Macroeconomics III

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Optimal Monetary Policy in the New Keynesian Framework: a Linear Quadratic Approach

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- **Efficient Allocation**

- Efficient allocation that would be chosen by a **social planner** in the absence of frictions

- The planner chooses $\{C_t,\ N_t,\ C_t(i),\ N_t(i)\}$ for all i and t to maximize:

$$U(C_t, N_t) \text{ s.t.}$$

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

$$C_t(i) = A_t N_t(i) \tag{2}$$

$$\int_0^1 N_t(i) di = N_t \tag{3}$$

Define by λ_t, ψ_t, γ_t the multipliers on constraints (1), (2), (3) respectively–> FOCs

$$U_{c,t} = \lambda_t$$

$$\lambda_t \left(\frac{C_t(i)}{C_t} \right)^{-\frac{1}{\varepsilon}} = \psi_t$$

$$U_{n,t} = \gamma_t$$

$$\psi_t A_t + \gamma_t = 0$$

Combining to eliminate the multipliers we obtain

$$\left(\frac{C_t(i)}{C_t}\right)^{-\frac{1}{\varepsilon}} = \frac{-U_{n,t}/U_{c,t}}{A_t} \tag{4}$$

$\longrightarrow$Notice that condition (4) holds for all varieties $i \in [0, 1]$, since the right hand side depends only on aggregate variables.

$\longrightarrow$Equation (4) implies:

$$C_t(i) = C_t \text{ for all } i \in [0, 1] \tag{5}$$

In other words, the planner wishes to allocate consumption (and therefore labor) **symmetrically** across varieties.

$\longrightarrow$Condition (5) implies also

$$P_t(i) = P_t \text{ for all } i \in [0, 1] \tag{6}$$

and therefore $D_t = 1$ for all t.

- The planner wishes to **minimize relative price dispersion** across varieties.

- This is a consequence of the **concavity** of the utility function, i.e., of diminishing marginal utility. With price dispersion in place, in fact, the utility gain of consuming more of the cheaper varieties is more than offset by the utility loss of consuming less of the more expensive ones.

Condition (5) therefore implies

$$-\frac{U_{n,t}}{U_{c,t}} = A_t \tag{7}$$

Hence the planner equates the marginal rate of substitution between consumption and leisure to the marginal rate of transformation, which coincides with the marginal product of labor.

- **Sources of Inefficiences in the Decentralized Equilibrium**

1. **Market power** $\rightarrow$average level of output **inefficiently low**.

- In particular the fact that prices are set as a (constant) markup over the marginal cost implies that the marginal cost of producing an additional unit of each variety of good is below the utility benefit of that additional unit.

$\longrightarrow$Steady-state real marginal cost is

$$mc = \mu^{-1} = \frac{W}{P} \tag{8}$$

where we have assumed (without loss of generality) that A is normalized to 1. In such a steady-state we also have that $Y = C = N$.

From the household's labor supply efficiency condition this implies that

$$\frac{N^{\gamma}}{C^{-\sigma}} = \frac{W}{P} = Y^{(\varphi+\sigma)} \tag{9}$$

By combining the two equations above we obtain

$$Y^{ic} = \left(\frac{1}{\mu}\right)^{\frac{1}{\varphi+\sigma}} \tag{10}$$

where Y^{ic} is the steady-state level of output that obtains under imperfect competition. The **efficient** level of output would obtain in the case of unitary markup $\mu = 1$.

$\longrightarrow$Let's define the level of output that would prevail under **perfect competition** as Y^{pc}. From (10) we have

$$Y^{pc} = \mathbf{1} > Y^{ic} = \left(\frac{1}{\mu}\right)^{\frac{1}{\varphi+\sigma}} \tag{11}$$

To understand how monopolistic competition induces a **violation of the efficiency conditions** that characterize the planner's problem, notice that the optimal price setting condition under flexible prices should read

$$P_t(i) = P_t = \mu MC_t = \mu \frac{W_t}{A_t}$$

Rearranging we obtain:

$$\frac{-U_{n,t}/U_{c,t}}{A_t} = \mu^{-1} \quad (12)$$

which violates (7), since $\mu > 1$.

