Monetary Transmission in the Euro Area: A Factor Model Approach

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Abstract

This paper studies the transmission of common monetary shocks across European countries by using a dynamic factor model (Forni-Reichlin (1998)). This technique allows to extract the common European monetary shock (identified with that of Germany) and to compute the country-specific responses. Our identification employs rotations of the shocks space and a loss function (Uhlig (1999)). European countries display responses in line with the predictions of a broad set of theoretical models and are characterized by quantitatively different responses. Spain and Germany are the most sensitive countries to common monetary shocks, while France, Italy and the Netherlands are the least.

Keywords: factor models, monetary transmission, shocks identification. JEL classification: C33, F42, E52, E58.

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$1 \quad {\rm Introduction^1}$

The country-specific effects of a centralized monetary policy are a crucial issue for the smooth functioning of a monetary union.

When countries fix the nominal exchange rate with respect to the other participants to the union, they lose the control of their national monetary policy. Given that specific features of national economies may require different policy interventions, the loss of the monetary tool may undermine the stability of the union, by creating tensions across countries about the preferred conduct of monetary policy.

The empirical literature has produced many works studying the incidence of asymmetric shocks and analysing the synchronization of business cycles. The analysis of the differences in the transmission of monetary policy shocks across countries participating to the union has on the contrary received less attention until few years ago.

In this paper we propose a new approach to the study of the effects of monetary policy in a monetary union. We take seriously the idea that in order to understand the asymmetries in the transmission of monetary policy, the focus has to be concentrated on the identification of the effects of a *common* monetary policy shock.

We use dynamic factor models (see Forni and Reichlin (1996 and 1998)), a technique novel in this literature. We will argue that this method is able to overcome many of the limitations of country-specific VARs, the method more commonly used in previous works.

Factor models have three main advantages with respect to other approaches:

- 1. They concentrate on the identification of the effects of *common* monetary shocks, which are the true object of interest, and not on the countryspecific shocks, as usually done in the VAR literature.
- 2. The number of restrictions needed for identification of the $structural$ common shocks is reduced, thanks to the reduction of dimensionality of the common shocks space.
- 3. They allow to pool all the countries under examination together in the same model.

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Concerning the first point, we identify the *common* European monetary shock with the German monetary shock, by recalling that Germany was the only country able to conduct an independent monetary policy during the EMS phase. The other European countries had to follow the decisions of the Bundesbank in order to maintain the peg with the Deutsche Mark: as reported in Dornbush, Favero and Giavazzi (1998), "Europe was on the Buba standard".

Concerning the second point, we will show below that the problem of identification of *structural* shocks in factor models generates the same kind of indeterminacy found in VAR models, but that the number of restrictions needed to reach identification is much lower.

We employ an identification criterion that does not impose any a priori zero restriction (the kind of restrictions typically used in the VAR literature) on the parameters of the model.

We choose the identification scheme that minimizes the distance between the impulse response functions (IRF) estimated for Germany using the factor model and the IRF estimated from a "benchmark" German-only VAR (specified \dot{a} la Bernanke and Mihov (1997)), so to give a precise formal meaning to the assumption that German monetary policy was the common European monetary policy.

Our results shed light on the pre-EMU period and infer what will happen under the EMU regime. The responses of 8 European countries participating to the Euro project to a common monetary shock are consistent with what predicted by a large class of theoretical models. We show that under the EMU asymmetric regime, Germany was acting as the nominal anchor and was putting the adjustment costs on the other countries. Under the ECB (symmetric) regime Germany appears more sensitive to common monetary shocks, as in Wieland (1996).

Under the ECB regime, Germany and Spain display strong sensitivity to monetary shocks, while the contrary is true for France, the Netherlands and especially for Italy.

We relate our ranking to proxies of various channels of monetary transmission and conclude that the interest rate channel is significant in explaining the asymmetries, while the credit channel does not play a significant role, as shown by Carlino-DeFina (1998a) for US states.

The paper is organized as follows. Section 2 discusses some limitations of the existing literature. Section 3 introduces the econometric model and the estimation strategy. Section 4 discusses the results for the Euro area countries and explains in greater detail the identification strategy. Section 5 studies the new environment in which the ECB operates and relates the results to the previous literature. Section 6 concludes.

2 A (Quick) Look at the Previous Literature

The empirical literature has recently produced many works dealing with the asymmetric effects of monetary policy on European countries. Different techniques have been employed, from small scale country-specific VARs to large scale multi-country models. It is not our objective to review this large literature here (for a comprehensive survey, see Angeloni, Kashyap, Mojon and Terlizzese (2002)).

In this section we discuss some limitations of VARs. This will give us the opportunity to motivate better our modelling choice: we believe that given the problem at hand, factor models are a way to overcome some of these drawbacks.

May studies estimate country-specific VARs (among the others, Gerlach and Smets (1995), Barran, Coudert and Mojon (1997), Kouparitsas (1999), Ramaswamy and Sloeck (1997), Ehrmann (1998)) and conduct impulse response analysis: how an unexpected shock to one of the variables (typically, a shock to the interest rate, assumed to be the monetary policy instrument) affects the other variables in the system.

While this line of research have provided interesting results in single country studies (Christiano, Eichenbaum and Evans (1999)), we believe it is not the most appealing strategy if the objective is to understand the effects of common monetary policy across countries.

