Optimal Welfare-to-Work Programs*

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Abstract

A Welfare-to-Work (WTW) program is an integrated scheme of government expenditures on "passive" (unemployment compensation, social assistance) and "active" (job search monitoring, training, wage taxes/subsidies) labor market policies targeted to the unemployed. This paper studies the optimal WTW program as a recursive contract between a cost-minimizing government and a worker whose human capital (skills) depreciates during unemployment, and whose search/training effort is private information. We provide a characterization of the optimal sequence of policies along the unemployment spell. Within each policy-phase, we characterize the optimal time-profile of benefits and the optimal use of subsidies vs. taxes upon re-employment. In a calibration exercise based on the U.S. labor market and on the evidence from several pilot programs, we use our framework to analyze quantitatively the features of the optimal WTW program for the U.S. economy and compare it to the existing welfare system.

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1 Introduction

Government expenditures on labor market policies targeted to the unemployed exceeded 3% of GDP, across OECD countries (Martin, 2002). Two thirds of these expenditures are allocated to "passive" policies, mainly unemployment insurance and social assistance policies providing income support of last resort once unemployment benefits have expired. The remaining third is allocated to "active" policies, like job-search monitoring, training, and wage subsidies. Typically, job search monitoring programs pair the unemployed worker with a public employee (the "mentor") who verifies her job-search activity, and often helps improving interviewing skills and selecting among available job-vacancies. Training programs tend to be of one of two types: basic education (brush-up courses for individuals with poor literacy and numerical skills, preparation for high-school level diplomas), and vocational training (classroom training in specific occupational skills). The share of expenditures on active labor market programs has risen substantially over the past 10 years and this type of government intervention is now a pivotal ingredient of social welfare policies.

Throughout OECD countries, governments use a mix of both passive and active policies. For example, in the United States at least since 1935 there exists an unemployment insurance system with vast coverage and, upon expiration of the unemployment compensation (usually after 26 weeks), several social assistance benefits become available. The Food Stamps program is, arguably, the most notable example. With the *Balanced Budget Act* of 1997, the federal U.S. government imposed strict participation requirement to active labor market programs to welfare recipients and allocated $3 billion in grants to states and local communities that put in place training and job-search monitoring policies. The Earned Income Tax Credit, introduced by the federal government in 1975, represents a large-scale wage subsidy program for low-income workers.

A Welfare-to-Work (WTW) program is precisely a government expenditure program that combines together passive and active policies.[1] Clearly, every WTW program im-

[1]In the United States the government expenditures on active and labor market policies are not systematically organized. An example of a very structured WTW program is the U.K. *New Deal for Young People,* a mandatory program for all the unemployed workers between 18-24. Formally, the "New Deal" is structured in four sequential stages. Stage 1 consists in a standard unemployment insurance policy that lasts up to 6 months. In stage 2 (the "Gateway"), a personal adviser meets the workers at least once

plicitly promises a certain level of ex-ante welfare to the unemployed agent. An *optimal* WTW program is an integrated scheme that maximizes the expected discounted utility of the unemployed agent, subject to not exceeding a given level of government expenditures.

The first objective of this paper is to develop a theoretical framework that allows to study the key features of optimal WTW programs. The point of departure of our theoretical analysis is the literature characterizing the optimal unemployment insurance contract in presence of a repeated moral hazard problem: the risk-neutral principal (planner/government) cannot observe the risk-averse unemployed agent's job search effort (hidden action). The objective of the government is insuring the unemployed agent through transfers of income which must be compatible with the individual search incentives. Following the seminal work by Shavell and Weiss (1979), several papers have advanced our understanding of the optimal solution to this key trade-off between insurance and incentives (Hopenhayn and Nicolini, 1997; Zhao, 2001; Pavoni, 2003a; see also Karni, 1999, for a survey). We follow the most recent contributions and exploit the recursive representation of the planner's problem where the expected discounted utility U promised by the contract to the unemployed agent becomes a state variable.

We extend this standard framework in two directions. First, following Pavoni (2003b) we allow workers' productivity and their job finding probabilities to depend on *human capital* (skills) and allow human capital to depreciate along the unemployment spell. Human capital h is our second key state variable in the recursive representation. Skill depreciation is a key candidate to explain the overwhelming evidence on unemployment duration dependence and wage loss upon displacement. Machin and Manning (1999) report a number of studies on hazard rate duration dependence in various OECD countries.[2] Keane and Wolpin (1997) estimate from NLSY data an average annual human capital depreciation rate for U.S. workers around 20% per year. In addition, many authors consistently find that displaced U.S. workers face large and persistent earning losses upon reemployment between 10% and 25% compared with continuously employed workers (Bartel and Borjas,

every two weeks to assist/enforce job-search. It lasts up to 4 months. In Stage 3 (the "Options") there are two training options targeted to augmenting the workers' skills. Then Stage 4 (the "Follow-Through") is again a job-search assistance/monitoring program, which lasts up to 3 months.

[2]For example, van den Berg and van Ours (1996) conclude that in the U.S. the exit probability from unemployment falls by 30% after 3 months of unemployment. For the U.K., Nickell (1979) finds a 50% decrease in the hazard rate after 15 months of unemployment, and van den Berg and van Ours (1994) report a decrease by 20% after 3 months, and by over 30% after 6 months of unemployment.

1981; Ruhm, 1987; Jacobson et al., 1996; for a survey, see Fallick, 1996).

Second, in accordance with actual Welfare-to-Work schemes, we enlarge considerably the space of policy instruments and allow the planner to choose among four distinct policies: 1) standard unemployment insurance, 2) job search monitoring, where the planner can observe the search effort upon payment of a cost, 3) a training program that requires the unobservable agent's effort as input into a human capital accumulation technology with stochastic outcome, and 4) social assistance, defined as an income-assistance program of "last resort" where the planner induces zero search effort and simply insures the worker. Moreover, we let the planner choose wage taxes and subsidies upon re-employment.

Within the (U, h) space, we identify the regions where each policy is likely to emerge as optimal. Given the evolution of U and h over time, we can provide a characterization of the optimal sequence of policies along the unemployment spell. Within each policy phase, we characterize the optimal time-profile of benefits and the optimal use of subsidies vs. taxes upon re-employment.

For expositional simplicity, in the benchmark we make the usual assumption in this literature that the planner fully controls the consumption stream of the agent. This precludes the occurrence of self-insuring trades in the asset market. In an extension of the benchmark model where the agent can hide her savings from the planner but cannot borrow, we show that the same optimal WTW program can be implemented with the help of one additional instrument: a linear interest tax (see Werning 2002, Kocherlakota 2003a, and Shimer and Werning, 2003, for models of optimal unemployment insurance with hidden savings, and Abraham and Pavoni, 2004a for a general moral hazard model with hidden access to the credit market).

The second objective of the paper is to study quantitatively the features of the optimal WTW program for the typical welfare recipient in the U.S. economy. We start by calibrating the parameters of our model to match some key labor market statistics. In so doing, we exploit information from the evaluation of several recent U.S. active labor market programs. Next, we solve numerically for the optimal program and, by simulation, derive the optimal sequence of policies, their duration, the pattern of optimal benefits, taxes and subsidies. Finally, we calculate the welfare gains for the worker and the budget savings for the government of shifting from the current scheme to the optimal scheme.

The rest of the paper is organized as follows. Section 2 presents the economic environment and studies the autarky benchmark. Section 3 describes the contractual relationship between planner and agent, and presents the recursive formulation of the planner's problem. Section 4 characterizes the key features of the optimal WTW program. In Section 5 we analyze the implementation of the optimal contract with hidden savings. Section 6 develops the quantitative analysis applied to the U.S. labor market. Section 7 concludes the paper.

2 The Economy

Preferences: Workers are risk-averse and discount the future at rate $\beta \in (0,1)$. In any given period the worker has preferences of a separable form over consumption c and effort a :

$$u(c) - \nu_z(a),$$

where we allow the disutility of effort to depend on the employment status z. The effort level $a \in \{0, e\}$, with $e > 0$. Moreover, we impose that $c \geq 0$, and that $u(\cdot)$ is strictly increasing, strictly concave and smooth, with $\lim_{c \to \infty} u'(c) = 0$. A technical assumption that will prove useful in our characterization is that u^{-1} has positive third derivative. This condition is satisfied by a large class of utility functions, including CARA and CRRA with risk-aversion parameter larger than one half.

Employment status: The agent can be either unemployed ($z = z^u$), or employed ($z = z^e$). Employment is defined as an absorbing state where the agent works and produces. During unemployment, the worker can either search or train (with low/high effort); the two activities are mutually exclusive within a period. Without loss of generality, set $\nu_{z^u}(0) = 0$, $\nu_{z^u}(e) = e$, and $\nu_{z^e}(e) = e_w$.

Human capital: Workers are endowed with a time-varying stock of human capital (skills) $h \geq 0$. Let $Q^y(H; h) = \Pr\{h' \in H; h, y\}$ denote the law of motion for human capital, contingent on the outcome y of the worker activity (search/train), with $y \in \{s, f\}$ where s denotes "success", and f denotes "failure".

