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Political intergenerational risk sharing

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ABSTRACT

In a stochastic two-period OLG model, featuring an aggregate shock to the economy, ex-ante optimality requires intergenerational risk sharing. We compare the level of intergenerational risk sharing chosen by a benevolent government and by an office-seeking politician. In our political system, the transfer of resources across generations is determined as a Markov equilibrium of a probabilistic voting game. Low realized returns on the risky asset induce politicians to compensate the old through a PAYG system. This political system typically generates an intergenerational risk sharing scheme that is (i) larger, (ii) more persistent, and (iii) less responsive to the realization of the shock than the social optimum. This is because the current politician anticipates her transfers to the elderly to be compensated by future politicians through offsetting transfers, and hence overspends.

1. Introduction

In times of financial troubles, public PAYG pension systems come back into fashion. Large reductions in housing prices, losses in private pension funds, and increased volatility in the stock market have large negative effects on the private wealth of the elderly, and ultimately on their consumption. Real annuities–such as public pension benefits–may instead guarantee stable old age consumption, albeit typically at the cost of lower average returns on the contributions paid during the working period. In other words, PAYG pension systems entail an important intergenerational risk sharing component that proves crucial in periods of high financial instability.

This paper focuses on the role of the intergenerational risk sharing as a crucial motivation for the existence of social security systems. We characterize the optimal risk sharing mechanism introduced by a benevolent government and compare it to the social security system designed by office-seeking politicians, who choose the current risk sharing policy in order to win the elections — but cannot commit to future policies. We show that election-minded politicians typically prefer more spending in social security and introduce more persistent policies.

Since early contributions by Enders and Lapan (1982), Merton (1983), and Gordon and Varian (1988), PAYG social security systems have been recognized as an important instrument of intergenerational risk sharing. The demand for risk sharing stems from the uncertainty that is usually associated with endowments, wages and/or rate of returns on savings. Individuals would typically like to insure against bad realizations during their lifetimes, before they are even born.[1] If there exists a long term player, such as a benevolent government that can bind current and future policies on the behalf of yet-to-be-born generations, intergenerational risk sharing through Social Security may arise.

A parallel, but less sympathetic literature provides instead evidence on the inefficiencies and the costs of the existing, generous social security systems. Large reductions in the employment rate among middle aged and elderly workers, rising labor cost, and the crowding out effect on the stock of capital induced by the reduction in private savings are only some of the by-products of these pension systems, which have been largely criticized. The upshot of this literature is that social security spending is inefficiently large.

Bohn (2003), and more recently Krueger and Kubler (2006), took a more comprehensive approach, and consider both these costs and benefits of PAYG schemes. They suggest that the crowding out effect on the private savings may be so severe as to overweight the positive role of intergenerational risk sharing. Storesletten et al. (1999) analyze the risk sharing properties of social security systems vis-à-vis idiosyncratic risks, such as wage fluctuations and mortality risk, and reach similar conclusions. Clearly, additional considerations on the labor market distortion induced by social security would only

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[1] Even in a dynastic environment in which parents care about the well being of their kids, and thus would like to purchase some insurance on their account, private markets may fail to work, to the extent that legal contracts signed by the parents do not bind their offsprings.

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strengthen this argument. Yet, in a more general setting, which–together with capital–features a long-lived asset (such as land), Gottardi and Kubler (2006) provide more optimistic results on the existence of a (ex-ante) Pareto improving social security system. Hence, the jury is still out.

In this paper, we concentrate on the intergenerational risk sharing property of a PAYG system vis à vis aggregate shocks, and abstract from its many distortionary aspects. This approach may be relevant to understand the historical experience of the US, where the introduction of the social security system followed the 1929 stock market crash, and of several countries, such as Belgium, France, Germany, and Italy, where episodes of hyperinflation wiped out the value of the bonds issued in nominal terms and called for government's intervention to transform the existing (funded) systems into PAYG schemes (see Flora, 1987). We show that both a benevolent government and an office-seeking politician have an incentive to introduce an unfunded system if a financial market crisis that wipes out the private wealth of the elderly occurs.

We then turn to examining the main features of these risk sharing instruments, and we ask whether electoral constraints may lead politicians to choose "too much" social security spending. In our two-period stochastic overlapping generation economy, the risk comes from an aggregate shock to the stock market, which affects the net private wealth, and thus the consumption of the agents in their old age. A crucial feature of our setting is that individuals benefit from intergenerational risk sharing, but redistributive motives are absent. A benevolent government thus sets a risk sharing policy that spreads current shocks forward into the future by trading off the well being of current and future generations. The optimal linear risk sharing policy features a transfer (typically) from the young to the old, which depends negatively on the realization of the net private wealth of the elderly.

Politicians' decisions are instead driven by electoral considerations. We introduce a probabilistic voting environment in which politicians choose the current social security policy by trading off the well being of the currently alive generations of young and old individuals. We concentrate on Markov perfect equilibria of this probabilistic voting game, in which the equilibrium policy depends only on the state of the economy. A specific feature of this political equilibrium is that voting is dynamic: rational young voters anticipate that current policies affect future policy decisions by inducing changes in the state of the economy that shapes the incentives of the future politicians and voters. As in the case of a benevolent government, this mechanism allows for intergenerational risk sharing–as current shocks are spread forward into the future–although in this case the intergenerational tradeoff is driven by electoral considerations.

In our political setting, a PAYG system is more likely to be introduced during an economic crisis, and to persist in future periods. Its size depends crucially on the electoral strength–as measured by the relative share of swing (or undecided) voters among the elderly–of the old generation, who happens to face the crisis. In other words, after a financial crisis office-seeking politicians are urged by their electoral constraints to "bail out" the elderly through the provision of generous public pensions. The politicians' incentives to intervene in case of a negative shock effectively create a quasi asset–the PAYG social security–whose returns are negatively correlated to stock market returns. Interestingly, this policy turns out to be quite persistent, since less disposable income for the current young generation leads to lower net private wealth in their old age and thus to more future government intervention.

We show that this political mechanism typically produces more intergenerational risk sharing than the social optimum. Overspending stems from the strategic behavior of the politicians under dynamic voting. They exploit the expectations by current young voters, who anticipate that their current transfers will be compensated by offsetting transfers provided by future politicians. This strategic effect lowers the electoral cost to the politicians. They hence have an incentive to overspend in social security to please the current elderly voters. In other words, politicians play strategically by "bequeathing" more than optimal risk sharing on the future generations. Furthermore, these transfers are more persistent and less responsive to the realization of the shock than the optimal policy would require.

This paper relates to different strands of the literature. Several contributions have established important results on the Pareto optimal intergenerational risk sharing properties of social security both ex-ante, i.e., behind a veil of ignorance, and ad interim, that is, when the realization of one (or many) shock has occurred. The structure of our economy is closely related to Gordon and Varian (1988). Additional key contributions carried out in partial equilibrium framework include Allen and Gale (1997), Shiller (1999), Demange (2002), Matsen and Thogersen (2004) and Ball and Mankiw (2007). Instead Bohn (2003), Krueger and Kubler (2006), Gottardi and Kubler (2006) and Olovsson (2004) consider also the general equilibrium effects arising from the introduction of a social security system, such as the crowing out of the private savings.