Eliminating the distortion induced by market power entails subsidizing the cost of labor, and therefore employment, in a lump-sum fashion. Hence there should exist a subsidy τ satisfying:

$$\frac{W_t(1-\tau)}{P_t A_t} = \frac{\varepsilon - 1}{\varepsilon}$$

The optimal subsidy τ^* is such that

$$\frac{(\varepsilon - 1)/\varepsilon}{(1-\tau^*)} = 1$$

which entails:

$$\tau^* = \frac{1}{\varepsilon}$$

$\longrightarrow$We assume henceforth that the optimal subsidy scheme is in place

2. **Price Stickiness and Relative Price Distortion**.

→Price stickiness in the form of Calvo staggering generates **relative price dispersion** between adjusting and non-adjusting firms.

Each firm would optimally choose a markup policy such that:

$$\mu_t(i) = \frac{P_t(i)}{W_t/A_t} = \frac{(P_t(i)/P_t)\,A_t}{W_t/P_t}$$

Using (12) the latter condition becomes:

$$\mu_t(i) = \left(\frac{P_t(i)}{P_t}\right)\mu$$

Hence **individual** markups will deviate from the **desired constant markup** to the extent that price dispersion is in place.

Recall that relative price dispersion and inflation are related as

$$D_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\varepsilon-1}}{1-\alpha}\right)^{\frac{-\varepsilon}{1-\varepsilon}} + \alpha\Pi_t^{\varepsilon}D_{t-1}$$

This implies:

$$\begin{aligned} \Pi_t &= 1 \Rightarrow D_t = 1 \text{ for all } t \\ \alpha &= 0 \Rightarrow D_t = 1 \text{ for all } t \end{aligned}$$

Hence the relative price distortion is eliminated either in the presence of *zero (net) inflation* or when prices are *flexible* ($\alpha = 0$).

- **Optimal Monetary Policy**

1. Specification of the model: canonical NK model

2. Monetary policy: maximize households' welfare

→Canonical model

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) \tag{13}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t \tag{14}$$

→Need to sepcify behavior of monetary authority

- **A Second-Order Approximation to Household's Utility**

1. Optimal subsidy τ^* is in place.

2. This implies that a *zero inflation* steady-state coincides also with the *efficient* one. It will therefore bear particular meaning to consider percentage deviations around that steady-state (either with flexible or with sticky prices).

3. A second order approximation of the household's utility function around the zero-inflation steady state.

Obtain

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{U_t-U}{U_cC}\right)\right\}=-\frac{1}{2}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_t^2+(\sigma+\varphi)\,x_t^2\right]\right\} \qquad (15)$$

- Observations

1. Welfare losses depend on the **variability** of both the **output gap** and the **rate of inflation** (noticeably not on the variability of the price *level*).

2. The **time variance** of the output gap matters for welfare because the household wishes to keep output at the efficient level (which corresponds here to the flexible price level of output).

3. The **cross-sectional dispersion** of output matters as well, due to the presence of relative price distortion. We have shown above that this is proportional to the cross-sectional variation in prices, which in turn is proportional to squared inflation.

4. Inflation variability matters in both its forecastable and unforecastable component. This follows from the forward-looking feature of the NKPC relative to a New-Classical Phillips Curve a la Lucas (which would imply that only surprises in inflation would matter for welfare).

5. Stabilization of the output gap, by implying a simultaneous stabilization of inflation, would minimize the welfare loss. Hence **no intrinsic tradeoff** characterizes the monetary policy conduct as such. This motivates the analysis in the following section.

The Gains from Monetary Policy Commitment

- We will analyze optimal monetary policy under two cases, full commitment and discretion.

- Under **commitment** the CB can credibly commit to a certain future path of the output gap at some generic time, and thereby is able to affect private sector's expectations.

- In the case of **discretion** the CB takes private sector's **expectations as given** and chooses its instrument optimally period by period.

Optimal Policy under Commitment

- The problem of the monetary authority can be written as the one of choosing a path $\{x_t, \pi_t\}_{t=0}^{\infty}$ to minimize (15) subject to the period-by-period constraint:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t \tag{16}$$

where u_t is a supply (**cost-push)** shock that forbids the CB from being able to simultaneously stabilize inflation and the output gap.

$\rightarrow$In the following we assume that $\{u_t\}$ follows an AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t$$

where ε_t is an iid shock with mean zero and variance σ_ε^2.

General foundations of the cost-push shock

Let

$$y_t^n \equiv \text{flex price level of output}$$

$$y_t^e \equiv \text{efficient level of output}$$

Hence the primitive Phillips curve reads:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \underbrace{(y_t - y_t^n)}_{x_t} \tag{17}$$

Then we have:

$$x_t = (y_t - y_t^e) + (y_t^e - y_t^n)$$

Rewrite:

$$x_t = x_t^e + u_t$$

where x_t^e is the **welfare-relevant** output gap, and

$$u_t \equiv y_t^e - y_t^n$$

Hence the term u_t generally captures variations in the gap between the efficient and the natural level of output.