The transition function $Q^y(\cdot; h)$ satisfies the following properties. First, $Q^y(\cdot; h)$ dominates in the first-order sense $Q^y(\cdot; h^*)$ for any $h \geq h^*$ and $y = s, f$, this for each h^* (i.e.

Q^y is monotone in h). Second, for any $h_0 \in H$, if $h \in \textit{support}\left\{Q^f(\cdot;h_0)\right\}$ then $h \leq h_0$. We label this property "non-overlapping support". Similarly, $Q^s(\cdot;h)$ dominates $Q^f(\cdot;h)$ in the first-order sense. Thus, it is natural to think of Q^s as a human capital accumulation technology, and of Q^f as a human capital depreciation technology. Moreover, let $Q^s(\cdot;0) \equiv Q^f(\cdot;0)$, i.e. human capital accumulation needs a positive input of human capital to be effective. Finally, we assume that $Q^y(\cdot;h)$ has the usual Feller property and that it is atomless for all $h > 0$.

Note that during unemployment the agent is subject to two stochastic events: the outcome of its activity y and the consequent realization of human capital h'. During employment, instead, $y = s$ by definition and human capital always follows Q^s.

Search technology: During search, both effort a and human capital h affect the job finding probability of an unemployed worker. Denote the unemployment hazard rate as $\pi(h,a)$. We assume that $\pi(h,0) \equiv 0$ and that $\pi(\cdot,e) \in (0,1)$ is continuous and increasing. These monotonicity properties have the interpretation of complementarity between the stock of human capital h and the effort level a in the search technology.

Training technology: The unemployed worker can choose to forego the search option and operate a training technology to accumulate human capital, upon payment of a cost κ^{TR}. The training technology is stochastic. With probability $\theta(a)$, where $\theta(e) > \theta(0) = 0$, training is successful, and the worker's human capital next period accumulates according to Q^s. Upon failure, human capital depreciates according to Q^f.

Wage function: When a worker of type h becomes employed, she earns a gross wage (before taxes/subsidies) $\omega(h)$. We assume that $\omega(\cdot) \in [0, \omega_{\max}]$ is continuous and increasing, with $\omega(0) = 0$.

Markets: In the baseline model we assume that the worker has no access to credit/storage and to insurance markets. In section 5, we will relax this assumption.

2.1 Autarky

In Appendix A, we study the problem of an agent who operates in isolation (autarky), without access to credit/insurance markets, and without the government intervention.

The agent is endowed with zero initial wealth. We illustrate formally that government-provided insurance against human capital and employment shocks, and government-provided credit towards the use of the training technology improve the worker's welfare upon what she can achieve in autarky. Moreover, we prove a useful result on the optimal choice of the search effort level.

Lemma 1 (Autarky): *In autarky, (i) the agent never uses the training technology; (ii) If the agent chooses search effort $a = 0$ in any given period, she will always do so thereon.*

Proof: *See Appendix A.*

The intuition is that, as time goes by, human capital tends to depreciate and the returns to job search decline (recall that both $\pi(\cdot)$ and $\omega(\cdot)$ are increasing in h), but the disutility of search effort is constant. Interestingly, this result will have a natural counterpart in the optimal WTW program: if at some point in the optimal contract the planner chooses to recommend zero effort to the unemployed agent, it will keep making the same recommendation from then onward.

3 The Contractual Relationship

We now introduce a risk-neutral planner/government (principal) who, at time $t = 0$, offers an insurance/credit contract to the unemployed worker (agent) that maximizes the expected discounted flow of net revenues for the planner and guarantees to the agent at least an expected discounted utility level U_0, exogenously given. The planner has the same discount factor β as the agent.

Information structure: The planner can perfectly observe the level of human capital h, the employment status z, whether the unemployed worker is searching or training, and the outcome y of the latter activity.[3] However, the agent's effort choice a during both

[3] With respect to the observability of human capital, note that if h depreciates deterministically, it is enough knowing the law of motion of h and the pre-displacement wage to recover the level of human capital at every unemployment duration. Our stochastic depreciation assumption is used only to "convexify" the problem. As we explain in section 4.2, the same convexification can be achieved with payoff-irrelevant shocks, in the case of deterministic depreciation.

search and training is private information of the agent, so the planner faces a moral hazard problem.

A search-effort monitoring technology is available to the planner when the worker seeks job opportunities: upon payment of a cost κ^{JM}, the job-search effort of the agent can be perfectly observed and enforced by the planner. The monitoring technology can be interpreted as the situation where the planner pays the services of a "mentor" who monitors closely the search activity of the worker.[4] Such technology is, by assumption, prohibitively costly during training.[5]

Contract: In each period t, the contract specifies transfers of resources to the worker, recommendations on search vs. training activities and on the search/training effort level to exert, and the choice of using the effort-monitoring technology, when search is suggested. The period-t components of the contract are contingent on all publicly observable histories up to t and, whenever the monitoring technology is not used, search-effort recommendations must be incentive compatible.

Appendix B describes the sequential formulation of the optimal contract and explains that, following the recursive contracts literature, the contract can be described by summarizing past histories through a state vector composed by the expected discounted utility U promised to the agent by the continuation of the contract, the level of human capital h of the worker, and the employment status z.

The components of the contract as policies of the Welfare-to-Work (WTW) program: The combination of un-monitored search, monitored search, training, together with the high and low effort recommendations configure six possible options. Notice first that the planner will never choose to pay the monitoring cost and suggest the minimal

[4] We could model the monitoring technology in a more general way, through a "stochastic monitoring" whereby the government observes the effort only with some probability q. The present version of the model can be interpreted as the limiting case where $q = 1$. However, notice that when there are no limits to the punishment the planner can inflict upon shirking (for example, when u is unbounded below), then any $q > 0$ will induce high effort with full insurance.

[5] While certain elements of the learning process, such as classroom attendance and home-work, are easily verifiable, there are other key components, like attention, focus and concentration, that are intrinsically "interior" and extremely hard to be verified by an external party. Although the assumption of infinitely large monitoring cost during training is made in order to simplify the analysis, it is reasonable to argue that learning is a far more complex activity than job-search and, as such, monitoring training effort is more costly.

effort level. The reason is that, since $\pi(0) = 0$, the observable realization of a successful search activity perfectly detects a deviation from the zero-effort recommendation at no additional cost.[6] Moreover, the planner will never choose to pay the training cost and suggest zero effort since $\theta(0) = 0$ and the cost would be wasted. As a result, the planner is left with four options, which we denote as "policy instruments" of the WTW program, and we index with i.

We denote as "Unemployment Insurance" ($i = UI$) the joint recommendation of search activity and positive search effort. When positive search effort is suggested together with the use of the monitoring technology, the policy will be labeled "Job-search Monitoring" ($i = JM$). The zero-effort recommendation in the search activity denotes the "Social Assistance" policy ($i = SA$). A high-effort recommendation with the use of the training technology describes the "Training" option ($i = TR$). Finally, during employment, the difference between the wage and the planner's transfer defines implicitly the employment tax (if positive) or subsidy (if negative).

Timing: Exploiting the recursive representation of the contract, consider an unemployed worker who enters the period with state (U, h). At the beginning of the period the planner chooses the policy instrument $i(U, h)$ –hence, an effort recommendation $a(U, h)$–, the transfer $c(U, h)$, and the continuation utilities $U^y(U, h)$ conditional on the outcome y of the selected policy i. The transfer, the effort recommendations, and the continuation utilities must deliver to the agent a promised expected discounted utility level U.

Next, the outcome y of the policy is revealed, which identifies the relevant transition function for human capital Q^y. Last, the planner delivers the promised utility U^y by choosing next period continuation utilities contingent on the realization h' of the human capital shock, which occurs at the end of the period. The precise timing implied by this recursive representation is depicted in Figure 1.

[6] Put differently, the incentive-compatibility constraint associated to the zero effort recommendation is a trivial one, since the planner can punish without limits the worker upon finding a job, an outcome which is off the equilibrium induced by the optimal contract.

3.1 The Planner Problem

We now describe in detail the recursive representation of the planner problem, starting from the situation where the worker is employed.

Employment: Recall that employment is an absorbing state. Let $W(U,h)$ be the optimal planner's net return in case the worker of type (U,h) is employed, then the planer solves

$$\begin{aligned} W(U,h) &= \max_{c,U^s} \omega(h) - c + \beta \mathbf{W}(U^s,h) \\ s.t. &: \\ U &= u(c) - e_w + \beta U^s, \end{aligned} \tag{1}$$

where the expected return during employment is

$$\mathbf{W}(U,h) = \int W(U,h') Q^s(dh';h), \tag{2}$$

where we used the fact that since employment is an absorbing state without informational asymmetries, the planner will fully insure the agent against human capital shocks, thus promised utility is constant over time and across states. From the promise-keeping constraint, the optimal transfer c^e is invariant with respect to h, with $c^e(U) = u^{-1}((1-\beta)U + e_w)$. The magnitude

$$\tau(U,h) = \omega(h) - c^e(U) \tag{3}$$

is the implicit tax (or subsidy, if negative) the government imposes on employed workers. State-contingent taxes and subsidies are a key component of an optimal WTW plan.