A recent literature has analyzed the dynamics of public policies[2] under Markov perfect equilibria. In most of this literature (see, for instance, Klein et al., 2008), Markov perfection allows to capture the effect of the current governments' policies on the current economic decisions by the private agents, and indirectly on the future state of the economy, and thereby on the future policy decisions. In our environment with an aggregate shock, Markov perfection captures instead the direct impact of the current intergenerational risk sharing policy on the future state variable, and thus on the future policy, which in turn has an effect on the voting behavior of the current young (see also Grossman and Helpman, 1998).

Yet, the closer literature is perhaps the one on the political support for intergenerational risk sharing. As Orszag and Stiglitz (2001, see myth 9), we in fact recognize that, if a negative shock occurs, office-seeking politicians may decide to "bail out" the elderly. Rangel and Zeckhauser (2001) present several arguments suggesting that neither the market nor politicians are typically able to provide the optimal level of intergenerational risk sharing. Our political environment is similar to Gonzales-Eira and Niepelt (2008), who study the effect of demographic transition on Markov equilibrium social security transfers using a probabilistic voting model.[3] In the presence of demographic risks, but no uncertainty on the assets' returns, their political equilibrium features some degree of intergenerational risk sharing and redistribution, which they compare to the Ramsey allocation (i.e., with commitment). Our contribution considers instead uncertainty in the financial markets and focuses on the strategic behavior of the politicians under dynamic voting. Finally, Demange (2009) characterizes the conditions for a PAYG social security system to have political support in absence of commitment on future policies. She finds that–besides the redistributive elements often embedded in these pension systems[4]–political support depends on the degree of risk aversion of the decisive voter and on the availability of financial assets. Our results are in line with her sustainability conditions, although we abstract from redistributive motives. In addition, we are able to characterize and to compare the risk sharing policy chosen by a benevolent government and by the politicians.

The paper is organized as follows: Section 2 describes the model. Section 3 and 4 analyze respectively the equilibrium policy chosen by a benevolent government, who cares about the current and future generations, and by office-seeking politicians in a probabilistic voting model. Section 5 compares these results and provides some

[2] See Grossman and Helpman (1998), Hassler et al. (2003), but also Azzimonti (2005) and Battaglini and Coate (2008), among many others.

[3] See also Song (2008) among others.

[4] On this point, see also Casamatta et al. (2000) and Conde-Ruiz and Profeta (2007).

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comparative statics. Section 6 concludes. All the proofs are in the Appendix A.

2. A simple stochastic economy

We consider a two-period overlapping generations model of a small open economy. Every period two generations are alive: young and old. Population grows at a constant rate n. Agents are endowed with one unit of labor in youth, which they supply inelastically to receive a wage, w. Agents evaluate old age consumption only, according to an increasing and concave utility function: $U\ (c^t_{t+1})$, with $U'(.)>0$, $U''(.)<0$, where c^t_{t+1} represents consumption at time $t+1$, i.e., in old age, by an agent born at time t.

Output in the economy is given by

$$y_t = wL_t + R_tK_t \tag{2.1}$$

where K_t represents the stock of capital, i.e., the amount of savings, in the economy. Capital fully depreciates at every period. The return on capital is stochastic. Claims to capital represent the only (risky) asset in this economy, which pays a real return R_t distributed according to a cumulative function $G(R_t)$, with mean $E[R_t]=R$, variance $\mathrm{Var}[R_t]=\sigma^2$ and no serial correlation $E(R_tR_{t+1})=R^2$. Limited liability applies in the stock market to the risky asset, which also features an upper bound $\overline{R}$ on its returns, $R_t\in[0,\overline{R}]\ \forall t$. The wage is deterministic and assumed to be unitary, $w=1$.

The distribution of the stochastic returns represents a crucial element in a model that analyzes intertemporal risk sharing. Instead of recurring to a specific distribution function, we choose to consider distributions that obey to two criteria. First, we assume that the average return from the risky asset is higher than the return from a PAYG social security system, $R>1+n$. Second, we assume that the distribution is rather spread out, so that the coefficient of variation, σ/R, is greater than one. Thus, we have the following assumption.

Assumption 1. $\sigma>R>(1+n)$.

Agents save their entire net endowment for old age consumption using the risky asset, so the budget constraint of an individual born at time t is:

$$c_{t+1} = R_{t+1}(1-T_t) + P_{t+1} \tag{2.2}$$

where $T_t(<1)$ is the amount of taxes paid by the young, which is used to provide a transfer to the current old — as in a PAYG social security system, and $P_t=(1+n)T_t$ is the amount received by the old. It is also convenient to define the net private wealth of the elderly at time $t+1$ as $\omega_{t+1}=R_{t+1}(1-T_t)$.

In most of the paper, agents are assumed to have quadratic preferences, which gives rise to a mean-variance representation:

$$U(c_{t+1}) = -\frac{1}{2}(c_{t+1}-\gamma)^2 \tag{2.3}$$

where the parameter γ plays a double role. In the deterministic formulation that applies to the consumption of the elderly, once the shock is realized, γ determines the marginal utility of consumption. In the expected utility formulation that applies instead to the young, the parameter γ measures the degree of risk aversion: a lower γ characterizes a more risk averse individual.

For this utility function to feature positive marginal utility, we need to have that $\gamma>c_t\ \forall t$. A sufficient condition is that $\gamma>\overline{R}+(1+n)$. This amounts to require that the marginal utility of consumption is positive even when individuals pay no contributions in youth, obtain the largest possible return, $\overline{R}$, from their savings–consisting on their entire labor income–and receive the largest possible pension transfer, $P_{t+1}=(1+n)$, in old age.[5] We choose to impose an additional restriction on γ to guarantee that a policy consisting of a positive contribution rate imposed on the young at time t, with no corresponding pension benefit at $t+1$, is associated with a negative expected marginal utility for the young. This occurs for $\gamma>S/R$ where $S=\sigma^2+R^2$. We thus have our next assumption.

Assumption 2. $\gamma>\mathrm{Max}\{\overline{R}+(1+n), S/R\}$.

Notice that in this economy, in absence of a social security system, consumption in old age is simply equal to the realization of the return on the risky asset: $c_{t+1}=R_{t+1}$. Hence, the expected consumption at time t corresponds to the average return $E_t[c_{t+1}]=E_t[R_{t+1}]=R$, and similarly for the variance $\mathrm{Var}_t[c_{t+1}]=\mathrm{Var}_t[R_{t+1}]=\sigma^2$. Clearly, the coefficient of variation for the consumption is $\sigma/R>1$.