First, country-specific VARs employed in the context of the EMU are affected by an endogeneity problem. The European Monetary System was designed to allow a certain degree of symmetry among countries, but in practice, it became an asymmetric system in which Germany was playing the role of the nominal anchor. The European-wide monetary policy was run by the Bundesbank with the other countries struggling to maintain the exchange rate parity with the Deutsche Mark. Given these premises, country-specific VARs that do not control for the leading role of Germany can easily mix up endogenous responses of, say, French monetary policy to a German policy shock with the truly exogenous French monetary policy shock. The fact of not controlling for the German leading role in the EMS causes the system to be misspecified. As shown in Clarida, Gali and Gertler (1998) for instance, the policy rule for France and Italy contains as a significant explanatory variable the German policy rate. If this is the case, inference drawn from a French-specific VAR will lead to wrong conclusions about the French transmission mechanism. Even in the case in which the country-specific transmission was identified correctly by controlling for all the exogenous effects, this is not the main focus of analysis. The objective, once again, is to measure the effects of a *common* monetary policy on heterogeneous units.

Second, country-specific VARs do not include any foreign variables. The covariance structure and the interdependences across countries are not exploited. Given that European countries are clearly interrelated, it seems sensible to have methods suited to analyze all the countries together, allowing for a reduction in the number of parameters to be estimated, but at the same time allowing for

heterogeneity in the propagation mechanism of shocks.

A third remark comes from the observation that the size of the impulse response functions (IRFs) and the comparison across countries depend crucially on the size of the shock. IRFs have been typically computed in response to a one-percent shock equal across countries, or to a country-specific one standard error shock. The shape of the responses will be the same, but the magnitudes will change. If the VAR is used to study the transmission in a single country, this does not create any problem: it is normal that changing the size of the impulse, will also modify the size of the responses. What happens when country-specific VARs are combined to draw cross-country inference?

Suppose one uses uniformly across countries the one standard error shock IRFs. As the standard errors of monetary shocks are in general different across countries, it will not be possible to tell apart the differences in the shocks from the differences in the transmission. A comparison based on these premises will be flawed, because it will not be able to isolate the origin of the movements from the induced movements themselves. Is the bigger response in country A than in country B caused by a bigger initial shock or is it a consequence of underlying structural differences in the transmission?

Suppose alternatively that the researcher decides to use the one-percent shock IRFs. In this case the shocks will be equalized across countries, but the relative magnitudes will be modified. Again, this will not allow a meaningful comparison: a one-percent shock may be a "normal" episode in country A, while it may be a completely unprecedented episode in country B.

Another related point is that also the dynamic path of the interest rate is in general different across countries, once again complicating the comparison.

Finally, the identification assumptions for different countries are often difficult to compare and to justify.

Recently, a number of papers have addressed some of the criticism exposed above.

Mihov (2001), Mojon and Peersman (2001) and Clements, Kontolemis and Levy (2001) estimate country-specific VARs, controlling for the leading role of Germany. The first two papers compute IRF to country-specific monetary shocks, the third computes IRF in response to German shocks.

Recognizing the existence of interdependencies across European countries, Ciccarelli and Rebucci (2003) estimate a three-countries Bayesian VAR and study whether the monetary transmission mechanism has changed across time. Kieler and Saarenheimo (1998) nest together 3 VARs for Germany, France and UK and estimate the system with SUR. A careful identification exercise is performed, monetary shocks are identified in each country, but they are left correlated across countries: it is then difficult to disentangle the effect of a German shock from a French shock and to identify the common shock.

A different approach is followed by Carlino and DeFina (1998b)). They draw from their previous work (Carlino and DeFina $(1998a)$) on state-specific effects of monetary policy in the US. They first estimate separate VARs for each state and identify the response of state personal income to a US-wide monetary shock. Second, they perform a cross-sectional regression of the state-specific responses

on a set of explanatory factors, proxies for different channels of the monetary transmission. Third, they build an index of sensitivity of European countries to monetary shocks by multiplying the regression coefficients estimated for the US by the values of the corresponding explanatory factors for each European country.

It is worth to recall that Kieler and Saarenheimo (1998), recognizing some of the limitations we have listed above, suggest the kind of modelling strategy we use in this paper as an interesting direction for future research. They write: "One could imagine decomposing the effects to, say, international monetary shocks, country-specific shocks, etc.. $[\dots]$ We think that structural identification at the multi-country level might be an interesting path for future work".The same view is shared by Guiso et al. (1998). They write: "there has been very little work on [identifying the responses to monetary shocks] for multiple countries using a common framework. [...] Our reading of the literature is that this kind of study has yet to be done".

This paper is precisely trying to fill this gap.

3 Factor Models

3.1 Why Factors?

Before going into the details, it is useful to understand the modeling philosophy underlying a factor model. A factor model is an unobservable components model. Each time-series is assumed to be composed by two parts: one, the common component, is driven by a small number of shocks that are common to the entire panel, the other, the idiosyncratic, is driven by a series-specific shock, with no interactions with the rest of the panel. The idea is that there are only few random forces generating the comovements in the economy.

As it will be clearer in the following, the intuition behind the estimation is to find ways to "kill" the idiosyncratic component, so to concentrate on the common part..

Why do this kind of modelling approach is better suited for the problem at hand? First, it allow us to use all the cross-country information at the same time. Second, it allow to identify the *common monetary policy shock* and not only the country-specific. Third, by reducing the dimensionality of the pervasive shocks space, it reduces the number of identifying restrictions. Fourth, it still allows for complete heterogeneity in the IRFs to a common shock.

3.2 The Econometric Model

Let us now introduce the econometric model².

Suppose we have a number of countries, indexed by i. Suppose that each of them can be represented by a structural equation of the form:

$$
y_t^i = A^i(L)u_t + \varepsilon_t^i \tag{1}
$$

 2 This section is based on Forni and Reichlin (1996 and 1998)

where y_t^i is a $(m \times 1)$ zero-mean covariance-stationary vector stochastic process $y_t^i = (y_t^{1i}, y_t^{2i}, ..., y_t^{mi})'.$

 $\varepsilon_t^i = (\varepsilon_t^{1i}, \varepsilon_t^{2i}, ..., \varepsilon_t^{mi})'$ is a $(m \times 1)$ vector of country-specific, idiosyncratic shocks, possibly autocorrelated, but mutually orthogonal at all leads and lags across countries, with variances bounded above by the real numbers σ_s^i , $s =$ 1...m.