Lagrangian problem

$$\mathcal{L} = -\frac{1}{2}E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\left(\omega x_t^2 + \pi_t^2 \right) + 2\phi_t \left(\pi_t - \beta E_t\{\pi_{t+1}\} - \kappa x_t - u_t \right) \right] \right\}$$

where ϕ_t is the Lagrange multiplier associated to constraint (16).

The FOCs for $t \geq 0$ with respect to π_t and x_t are respectively

$$\beta^t \pi_t + \beta^t \phi_t - \beta^{t-1} \phi_{t-1} \beta = 0$$

$$\beta^t \omega x_t - \beta^t \kappa \phi_t = 0$$

These can be rewritten more simply as:

$$\pi_t + \phi_t - \phi_{t-1} = 0 \tag{18}$$

$$\omega x_t - \kappa \phi_t = 0 \tag{19}$$

along with the initial condition

$$\phi_{-1} = 0 \tag{20}$$

By eliminating the multiplier ϕ_t we can write

$$\pi_t + \frac{\omega}{\kappa}(x_t - x_{t-1}) = 0 \quad \text{for t} > 0 \tag{21}$$

while in $t = 0$ we must use (20) and write

$$\pi_0 + \frac{\omega}{\kappa} x_0 = 0 \quad \text{for t} = 0 \tag{22}$$

- **Time Inconsistency** of the Optimal Plan

Equations (21) and (22) imply that the system of first order conditions characterizing the optimal plan is not time invariant.

In time $t = 0$ condition (18) implies

$$\pi_0 = -\phi_0$$

with the promise of setting

$$\pi_t = -(\phi_t - \phi_{t-1}) \tag{23}$$

for all periods $t > 0$.

Yet suppose the policy authority were allowed to reoptimize at $t = 1$. Then it would choose to set

$$\pi_1 = -\phi_1$$

which would contrast with the promise formulated at $t = 0$ and consistent with (23) of setting

$$\pi_1 = -(\phi_1 - \phi_0) \tag{24}$$

This is the *time inconsistency* problem of the optimal program.

- From now on will assume, though, that the system of first order conditions (18)-(19) applies for all periods $t \geq 0$.

- This corresponds to looking at optimal policy from a *timeless* perspective, as though the period $t = 0$ were to bear no particular meaning.*

*See Woodford (2003) for a discussion of timeless perspective concept.

- **The History Dependence of Policy**

If we substitute (21) into (16) we obtain a second order difference equation in the output gap

$$Ax_t = \beta E_t\{x_{t+1}\} + x_{t-1} - \frac{\kappa}{\omega}u_t \tag{25}$$

where $A \equiv \left(1 + \beta + \frac{\kappa^2}{\omega}\right)$.

We can solve 25) by using the method of **undetermined coefficients**. We guess the solution

$$x_t = a_x x_{t-1} + b_x u_t \tag{26}$$

This implies

$$\begin{aligned} E_t\{x_{t+1}\} &= a_x x_t + b_x \rho u_t \\ &= a_x^2 x_{t-1} + (a_x + \rho)\, b_x u_t \end{aligned}$$

Substituting into (25) and rearranging:

$$x_t = \left(\frac{1 + \beta a_x^2}{1 + \beta + \frac{\kappa^2}{\omega}} \right) x_{t-1} + \left(\frac{\beta \left(a_x + \rho \right) b_x - \kappa / \omega}{1 + \beta + \frac{\kappa^2}{\omega}} \right) u_t \tag{27}$$

Equating the coefficients in (26) and (27) yields

$$a_x = \frac{1 + \beta a_x^2}{A} \tag{28}$$

$$b_x = - \frac{\kappa}{\omega \left[1 + \beta (1 - a_x - \rho) \right] + \kappa^2} \tag{29}$$

Expression (28) is a quadratic equation in a_x with roots

$$a_x^{1,2} = \frac{A\left(1 \pm \sqrt{1 - \frac{4\beta}{A^2}}\right)}{2\beta}$$

Since we wish to focus on the unique bounded solution to (25), we will henceforth assume that $a_x = a_x^1 < 1$.