2.1. Individual demand for intergenerational risk sharing

In this simple economic setting, individuals have no mechanism to ensure against the risk of a negative stock market shock. In absence of intergenerational transfers, a low realization of the return on their assets affects their private wealth, and hence their consumption. With limited risk diversification our simple model economy is doomed to be ex-ante Pareto inefficient; i.e. there exist a set of intergenerational transfers that are ex-ante Pareto improving, provided that agents exhibit large enough risk aversion. Agents may hence be willing to pay a tax when young in order to receive a transfer from the next generation of individuals when old. As a simple benchmark, consider the individual demand by a member of the young generation for a hypothetical asset that promises to pay $1+n$ in every future state of the world in exchange for a unit of resource today. Call α the share of the youth unitary endowment invested in this asset. The optimization problem is:

$$\begin{aligned} &\underset{\{\alpha\}}{\mathrm{Max}} -\frac{1}{2}E_t(c_{t+1}-\gamma)^2 \\ &c_{t+1} = R_{t+1}(1-\alpha) + \alpha(1+n) \end{aligned} \tag{2.4}$$

As the following first order condition shows, this amounts to a simple portfolio decision problem in which agents have to choose how to divide their savings between a safe asset that provides a return $(1+n)$, and a risky asset with a stochastic return R_{t+1}:

$$E_t(c_{t+1}-\gamma)(R_{t+1}-(1+n)) = 0 \tag{2.5}$$

The share of savings allocated to intergenerational risk sharing is

$$\alpha^* = \frac{\sigma^2+(R-(1+n))(R-\gamma)}{\sigma^2+(R-(1+n))^2}.$$

Depending on their degree of risk aversion, the young may have preferences for some intergenerational risk sharing. In particular, $\alpha^*>0$ for a large enough degree of risk aversion, $\gamma<\frac{\sigma^2+R[R-(1+n)]}{R-(1+n)}=\frac{S-(1+n)R}{R-(1+n)}$, where $S=\sigma^2+R^2$. To consider an environment in which intergenerational risk sharing plays a role, we thus set our next assumption.[6]

[5] In what follows, we will allow the benevolent government and the politicians to choose a negative transfer–that is, a transfer of resources from the elderly to the young–at least for some realization of the shock. If this occurs, the savings of the young may exceed their labor income. However, it is straightforward to show that the consumption level implied by the equilibrium policy chosen by the benevolent government and the politicians (see Sections 3 and 4) never exceeds $\overline{R}+(1+n)$ The sufficient condition at Assumption 2 hence holds also for the following sections.

[6] It is worth noticing that Assumption 3 is consistent with Assumption 2 for distribution functions that have a sufficiently high variance, that is, if $\sigma^2>(R-(1+n))(\overline{R}-R+(1+n))$.

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Assumption 3. $\gamma < \frac{S-(1+n)R}{R-(1+n)}$.

It is immediate to see that under Assumption 3 any α such that $0<\alpha\leq\alpha^*$ implements an intergenerational risk sharing transfer scheme that is ex-ante Pareto improving.[7]

Not surprisingly, in the presence of this intergenerational risk sharing transfer, the expected consumption in old age drops, $E_t[c_{t+1}] = R-\alpha(R-(1+n))$, but also the variance decreases, $\mathrm{Var}_t[c_{t+1}] = (1-\alpha^*)^2\sigma^2$. As a result, the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing.

However, who is going to provide such a valuable (quasi-)asset? If financial markets are incomplete, for instance because a risk-free asset fails to exist, a scheme of intergenerational risk sharing may be provided by a benevolent government or by office-seeking politicians, even in absence of redistribution.[8] We will turn to the design of these programs, respectively by a benevolent government and by the politicians, in the next two sections.

3. Intergenerational risk sharing by a benevolent government

In this section, we consider the intergenerational risk sharing decision of a benevolent government which cares about the well being of current and future generations. In every period, after the realization of the shock has occurred, and hence the net private wealth of the elderly has become known, a benevolent government decides whether to transfer resources from the young to the elderly (or vice versa).

The benevolent government optimization problem at time t is thus

$$\underset{\{T_{t+i}\}_{i=0}^{\infty}}{\mathrm{Max}}\ U(c_t) + \delta(1+n)E_tU(c_{t+1}) + \delta^2(1+n)^2E_tU(c_{t+2}) + \dots \tag{3.1}$$

subject to the budget constraint at Eq. (2.2), where $\delta<1/(1+n)$ represents the benevolent government's discount rate, and the utility function is at Eq. (2.3). Individual agents take no economic decisions, while the government decision variable is the policy, T_t. The state variable is $\omega_t = R_t(1-T_{t-1})$, which characterizes the net private wealth of the elderly at time t. We can thus use the following recursive formulation:

$$V(\omega_t) = \underset{\{T_t\}}{\mathrm{Max}}\left\{U(\omega_t, T_t) + \delta(1+n)E_tV(\omega_{t+1})\right\}. \tag{3.2}$$

The first order condition of this optimization problem with quadratic preferences as at Eq. (2.3) is:

$$-(\omega_t + (1+n)T(\omega_t) - \gamma) + \delta E_t(\omega_{t+1} + (1+n)T(\omega_{t+1}) - \gamma)R_{t+1} = 0. \tag{3.3}$$

The former term represents the marginal utility for the elderly of an increase in their consumption due to the intergenerational transfer, whereas the latter represents the expected reduction in marginal utility for the young from lower future consumption. To solve this optimization problem, we guess a linear policy, $T(\omega_t) = A + B\omega_t$, and verify that it satisfies Eq. (3.3). Recall that $S = \sigma^2 + R^2$. The next proposition characterizes the optimal interior linear policy function.

Proposition 3.1. *If $\delta\in\Lambda = \left(\frac{1}{R}, \frac{1}{1+n}\right)$, there exists a linear policy function $T^G(\omega_t) = A + B\omega_t$, that solves the benevolent government problem at Eq. (3.1), with $T^G(\omega)<1\ \forall\omega$, and $T^G(\omega)>0$ for some ω, where*

$$B = -\frac{1}{\delta S}$$
$$A = \frac{\delta[S-R\gamma] + (\gamma-(1+n))}{\delta[S-R(1+n)]}.$$

This proposition shows that if a benevolent government cares sufficiently about the future generations, i.e., if $\delta\in\Lambda$, it will implement a linear interior intergenerational risk sharing mechanism, which provides the elderly with a transfer consisting of a constant share, A, which is reduced of a proportion B according to the realization of the state of the world, ω. In the worst case scenario, in which the elderly have zero private wealth, $\omega_i = 0$ — for instance because of a very bad stock market shock, $R_i = 0$ — the benevolent government imposes a positive, large transfer on the young, $T^G(\omega_i) = A\in(0,1)$. Better realizations of the rate of return, and hence higher private wealth for the elderly, are associated with lower transfers from the young.

A brief discussion of the restrictions on the benevolent government's discount factor is in order. A low weight on the future generations, $\delta<1/R$, may lead, in the occurrence of a particularly negative shock on the returns, to a complete transfer of resources from the young to the elderly, $T^G(\omega_i) = 1$. Equilibria with full expropriation have the unpleasant feature of representing an absorbing state. Indeed, the young generation on which a 100% tax rate is imposed reaches old age with zero private wealth, $\omega = 0$, which will in turn command a 100% tax rate on the young and so on. In the remaining of the paper, we will disregard these full expropriation equilibria and concentrate on interior equilibrium solution, thereby assuming that $\delta>1/R$. If the weight on the future generations is too large, however, this would call for a transfer from the old to the young even in the worst case scenario, in which $\omega = 0$, which is clearly unfeasible. As shown in the Appendix A, this case is ruled out for $\delta<1/(1+n)$.

