theta_y a_t
\end{aligned}
$$

where

$$
\Theta_y \equiv \frac{(1+\varphi)}{(\sigma+\varphi)} \left(\frac{1}{1 + \frac{\sigma(1-\beta\rho^a)(1-\rho^a)}{\kappa(\phi_\pi - \rho^a)}} \right) > 0 \tag{62}
$$

$\longrightarrow$Hence output **rises** in response to a positive technology shock, similarly to what happens in a RBC model.

- Role played by price stickiness.

For $\kappa \to \infty$ (flexible prices) we have:

$$\Theta_y \equiv \Theta_y^{RBC} = \frac{(1+\varphi)}{(\sigma+\varphi)}$$

- From (62) we see that a higher degree of price rigidity (smaller κ) **dampens** the effect of technology shocks on output:

$$\begin{aligned} \Theta_y &< \Theta_y^{RBC} \text{ for } \kappa < \infty \\ &= \Theta_y^{RBC} \text{ for } \kappa \to \infty \end{aligned}$$

- Impact effect of technology shocks on **employment**.

$$\begin{aligned} n_t &= y_t - a_t \\ &= (\Theta_y - 1)\, a_t \end{aligned}$$

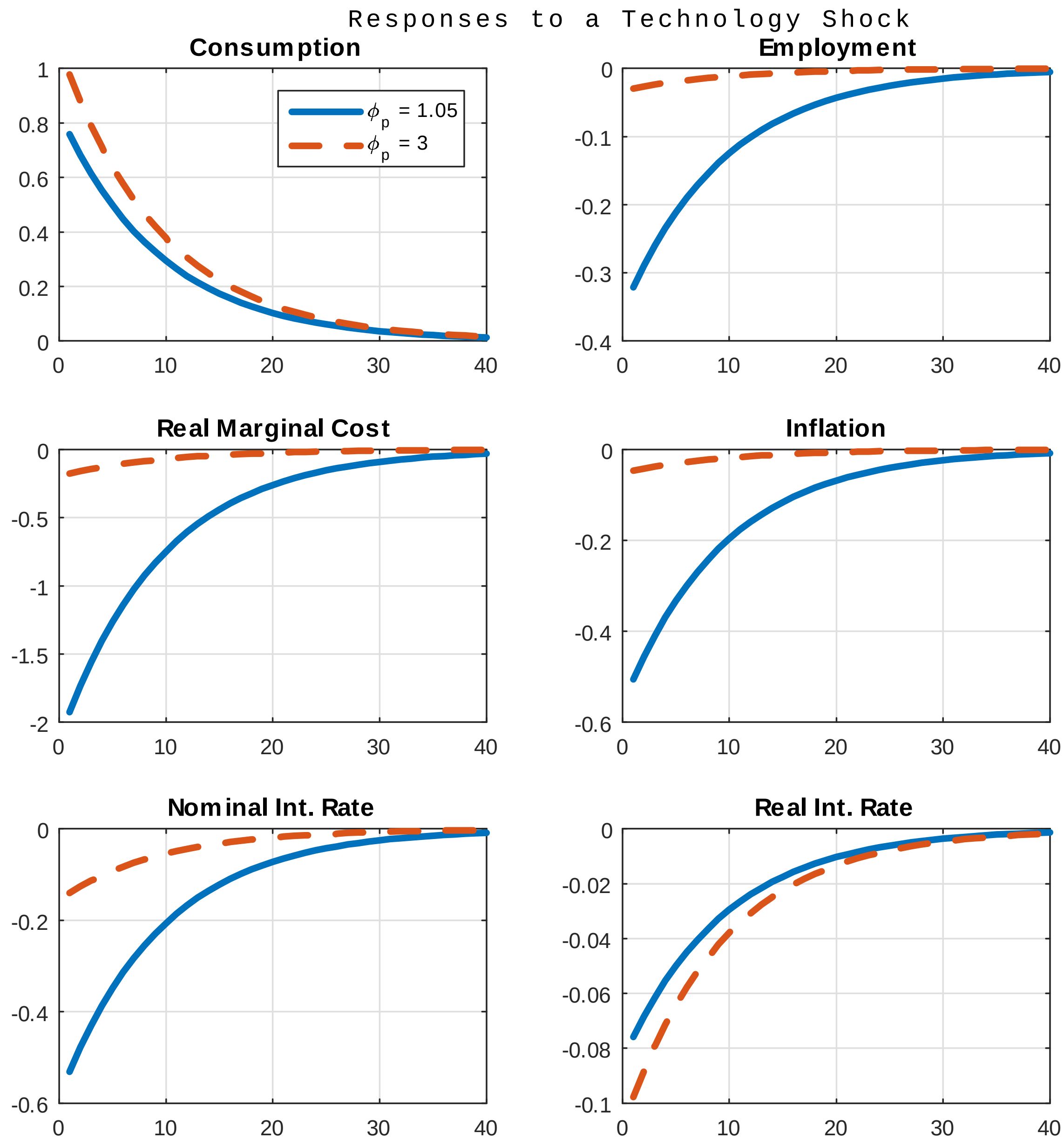
For employment to fall in response to a technology shock it is required that:

$$\left(\frac{1-\sigma}{\sigma+\varphi}\right)\kappa(\phi_\pi - \rho^a) < \sigma(1-\beta\rho^a)(1-\rho^a) \tag{63}$$

- Condition (63) is easily satisfied, e.g., in the case of log-consumption utility ($\sigma = 1$) for any $\kappa < \infty$, i.e., to the extent that price stickiness is present.

- In the case of fully rigid prices ($\kappa = 0$), the same condition is always satisfied for any value of σ.

The role of the monetary policy rule in shaping the response to shocks


Responses to a Technology Shock
Consumption
ϕ_p = 1.05
ϕ_p = 3
Employment
Real Marginal Cost
Inflation
Nominal Int. Rate
Real Int. Rate

Existence and uniqueness of a RE equilibrium

- **Existence and Uniqueness of a RE Equilibrium**

- The characteristic polynomial of $\mathbf{M}$ can be written

$$P(\xi) = \xi^2 - tr(\mathbf{M}) + \text{det}(\mathbf{M})$$

where

$$tr(\mathbf{M}) = \frac{\sigma + (\sigma\beta + \kappa)}{\sigma + \kappa\phi_\pi}$$

and

$$\text{det}(\mathbf{M}) = \frac{1}{(\sigma + \kappa\phi_\pi)^2}(\sigma^2\beta + \sigma\kappa\beta\phi_\pi)$$

- Conditions for existence and uniqueness of an equilibrium are that both roots lie inside the **unit circle**.

- We know that the roots μ_1 and μ_2 must obey:

$$\begin{aligned} \mu_1 + \mu_2 &= \text{tr}(M) \\ \mu_1 \mu_2 &= \det(M) \end{aligned}$$

- Alternatively, the same conditions for uniqueness can be stated as follows:*

$$|\,\mathbf{det}(\mathbf{M})| < \mathbf{1} \tag{64}$$

$$|-tr(\mathbf{M})| < \mathbf{1} + det(\mathbf{M}) \tag{65}$$

As for condition (64) we can verify that

*See for instance, Bullard and Mitra (2000) and references therein.

$$
\begin{aligned}
|det(\mathbf{M})| &= \left|\frac{1}{(\sigma+\kappa\phi_\pi)^2}\sigma^2\beta\left(1+\frac{\kappa\phi_\pi}{\sigma}\right)\right| \\
&= \left|\frac{\sigma\beta}{(\sigma+\kappa\phi_\pi)}\right|
\end{aligned}
$$

which requires that

$$
\beta < 1 + \frac{\kappa\phi_\pi}{\sigma}
$$

It is clear that this is verified for any value of $\phi_\pi \geq 0$.

On the other hand condition (65) requires

$$\begin{aligned}
\frac{\sigma\beta + \kappa + \sigma}{\sigma + \kappa\phi_\pi} &< 1 + \frac{\sigma\beta}{\sigma + \kappa\phi_\pi} \\
&= \frac{\sigma\beta + \kappa\phi_\pi + \sigma}{\sigma + \kappa\phi_\pi}
\end{aligned}$$

which is satisfied if and only if $\phi_\pi > 1$.

Equilibrium uniqueness under the simple interest rate rule