 $u_t = (u_t^1, u_t^2, ..., u_t^q)'$ is a $(q \times 1)$ vector of unit variance white noises, the *common structural shocks*, mutually orthogonal and orthogonal to ε_t^i for all i, and with $q < m$.

 $A^{i}(L)$ is a $(m \times q)$ matrix of rational functions in the lag operator L.

The intuition behind the method is that by using a Law of Large Numbers argument, thus by averaging across sectors, the idiosyncratic component vanishes with respect to the common. This implies that we will be able to find aggregates Y_t generated only by the common shocks u_t^3 :

$$
Y_{t} = \begin{pmatrix} \sum_{i=1}^{n} \omega^{1i} y_{t}^{1i} / \sum_{i=1}^{n} \omega^{1i} \\ \sum_{i=1}^{n} \omega^{2i} y_{t}^{2i} / \sum_{i=1}^{n} \omega^{2i} \\ \vdots \\ \sum_{i=1}^{n} \omega^{mi} y_{t}^{mi} / \sum_{i=1}^{n} \omega^{mi} \end{pmatrix} = A(L)u_{t}
$$
 (2)

where Y_t is a $(m \times 1)$ vector, u_t is a $(q \times 1)$ vector, and the matrix $A(L)$ have dimension $(m \times q)$.

3.3 How to Determine the Number of Factors

Define the $(m \times 1)$ vector Y_t of aggregates, as in equation (2), $Y_t = A(L)u_t$, where the weights ω^{hi} are set equal to $1/\sigma^{hi}$, $\sigma^{hi} = Var(y^{hi})^4$.

The dimension of the common shocks space can be recovered in the following way. From equation (2), recall that the $(m \times 1)$ vector Y_t is generated by q shocks $(q < m)$.

This in turn means that the $(m \times m)$ spectral density matrix of Y_t , $f_Y(\lambda)$ (with $\lambda \in [0, \pi]$) will have reduced rank q.

We can then decompose the matrix $f_Y(\lambda)$ in terms of its dynamic eigenvectors and eigenvalues⁵^{,6}: $f_Y(\lambda) = P(\lambda)\Lambda(\lambda)P(\lambda)'$. $\Lambda(\lambda)$ is a diagonal matrix containing the dynamic eigenvalues(DE) sorted according to their magnitude

³The proof can be found in Forni and Reichlin (1998), Proposition 1.

⁴The problem of selecting the weights ω^{hi} is relevant when the cross-sectional dimension is not very large, as it will be in our case. The weights minimizing the variance of the local components are given by $\omega^{hi} = 1 / Var(\varepsilon^{hi})$. As $Var(\varepsilon^{hi})$ is not known, then it is reasonable to assume $\omega^{hi} = 1 / Var(y^{hi})$. This is similar to what is done in GMM estimation by using the "optimal weighting matrix", which is nothing else than the inverse of the variance-covariance matrix.

⁵This is the standard decomposition of a matrix in terms of eigenvalues and eigenvectors. One more dimension is added here: the decomposition is performed frequency-by-frequency.

 6 For a detailed discussion of the properties of dynamic eigenvalues and eigenvectors and for the related concept of dynamic principal components, see Brillinger (1981).

from the biggest to the smallest, for each frequency λ . The columns of $P(\lambda)$ contain the dynamic eigenvectors associated with each eigenvalue.

The rank of $f_Y(\lambda)$ and the number of common shocks q correspond to the number of eigenvalues different from zero at each frequency⁷.

In practice, the dimension of the vector u_t can be recovered by a informal test, by checking graphically the rank of the spectral density matrix, or in other words, by checking how many DE are needed to capture most of the trace of $f_Y(\cdot)$ across frequencies⁸.

3.4 Estimation of the Common Component

Having determined the number of factors q , we can estimate the common component as follows.

Consider q of the m weighted averages⁹ in Y_t , call this vector Y_t^q . We can then rewrite: $Y_t^q = A^q(L)u_t$, where $A^q(L)$ is $(q \times q)$. From $Y_t^q = A^q(L)u_t$, under the assumptions of non-singularity of Y_t^q and fundamentalness¹⁰ of u_t , there exists a finite VAR representation of the form $u_t = A^q(L)^{-1}Y_t^q$. We can then substitute in (1) to get: $y_t^i = A^i(L)A^q(L)^{-1}Y_t^q + \varepsilon_t^i$. From this, one can see that the common components of all the series y_t^i lie in the space spanned by the present and past values of Y_t^q . The common component can then be consistently estimated equation-by-equation by OLS, using as regressors Y_{t-k}^q , $k = 0, ..., K$.

It can be easily proved that the weights ω^{hi} minimizing the variance of the aggregate local component are given by $1/Var(\varepsilon^{hi})^{11}$. As ε^{hi} is unknown before estimation, we can follow an iterative procedure:

- 1. Start with $\omega^{hi} = 1/Var(y^{hi})$, and compute Y_t^q as in (2).
- 2. Regress y_t^{hi} on Y_{t-k}^q , for $k = 0, ..., K$, get the regression residuals $\hat{\epsilon}^{hi}$, compute $\omega^{hi} = 1/Var(\hat{\varepsilon}^{hi})$, and a new Y_t^q .
- 3. Iterate until convergence of all Y_t^q is achieved. At the end of the procedure final estimation of common and idiosyncratic components will be obtained.

 7 The rank of a matrix is given by the number of its eigenvalues different from zero. Recall also that: $Trace(f_Y(\lambda)) = Trace(\Lambda(\lambda))$ at any λ .