Policy choice during unemployment and "randomization": When the unemployed worker with state (U,h) enters the period, the planner chooses which policy instrument i to use, by solving

$$V(U,h) = \max_{i \in \{JM,SA,TR,UI\}} V^i(U,h). \tag{4}$$

After the realization of the outcome y of the selected policy, the planner can choose the next-period continuation utility contingent on the end-of-period observable realization of

h' that will take place through Q^y. The value function for the planner at this stage of the maximization solves

$$\begin{aligned} \mathbf{V}^y(U,h) &= \int \max_{U(h')\in D} V(U(h'),h')Q^y(dh';h), \\ s.t. &: \\ U &= \int U(h')\, Q^y(dh';h). \end{aligned} \tag{5}$$

The integral constraint says that the planner needs to deliver to the agent utility U in (ex-ante, with respect to h') expected value terms. We will explain later that this "randomization" on continuation utilities may be used in the optimal contract to convexify the planner's problem and, thus, enhance welfare.[7]

We now describe the values of the individual policies, one by one.

Social Assistance (SA): In social assistance, the worker is "released" by the planner, in the sense that the planner does not ask her high (search or training) effort, but simply transfers some income to the worker. In section 4.1 we will prove that if at any point during the contract the planner makes the "zero effort" recommendation, it is optimal to do so from that point onward: SA is an absorbing policy.

To simplify the notation, we exploit this result in writing down the planner's problem under this policy. Since $\pi(h,0)=0$, and because of its absorbing nature, the value of SA does not depend on h and solves

$$\begin{aligned} V^{SA}(U) &= \max_{c,U^f} -c+\beta V^{SA}\left(U^f\right) \\ s.t. &: \\ U &= u(c)+\beta U^f. \end{aligned}$$

It is easy to see that the agent will be fully insured and that the value of social assistance can be written as

$$V^{SA}(U) = -\frac{c^{SA}(U)}{1-\beta}, \tag{6}$$

[7]It should be noted that, loosely speaking, these ex-ante lotteries across policies used by the planner are equivalent to inducing the agent to use different technologies (i.e. search and training) for a fraction of the time endowment, within a given period. However, it is well know that the presence of incentive constraints might induce non convexities even when the planner can choose continuously between the different alternatives.

where $c^{SA}(U) = u^{-1}((1-\beta)U)$ is the constant benefit paid to workers in SA by the planner. Note that V^{SA} is decreasing and concave in U. In light of this characterization, it is natural to think of SA as a pure income-assistance program of last resort.

Unemployment Insurance (UI): When the worker is enrolled by the planner in the unemployment insurance scheme, the problem of the planner is

$$\begin{aligned} V^{UI}(U,h) &= \max_{c,U^f,U^s} -c + \beta\left[\pi(h)\mathbf{W}(U^s,h) + (1-\pi(h))\mathbf{V}^f(U^f,h)\right] \\ s.t. &: \\ U &= u(c) - e + \beta\left[\pi(h)U^s + (1-\pi(h))U^f\right], \\ U &\geq u(c) + \beta U^f, \end{aligned} \tag{7}$$

where $\left(U^s, U^f\right)$ are the pair of lifetime utilities promised by the planner contingent on the outcome of search (s denotes success and f failure of the search activity). Given the observability of the employment status, the outcome of search is verifiable. For notational simplicity we have denoted $\pi(h,e)$ as $\pi(h)$.

The expressions for $\mathbf{W}$ and $\mathbf{V}^f$ are given by equations (2) and (5), respectively. The first constraint describes the law of motion of the state variable U (promise-keeping constraint), and the second constraint states that payments have to be incentive-compatible.

Job Search Monitoring (JM): The problem of the planner that chooses to monitor the search effort of the agent is

$$\begin{aligned} V^{JM}(U,h) &= \max_{c,U^f,U^s} -c - \kappa^{JM} + \beta\left[\pi(h)\mathbf{W}(U^s,h) + (1-\pi(h))\mathbf{V}^f(U^f,h)\right] \\ s.t. &: \\ U &= u(c) - e + \beta\left[\pi(h)U^s + (1-\pi(h))U^f\right]. \end{aligned} \tag{8}$$

Notice the similarity between problem (JM) and problem (UI): the former is identical to (UI) except for the fact that there is no incentive-compatibility constraint in exchange for the additional per period cost κ^{JM}. This cost can be interpreted as the salary of the government employee ("mentor") who monitors and enforces the search activity of the unemployed worker, plus the additional administrative expenditures associated to this task.

Training (TR): We think of TR as a situation where the planner operates a costly stochastic skill accumulation technology requiring the unobservable agent's effort as input. The planner's problem when the worker is enrolled in training is defined as

$$\begin{aligned} V^{TR}(U,h) &= \max_{c,U^s,U^f} -c-\kappa^{TR}+\beta\left[\theta\mathbf{\Omega}(U^s,h)+(1-\theta)\mathbf{V}^f\left(U^f,h\right)\right] \\ s.t. &: \\ U &= u\left(c\right)-e+\beta\left[\theta U^s+(1-\theta)U^f\right], \\ U &\geq u(c)+\beta U^f, \end{aligned} \tag{9}$$

where we have simplified the notation for the success rate of training $\theta\left(e\right)$ as θ. This formulation accommodates the two most typical examples of training programs. The first interpretation of the training option is *formal training*, obtained by setting $\mathbf{\Omega}=\mathbf{V}^s$ in (9). During formal training, workers improve their literacy/numerical skills (basic training), or learn some occupational-specific skills (vocational training) in the classroom. The probability θ denotes the likelihood of the worker passing the examination or attaining the degree in any given period. According to this interpretation, the cost of the training technology κ^{TR} becomes the per-period/per-head cost of administering the (basic or vocational) course.

Second, one can easily generate *on-the-job training* by setting $\mathbf{\Omega}=\mathbf{W}$ in (9) to allow for the possibility that a worker trained in a private firm is retained and hired permanently by the firm itself, with probability θ, at the end of each period. The cost κ^{TR} has the interpretation of a wage subsidy paid to the firm hosting the worker.

Finally, note that the outcome of the training program is always observable to the planner. Formal training programs award official degrees to those who have satisfactorily passed the final exam, and the success of on-the-job training programs can be simply measured by whether the worker is retained by the firm or let go at the end of its internship.

3.2 Properties of the Value Functions

We now study some technical properties of the value functions that will be to be useful in the characterization of the optimal WTW contract that we offer in the next section.

Proposition 1 (Value functions): *(i)* $\mathbf{V}^y(U,h)$ *is bounded, continuous in* (U,h) *and concave in* U*; (ii) If* u *is unbounded below, then* $\mathbf{V}^y(U,h)$ *is decreasing in* U*; (iii) If*

$e_w = 0$, *then* $\mathbf{V}^y(U,h)$ *is increasing in* h*; (iv) if* Q^y, ω *and* π *are differentiable then* $\mathbf{V}^y$ *is differentiable in* h*; (v)* $\mathbf{W}$ *satisfies all the properties of* $\mathbf{V}^y$ *stated in (i)-(iv).*

Proof: *See Appendix C.*

The properties of $\mathbf{W}$ can be derived by inspection, since the value of employment has the following separable form

$$\mathbf{W}(U,h) = \frac{\mathbf{E}\left[\omega\left(h'\right); Q^s\left(\cdot, h\right)\right]}{1-\beta} - \frac{u^{-1}\left(\left(1-\beta\right)U + e_w\right)}{1-\beta}, \tag{10}$$

where the first term is the expected discounted stream of gross wages, which are increasing over time due to the accumulation function Q^s; the second term is the present value of the constant level of benefits guaranteed by the planner to an employed worker.

Most of the properties of $\mathbf{V}^y$ are obtained as applications of fairly standard results in dynamic programming, except for the concavity in U, which is derived by extending the result in Aumann (1965). To prove concavity, we exploit heavily the end-of-period randomization over human capital shocks in (5). Note that, in our model, this randomization is performed over a state variable rather than over a payoff-irrelevant variable, as typically done in the repeated-games literature and, recently, in the optimal taxation literature (Phelan and Stacchetti, 2001).

It is useful to notice that, as a by-product of the main proof of Proposition 1, we obtain that the properties of $\mathbf{V}^y$ are inherited by the value functions of every single policy, which allows us to state

Corollary 1: *(i) Under the assumptions of Proposition 1, the functions* V^i *with* $i = JM, SA, TR, UI$*, satisfy the properties of* $\mathbf{V}^y$*: they are bounded, continuous, concave in* U*, and each* $V^i(U,h)$ *is decreasing in* U *and increasing in* h; *moreover, they are strictly concave and differentiable in* U*; (ii)* V^{SA} *is constant in* h; *(iii) if* Q^y, ω *and* π *are differentiable then* V^i *are differentiable in* h*. (iv)* $\mathbf{V}^y(U,h)$ *is differentiable in* U*.*

4 Characterization of the Optimal WTW Program

To characterize the optimal WTW program, we proceed in steps. We start by listing the key economic forces that shape the trade-offs across the four policies of the WTW program. Next, we prove that social assistance is absorbing. We then study the stationary benchmark (with constant human capital), useful to understand what are the features of the optimal WTW program that are independent of human capital dynamics. Finally, we move to the general framework with human capital depreciation and accumulation: we start without training policies, and then we let the planner finance a training program (TR) for the worker.