This former restriction on δ plays another important role: it allows us to abstract from the redistributive motive that may lead the benevolent government to set a transfer from the young to the old. In fact, as shown in the following Lemma, in the deterministic version of our model economy (i.e., for $\sigma\to 0$, $R_t = R$), in which no intergenerational risk sharing motive can be in place, a benevolent government that cares sufficiently about the young, that is, if $\delta>1/R$, would not transfer resources to the elderly.

Lemma 3.2. *In the deterministic environment associated to our model economy, in which $R_t = R$ for any t and $\sigma^2 = 0$, it holds $T^G\leq 0$ whenever $\delta\geq 1/R$.*

The next proposition further characterizes this interior equilibrium risk sharing policy by presenting the results of some comparative statics.

Proposition 3.3. *For $\delta\in\Lambda$, an increase in (i) the discount factor, δ ; or in (ii) average rate of return, R, reduces the fixed component, A, of the linear policy function $T^G(\omega_t)$, and makes the transfer less responsive to the shock. An increase in the variance of the shock, σ^2, increases the linear policy function $T^G(\omega_t)$.*

The intuition is straightforward. Recall that for any given realization of the shock, providing a transfer to the current elderly comes at the cost of lower expected utility for the young generations, because of the opportunity cost of using a PAYG system for risk sharing — given that $R>1+n$. Hence, the higher the weight placed on these future generations, or the higher the average return of the risky asset–and hence the opportunity cost–the lower the fixed component of the transfer, which becomes also flatter. On the other hand, higher

[7] See also Demange (2009). This set of transfers is also considered in Gordon and Varian (1988).

[8] Notice that even in the presence of a financial risk-free asset paying a safe return, $r>n$, state contingent intergenerational risk sharing schemes may be adopted by a benevolent government and by the politicians that condition the transfer on the magnitude of the shock. See, for example, D'Amato and Galasso (2002) and Bossi (2008).

volatility of the returns clearly requires more risk sharing, and hence $T^G(\omega_t)$ increases.

4. Intergenerational risk sharing by office-seeking politicians

In the political system, intergenerational risk sharing may arise because office-seeking politicians choose to transfer resources from the young to the old (or vice versa) in order to improve their electoral perspective. Politicians act after the stock market shock has occurred — and hence the return on the risky asset, R_t, is realized.

Formally, we consider a probabilistic voting model, (see Persson and Tabellini, 2000; Hassler et al., 2003, for a framework with dynamic voting). Two political candidates compete in a majoritarian election. Each candidate determines her political platform, which is represented by the contribution, T, in order to maximize her probability of winning the election. The candidate who wins the election becomes the policy-maker, and implements the proposed policy. Elections take place every period, after the realization of the stochastic return on the assets of the current old. Hence, political candidates can condition their intergenerational risk sharing policy on the realized state of the world.

At every election, individual's voting decisions depend on the policy chosen by the political candidates — and thus on how this affects their utility, on the individual's political ideology towards the two candidates, and on a common popularity shock that may hit the candidates before the election. Political candidates will use the intergenerational risk sharing policy to target the young and/or the old, in an attempt to increase their probability of winning the election, but they cannot affect the voters' ideology or their own popularity. Within each age group, all individuals share the same economic preferences, thereby being equally affected by the candidates' platforms. Elderly care only about the current transfer. Instead, the preferences of the current young–and thus their voting behavior–depend also on the expected future policy. If the young were myopic, they would only consider the direct effect of the current payroll tax on their endowment and thereby on their future consumption. Young workers however do realize that a current tax makes them more likely to be poorer tomorrow, and this may modify the future politicians' behavior. Current young electors hence need to understand and evaluate how the decisions of the current politicians may affect the future politicians' policy choice. We choose to consider a Markov policy, in which intergenerational risk sharing transfers depend only on the current state of the economy, in order to emphasize the absence of commitment to future policies by the politicians.

Besides the utility provided by the economic policy, individuals care also about the political ideology, with some people feeling ideologically closer to one candidate or another. The distribution of ideology within each age group affects the candidate policy decision by determining the size of the swing voters, i.e., of the non-ideological voters who can be convinced to vote for a candidate if targeted with the appropriate policy. It is convenient to assume that each age group has a uniform distribution of ideology across agents.

In this environment, the two political candidates face the same optimization problem, and thus their political platforms converge, i.e. both candidates choose the same contribution, T. Maximizing the probability of winning the election at time t is equivalent to maximizing the following expression, which may also be interpreted as the welfare function of the policy-maker at time t:

$$W_t = \phi_o U(c_t) + (1+n)\phi_y E_t U(c_{t+1}) \tag{4.1}$$

where ϕ_o and ϕ_y represent the density of the uniform ideology distribution function in the two groups, respectively old and young. We normalize $\phi_o = 1$ and define $\phi = \phi_y \geq 0$ as the relative importance of non-ideological, or swing, voters among the young generation. This can be interpreted as a measure of how fiercely the young generations pursue their interests in the political arena.

Eq. (4.1) shows that political competition, as modelled in this probabilistic voting framework, entails a tradeoff between providing state contingent transfers (and utility) to current retirees and providing current negative transfers, but expected positive transfers (and utility) to current workers. Hence, the voting behavior of the young depends on the policy chosen by the current politician, as well as on its impact on tomorrow's policy. To model this intertemporal link, we focus on stationary Markov perfect equilibria, in which each politician's policy decision is contingent on the current state of the economy. At any time t, the state variable is the amount of old age consumption that can be financed out of the private assets ω_t. This clearly depends on the young's savings (or net income) and on the outcome of the stock market. Thus, past policies directly contribute to defining the state of the economy. Clearly, each politician anticipates that its current choice will affect the incentive faced by the future politicians and, therefore, the future level of the intergenerational risk sharing.

The optimization problem of a policy-maker at time t is thus

$$\underset{\{-\omega \leq T(\omega) \leq 1\}}{\text{Max}} U(c_t) + \phi(1+n)E_t U(c_{t+1}) \tag{4.2}$$

where the Markov strategy is $T_t = T(\omega_t)$, the state variable is defined as $\omega_t = R_t(1-T_{t-1})$. Consumption can be written as $c_t = \omega_t + (1+n)T(\omega_t)$. Notice that $c_{t+1} = R_{t+1}(1-T_t) + (1+n)T_{t+1}$ where T_{t+1} is the expected strategy played by future governments.

We can now formally define the linear Markov policy analyzed in this section.