⁸A similar test has been proposed by Phillips and Ouliaris (1988). They propose a formal cointegration test based on the rank of the spectral density matrix at frequency zero, conducted by using the same decomposition.

⁹As the rank of Y_t^q has to be equal to q, the component have to be chosen in such a way as to differentiate the aggregates as much as possible.

¹⁰If u_t are non-fundamental, the common components are spanned by the past, present and future of the u_t . Only under the assumption of fundamentalness of u_t we can restrict our attention to the present and the past.

 11 The intuition is clear: as the weighted average must be such that the idiosyncratic component vanishes, the series with the higher idiosyncratic component must receive a small weight in the aggregation. The proof can be found in Forni and Reichlin (1996).

3.5 Recovering the Structural Shocks

Up to now we have estimated two unobserved components: the common and the idiosyncratic. The last step is then to estimate the structural common shocks u_t .

As we have seen, Y_t^q spans the same space as the *structural* common shocks u_t , so that we can rewrite it in VAR form: $A^q(L)^{-1}Y_t^q = u_t$.

We can then estimate a finite order VAR and apply the identification techniques developed for VARs.

More precisely, we can follow the standard steps:

- 1. Estimate the *reduced form* $B^{q}(L)^{-1}Y_t^q = v_t$, where $Cov(v_t) = \Sigma$.
- 2. Orthogonalize the shocks, using the Cholesky decomposition: $\tilde{u}_t = C v_t$, where $Cov(\tilde{u}) = I_q$
- 3. Pick among the infinite orthogonal rotation matrices R such that $RR' = I$ the one that satisfies our identification assumptions: $u_t = R\tilde{u}_t$, where $Cov(u) = I_q$

Once the matrix R has been chosen, we can construct the IRF of all the variables y_t^i to the *common* shocks u_t .

4 European Monetary Policy

We are now ready to discuss the analysis for European countries.

We use monthly data on the sample 1985:01-1998:12. The choice of this sample is motivated by the fact that only in the mid-eighties exchange controls, which might have affected the transmission of monetary policy, were lifted (Dornbush, Favero and Giavazzi (1998)). We consider eight European countries: Austria, Belgium, France, Germany, Italy, the Netherlands, Portugal and Spain. For each country we use the following set of variables: $x_t^i =$ $\{IP_t^i, CPI_t^i, INT_t^i, NOM_t^i\}.$ IP is the logarithm of the Industrial Production Index, CPI is the logarithm of the consumer price index¹², INT is the shortterm nominal interest rate, NOM is the nominal exchange rate with respect to the US dollar, defined in units of national currency per one dollar.

¹²Raw data on prices are not deseasonalized. The filter $(1 - L^{12})$ is applied to CPI_t to remove the seasonal component.

Table 1: Unit Root ADF Test on NOM_t					
Country	ADF Test Statistic $(\# \text{ lags})$				
\rm{AUT}	$-3.76***$ (4)				
BEL	$-3.63***$ (4)				
FRA	$-3.70***$ (10)				
GER	$-3.81***$ (4)				
ITA	$-2.74*$ (12)				
NED	$-3.78***$ (4)				
POR	$-2.96**$ (12)				
SPA	$-2.68*$ (12)				
*: significance at $10\%,$ **: $5\%,$ ***: 1%					

According to standard unit-root tests (summarized in Table 1 below), the variables IP_t^i , IN F_t^i and INT_t^i are difference-stationary, while NOM_t^i is $I(0)$ in the sample considered. For each country we then consider the vector: $y_t^i =$ $\{\Delta IP_t^i, \Delta INF_t^i, \Delta INT_t^i, NOM_t^i\}$, with all the series standardized.

4.1 Determining the Number of Factors

The first step is to determine the number of factors. We then construct the 4 aggregates Y_t , where Y_t is defined as in equation (2), using weights $\omega^{hi} =$ $1/Var(y^{hi})$. The cumulated Dynamic Eigenvalues (DE) of the spectral density matrix of the standardized version¹³ of Y_t are shown in Figure 1. The frequencies $[0, \pi]$ are reported on the horizontal axis, the percentage of variance explained by the first n DE is reported on the vertical axis.

As there is no formal test for the rank of the spectral density matrix, we have to use a graphical and heuristic criterion: we conclude that our data are characterized by q factors if q DE are sufficient to explain 90% of the variance across frequencies. We conclude that our dataset is characterized by 3 common factors.

[Insert Figure 1 about here]

4.2 The Common Component

As we have seen, the common components can be consistently estimated by OLS equation-by-equation.

We use the iterative procedure explained above, using as regressors the components of the vector $Y_t^{\bar{q}} = \{Y_t^{IP}, Y_t^{\bar{I}NT}, Y_t^{NOM}\}.$

A finite VAR representation for Y_t^q exists only under the assumption of its non-singularity. We check if this assumption is satisfied by the components chosen, by computing the squared coherence between the elements of the vector Y_t^q . Squared coherence is a sort of dynamic R^2 for bivariate stochastic

¹³Standardization is necessary because otherwise the series with the higher variance will receive a big weight in the construction of the DE, biasing the results towards a smaller number of factors than the true one.

processes. It is defined for each frequency λ , as: $\left[\Xi_{12}(\lambda)\right]^2$, where: $\Xi_{12}(\lambda) =$ $f_{12}(\lambda)/[f_{11}(\lambda) f_{22}(\lambda)]^{1/2}$. $f_{12}(\lambda)$, $f_{11}(\lambda)$ and $f_{22}(\lambda)$ are respectively, the offdiagonal and the diagonal elements of the spectral density matrix of the vector stochastic process.

[Insert Figure 2 about here]

High values of squared coherency at any frequency would be an indication of stochastic singularity. It is clear from Figure 2 that in our case there is no evidence of any rank reduction at any frequency.