Hence the solution to (25) **under commitment** reads:

$$x_t = a_x x_{t-1} - B_x u_t \tag{30}$$

where

$$B_x \equiv \frac{\kappa}{\omega\left[1+\beta(1-a_x-\rho)\right]+\kappa^2} > 0$$

- I*f the supply shock* u_t *is i.i.d*, it is not only the *current* output gap that must be contracted in response to the current shock, but also the output gap in the subsequent periods.

- Notice in fact that (30) can be rewritten as

$$x_t = -B_x \sum_{j=0}^{\infty} a_x^j u_{t-j} \tag{31}$$

Hence the output gap path must depend (negatively) on the distributed lags of the (cost-push) shock. This feature of **history-dependence** critically distinguishes the optimal solution under commitment from the one under discretion (see below).

Combining (30) with (21) we obtain a **solution for inflation**

$$\pi_t = \frac{\omega(1 - a_x)}{\kappa} x_{t-1} + \frac{\omega}{\kappa} B_x u_t \tag{32}$$

Thus the feature of serial correlation characterizes also the path of inflation, with this holding even in the case of iid serially uncorrelated cost shocks ($\rho = 0$).

- **Stationarity of the Price Level**

Let's define p^* as the (log) price level target adopted by the CB in the period preceding the adoption of the full commitment policy at time t (it could be $p^*_{-1} = 0$). The first order condition (21) can be rewritten as a relationship between the price level and the output gap as

$$\widetilde{p}_t = -\frac{\omega}{\kappa} x_t \tag{33}$$

where $\widetilde{p}_t \equiv p_t - p^*$.

By substituting (33) into (16) one obtains a second order difference equation in the price level

$$\widetilde{p}_t A = \beta E_t\{\widetilde{p}_{t+1}\} + \widetilde{p}_{t-1} + u_t \tag{34}$$

where once again $A \equiv (1 + \beta + \frac{\kappa^2}{\omega})$.

Solution to this equation is

$$\widetilde{p}_t = \zeta_1 \widetilde{p}_{t-1} + \zeta_1 u_t \tag{35}$$

Hence we see that under commitment the deviations of the (log) price level from the target must follow a **stationary** process. Any deviation induced by the cost-push shock must be subsequently undone by an appropriate setting of the evolution of the output gap.

- **Optimal Policy under Discretion**

- Case in which a once and for all commitment is **not feasible** the monetary authority will not be able to affect private sector's expectations.

- We will focus below on a **Markov perfect equilibrium**.

The monetary authority will maximize:

$$-\frac{1}{2}\left(\pi_t^2 + \omega x_t^2\right) + F_t$$

subject to

$$\pi_t = \kappa x_t + u_t + f_t \tag{36}$$

with F_t and f_t being defined respectively

$$\begin{aligned} F_t &\equiv -\frac{1}{2}E_0\left\{\sum_{t=1}^{\infty}\beta^j\left(\omega x_t^2 + \pi_t^2\right)\right\} \\ f_t &\equiv \beta E_t\{\pi_{t+1}\} \end{aligned}$$

and being treated parametrically in the maximization problem.

The first order condition of this problem, holding for all t, reads:

$$\pi_t + \frac{\omega}{\kappa} x_t = 0 \tag{37}$$

This can be usefully contrasted with equation (21). Notice that, under discretion, current inflation depends only the current value of the output gap (as opposed to its rate of change as it happens under commitment).

Substituting (37) into (36) we obtain a first order difference equation in the output gap

$$\left(1 + \frac{\kappa^2}{\omega}\right) x_t = \beta E_t\{x_{t+1}\} - \frac{\kappa}{\omega} u_t \tag{38}$$

We guess the solution:

$$x_t = a_x^d u_t \tag{39}$$

which implies

$$E_t\{x_{t+1}\} = a_x^d \rho u_t$$

Substituting into (38) and rearranging:

$$x_t = \left(\frac{\beta a_x^d \rho - \kappa/\omega}{1 + \frac{\kappa^2}{\omega}} \right) u_t \tag{40}$$

Equating with the coefficient in (39) and solving for a_x^d we obtain

$$a_x^d = -\left(\frac{\kappa}{\omega(1 - \beta\rho) + \kappa^2} \right) u_t$$

Hence the solutions for output gap and inflation under discretion read

$$x_t^d = -\left(\frac{\kappa}{\omega(1 - \beta\rho) + \kappa^2} \right) u_t \tag{41}$$

$$\pi_t^d = \left(\frac{\omega}{\omega(1 - \beta\rho) + \kappa^2} \right) u_t \tag{42}$$

Notice that, under discretion, the output gap and inflation are simply proportional to the path of the shock, and therefore do not exhibit any feature of serial correlation.