Definition 4.1. A policy $T^P(\omega) = \theta + T'\omega$, where θ and $T' = \frac{\partial T^P(\omega_{t+1})}{\partial \omega_{t+1}}$ are constant parameters, is a linear Markov perfect equilibrium of the intergenerational risk sharing game if it is a fixed point of the mapping from $T^e(.)$ to $T^P(.)$, where $T^e(.)$ is the expected policy function,

$$T^P(\omega_t) \in \arg\max_{T(\omega_t)} U(\omega_t + (1+n)T(\omega_t)) + \phi(1+n)E_t U\Big((1-T(\omega_t))R_{t+1} + (1+n)T^e(\omega_{t+1})\Big)$$

and $T^P(\omega_t) = T^e(\omega_t)$.

In what follows, we will characterize this equilibrium policy outcome for any well behaved utility function with $U'>0$ and $U''<0$. We will return to the quadratic utility function later in this section.

The first order condition for the politician's problem is

$$U'(c_t) + \phi \frac{\partial E_t U(c_{t+1})}{\partial T^P(\omega_t)} = 0 \tag{4.3}$$

where–for $T_t^P>0$–the former term represents the marginal utility of increasing the consumption of the current old, while the latter defines the expected marginal disutility to the current young from imposing this risk sharing policy. This marginal cost can be decomposed as follows:

$$\frac{\partial E_t U(c_{t+1})}{\partial T^P(\omega_t)} = E_t U'(c_{t+1})\left[1 + (1+n)\frac{\partial T^P(\omega_{t+1})}{\partial \omega_{t+1}}\right]\frac{\partial \omega_{t+1}}{\partial T^P(\omega_t)}.$$

Notice that the impact of today's policy on tomorrow net private wealth is $\frac{\partial \omega_{t+1}}{\partial T^P(\omega_t)} = -R_{t+1}$ and define $\frac{\partial T^P(\omega_{t+1})}{\partial \omega_{t+1}} = T'$. The first order condition of the maximization problem at Eq. (4.2) becomes:

$$U'(c_t) - \phi\left[1 + (1+n)T'\right]E_t U'(c_{t+1})R_{t+1} = 0. \tag{4.4}$$

Thus, if an interior (linear) Markov equilibrium policy $T^P(\omega_t)$ exists, it must satisfy[9] $-1/(1+n) < T'(\omega_t) \leq 0$. The above expression

[9] To see that $T'(\omega) \leq 0$, consider the impact of a small increase in ω_t on Eq. (4.4). If $T'(\omega)$ were positive, c_t would increase and c_{t+1} decrease, so that Eq. (4.4) would no longer hold with equality.

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provides a first insight on this political intergenerational risk sharing. This policy is shaped by the political tradeoff between bailing out the current old from a negative stock market shock and imposing an expected cost on current young. In an interior Markov equilibrium, the intergenerational risk sharing agreement[10] features a transfer from the young to the old that is inversely related to the outcome of the stock market. It is important to notice that the political discretion by policy-makers in setting an intergenerational transfer policy creates a quasi asset, whose returns are negatively correlated to stock market returns. Furthermore, by increasing T^P, the current politician reduces, for any future realization of the stock market R_{t+1}, the level of private wealth of the current young, thereby requiring larger future intervention. This property creates a strategic effect that induces persistence in the policy. In this model, a large current political intervention creates its own constituency for future large political interventions. Although this is commonly thought as the root of the persistence of inefficient policies (see Coate and Morris, 1999; Conde-Ruiz and Galasso, 2003), in our context the tension between persistence and efficient allocation of risk is more subtle. The essence of intergenerational risk sharing is to spread current shocks on to future generations (see also Gordon and Varian, 1988; Ball and Mankiw, 2007), i.e., persistence is a crucial ingredient of an efficient risk sharing policy. By transferring the burden of current negative shock to current workers, the politician triggers a reaction by all future politicians, who keep transferring this shock into the infinite future.

To obtain further insights on the intergenerational risk sharing policy chosen by office-seeking politicians, we continue our analysis using the quadratic utility function described at Eq. (2.3). The first order condition of the maximization problem at Eq. (4.2), which describes the stationary Markov policy chosen by the politician a time t, becomes:

$$-\left[\omega_t + (1+n)T^P(\omega_t) - \gamma\right] + \phi\left(1 + (1+n)T'\right)E_t \times\left[\omega_{t+1} + (1+n)T^P(\omega_{t+1}) - \gamma\right]R_{t+1} = 0. \quad (4.5)$$

subject to the linear policy $T^P(\omega) = \theta + T'\omega$. The next proposition characterizes the equilibrium linear policy function.

Proposition 4.2. *If $\phi \in \Phi = \left(\frac{S/R}{S-R(1+n)}, \frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}\right)$, there exists a linear Markov perfect policy function $T^P(\omega_t) = \theta + T'\omega_t$, with $T^P(\omega) < 1\ \forall\omega$, and $T^P(\omega) > 0$ for some ω, where*

$$T' = -\frac{1}{2(1+n)}\left(1 - \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)$$

$$\theta = \frac{2(\gamma-(1+n)) - \phi(\gamma R - S)\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)}{\phi[S - R(1+n)]\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)}$$

This proposition characterizes the behavior of the sequence of office-seeking politicians under a Markov perfect equilibrium when individual preferences have a mean-variance representation. Analogously to the case of a benevolent government, the conditions on ϕ make sure that the young generation has sufficient relative electoral weight to avoid equilibrium sequences featuring full expropriation of the young, in the occurrence of negative stock market shocks. Also in this case, full expropriation would become an absorbing state, since it would lead the young to have zero private wealth in old age, $\omega = 0$, and thus trigger further full expropriation by future old generations.

For a higher electoral weight of the young, office-seeking politicians will still introduce a linear intergenerational risk sharing scheme, featuring a positive constant component, θ, which is reduced by a share T' according to the realization of the state of the world, ω. In the worst case scenario, $\omega = 0$, the elderly obtain the largest transfer $T^P(\omega) = \theta \in (0, 1)$. Higher levels of private wealth, ω, command lower transfers. Finally, if the relative electoral weight of the young is large enough, that is, if ϕ is above the upper limit of Φ, politicians would refrain from introducing a (PAYG) intergenerational risk sharing system, even when the worst case, $\omega = 0$, occurs.

As with the benevolent government, the restrictions on ϕ, and in particular the fact that the political weight of the young is above the lower threshold of Φ, guarantee that no redistributive motive shapes the incentives of the politicians when setting the transfer policy from young to old. The following lemma in fact establishes that in the deterministic version of our economy, no transfer from the young to the old would take place if $\phi \in \Phi$.

Lemma 4.3. *In the deterministic environment associated to our model economy, with $R_t = R$ and $\sigma^2 = 0$ for any t, $T^P \leq 0$ whenever $\phi \in \Phi$.*

The next proposition provides some results on the comparative statics.

Proposition 4.4. *For $\phi \in \Phi$, an increase in the average rate of return, R, or in political weight of the young, ϕ, modifies the linear policy function $T^P(\omega) = \theta + T'\omega$ by decreasing its fixed component θ and by making it less responsive to the realization of the state variable, ω.*

The intuition is straightforward. A lower average return reduces the (opportunity) cost–recall that $R > 1 + n$–of using a PAYG system to provide risk sharing, while an increase in the political weight of the young, ϕ, increases the electoral cost of introducing risk sharing. In both cases, the fixed component of the system thus shrinks, and the system becomes less responsive to the shocks. In other words, the young prefer less risk sharing with a flatter schedule.