We then used a constant, the contemporaneous value and $p = 2$ lags of Y_t^q in the estimation¹⁴. The initial weights were set to $\omega^{ih} = 1/Var(y^{ih})$.

Convergence is achieved after few iterations and the final weights, reported in Table 2, have interesting economic interpretations:

				BEL FRA GER ITA NED POR SPA	
				0.11 0.11 0.15 0.11 0.14 0.14 0.10	0.14
				INT 0.12 0.09 0.16 0.12 0.18 0.12 0.12	-0.09
$NOM = 0.34$		$0.02 \qquad 0 \qquad 0.32$		0 0.32	

Table 2: Optimal weights after the iterative procedure

The only countries with a significant positive weight on the exchange rate are Germany, Austria and the Netherlands. Austria and the Netherlands were the countries with an exchange rate closely pegged to the DM during the whole sample, in other words, with a "more common" exchange rate. The weight on the other countries is zero because of the various realignments of the exchange rate that increased their idiosyncratic variance. It is true that in some historical episodes, namely during the September 1992 crisis, the realignments have been "common", as many countries went out of the EMS at the same time and one would expect the weights to reflect this "commonality". Nevertheless, the weights are constructed not only as cross-sectional averages. They take also into account the time-series properties of the data as they are obtained via the OLS regression. They reflect the average behavior during the whole sample period.

4.3 Identifying the Structural Shocks

The last step is the identification of the monetary shock in the structural VAR: $A^{q}(L)^{-1}Y_{t}^{q} = u_{t}$, where $Cov(u_{t}) = I_{q}$.

We estimate the reduced form $B^{q}(L)^{-1}Y_{t}^{q} = v_{t}$, with $Cov(v_{t}) = \Sigma$, using 12 lags.

Notice that the rank reduction $q = 3$ reduces the number of restrictions needed. Indeed, we only need $3 = q(q-1)/2$ restrictions to identify the whole model for all European countries¹⁵.

¹⁴Results are robust to the choice of p and to the exclusion of the constant.

 15 If we used a typical 4-variables country-specific VAR for each of the 8 countries, we would have had to impose $8(4(4-1)/2) = 48$ zero restrictions!

As we do not have any clear view about how to identify directly the VAR for the aggregates and as we do not want to impose any a priori zero-restriction, we use an agnostic criterion (for a similar approach, see Lippi and Reichlin (1994a) or Uhlig (1999)), based on orthonormal rotations of the shocks space and on a minimization criterion.

Our assumption is that the common monetary shock during the EMS period in Europe can be identified with the German one, so we use this information in the identification strategy.

First, we fit a VAR for Germany alone, following the specification presented in Bernanke and Mihov (1997). The two authors estimate a VAR with $Y_t =$ $\{IP_t, log(WORLD_t), log(CPI_t), INT_t\}$, where the variable $WORLD$ is the Commodity Price Index¹⁶.

Second, we search for the rotation matrix R that minimizes the distance between the IRF of the German interest rate INT^{GER} to the monetary shock identified from the German-only VAR \dot{a} la Bernanke and Mihov and the IRF of the same variable INT^{GER} to one of the three common shocks u_t generating the factor model.

Let us explain in detail our procedure by going one step back. Recall that to identify a VAR we can proceed in the following way.

Orthogonalize the shocks, using the Cholesky decomposition: $\tilde{u}_t = Cv_t$, where C is s.t. $Cov(\tilde{u}) = I_q$. Choose among the infinite orthogonal rotation matrices R such that $RR' = I$ the one that satisfies our identification assumptions: $u_t = R\tilde{u}_t$, where $Cov(u) = I_q$.

For an m -dimensional system, any rotation matrix R can be parameterized as function of $m(m-1)/2$ parameters. In our 3 shocks case, any rotation matrix R can be parameterized as follows:

$$
R(a, b, c) = \begin{pmatrix} \cos(a) & \sin(a) & 0 \\ -\sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(b) & 0 & \sin(b) \\ 0 & 1 & 0 \\ -\sin(b) & 0 & \cos(b) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(c) & \sin(c) \\ 0 & -\sin(c) & \cos(c) \end{pmatrix}
$$

with $(a, b, c) \in [0, 2\pi]$.

We look for the matrix $R(a, b, c)$ that minimizes the distance between the IRF of INT^{GER} to the German monetary policy shock obtained from the Bernanke and Mihov VAR, call it $(u^{VAR} \rightarrow INT^{GER})(t)$ and the IRF of INT^{GER} to the shocks $u_i^{FACTOR}(a, b, c)$, $i = 1, ..., 3$, obtained from the factor model, call it $(u_i^{FACTOR}(a, b, c) \rightarrow INT^{GER})(t)$, by using a quadratic loss criterion:

$$
\Pi(a,b,c,i) = \sum_{t=1}^{hor} [(u^{VAR} \rightarrow INT^{GER})(t) - (u_i^{FACTOR}(a,b,c) \rightarrow INT^{GER})(t)]^2
$$
\n(3)

The shock identified by the parameters (a, b, c, i) such that the function $\Pi(\cdot)$ reaches its minimum will be denoted as the *common European monetary policy*

 16 We identify the model with a simple Choleski decomposition, while Bernanke and Mihov (1997) use a more "refined" identification, based on information on the institutional framework in which the Buba conducts its monetary policy. The results are essentially the same.

 $shock$ ¹⁷. In Figure 3 we report the IRF obtained from the VAR and the one from the factor model such that $\Pi(a, b, c, i)$ reaches its minimum.

[Insert Figure 3 about here]

Having identified the *common shock*, we can now compute the responses to it of all the variables in the system.

Results for each of the 4 variables in the system are reported in Figure 4. The response labeled EUROPE represents a weighted average¹⁸ of the countryspecific responses.