- **Gains from Commitment**

- *Figure 1* we show indifference curves for the CB in the output gap-inflation space specified by the (expectation augmented) Phillips curve

- A positive supply shock shifts the Phillips curve upward from the (0,0) locus, where inflation expectations are given at zero.

- Under **discretion** the monetary authority reaches the point *DIS*, at unchanged inflation expectations.

- However it is clear that the CB can improve welfare by affecting inflation expectations, namely by generating (after the initial rise) expectations of subsequent future deflation. This would allow her to reach the point *COMM*.

Dynamics of inflation and output gap

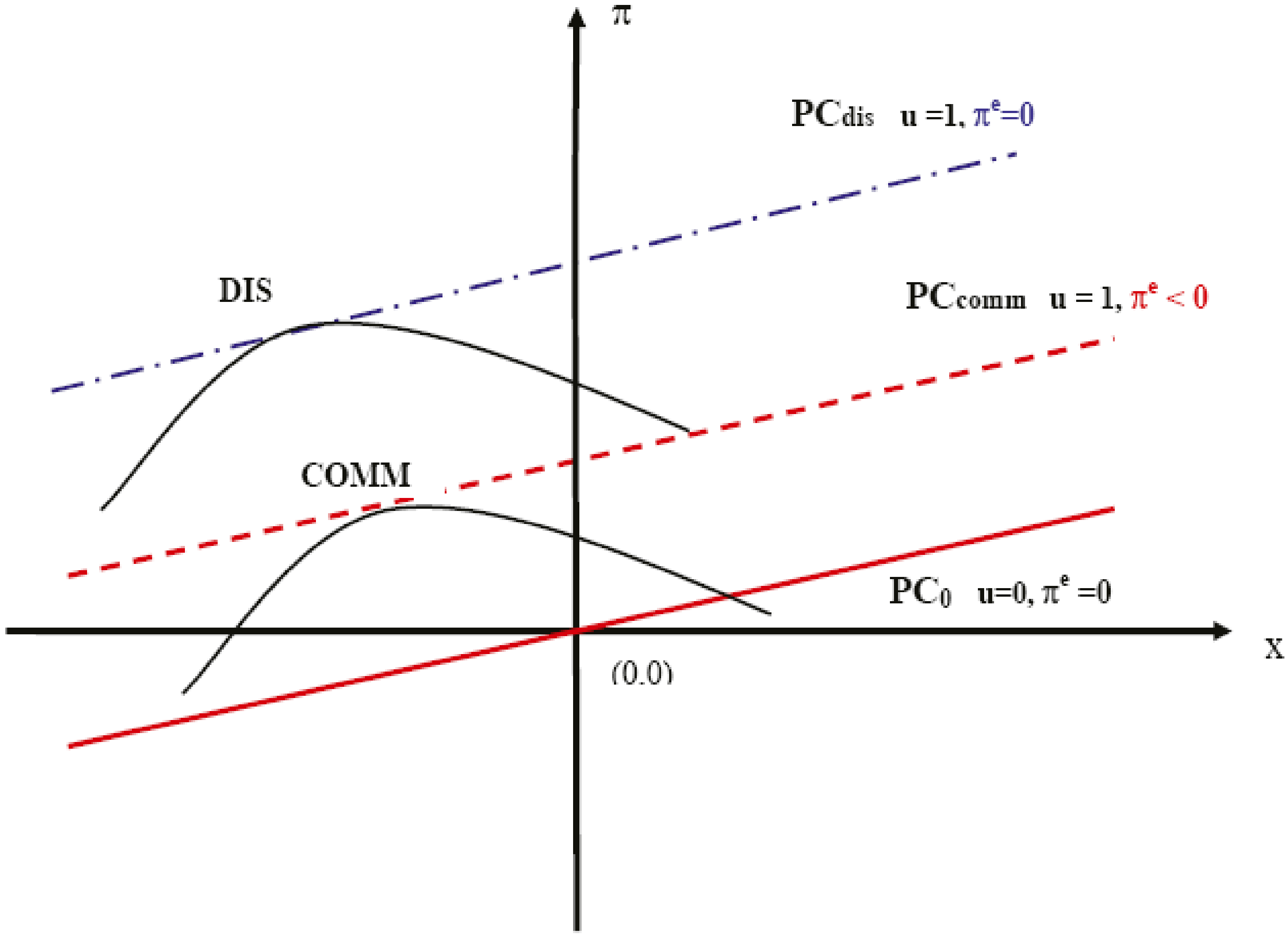

Figure 1: Cost-Push Shock: Welfare under Discretion vs. Commitment.