5. How well do politicians do?

Both office-seeking politicians and a benevolent government would provide intergenerational risk sharing in the stochastic environment[11] introduced at Section 2. Moreover, the linear equilibrium policies share similar properties in the two cases. As characterized at Propositions 3.1 and 4.2, both policies consist of a constant component (A for a benevolent government and θ for the politicians), which is transferred to the elderly in the worst case scenario, i.e., for $\omega = 0$, and of a proportion–respectively B and T'–which reduces the maximum transfer in accordance to the realization of the state variable ω. Propositions 3.3 and 4.4 push these similarities even further, as they suggest that the steady state properties of the two policy functions are comparable.

We now examine under which conditions politicians aiming at being elected behave exactly as a benevolent government. In other words, when is the interior linear Markov equilibrium policy chosen by office-seeking politicians optimal? The next proposition characterizes when the interior linear Markov equilibrium policy $T^P(\omega)$ coincides with the optimal policy, $T^G(\omega)$. A graphic representation is at Fig. 1.

[10] The structure of the problem faced by these office-seeking politicians shares some structural features with the problem of optimal bequests strategies in altruistic economies where the current generation cares about the utilities of their immediate successors (see Phelps and Pollak, 1968; Bernheim and Rey, 1989, and, more recently, Nowak, 2006, and references therein). The main difference is that in our political environment the weight on different generations depends on the relative share of non-ideological (swing) voters in each age group; whereas in the former class of models the relative weight between (state contingent) utility to ancestors and expected utility to descendants is dictated by altruism and other ethical considerations.

[11] Neither one would however transfer resources from the young to the old for redistributive motives in the deterministic version of our economic environment.

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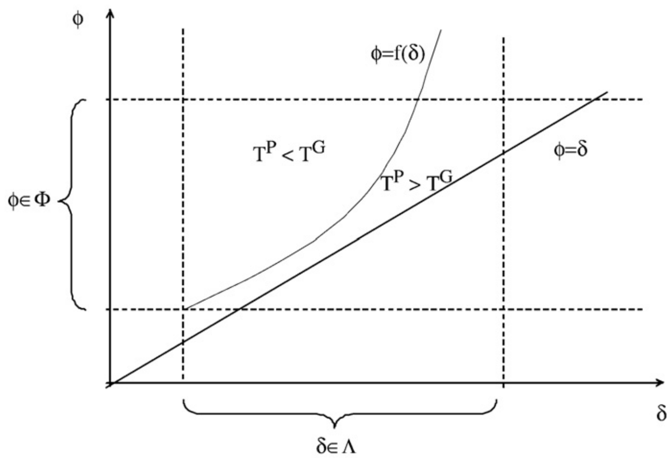

Fig. 1. comparing intergenerational risk sharing by a benevolent government (G) and by politicians (P).

Proposition 5.1. *For $\delta \in \Lambda$ and $\phi \in \Phi$, if $\phi = f(\delta) = \delta\left[1 - \frac{1+n}{\delta S}\right]^{-1}$, the interior linear Markov equilibrium policy chosen by office-seeking politicians correspond to the optimal policy, i.e., $T^P(\omega) = T^G(\omega)\ \forall\omega$. For $\phi < f(\delta)$, $T^P(\omega) > T^G(\omega)\ \forall\omega$, with $T' > B$ and $\theta > A$. For $\phi > f(\delta)$, $T^P(\omega) < T^G(\omega)\ \forall\omega$, with $T' < B$ and $\theta < A$.*

According to this proposition, a Markov game among successive generations of office-seeking politicians may deliver the optimal policy only if the relative electoral weight of the young is larger than the weight assigned by a benevolent government to the future generations, since $\phi = f(\delta) > \delta$. For equal weights, $\phi = \delta$, office-seeking politicians will provide larger transfer than the social optimum. This transfer is characterized by a larger than optimal fixed component, $\theta > A$; and by a lower than optimal reduction associated to the state of nature, $T' > B$. In other words, political intergenerational risk sharing policy is too generous, and too persistent, that is, not enough responsive to the state variable. These distortions come from the politicians' strategic behavior. In their decisions over the transfer policy, current politicians anticipate that future politicians will compensate the current young in their old age for their current social security contributions. This stems from the fact that higher taxes on today's environment lead to a lower private wealth in old age–that is, to a lower state variable in the following period–thereby triggering more transfers by the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly, and leads to overspending — unless the young enjoy an unusually large political power, i.e., $\phi > f(\delta)$.

These two intergenerational risk sharing policies have different implications for the consumption in old age. In both cases, old age consumption depends on the realization of the shock to the returns of the risky assets. However, for $\phi < f(\delta)$, that is, when the politicians are more generous than optimal in their risk sharing policy, they guarantee a higher than optimal expected consumption in old age, but at the cost of introducing also a higher than optimal variance of consumption. By transferring too many resources to old age, and by failing to have these transfers depending more on the realization of the state variable, the politicians fail to provide the optimal risk sharing policy.

6. Concluding remarks

The risk sharing properties of social security have long been recognized in the literature. In several stochastic environments, individuals would benefit from insuring against aggregate shocks before they are even born. Clearly, this is not contractible upon. Yet, once they are born, and uncertainty is realized, there is no more room for risk sharing. Establishing a PAYG system thus seems to require the existence of a long term player, who can bind future, yet-to-be-born generations to carry out the risk sharing policy.

We show that both a benevolent government, who can bind future generations, and office-seeking politicians, who cannot, choose to adopt a state contingent social security system with analogous features. The amount of resources transferred to the elderly by the working generation depends *negatively* on the elderly private wealth — and therefore on the realization of the aggregate shock to the returns of the risk asset. This state contingent social security thus constitutes a quasi asset, whose returns are negatively related to the market returns. This result is in line with Ball and Mankiw (2007) who propose an optimal intergenerational risk sharing plan featuring a negative correlation between social security benefits and asset returns.

Despite these similarities, the intergenerational risk sharing schemes proposed by a benevolent government and by the politicians may also differ. Office-seeking politicians are more likely to provide generous transfers that are less responsive to the aggregate shock, and hence more persistent. While persistence is typically at the root of efficient intergenerational risk sharing policies, since it allows to spread the risk over time and hence over several generations, office-seeking politicians have an incentive to overplay this feature. In fact, politicians are willing to tax more heavily current workers and to provide generous transfers to the current elderly, because they anticipate that future politicians–facing elderly individuals with low net wealth, due to the large contributions they had to pay in their youth–will compensate them with generous pension transfers. This mechanism thus reduces the electoral cost among the young voters of providing large transfers and leads to generous, persistent pension systems.[12]

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Appendix A

Proof of Proposition 3.1. Consider the optimization problem of the benevolent government at Eq. (3.1). Its recursive formulation yields the following first order condition:

$$\frac{\partial U(c_t)}{\partial T_t} + \delta E_t \frac{\partial U(c_{t+1})}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial T_t} = 0$$

where $\omega_t = R_t(1 - T_{t-1})$ defines the state variable at time t. Using the utility function at Eq. (2.3), the above expression can be written as Eq. (3.3). To solve for interior equilibrium policies, we guess a linear solution: $T^G(\omega_t) = A + B\omega_t$. Using simple algebra, from Eq. (3.3) we obtain the following expression:

$$T^G(\omega_t) = -\frac{\omega_t}{1 + n + \delta S(1 + B(1 + n))} + \frac{\gamma(1 - \delta R) + \delta S(1 + B(1 + n)) - \delta(1 + n)AR}{1 + n + \delta S(1 + B(1 + n))}.$$

[12] This result for example is consistent with Bohn (2003) findings that the current US social security system does not provide the optimal level of risk sharing, since it is too generous with the elderly and shifts most of the burden of risk sharing on to future generations.