[Insert Figure 4 about here]

The responses of IP are in line with what predicted by theory: a monetary contraction causes a reduction in industrial production after about 6 months in all countries. It is interesting to note that the European response to a common monetary shock virtually coincides with the German response, thus confirming indirectly our identification strategy concerning the leading role of Germany. There are nevertheless differences in the responses across countries. In order to understand these wide differences we move to analyze the response of the other variables.

Our experiment allows different endogenous responses of the interest rates. We can see that all the countries increase their interest rates and that all of them have to perform an endogenous monetary restriction stronger than Germany. This behavior can be explained by the need for all countries to show their tightness in the conduct of monetary policy during the EMS period, in order to gain credibility and reduce inflation expectations. Italy and France are the countries characterized by the stronger increases.

The same kind of behavior is evident in the responses of the inflation rates. All countries display a deeper reduction in inflation than Germany.

Interesting results come from the analysis of the exchange rates. Without imposing any predeterminedness assumptions between interest rates and exchange rates, we see that exchange rates appreciate on impact and they then depreciate to the old equilibrium, as predicted by the Dornbush (1976) overshooting model. We can notice a clear distinction between the peripheral countries, Italy, Portugal and Spain, the countries that adjusted more often the exchange rate parity during the EMS period, and the rest of Europe.

5 The New Environment for the ECB

In the new environment, monetary policy will not be decided taking Germany as the leader. It is likely that the ECB will target a weighted average of countryspecific indicators.

¹⁷The identifying assumption is really "light": we just ask that as a consequence of a monetary shock (an increase in the interest rate), the interest rate indeed increases on impact and then decreases (of course following the German response).

 18 The weights are given by the share of each country in the European GDP.

In order to shed light on the conduct of monetary policy by the European Central Bank, we perform a counterfactual experiment, constraining the interest rate responses of all countries to follow the European average response.

Given this constrained interest rate response we recomputed the IRFs for the Industrial Production. Results are shown in Figure 5. We can notice that Italy and to a lesser extent, France and the Netherlands are now characterized by a smaller-than-average response, while Germany and Spain are characterized by a very large response in comparison to the other countries. The strong endogenous increase in the interest rate in France and Italy in the pre-EMU regime was responsible for much of the restrictive effect of monetary policy on output. The other countries will not be far away from the European average.

[Insert Figure 5 about here]

We follow Carlino and DeFina (1998a,b) and we relate the cumulative responses of IP to proxies of different theories of the transmission mechanism. Having only 8 observations, we cannot add more than one regressor at time. Given the limited amount of degrees of freedom, our results are to be interpreted with caution; nevertheless, they provide interesting insights.

Our analysis is similar to the one performed by Mihov (2001). He finds that significant explanatory variables for the strength of monetary policy in his country-specific VAR analysis are the share of the manufacturing sector and some indicators of the credit channel. It is important to notice that his sample is composed of 10 countries, 5 of which are non-Euro countries. Some of his results may be driven by the inclusion of these countries in the analysis.

We run univariate regressions of the three indexes discussed above, our IRFs computed 24 months after the shock (called Factor), the Mihov index and the Carlino and DeFina index, on various indicators of the interest rate channel and of the credit channel. We use as proxies: TOT, the share of the manufacturing + construction sector (from Carlino and DeFina (1998b)), LOANS1, the ratio of bank loans to total liabilities (from Mihov (2001)), LOANS2, bank loans as a percentage of all forms of finance (from Cecchetti (1999)), SMALL, the percentage of small firms (from Carlino and DeFina (1998b)) THOM, the Thomson Index of bank health (from Cecchetti (1999)), CONC, the concentration ratio of the three largest commercial banks (from Carlino and DeFina (1998b)) and EF-FECT, the predicted effectiveness of monetary policy (from Cecchetti (1999)). The figures are reported in Table 3.

Table 3: Proxies for the transmission channels

	AUT	BEL	FR A	GER.	ITA	NED	POR.	SPA
TOT	0.29	0.26	0.27	0.37	0.30	0.24	0.33	0.32
LOANS1	0.14	0.08	0.08		0.09	0.04		0.1
LOANS ₂	0.65	0.49	0.49	0.55	0.5	0.53	0.62	0.58
SMALL		0.55	0.56	0.57	0.78	0.61	0.79	0.74
THOM	2.38	$\mathcal{D}_{\mathcal{A}}$	2.28	1.97	2.57	2.1	2.3	1.79
CONC	0.17	0.44	0.33	0.24	0.28	0.60		0.34
EFFECT	2.67	$1.33\,$	2.33	2.33	2.67	1.67	2.33	$\mathcal{D}_{\mathcal{L}}$

Sources: see text

Table 4: Regression Analysis

	CdF	Mihov	Factor
TOT	3.77(1.83)	11.02(5.08)	2.38(2.03)
LOANS1	2.03(0.49)	8.88 (3.08)	0.58(0.26)
LOANS2	1.40(0.85)	4.06(0.91)	0.80(0.82)
SMALL	0.25(0.22)	0.05(0.01)	-0.18 (-0.27)
THOM	$-0.71(-2.42)$	$-0.46(-0.34)$	-0.49 (-3.57)
CONC	-0.28 (-0.33)	-2.55 (-1.87)	$-0.19(-0.36)$
EFFECT	-0.16 (-0.79)	0.72(1.08)	-0.04 (-0.34)

t-ratios in brackets

In Table 4 we show results for univariate regressions. Regressions with the Mihov index contain at maximum 5 observations.

The only variable that turns out to be significant consistently across studies is the share of the (manufacturing $+$ construction) sector. We thus find evidence for a role of the interest rate channel in explaining the asymmetries. We do not find any significant role for the credit channel and if any, the evidence goes in the opposite direction, as healthier banking systems (measured by low values of the Thomson Index) appear to be very sensitive to monetary policy and the same is true for the Carlino and DeFina index.