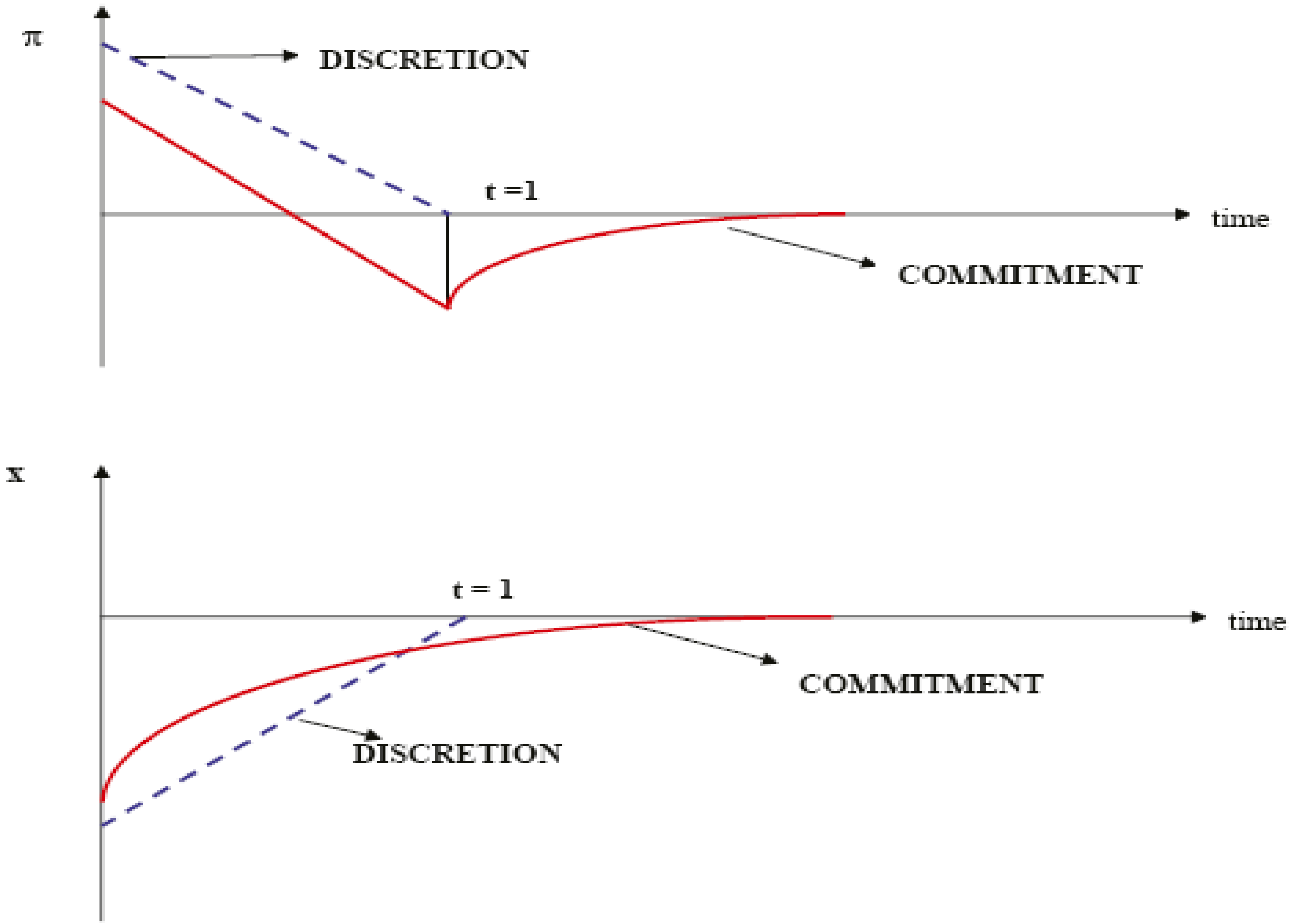

Figure 2: Impulse Responses to a Cost-Push Shock (iid): Discretion vs. Commitment.