Hence, we have

$$B = -\frac{1}{1+n+\delta S(1+B(1+n))} \tag{6.1}$$

$$A = \frac{\gamma(1-\delta R)+\delta S(1+B(1+n))-\delta(1+n)AR}{1+n+\delta S(1+B(1+n))} \tag{6.2}$$

For $1+B(1+n)\neq 0$, we obtain

$$B = -\frac{1}{\delta S} \text{ and} \tag{6.3}$$

$$A = \frac{\gamma-(1+n)+\delta(S-\gamma R)}{\delta(S-(1+n)R)} \tag{6.4}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, that is, $T^G(\omega)<1\,\forall\omega$. To guarantee that this condition holds, we need to impose $T^G(\omega)<1$ for $\omega=0$, i.e., $A<1$. Simple algebra yields $\delta>1/R$. Additionally, we require some risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The most favorable case for this transfer to occur is for $\omega=0$. We hence need to have $A>0$. Simple algebra shows that, since $\gamma>S/R$ (Assumption 2), a sufficient condition for $A>0$ is $\delta<\frac{\gamma-(1+n)}{R\gamma-S}$. However, using Assumption 3, it is easy to see that $\delta<\frac{1}{1+n}$ implies $\delta<\frac{\gamma-(1+n)}{R\gamma-S}$, and thus $A>0$. Q.E.D.

Proof of Lemma 3.2. A stationary Markov policy T for $R_t=R$ and $\sigma^2=0$ satisfies $T=A+B(1-T)R$. Downward transfers from old to young occur if $A+BR\leq 0$. Substituting the expressions for A and B from Proposition 3.1, we have that $\delta\geq 1/R$.

Proof of Proposition 3.3. For $\delta\in\Lambda$, recall that the linear policy function is $T^G(\omega_t)=A+B\omega_t$ with A and B defined in Proposition 3.1.

(i) consider a change in the discount factor, δ. Simple algebra shows that $\frac{\partial B}{\partial\delta}=\frac{1}{\delta^2 S}>0$ and $\frac{\partial A}{\partial\delta}=-\frac{\gamma-(1+n)}{\delta^2(S-(1+n)R)}<0$.

(ii) consider a change in the average rate of return, R. We have that $\frac{\partial B}{\partial R}=\frac{2R}{\delta S^2}>0$ and $\frac{\partial A}{\partial R}=-\frac{(\gamma-(1+n))\left[\delta\left(\sigma^2-R^2\right)+2R-(1+n)\right]}{\delta(S-(1+n)R)^2}<0$, since $\sigma^2>R^2$ (Assumption 1).

Finally, (iii) consider a change in the variance of the shock, σ^2. It is easy to see that $\frac{\partial B}{\partial\sigma^2}=\frac{1}{\delta S^2}>0$ and $\frac{\partial A}{\partial\sigma^2}=\frac{(\delta R-1)[\gamma-(1+n)]}{\delta(S-(1+n)R)^2}>0$. Hence, $\frac{\partial T^G(\omega)}{\partial\sigma^2}>0$. Q.E.D.

Proof of Proposition 4.2. Consider the first order condition at Eq. (4.5), which describes the stationary Markov policy chosen by the politician a time t. Recall that the state variable is defined as $\omega_t=R_t(1-T^P_{t-1})$ $\forall t$, and that $T^P(\omega_t)=\theta+T'\omega_t$. Moreover, define $Q=1+(1+n)T'$. We need to obtain the value of the parameters T' and θ, which solve this FOC. Using simple algebra, from Eq. (4.5) we obtain the following expression:

$$T^P(\omega_t) = -\frac{\omega_t}{1+n+\phi SQ^2}+\frac{\gamma+\phi SQ^2-\phi QR(\gamma-\theta(1+n))}{1+n+\phi SQ^2}.$$

Hence, we have

$$T' = -\frac{1}{1+n+\phi SQ^2} \tag{6.5}$$

$$\theta = \frac{\gamma+\phi SQ^2-\phi QR(\gamma-\theta(1+n))}{1+n+\phi SQ^2} \tag{6.6}$$

Since $Q=1+(1+n)T'$, we solve the expression at Eq. (6.5) for T' to find two solutions:

$$T'_A = -\frac{1}{2(1+n)}\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right) \tag{6.7}$$

$$T'_B = -\frac{1}{2(1+n)}\left(1-\sqrt{1-\frac{4(1+n)}{\phi S}}\right). \tag{6.8}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ω. That is, we require $T^P(\omega)<1\,\forall\omega$. To guarantee this condition for these two candidate solutions of T', we need to impose the first order condition of the politicians (see Eq. (4.5)) evaluated at $\omega=0$ to be negative when $T=1$. Substituting $\omega=0$ and $T=1$ in Eq. (4.5), and imposing the expression to be negative yields the following inequality:

$$\phi\left(1+(1+n)T'\right)R>1 \tag{6.9}$$

Let's begin investigating the candidate solution $T'=T'_A$. Substituting T'_A in Eq. (6.9) yields the following inequality

$$\phi R\sqrt{1-\frac{4(1+n)}{\phi S}}<\phi R-2.$$

Clearly, for $\phi<2/R$, the above inequality is not satisfied, and thus T'_A is not part of an interior equilibrium solution. For $\phi>2/R$, we can elaborate on the above expression to obtain the following inequality: $\phi>\frac{S/R}{S-R(1+n)}$. Simple algebra shows that for $\sigma^2/R^2>1$ (see Assumption 1), this inequality cannot hold for $\phi>2/R$. Hence, candidate solution $T'=T'_A$ cannot be part of an interior equilibrium solution.

Let's now turn to the candidate solution $T'=T'_B$. Substituting T'_B in Eq. (6.9) yields the following inequality

$$\phi R\sqrt{1-\frac{4(1+n)}{\phi S}}>2-\phi R.$$

Clearly, for $\phi>2/R$, the above inequality is always satisfied, and thus T'_B can be part of an interior equilibrium solution. For $\phi<2/R$, the above inequality can be rewritten as $\phi>\frac{S/R}{S-R(1+n)}$. Notice that for $\sigma^2/R^2>1$ (see Assumption 1) $\frac{S/R}{S-R(1+n)}<2/R$. Hence, candidate solution $T'=T'_B$ is part of an interior equilibrium solution if $\phi>\frac{S/R}{S-R(1+n)}$.