Our findings extend their results for US states to European countries. Once we control for the limitations in the country-specific VARs outlined above, the credit channel does not seem to play a first-order effect in explaining the asymmetries. In Table 5 we compare our results with those in CDF. It is evident that the ranking provided by the different methods are very similar, particularly for the "extreme" countries. Regression of Factor on CDF gives a coefficient of 0.45 with a t-statistic of 2.89.

Ranking in brackets

We conclude that in the new environment in which the ECB is operating Spain and Germany will be the countries more sensitive to monetary policy, while Italy and the Netherlands and France will be the less. Results for the other countries show that they will be in the European average.

Interesting observations can be drawn by comparing our results with those in Wieland (1996). In that paper a large-scale macroeconometric model is estimated for Germany, France and Italy. It is then shown that an asymmetric monetary regime with Germany as leader, as the one that was functioning during the EMS period, allowed the Bundesbank to reduce output and inflation volatility, the other 2 countries had to bear the burden of the adjustment. Similar conclusions can be drawn from our results in Figure 3. On the other side, a symmetric regime in which European averages are taken as targets (similar to the one we have simulated by constraining the interest response in Figure 4) reduces variability in France and Italy at the expenses of a higher variability in Germany (and this is what happens in our estimates as well: the response of Germany is much stronger than that of France and Italy under the symmetric rule).

6 Conclusions

In this paper we propose a new way to identify common European monetary policy shocks.

The econometric technique, a dynamic factor model, allows us to find the number of common factors in the data. We showed that our database is characterized by three common factors. The identification assumption was that European monetary policy followed Germany during the EMS period. We develop a method to identify the common shocks suited to give formal content to the informal statement about the leading role of Germany in the conduct of monetary policy.

Results are in line with the theoretical literature about the effects of monetary policy. The dynamic responses of European countries show that they will be hit asymmetrically by the same shock.

We conclude that Spain and Germany will be the countries the more sensitive to monetary policy, while the Netherlands, France and Italy will be the less. The interest rate channel is significant to explain the differential responses, while we find no role for the credit channel.

A caveat and a proposal for further research are necessary at this point. Our analysis relies on past data. Given that the establishment of the ECB represents a clear change in regime, we are fully aware of the risks of drawing conclusions regarding the future by looking backwards.

On the other side, we believe that the factors influencing the monetary transmission will not change immediately across Europe. A challenging research agenda should then try to study how agents and institutions' behavior will endogenously adapt to the new environment.

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7 Appendix 1: Common Factors and Common Trends

This appendix provides a link between the Factor Model we are using and the Common Trends model \dot{a} la Stock and Watson (1988). Let us briefly recall the Stock and Watson model.

Suppose X_t is a $(n \times 1)$ vector of $I(1)$ processes, whose first differences have the Wold representation: $\Delta X_t = C(L)\varepsilon_t$, with $Cov(\varepsilon) = \Sigma$. This can be rewritten in terms of orthogonal shocks as: $\Delta X_t = C(L)\Sigma^{1/2} \nu_t$, where $Cov(\nu)$ = I.

Stock and Watson proved that in presence of $n - q$ cointegration relations among the X_t , the system can be rewritten in the common trends representation:

$$
X_t = A\tau_t + D(L)\nu_t \qquad \tau_t = \mu + \tau_{t-1} + \eta_t \qquad (A1.1)
$$

where A is a $(n \times q)$ matrix of loadings, τ_t is a $(q \times 1)$ vector of common trends, random walks generated by the η_t , which are in turn function of the original shocks ν_t^{19} .

In this representation, the vector X_t is decomposed in two parts. The first, $A\tau_t$, is driven by q common factors τ_t , random walks with permanent effects on the X_t . The second, $D(L)\nu_t$, is driven by n white noises and has transitory effect on X_t .

The factor model we use can be compactly rewritten as:

$$
\begin{pmatrix} x_t^1 \\ x_t^2 \\ \vdots \\ x_t^n \end{pmatrix} = \begin{pmatrix} A^1(L) \\ A^2(L) \\ \vdots \\ A^n(L) \end{pmatrix} u_t + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \vdots \\ \varepsilon_t^n \end{pmatrix} \Rightarrow X_t = A(L)u_t + \varepsilon_t \qquad A1.2
$$

where X_t is a $(nm \times 1)$ vector that stacks the n cross sectional units, each represented by the $(m \times 1)$ vector x_t^i , $A(L)$ is a $(nm \times q)$ polynomial matrix. As we have seen, the common shocks u_t are not constrained to affect only the long-run behavior of the system, as in the common trends model, but they generate transitory dynamics.

It is then possible that while q common shocks are present in the system, only $k < q$ of them have permanent effects and are thus the equivalent of the τ_i 's in equation $(A1.2)^{20}$.

The link between the two models can be then formulated by analyzing the spectral density matrix $f_X(\lambda)$ at frequency zero.

¹⁹This decomposition does not imply automatically a separation of the orthogonal shocks ν_t between transitory and permanent. Cointegration just tells us that some linear combinations are transitory and others are permanent. We need to impose more identifying restrictions in order to separate between permanent and transitory shocks among the ν' s (see Blanchard and Quah (1989), King et al. (1991) or Warne (1993)).

²⁰This is true under the additional assumption that the idiosyncratic components of all the series are $I(0)$.

Cointegration can be studied in the time and in the frequency domain. The analysis of the rank of the spectral density matrix of $f_X(\lambda)$ at zero frequency provides the same information as a time-domain cointegration test on X_t . Given that in our case the dimension of the system can be very large and traditional time-domain cointegration tests are unfeasible, a valid alternative is to work in the frequency domain. The rank of $f_X(\lambda)$ at frequency zero will then equal the number of common stochastic trends driving the system²¹.