4.1 Economic Forces in the Choice of Policies

Within our model, the key economic forces that induce the planner to select one particular policy over the other three can be identified as follows:

Direct cost: A planner who wants to implement JM or TR will have to incur in certain direct expenses associated to the administration of the job search monitoring and training programs (respectively, κ^{JM} and κ^{TR}). The larger these costs are, the less attractive are these two policies compared to UI and SA.

Incentive cost: By using the promise-keeping constraint, the incentive compatibility constraint during unemployment insurance can be conveniently reformulated (independently of the unemployment benefit c), as

$$U^s - U^f \geq \frac{e}{\beta\pi(h)}. \tag{IC1}$$

The difference between the state-contingent utilities U^s and U^f is increasing as h falls, through the hazard rate $\pi(h)$. Since the agent is risk-averse, in order to compensate the agent for the wider spread of payments across states, the planner has to deliver the agent a higher average transfer. In other words, incentive costs for the planner (i.e., resource costs of satisfying the incentive-compatibility restriction during UI) increase as human capital h depreciates. Note that this cost is absent in JM and is independent of h in TR (because θ does not depend on h).

Satisfying the IC constraint in UI or TR requires state-contingent benefits (i.e. a consumption lottery). When the inverse of the marginal utility is convex, the cost of providing this lottery, in terms of consumption payments of the planner, increases with

U. Hence, the incentive costs (during both UI and TR) increase with the level of promised utility U.

Effort compensation cost: Since u is concave, the higher is the promised utility U the lower is the marginal utility of consumption. Hence, the larger must be the benefits paid by the planner necessary to compensate the worker for the fixed disutility of the search/training effort cost e. This force makes SA more attractive, compared to UI, JM and TR, for high enough levels of U.

Returns to search/training: The returns to search, in terms of job finding rate and earnings once employed are increasing in h. The returns to human capital accumulation due to training are of three types: a higher level of human capital h increases earnings during employment, increases the worker's future returns to job search in UI and JM, and reduces the future incentive costs of UI. Finally, since training excludes job search, in addition to the direct cost κ^{TR} the training activity also faces an opportunity cost which increases with h $(\pi(h))$.

These economic forces help understanding the following result.

Proposition 2 (SA absorbing): *Assume that $e_w = 0$. Then, SA is absorbing: if it is chosen at any period t, it is optimal to chose it thereafter.*

Proof: *See Appendix C.*

During SA, given the absence of IC constraints, the planner offers full insurance to the agent, hence U is constant. Because of depreciation, however, h declines over time. As h gets smaller, the incentive costs rise and the returns to search and training fall, hence any other alternative program becomes less attractive compared to SA, which reinforces its optimality.

Interestingly, Proposition 2 establishes already one key property of the optimal sequence of policies in a WTW program, as it rules out programs where incentive-provision or monitoring is offered after a spell of social assistance.

4.2 The Stationary Economy

The stationary benchmark is a particular case of the general model where human capital is not a state variable (i.e. it remains always constant), thus $\pi(\cdot)$ and $\omega(\cdot)$ do not depend on h. We assume $\pi, \omega > 0$. The planner's values of each policy are defined as in Section 3.1, without dependence on h.[8] Since there is no human capital variation, there is no role for training policies.

In the next Proposition, we show that the structure of an optimal WTW program in an economy without human capital depreciation is very simple: each program is "absorbing", i.e. once the planner selects the initial program, it will never switch out of it.

Proposition 3 (Stationary economy – Policies): *Every policy (JM, SA, UI) is absorbing: if it is chosen at the beginning of the program, it is optimal to choose it thereafter.*

Proof: *See Appendix C.*

Consider the problem of a planner facing an agent with initial utility entitlement equal to U_0. For U_0 high enough, the search effort compensation cost is prohibitively high and planner will release the agent immediately into social assistance, which is absorbing.

Suppose now that U_0 is such that the planner decides to require the agent to supply positive search effort: the choice would be either facing the IC constraint or paying κ^{JM} to monitor the agent's effort perfectly. As the utility entitlement falls, the IC constraint becomes "cheaper" to satisfy, so for low enough initial levels of U_0, the planner will begin by enrolling the agent in UI, while for intermediate values of U_0 the planner will choose JM as its initial policy.

[8] In this case, the randomization is *payoff-irrelevant* (similar to sunspots*)*, hence it has no particular economic meaning, but it is a useful technical step to convexify the value function of the planner. Problem (5) becomes

$$\begin{aligned} \mathbf{V}^z(U^z) &= \int \max_{U^z(x)\in D} V(U^z(x))dx, \\ s.t. &: \\ U^z &= \int U^z(x)\,dx, \end{aligned}$$

where we have denoted the payoff-irrelevant random variable by x.

During UI, because of incentive compatibility, the state variable U is decreasing which reinforces the optimality of UI compared to the other available policies. Finally, under strict concavity of $\mathbf{V}$ in a neighborhood of the initial utility entitlement U_0, it is easy to show that during JM the agent is fully insured and the promised utility remains constant over time, hence the planner will never switch out of JM.

The next result regards the relative slopes of the value functions, and it is directly obtainable from the line of proof adopted in Proposition 3.

Corollary 2 (Slopes of the value functions with respect to U): *The (negative) slopes of the value functions with respect to U satisfy*

$$V_U^{SA}(U) \geq V_U^{JM}(U) \geq V_U^{UI}(U),$$

where the first inequality holds for any U, whereas the second inequality holds at the crossing-point, i.e. at the unique U (if any) where $V^{JM}(U) = V^{UI}(U)$.

In the top panel of Figure 2, the value of unemployment insurance for the planner V^{UI} falls more steeply than JM with respect to U because of the incentive cost, and V^{JM} is steeper than V^{SA} because of the effort-compensation cost.

We now turn to the characterization of benefits and taxes/subsidies during the optimal WTW program of a stationary economy.

Proposition 4 (Stationary economy – Payments): *(i) During unemployment insurance (UI), benefits are decreasing and the wage tax is increasing over time; (ii) During job search monitoring (JM), both the benefits and the wage tax (or subsidy) are constant; (iii) During social assistance (SA) benefits are constant.*

Proof: *See Appendix C.*

Benefits are constant in SA and JM because, within these policies, the absence of incentive problems allows the planner to implement full insurance. The result on the structure of payments and taxes during UI is a re-statement of Hopenhayn and Nicolini (1997) specialized to our environment. A direct consequence of *(i)* is that wage subsidies are either paid at the beginning of the unemployment spell (for particular combinations of high U_0 and low h), or otherwise they will be never used. The government will never switch from a wage tax to a wage subsidy during the program.

Moreover, it is easy to see that in a stationary economy without human capital depreciation, informational constraints do not play any role in shaping the *sequence* of policies in the optimal WTW plan. Consider the problem of a planner who can perfectly observe search effort at no additional cost. Clearly, in this case there is no reason for JM programs. Due to the absence of incentive problems, both consumption c and utility U are constant and the agent is fully insured. Hence, once again, both SA and UI are absorbing.

4.3 Optimal WTW Program without Training

In this section we begin the characterization of the optimal WTW scheme in presence of human capital dynamics. It is useful to start from the case where training is prohibitively costly and will never be chosen. In section 4.4, we enrich the analysis by introducing on-the-job and basic training.

Throughout the analysis, we will exploit a graphical representation in the (U, h) state space. In particular, reading the (U, h) state space as a phase diagram –whose dynamics are driven by the policy functions $U^y(U, h)$ describing the law of motion for the endogenous variable U, and by the exogenous laws of motion for human capital $Q^y(\cdot, h)$– we can then recover the sequence of policies within the optimal WTW program.

Finally, the policy functions $\{c^i(U, h), c^e(U)\}$, together with the laws of motion for the two states, fully describe the optimal sequence of unemployment benefits and wage taxes/subsidies during the optimal WTW program.

4.3.1 Representation in the (U,h) Space

In Corollary 2, we have established the relative slopes of the value functions with respect to U. The following proposition establishes a ranking on the slope of the value functions V^i across the different policies $i = JM, SA, UI$, with respect to human capital.

Proposition 5 (Slope of the value functions with respect to h): *If $\mathbf{V}^f$ is a sub-modular function, the slopes of the value functions $V^i(U, h)$ with respect to h satisfy*

$$V_h^{UI}(U, h) \geq V_h^{JM}(U, h) \geq V_h^{SA}(U, h) = 0.$$

Proof: *See Appendix C.*

The bottom panel of Figure 2 shows the typical shape of the value functions $V^i(U,h)$ for $i = JM, SA, UI$ with respect to h. V^{UI} is steeper than V^{JM} because of the incentive cost and V^{JM} is steeper than V^{SA} (invariant to h) because of the returns to search.