With $T'=T'_B$, we can now solve the expression at Eq. (6.6) for θ. Simple algebra shows that $\theta=\frac{2(\gamma-(1+n))-\phi(\gamma R-S)\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}{\phi[S-R(1+n)]\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}$.

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ω. That is, we require $T^P(\omega)<1\ \forall\omega$. To guarantee that this condition holds, we need to impose $T^P(\omega)<1$ for $\omega=0$, i.e., $\theta<1$. Simple algebra yields $\phi>\frac{S/R}{S-R(1+n)}$. Additionally, we require some risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The more favorable case for this transfer to occur is for $\omega=0$. We hence need to have $\theta>0$. Simple algebra shows that the denominator is always positive, while the numerator is positive for $\phi<\frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}$. Q.E.D.

Proof of Lemma 4.3. In the deterministic environment with $R_t = R$ and $\sigma^2 = 0$, the linear Markov perfect policy function at Proposition 4.2 satisfies $T = \theta + T'(1-T)R$, i.e.

$$T = \frac{\theta + T'R}{1 + T'R}$$

with $T' = -\frac{1-\sqrt{1-\frac{4(1+n)}{\phi R^2}}}{2(1+n)}$ and $\theta = \frac{2(\gamma-(1+n)) - \phi R(\gamma - R)\left(1+\sqrt{1-\frac{4(1+n)}{\phi R^2}}\right)}{\phi R[R-(1+n)]\left(1+\sqrt{1-\frac{4(1+n)}{\phi R^2}}\right)}$.

We study numerator and denominator of T separately.

Simple algebra shows that the numerator of T being positive, i.e., $\theta + T'R > 0$ can be reduced to

$$\frac{2}{\phi R} - 1 \geq \sqrt{1 - \frac{4(1+n)}{\phi R^2}}$$

A necessary condition for the numerator to be positive is $\frac{2}{\phi R} - 1 > 0$, i.e., $\phi < \frac{2}{R}$. Additionally, squaring both terms in the above inequality, we get that for the numerator is positive, for $\phi < \frac{2}{R}$ and $\phi < \frac{1}{R-(1+n)}$. While it is negative for $\phi > \frac{2}{R}$ and for $\frac{1}{R-(1+n)} < \phi < \frac{2}{R}$.

Notice that $\frac{1}{R-(1+n)} > \frac{2}{R}$ i.e. for $R < 2(1+n)$. Two cases arise:

for $R < 2(1+n)$, $\theta + T'R > 0$ if $\phi \leq \frac{2}{R} \leq \frac{1}{R-(1+n)}$
for $R > 2(1+n)$, $\theta + T'R > 0$ if $\phi \leq \frac{1}{R-(1+n)} \leq \frac{2}{R}$

Simple algebra shows that the denominator of T being positive, i.e., $1 + T'R > 0$ can be reduced to

$$1 > R\frac{1-\sqrt{1-\frac{4(1+n)}{\phi R^2}}}{2(1+n)} \text{ or } \sqrt{1-\frac{4(1+n)}{\phi R^2}} > \frac{R-2(1+n)}{R}$$

It is easy to see that the inequality is always satisfied for $R < 2(1+n)$. Whereas, for $R > 2(1+n)$, $1 + T'R > 0$ if $\phi > \frac{1}{R-(1+n)}$.

Hence, we have that:

for $R < 2(1+n)$, the denominator is always positive, but the nominator is negative for $\phi > \frac{2}{R}\left(\leq \frac{1}{R-(1+n)}\right)$, and hence $T < 0$ for $\phi \in \Phi$;

for $R > 2(1+n)$, the denominator is positive for $\phi > \frac{1}{R-(1+n)}$, but the nominator is negative for $\phi > \frac{1}{R-(1+n)}$, and hence $T < 0$ for $\phi \in \Phi$.

Hence, $T < 0$ for $\phi \in \Phi$. Q.E.D.

Proof of Proposition 4.4. For $\phi \in \Phi$, recall that the linear policy function is $T^P(\omega) = \theta + T'\omega$ with θ and T' defined in Proposition 4.2. Notice that we can write

$$\theta = \frac{2(\gamma-(1+n))}{\phi(S-R(1+n))\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)} - \frac{\gamma R - S}{S - R(1+n)}. \tag{6.10}$$

Consider an increase in the average rate of return, R. It is easy to see that $\frac{\partial T'}{\partial R} = \frac{2R}{\phi S^2\sqrt{1-\frac{4(1+n)}{\phi S}}} > 0$ and $\frac{\partial \theta}{\partial R} = -\frac{\theta(2R-(1+n))}{(S-R(1+n))} - \frac{\gamma - 2R}{(S-R(1+n))} - \frac{8(1+n)R(\gamma-(1+n))}{\phi^2(S-R(1+n))\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)^2} < 0$, since $\gamma R > S$ and $\sigma > R$ imply that $\gamma > 2R$.

Consider an increase in ϕ, it is easy to see that $\frac{\partial T'}{\partial \phi} = \frac{1}{\phi^2 S\sqrt{1-\frac{4(1+n)}{\phi S}}} > 0$. Notice that θ can be written as

$$\theta = \frac{\phi S Q^2 + \gamma(1-\phi Q R)}{\phi S Q^2 + (1+n)(1-\phi Q R)} \tag{6.11}$$

with $Q = \frac{1}{2}\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)$ and $\phi QR > 1$. Using the above expression, we have

$$\frac{\partial \theta}{\partial \phi} = -2\frac{(\gamma-(1+n))\left[2(1+n) + S\left(1+\sqrt{\Delta}\right)\sqrt{\Delta}\phi\right]}{S(S-(1+n)R)\left(1+\sqrt{\Delta}\right)^2\sqrt{\Delta}\phi^3} < 0$$

with $\sqrt{\Delta} = \sqrt{1-\frac{4(1+n)}{\phi S}}$. Q.E.D.

Proof of Proposition 5.1. Comparing the first order condition respectively for the benevolent government (Eq. (3.3)) and for the politicians (Eq. (4.5)), we have that, in an interior equilibrium, the benevolent government and the politicians will adopt the same policy if $\delta = \phi(1 + (1+n)T')$. Recall that interior equilibrium policies, involving risk sharing at least when $\omega = 0$, require respectively, $\delta \in \Lambda$ and $\phi \in \Phi$. Using the expression for T' at Proposition 4.2, the above expression can be rewritten as $\phi = f(\delta) = \delta\left[1 - \frac{1+n}{\delta S}\right]^{-1}$. Furthermore, it is trivial to see that for δ and $T^G(\omega)$ that solve the benevolent government problem (for an interior equilibrium), if $\phi > f(\delta)$, at $T^P(\omega) = T^G(\omega)$ the first order condition of the politicians is negative, so that $T^P(\omega) < T^G(\omega)$. And vice versa for $\phi < f(\delta)$.

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