Following the approach explained in Section 2.1, this implies that if the system is characterized by k common trends, then only k dynamic eigenvalues will be necessary to explain the trace of the spectral density at frequency zero, while q of them will be necessary at higher frequencies (for a formal test, see Phillips and Ouliaris (1988)).

As in the presence of cointegration a finite VAR representation for the differences does not exists, the procedure requires the estimation of a VECM for the vector of the q aggregates spanning the space of the common shocks u_t with the imposition of $n - k$ cointegration relations.

 21 Recall that cointegration implies a rank reduction in the matrix of long run coefficients C(1) of the Wold representation. The spectral density $f_X(\lambda) = \frac{C(e^{-i\lambda})^2 \frac{\sigma^2}{2\pi}}{2\pi}$ at frequency zero is: $f_X(0) = |C(1)|^2 \frac{\sigma^2}{2\pi}$. The rank of $C(1)$ is the magnitude through which cointegration can be studied both in time and in frequency domain.

8 Appendix 2: Partial Identification

In Uhlig (1998) , it is shown that any impulse vector²² can be characterized as:

$$
g = \sum_{i=1}^{m} (\alpha_i \sqrt{\lambda_i}) x_i
$$
 (A2.1)

where λ_i and x_i are respectively the eigenvalues and the eigenvectors²³ of Σ and $\alpha_i, i = 1, ..., m$ are coefficients such that: $\sum_{i=1}^{m} \alpha_i^2 = 1$. It is immediate to see that there are $m-1$ degrees of freedom in the choice of g.

Let us turn to the rotation approach. As we showed in the text, a complete identification scheme is composed by the product of two matrices CR , where C is the Cholesky factor of Σ and R is an orthonormal matrix for \mathbb{R}^m In the \mathbb{R}^3 case:

$$
R(a, b, c) = \begin{pmatrix} \cos b \cos c & -\cos a \sin c + \sin a \sin b \cos c & -\sin a \sin c - \cos a \sin b \cos c \\ \cos b \sin c & \cos a \cos c + \sin a \sin b \sin c & \sin a \cos c - \cos a \sin b \sin c \\ \sin b & -\sin a \cos b & \cos a \cos b \end{pmatrix}
$$

(A2.2)

One can immediately see that the first column is function only of two parameters or, more generally, of $m-1$ parameters, as in the Uhlig approach. If one wants to identify only the first shock, he or she has to choose only $m - 1$ parameters. Recalling the definition of the Cholesky factor $C = X\Lambda^{1/2}$, one can immediately see that the first column of CR can be parameterized as g in (A2.1) and that the first column of R satisfies the same restriction $\sum_{i=1}^{m} R_{i1}^2 = 1$. As the dimension of the system increases from $m-1$ to m , the number of restrictions required to obtain a complete identification increases by $(m - 1)$. while the number of restrictions needed to obtain a "one-shock" identification increases by 1.

Let us discuss an example of how to implement a long-run restriction $\dot{a} \, la$ Blanchard and Quah (1989), using the rotation approach. Suppose that our system is composed of difference-stationary time-series. We want to identify only the first shock and our assumption is that it has a transitory effect on the levels of the first variable. From the preceding analysis we have seen that we need $m-1$ restrictions.

The Wold representation of the reduced form is: $Y_t = B(L)v_t$.

The Wold representation of the structural system is: $Y_t = B(L)CRu_t$.

The identifying restriction is: $B_{1i}(1)(CR)_{i1} = 0$, where $B_{1i}(1)$ and $(CR)_{i1}$ are respectively, the first row of the long-run matrix $B(1)$ and the first column of the matrix product CR. Recall that $(CR)_{i1}$ is function only of $m-1$ parameters.

In a bivariate system, as in Blanchard and Quah (1989), this restriction exhausts the degrees of freedom in the choice of g : we obtain a "one-shock" identification. At the same time, it corresponds to an complete identification

²²A vector $g \in \mathbb{R}^m$ is called an *impulse vector* iff there is some matrix A, such that $AA' = \Sigma$, and such that g is a column of A .

²³Normalized to form an orthonormal basis of \mathbb{R}^m .

scheme. For $m = 2$, $m(m-1)/2 = m - 1$: a "one-shock" identification coincides with a complete identification.

In a three-dimensional system, this restriction is not sufficient to get a "one shock" identification and, of course, neither a complete identification, as we need $m(m-1)/2 > m-1$ restrictions. For the case of the "one-shock" identification, we can however reduce the dimension of the "admissible" rotations (those satisfying the restriction) to the choice of only one parameter, as the other has to satisfy a typically non-linear restriction of the kind $b = f(c)$ in (A2.2).

For the case of the complete identification, we still have a reduction of the admissible rotations space. It will be defined by an equation of the form: $a =$ $g(b, c)$.

9 Appendix 3: The Data

Industrial Production Index. OECD database, code: xx2027KSA (where xx denotes the country code).

Consumer Price Index. OECD database, code: xx5241K.

Interest Rate. Austria, Belgium, Germany, Italy, the Netherlands and Portugal: Call Money (Money Market) Rate, IMF database, code: xxx60B.., France, Spain: Call Money Rate < 24 hours), OECD database, code: xx6207D.

Exchange Rate. OECD database, code: xx7003D.

Figure 1. Variance explained by Dynamic Eigenvalues

Figure 3. The result of the minimization procedure e of INTEREST RATE to a monetary shock (starred: FAC TOR, line: VAR)

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Pored Spa EUROPE

−0.1 $\mathbf 0$ 0.1 0.2 0.3 0.4

Figure 5. The ECB regime: industrial production