Recall that in the twice-differentiable case submodularity means $\mathbf{V}^f_{Uh}(U,h) \leq 0$. The shape of $\mathbf{V}^f$ is generated by two contrasting forces. First, "within-policy" there is a tendency towards *super*modularity as an increase in h reduces the marginal cost of delivering a given level of utility U. However, a high h makes policies implementing active search (like JM or UI) more attractive, and Corollary 2 suggests that search-intensive policies have higher slopes with respect to U. This "between- policy" force tends to generate *sub*modularity of $\mathbf{V}^f$. The assumption in Proposition 5 holds whenever the second force dominates the first, for example for high rates of human capital depreciation.[9]

When the upper envelope $V(U,h) = \max_i V^i(U,h)$ is projected onto the (U,h) space, as done in Figure 3, we obtain immediately the regions in the state space where each policy emerges as optimal. The slopes of the value functions with respect to both states, characterized in Corollary 2 and Proposition 5 suggest that this is the only possible configuration of the state space. We start by interpreting Figure 3 as we move "horizontally" in the (U,h) space, i.e. we let U change for a given h. Next, we study the optimal policies as we move "vertically" through Figure 3, i.e. we change h for a given level of utility entitlement U.[10]

Moving horizontally (along U): Given any h, start from the highest utility level in the diagram. For high enough U, compensating the agent for the high effort is prohibitively costly, and SA is optimal. As we decrease U, the effort compensation cost falls and it becomes optimal to choose a program with high-effort requirement. For intermediate

[9]The case with i.i.d. shocks trivially satisfies submodularity, since $\mathbf{V}^f_{Uh}(U,h) \equiv 0$. General conditions on the primitives for $\mathbf{V}^f$ to be submodular are difficult to find. One technical reason is that the nature of the *max* operator is to preserve *super*modularity, but not necessarily *sub*modularity (e.g. see Hopenhayn and Prescott, 1992).

[10]It should be clear, at this point, that moving horizontally in the Figure 3 diagram corresponds to reading the top panel of Figure 2 from right to left, and moving vertically corresponds to reading the bottom panel of Figure 2 from right to left.

levels of U the incentive cost is still high and the value of JM dominates the value of UI. As we keep decreasing U, gradually the planner finds more profitable facing the incentive cost than paying the fix monitoring cost κ^{JM} and UI becomes optimal.

Moving vertically (along h): For high levels of h (i.e. high π), returns from search are high and incentive costs are low, so UI is optimal. As h falls, incentive costs increase and the planner finds optimal to pay the monitoring cost and implement JM. For very low levels of h, the returns to search are so low that the planner prefers to save the effort-compensation costs as well, and SA is the optimal program.

4.3.2 The Optimal Sequence of Policies

The optimal sequence of policies is dictated by the evolution of the state variables (U, h). Conditional on unemployment, given the assumption of non-overlapping supports for Q^f, h declines monotonically.

The evolution of U depends on the policy. Because of full-insurance, during SA the continuation utility U is constant. During JM, perhaps surprisingly, the utility entitlement of the agent U^f will tend to increase. The reason is that, as h decreases along the unemployment spell, the optimal program approaches the social assistance option for low levels of h. Recall that, because of full insurance, the benefits c are constant between JM and SA, and the socially assisted agent will also save the search effort cost e. Hence, the utility U is higher in SA, and U^f gradually increases during JM to approach the social assistance level.[11] Finally, as expected, during UI, the utility entitlement U promised by the planner to the unemployed worker tends to decline monotonically to satisfy the incentive constraint.

Putting all together, conditional on failure of search, the typical sequence of an optimal WTW program without training begins with UI followed by JM followed, in turn, by SA. When the search-effort monitoring cost κ^{JM} is too high for JM to be chosen, UI is eventually followed by SA; when monitoring is cheap, the optimal program might not include UI at all.

[11] Specifically, during JM, the continuation utility stays constant when h does not depreciate and rises if h depreciates, since the implementation of SA becomes more likely.

4.3.3 Optimal Benefits and Wage Taxes/Subsidies

It is straightforward exercise to verify that in an economy with capital depreciation Proposition 4 becomes:

Proposition 6 (Payments): *(i) During unemployment insurance (UI), benefits are decreasing and the behavior of the wage tax is in general ambiguous; (ii) During job search monitoring (JM), the benefits are constant and the wage tax is decreasing; (iii) During social assistance (SA) benefits are constant.*

Proof: *See Appendix C.*

There are two main differences with respect to the stationary case. First, the behavior of the wage tax during UI becomes a quantitative issue, which will be discussed below. Second, since the expected gross wage $\mathbf{E}\left[\omega\left(h'\right);h\right]$ decreases during unemployment and c^e is constant during JM the wage tax $\tau = \mathbf{E}\left[\omega\left(h'\right);h\right] - c^e$ must decrease.

In order to illustrate the key features of the benefits paid across the various policies, we use particular histories of human capital shocks simulated by the model.

The bottom-right panel of Figure 4 shows the gross re-employment wage at every period, hence it describes the characteristics of the specific history of human capital shocks we are considering; the top left panel shows the behavior of the UI benefits as a fraction of the initial wage, and the net wage (gross wage minus tax, or plus subsidy) that the unemployed worker would earn if she found a job in that period; the top-right panel depicts the implied tax/subsidy, as a fraction of the current wage; the bottom-left panel shows the dynamics of U^f.

As previously discussed, benefits (consumption during unemployment) decrease during UI and remain constant throughout JM and SA because of consumption smoothing. The net wage (consumption during employment) first decreases and then rises sharply as UI approaches JM. The reason for these dynamics is that in a multiperiod setting, the optimal incentive scheme is shaped by the tension between intra- and inter-period consumption smoothing. The planner can improve intra-period consumption insurance (across unemployment and employment states) by moving part of the punishment burden forward into the future. This is why, in a stationary model without human capital dynamics, benefits, net wage, and U_t^f never stop decreasing (and taxes never stop increasing) during unemployment, as shown by Hopenhayn and Nicolini (1997).[12] The emergence of JM and SA policies where U^f cannot decline shortens the effective time horizon of the UI problem, forcing the planner to design a scheme biased toward the static component of the incentives. As a result, the planner uses heavily wage subsidies in order to reward employment and widen the difference between UI payments and net wage upon job finding.

When the worker enters JM, there is complete insurance also across employment and unemployment states, hence the net wage and unemployment benefits coincide and remain constant. Hence, the behavior of the wage subsidy essentially mirrors that of the re-employment wage (and of human capital): a simple inspection of the bottom-right and the top-right panels shows that, indeed, once entered into JM the wage subsidy increases if and only if human capital depreciates.

4.4 Optimal WTW Program with Training

4.4.1 Representation in the (U,h) Space

On-the-job Training– It is useful to start from the addition of on-the-job training, as defined in Section 3.1, to the set of instruments available to the planner. When $\kappa^{TR} = 0$, this case is especially simple to analyze because a comparison with the UI problem in (7) illustrates immediately that this particular form of training is exactly like UI with success probability θ instead of $\pi(h)$, hence there is a critical level of human capital h^{TR} solving $\pi\left(h^{TR}\right) = \theta$ such that below that level TR is always strictly preferred to UI.

Figure 5 shows that on-the-job training emerges as optimal in the bottom-left region of the (U, h) space. As U increases, JM will be preferred to both, since paying the cost κ^{JM} to avoid facing the incentive costs present in both TR and UI becomes optimal.

Formal Training- Consider now a planner who has access to a "formal training" technology, as detailed in equations (9). Here, the comparison among policies is less

[12] We chose a history of human capital shocks where, for several periods (5 to 17), h is constant, as assumed by Hopenhayn-Nicolini, to illustrate that the features they emphasize arise as a particular case of our setup.

stark. Qualitatively, from the fact that $\omega(\cdot)$ and $\pi(\cdot)$ are bounded above and from the restriction $Q^s(\cdot;0) \equiv Q^f(\cdot;0)$, one can easily show that formal training will emerge only for intermediate values of h. Interestingly, as illustrated in Figure 6, the typical region of the state space where formal training arises as optimal is very close to that of on-the-job training, i.e. intermediate to low levels of h, and low levels of U. Another interesting regularity is that the training region is always *connected*, i.e. TR is never optimal in separate areas of the (U, h) state space. However, this feature is not necessarily true for the whole range of the parameters.

4.4.2 The Optimal Sequence of Policies

In general, the model does not put tight restrictions with respect to the position of training in the optimal policy sequence. In the case of Figures 5 and 6, JM can never be chosen optimally *before* TR (as U rises during JM), but for low enough values of the monitoring cost κ^{JM} it is easy to generate graphs where JM surrounds the TR area, and job search monitoring can optimally lead into a training phase.

Moreover, from Figure 6, it is clear that *after* a successful spell of TR, both JM and UI are possibly optimal. The reason is that, in this event, U can rise in order to satisfy the incentive compatibility constraint, i.e. $U^s > U$. This increase in continuation utility is accompanied by human capital accumulation and the agent moves "north-east" in the phase diagram. Only a quantitative analysis, case by case, can yield a sharper answer to this question.

4.4.3 Optimal Benefits and Wage Taxes/Subsidies with Formal Training

In Figure 7, we illustrate the typical time path of optimal benefits and wage taxes/subsidy with formal training. We chose a history where TR first fails for several periods and only later it starts becoming successful, as clear from the path of human capital in the bottom-right panel.

The most interesting features are two. First, unemployment benefits increase upon successful training as a reward to the unemployment agent. Second, when skills are rebuilt through successful training, both the gross re-employment wage and the continuation utility promised by the WTW program increase. The first force makes a tax upon re-employment more likely, while the second makes it less likely: as clear from the top-right panel, for the chosen parametrization the human capital effect dominates.

More in general, the less effective is the formal training technology (small success probability θ and/or negligible human capital gain from training), the more likely is the optimal wage tax (subsidy) to decrease (rise) after a spell of successful training. For a given gain in gross wage $\omega(h)$, a small value of θ will be associated with a higher value of U^s and hence a higher promised consumption level upon re-employment. For given U^s, a small increase in human capital during training is associated to a low rise in $\omega(h)$.

5 A Simple Implementation with Access to Credit Markets

Throughout our analysis we have assumed that the agent starts with zero wealth and does not have access to credit. In this section, we relax this assumption and allow the agent to save through credit markets at rate $R = \beta^{-1}$, but maintain that she faces a no-borrowing constraint. We show that with the help of an additional fiscal instrument, a linear interest tax, the planner can induce the agent not to save (she is pushed at the borrowing constraint) and, as a result, is able to fully control her consumption through the payments specified by the contract.

It is easy to demonstrate that during UI and JM the payments of the optimal contract satisfy the following condition for any period t of unemployment, and any human capital level h

$$\frac{1}{u'(c_t)} = \pi(h) \frac{1}{u'\left(c^s_{t+1}\right)} + (1 - \pi(h)) \frac{1}{u'\left(c^f_{t+1}\right)}, \tag{11}$$

where the superscripts s and f denote “success” and ‘failure” of search.[13] From (11) and Jensen’s inequality,

$$u'(c_t) \leq \pi(h) u'\left(c^e_{t+1}\right) + (1 - \pi(h)) u'\left(c^u_{t+1}\right),$$

with strict inequality each time $c^e_{t+1} \neq c^u_{t+1}$ (a typical situation under UI or TR where the allocations must be incentive compatible and the planner cannot offer full insurance).

[13] During TR, the equation holds with θ in place of $\pi(h)$, and during SA it holds trivially as consumption is constant over time.

The optimal payment scheme always forces the agent to under-consume next period, compared to its individual optimum: the agent would then choose to *save* at time t to increase consumption next period and induce her Euler equation to hold with equality.

Therefore, imposing only a borrowing constraint does not help to rule out a situation where individual consumption would diverge from the benefits paid by the planner, and the implementation of the contract would fail: the planner must prevent the agent from saving.

Assuming observable savings, Kocherlakota (2003b) and Golosov et al. (2003) argue that a simple linear tax that satisfies the Euler equation of the agent under the optimal contract, i.e. such that

$$u'(c_t) = \left(1-\tau^k\right)\left[\pi(h)\, u'\left(c^s_{t+1}\right) + (1-\pi(h))\, u'\left(c^f_{t+1}\right)\right],$$

does not guarantee that the agent would not be willing to save. Indeed, the relevant deviation for the agent is joint: the agent would reduce effort to zero and save at the same time. Because of the incentive constraint, typically $c^s_{t+1} > c^f_{t+1}$, hence a reduction in effort makes the consumption distribution shift towards the worst outcome, which in turn generates an additional incentive to save at t to finance consumption at time $t+1$.

We propose a simpler implementation mechanism which can be applied also in the case of hidden savings. Assume the agent enters the contract with no wealth ($k_0 = 0$) and faces a borrowing constraint of the form $k_t \geq 0$ thereafter. Consider a *linear interest tax* τ^k that, for any t, satisfies

$$u'(c_t) \geq \left(1-\tau^k\right) u'\left(c^f_{t+1}\right).$$

Clearly, the agent is never willing to save, not even considering the joint deviation "save and shirk". More precisely, in equilibrium (i.e., when the agent follows the effort recommendations of the contract), the agent would always be willing to borrow. However, because of the liquidity constraint, the planner maintains full control on her consumption stream and the optimal WTW contract characterized in the previous sections can still be implemented.[14]

[14] The proposed implementation is "anonymous", in the sense that it does not require observability of savings at the individual level, but it only demands control over the *aggregate volume* of savings. This requirement can be guaranteed, for example, by the presence of financial intermediaries which are allowed to maintain secrecy on the identities of the specific depositors, but whose aggregate volume of transactions is monitored for taxation proposes.

Some remarks are in order. First, it is easy to see that when utility is logarithmic in consumption, the interest tax is simply determined by

$$\tau^k = 1 - \min_t \left\{ \frac{c^f_{t+1}}{c_t} \right\},$$

or in other words, the tax is proportional to the steepest slope of the unemployment benefits along the optimal WTW program.[15]

Second, this implementation scheme can be adapted to situations where the initial (but observable) wealth is positive as long as it is not too large. In these cases, the agent must be forced immediately toward the borrowing limit with an appropriately chosen initial transfer. Assuming that unemployment benefits cannot be negative, it is easy to show that the optimal WTW program can still be implemented for initial wealth levels up to the payment specified by the optimal WTW program at time $t = 0$. In extreme cases, the optimal WTW program would require a *waiting period* without payment of benefits.[16]

Third, our implementation mechanism is totally anonymous, i.e. it does not require the direct observability of the individual savings, but it only demands control over the *aggregate volume* of savings.[17]

Finally, we acknowledge that this implementation of the optimal contract where the agent is not allowed to save and her consumption is fully controlled by the government is not too appealing for the design of an optimal taxation scheme in the aggregate economy. However, it fits well the case of low-income, low-wealth workers on the welfare rolls, which are the target of our study.

[15] It follows that, in an optimal WTW program where only full-insurance policies (JM and SA) are implemented, there is no need for an interest tax.

[16] Interestingly, in several states (e.g. Texas and California) UI benefits start to be paid one week after declaring the unemployment status. This rule corresponds exactly to a zero initial transfer to induce the agent to dissave.

[17] This requirement can be guaranteed, for example, by the presence of financial intermediaries which are allowed to maintain secrecy on the identities of the specific depositors, but whose aggregate volume of transactions is monitored for taxation proposes, and are required by the government to act as withholding agents, i.e. they deduct a withholding tax from all interest payments, and transfer the total revenues to the government.

6 Quantitative Analysis: The Optimality of the U.S. Welfare System

The first step of the quantitative analysis is the calibration of the model to match the salient features of the U.S. labor market and the current U.S. welfare system. Once we choose an initial level of skills h_0, the parameterization of the existing system allows us to simulate histories of unemployed workers in order to calculate the expected initial entitlement of utility $\bar{U}_0(h_0)$ associated to the current program. This is an essential ingredient of the exercise, since it establishes the initial conditions of the (U, h) phase diagram studied in the previous sections.

Next, we solve for the optimal WTW program in the U.S. and characterize the optimal sequence of policies, payments and taxes/subsidies corresponding to a representative unemployed worker with the same initial conditions $\left(\bar{U}_0, h_0\right)$ as in the actual program. We then compare the current and optimal programs and calculate the budget savings for the government and the welfare gains for the workers associated to switching to the optimal WTW scheme.

6.1 Calibration

The parameters we need to calibrate can be divided into three groups. First, the labor market parameters $\left\{w(h), Q^f, Q^s, \pi(h)\right\}$. Second, the set of parameters characterizing the current U.S. welfare system $\left\{\kappa^{JM}, \kappa^{TR}, \theta, Q^{tr}, \bar{c}^{UI}, \bar{c}^{JM}, \bar{c}^{TR}, \bar{c}^{SA}, \bar{d}^{UI}, \bar{d}^{JM}, \bar{d}^{TR}, \bar{\tau}\right\}$. Note that the first four parameters should be interpreted as technological parameters that we also use when studying the optimal program, whereas the remaining parameters represent the observed payments $(\bar{c})$ and the observed durations $\left(\bar{d}\right)$ of the actual U.S. scheme. Third, to parameterize preferences we need to choose a specification for intra-period utility $u(\cdot)$, and values for $\{\beta, e, e_w\}$. Below, we describe our calibration strategy, and in Table 1 we list the calibrated parameter values.

6.1.1 Labor Market Parameters

Wage function– We assume a linear (monthly) wage function $w(h) = h$, so that human capital can be interpreted as efficiency units of labor in a competitive labor market. Thus, changes in human capital map directly into observable wage changes. Our interest lies in the group of households who are more likely to become recipients of welfare benefits. Moffitt (2001, Table 5) reports that in 1999, 49% of welfare recipients were high-school drop-outs. Hence, it seems reasonable to focus in our analysis on workers with at most a high-school degree. The year 2000 U.S. Census reports that median monthly earnings for this group was $2,100.

Human capital depreciation– From our qualitative analysis, it is clear that the rate of depreciation of human capital is a key parameter of our model. Within a structural model, Keane and Wolpin (1997) estimate the annual earnings loss for males in the U.S. to be 9.6% for blue-collar workers and 36% for white-collars. In our benchmark analysis, we use 22%, the average value, but we also experiment with a 10% annual rate of skill loss. To parameterize the matrix $Q^f(h', h)$, we assume that workers can either keep their human capital level with probability q^f, or move down one step on the human capital grid, with probability $1 - q^f$. In order to have a constant depreciation rate for all levels of human capital, we set a geometrically-spaced grid.[18]

Human capital accumulation on the job- We chose a human capital accumulation rate during employment of 1% per year. This number is somewhat smaller than existing estimates to account for the fact that employment is an absorbing state in our model. To parameterize the matrix $Q^s(h', h)$, we assume that workers can either keep their human capital level with probability q^s, or move up one step on the human capital grid, with probability $1 - q^s$. Given the geometrically-spaced grid, q^s is set to match the estimated accumulation rate.[19]

Job finding probabilities– We postulate a logistic form for the hazard function

$$\pi(h) = \frac{1}{2}\frac{\exp(\lambda_1 h)}{\lambda_2 + \exp(\lambda_1 h)},$$

[18] If the grid spacing implies a monthly decay at rate Δ, and x^f is the estimated monthly depreciation rate of human capital, q^f solves

$$q^f = 1 - \frac{x^f}{\Delta}.$$

[19] If the grid spacing implies a monthly decay at rate Δ, and x^s is the estimated monthly accumulation rate of human capital, q^s solves

$$q^s = 1 - \frac{1-\Delta}{\Delta}x^s.$$

and we use duration data to estimate λ_1 and λ_2 by matching several moments of the hazard rate at different duration levels for the group of workers of interest (high-school graduates and below). Meyer (1990, Tables V and VII, specification (5)) estimates an average monthly hazard of .33 for a pool of U.S. workers with average education of 8.6 years. At the moment, we only use this moment condition. Within our grid, the parametrization generates a range of monthly hazard rates between $\pi(h_{\max}) = 0.50$ and $\pi(h_{\min}) = .006$.

6.1.2 The Current U.S. Welfare System

The U.S. do not have a fully structured Federal WTW program, but several pieces of legislation over the years have built a network of Federal and State government interventions. In Appendix D, we put together the major components of the U.S. welfare system and reconstruct the typical welfare-to-work program faced by an unemployed worker. In light of that description, it suffices here to explain that we model the current U.S. Welfare system as follows.

During unemployment, workers receive UI benefits with a replacement rate of 60% on their past earnings for the first 9 months. At the expiration of the UI benefits, workers enter the Temporary Assistance for Needy Families (TANF) regime and are subject to mandatory active labor market programs, which differ by state. Broadly speaking, there exist two types of programs.

First, Human Capital Development (HCD) programs where individuals spend a maximum of 24 months on basic training, with the features described above. Upon success, or at the end of the 24 months, workers move into a job-search monitoring activity with maximum duration of 6 months. Second, Labor Force Attachment (LFA) programs where individuals spend a maximum of 12 months on job-search monitoring, followed by 6 months in basic training. During the period in which they are enrolled in the HCD/LFA program, they receive welfare benefits and food stamps for a total of \$700 per month. The monthly costs of administering the programs are, respectively, $\kappa^{JM} = \$480$ for job-search monitoring, and $\kappa^{TR} = \$160$ for training, and training is successful with probability $\theta = .15$.

If at the end of the HCD/LFA program workers are still unemployed, they will continue receiving the same benefits, without being enrolled in any other active program up to 45 months from the date of displacement, after which we assume that their TANF benefits expire (that is, we assume that the transfers expire after 3 years on top of the initial 9 months of unemployment insurance), and workers only receive food stamps for an amount of \$280 per month. These welfare payments outside of active labor market programs should be interpreted as form of pure social assistance, in the context of our model.

In the event individuals become employed, they are subject to the Federal Unemployment Tax (FUTA) at a rate of 1.4% on the first \$583 earned monthly, and 0.6% above that threshold. Moreover, workers' earnings are subsidized exactly as indicated by the Earned Income Tax Credit (EITC) legislation.

6.1.3 Preference Parameters and Initial Conditions

We set the model's period to one month. We use a logarithmic intra-period utility function for consumption, i.e. $u(c) = \ln(c)$ and choose a value for the discount factor $\beta = .9957$ in order to match an interest rate of 5% on a yearly basis.

To calibrate the effort cost we choose a value for $e = 0.6$ corresponding to roughly $1/5$ of the utility associated to consuming the median monthly wage for our group of workers with at most high-school education.

The key inputs of the normative analysis are the initial utility entitlement promised implicitly by the actual U.S. welfare program to each worker, and the associated stream of expenditures of the U.S. government. Since both employment and social assistance are absorbing states, by backward induction it is easy to reconstruct the initial expected utility entitlement $\bar{U}_0(h)$ and the expected stream of expenditures $\bar{V}(h)$ for workers who enter unemployment with different levels of h and face the U.S. welfare system. For example, the net expenditures include the benefits and wage subsidies paid to the worker (during unemployment and employment, respectively) and the costs of operating training and job search monitoring programs, for the durations specified by the current U.S. system, net of the tax levied on earnings upon employment.

We chose to study the sequence of policies and payments for a worker with pre-displacement monthly wage of $h_0 = \$1,500$. For this worker, we compute that lifetime discounted government expenditure amount to $\bar{V} = \$29,000$ under the LFA program, and $\bar{V} = \$36,000$ under the HCD program, corresponding to an average expenditure of, respectively, \$125 and \$154 per month-per worker.

Table 1: Calibrated Parameters

Parameter	Value	Moment to Match
Δ	0.100	Rate of geometric decay for human capital grid
β	0.9957	Interest rate (Cooley, 1995)
e	0.600	1/5 of utility of average consumption (Pavoni, 2003b)
e_w	0.600	1/5 of utility of average consumption (Pavoni, 2003b)
q^f	0.927	Wage loss upon displacement (Keane and Wolpin, 1997)
q^s	0.963	Wage growth on the job (Violante, 2002)
q^{tr}	0.600	Wage gain associated to the degree in TR (NEWWS, 2002)
λ_1, λ_2	$0.21, 50$	Unemployment hazard function (Meyer, 1990)
κ^{JM}	\$480	Monthly cost of JSM (NEWWS, 2001)
κ^{TR}	\$160	Monthly cost of TR (NEWWS, 2001)
θ	0.150	Fraction of workers in TR receiving degree (NEWWS, 2001)
$\bar{c}^{UI}$	$0.60w(h_0)$	Benefit rule during UI (U.S. Department of Labor)
$\bar{c}^{JM}$	\$700	Benefits during JSM (NEWWS, 2001)
$\bar{c}^{TR}$	\$700	Benefits during TR (NEWWS, 2001)
$\bar{c}^{SA}$	\$280	Maximum allotment of Food Stamps (NEWWS, 2001)
$\bar{d}^{UI}$	9	Duration of UI in months (U.S. Department of Labor)
$\bar{d}^{JM}$	$LFA = 6, HCD = 6$	Duration of JSM in months (NEWWS, 2001)
$\bar{d}^{TR}$	$LFA = 3, HCD = 18$	Duration of TR in months (NEWWS, 2001)
$\overline{\tau}$	see text	FUTA and EITC (U.S. Department of Labor)
h_0	15	Monthly Earnings of \$ 1,500
U_0	$LFA = 508, HCD = 482$	Utility entitlement implied by actual U.S. program

6.2 Results

6.2.1 The Features of the Optimal WTW Program

Figure 7 summarizes the results of our first simulation exercise, where we computed $\bar{U}_0(h_0)$ based on the U.S. welfare system with the HCD program. Recall that in our model the evolution of h is stochastic, and both payments and policy assignments depend on h. In order to provide a general idea of the main quantitative features of the optimal WTW program, we generated 500 histories of human capital shocks, conditional on the worker always remaining unemployed. We then calculated sample averages of the planner's optimal transfers (worker's consumption), lifetime utilities, and re-employment wages.

Optimal sequence of Policies– The bottom-right panel displays the fractions of workers assigned to the different policies at each duration. All workers start in UI.

As human capital depreciate, workers begin to be moved to JM, hence the fraction of workers in UI decreases while that in JM steadily increases. After roughly 2.5 years of unemployment, also the fraction of workers in JM decreases as the flow from JM into SA more than counterbalances that from UI into JM. For sufficiently long durations, all unemployed workers end up in SA. From the figure, one can see that the average duration of both UI and JM is approximately 20 months.

A striking feature of the optimal program is that no worker is ever assigned to training. Simply put, TR is too expensive compared to the other programs. We calculate that, at the current cost κ^{TR}, TR would start emerging as optimal only if its success rate soared to 33% from the current 15% level. Alternatively, at the existing success rate, the cost of TR should fall from \$160 to $-$\$250 (i.e., the government should be paid) for TR to become optimal in a sizeable region of the state space. This result makes clear that the key reason why TR is never chosen by the planner is not that the technology has negative return per se, but it is the opportunity cost of foregoing the other policies (UI, JM) that is prohibitive.

Payments and Taxes/Subsidies– The upper left panel shows that the average optimal replacement ratio for unemployment/welfare benefits (the smooth line) is quite generous, at least compared to the actual scheme (represented by the step-shaped line). The optimal payments decrease from 85% to 65% of the pre-displacement wage, while the payments in the existing U.S. program never exceed 60%.

In the upper right panel, the steeply decreasing line represents the optimal average tax/subsidy upon re-employment, as a fraction of the initial re-employment wage, where negative numbers represent wage subsidies.[20] At low unemployment durations, –where as clear from the bottom-right panel, the vast majority of workers is assigned to UI– the actual system pays a more generous wage subsidy than the efficient scheme, whereas for long durations the opposite is true.

Combining the two panels we can conclude that, at the beginning of their unemployment spell, the existing U.S. WTW scheme exceeds in providing incentives (and therefore,

[20] If we let h_t be the human capital level at unemployment duration t, the tax rate τ_t reported in the figure solves

$$\tau_t = \frac{w(h_t) - c_t^e}{w(h_t)}.$$

delivers too little insurance) to unemployed workers. According to our calibration, agents could be motivated to actively search for new jobs in UI with much less consumption dispersion, and this would improve total welfare. Notice moreover that in the optimal scheme the wage tax decreases with duration, and after one year of unemployment the program provides a wage subsidy. This feature of the optimal program confirm the analysis in Pavoni (2003b) and is in sharp contrast with what Hopenhayn and Nicolini (1997) found for the stationary model without human capital depreciation. In section 4.4.3 we discussed the mechanism generating such result.

Continuation Utility– The bottom-left panel reproduces the sample-average lifetime utility, our key endogenous state variable, as a function of unemployment duration. The result confirms our previous discussion: despite the fact that incentive provision requires payments to decrease, worker's lifetime utility might eventually increase during unemployment. The reason is that human capital depreciation makes attractive to the planner the social assistance state, and SA is a policy where for each given consumption level, the worker's total utility is highest since the agent saves the effort cost e. Thus, as SA becomes more and more a likely outcome, the continuation utility has a tendency to rise.

Finally, when $\bar{U}_0$ is calibrated according to the LFA program (see Figure 8) the optimal program presents very similar features, with even more generous payments, in both states. This is simply due to the fact that the calculated initial utility entitlement under LFA is larger than that computed under the HCD program (not surprising, given that the training is a poor choice).

6.2.2 Welfare Gains and Budget Savings

Figure 9 summarizes our computations of the welfare gains and the government budget savings, as a function of the worker's initial level of human capital h_0 (her pre-displacement wage).

The welfare gains are computed by comparing the actual initial utility $\bar{U}_0\left(h\right)$ with the level $U_0\left(h\right)$ that the planner can deliver by spending exactly as much as the actual program in the optimal scheme, i.e. $U_0(h)$ solves the equation

$$\mathbf{V}\left(U_0(h), h\right) = \bar{V}(h),$$

and then expressed in terms of gain in lifetime consumption.

The budget savings are computed by comparing the actual expenditures $\bar{V}_0(h)$ to the expenditures $\mathbf{V}\left(\bar{U}_0(h), h\right)$ that the planner would incur by delivering utility $\bar{U}_0(h)$ under the optimal program.

The top-panel plots welfare gains in consumption-equivalent terms for the HCD and the LFA programs, as a function of initial human capital of the unemployed workers. Welfare gains are large, but vary a lot across workers. For the worker we considered (with initial $h_0 = \$1,500$), they are between 5% and 7% of lifetime consumption. Interestingly, welfare gains are low at the two extremes of the h distribution.

Budget savings of switching to the optimal scheme from the LFA program are of the order of \$130 dollars per month per worker, vis-a-vis a typical expenditure per month-per worker of the actual program around \$125. Thus, through tax revenues the government could implement an optimal WTW program guaranteeing the same welfare level as the LFA program on a balanced-budget basis.

The results for the *HCD* program are even more striking: the government could earn \$150 per month-per worker by implementing an optimal WTW program guaranteeing the same welfare level as the current HCD program.

7 Concluding Remarks

Welfare-to-Work programs combine passive and active labor market policies in an attempt to solve a very delicate trade-off between providing insurance to jobless workers and offering an incentive structure that will move them quickly among employment ranks.

In this paper we have provided a theoretical framework to study welfare-to-work programs from a pure normative standpoint. We see our work as a first step to answer a large set of important questions, such as: what is the optimal sequence of policies in an optimal WTW program? And, how long should each stage be? What is the optimal level and dynamics of payments in each phase of the program? Should wages upon re-employment be taxed or subsidized? Our theoretical characterization offers sharp answers to some of these questions, but only general guidelines to other questions. In this latter case, we showed how a numerical analysis based on the calibrated model does exhaustively the job.

The main qualitative features of the optimal WTW program can be summarized as follows:

- In an economy without human capital dynamics, the optimal WTW program does not contemplate switching between different policies at any point: each policy is absorbing.
- With human capital dynamics, when TR is not chosen, the typical policy sequence in the optimal WTW program starts from UI, switches into JM and then into SA, which is the only absorbing policy. The faster is human capital depreciation, the more rapidly the optimal WTW program switches between policies.
- Generally, TR emerges as optimal for intermediate levels of human capital h and low levels of promised utility U, in the state space.
- Unemployment benefits decrease during UI and during an unsuccessful spell of TR, remain constant during JM and SA, and increase after a successful spell of formal TR.
- In a phase of UI or unsuccessful TR, conditional on human capital not depreciating too fast, the wage tax rises with duration (Hopenhayn and Nicolini, 1997). As UI approaches JM, the tax tends to become a subsidy and the subsidy rises with unemployment duration during JM.
- The less effective is the formal TR technology (small success probability θ and/or negligible human capital gain), the more likely is the wage tax (subsidy) upon re-employment to decrease (rise) after a spell of successful TR.

When we used our theoretical framework to study the optimality of the current U.S. welfare system, we concluded that:

- The existing welfare system in the U.S. spends around $30,000 over the lifetime of a typical high-school dropout worker. With the same expenditures, the optimal program delivers a welfare gain equal to 6% of lifetime consumption for the worker.

- Compared to the current program, the optimal program would pay more generous benefits (with replacement rate over 80%), with flatter time-profile. At the same time, it would impose a higher tax upon re-employment for short durations (10% of wages after 1 month) and a more generous subsidy for long duration (10% of wages after 12 months).

- The optimal program keeps the worker in UI for about 6 months and then JM for an equal period, before switching into SA. At the observed level of effectiveness, formal TR policies are never part of an optimal WTW program.

This latter result agrees with a vast empirical evaluation literature that finds job-search monitoring policies much more effective than adult training (for surveys, see Heckman, LaLonde and Smith, 1999; Heckman and Carneiro, 2002).

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8 Appendix A: Autarky

Proof of Lemma 1:

(i) The timing is as follows: at the beginning of the period, the unemployed agent chooses between search and training; next, the agent chooses consumption and effort. Then, the random outcome y of the chosen activity is revealed. Since, in autarky, failure of search leads to $c = 0$ for a period, we allow $u(0) < -\infty$.[21]

The recursive formulation of this problem becomes

$$\begin{aligned} v(h) &= \max\left\{v^U(h), v^T(h)\right\}, \\ \text{where} &: \\ v^U(h) &= \max_{a\in\{0,e\}} u(0) - a + \beta\left[\pi(h,a)\,\mathbf{w}(h) + (1-\pi(h,a))\,\mathbf{v}^f(h)\right], \\ v^T(h) &= \max_{a\in\{0,e\}} u\left(-\kappa^{TR}\right) - a + \beta\left[\theta(a)\,\mathbf{v}^s(h) + (1-\theta(a))\,\mathbf{v}^f(h)\right]. \end{aligned} \tag{12}$$

The human capital shocks occur at the end of the period. Recall that the search and training outcomes $y \in \{s, f\}$ affect the realization of human capital shocks by determining the appropriate law of motion Q^y. The associated end-of-period value functions in employment and unemployment are respectively

$$\begin{aligned} \mathbf{w}(h) &= \int \left[u(\omega(h')) - e_w + \beta\mathbf{w}(h')\right] Q^s(dh';h), \\ \mathbf{v}^y(h) &= \int_H v(h')Q^y(dh';h). \end{aligned}$$

Since the agent has no income during unemployment, it is clear from (12) that whenever $\kappa^{TR} > 0$ the worker cannot afford to use the training technology, which may provide partial insurance against adverse human capital shocks. Thus, there is scope for government-provided credit towards the use of the training technology. The only available instrument for consumption smoothing is search effort. Hence, there is also scope for government-provided insurance against negative human capital shocks and against failure of job search.

[21] Alternatively, one can impose $u(0) = -\infty$, but at the same time allow the worker to access a form of home production $\underline{c} > 0$. In this case, we will require $\kappa^{TR} > \underline{c}$.

(ii) To simplify the exposition of this result, we assume that the depreciation technology is deterministic and the wage is constant.[22] For any h such that $a(h) = e$ we must have

$$\mathbf{w} \geq \mathbf{v}^f(h) + \frac{e}{\beta\pi(h,e)}. \tag{13}$$

We want to show that if (13) holds, then in the previous period, when $h_0 \geq h$, condition (13) was still true at h_0: if an agent chooses high effort in a given period, she must have also chosen high effort in the previous period. Note that:

$$\begin{aligned}
\mathbf{w} - \mathbf{v}^f(h_0) &= \mathbf{w} - v^U(h) \\
&= \mathbf{w} - \left\{u(0) - e + \beta\left[\pi(h,e)\mathbf{w} + (1-\pi(h,e))\,\mathbf{v}^f(h)\right]\right\} \\
&= (1-\beta\pi(h,e))\,\mathbf{w} - u(0) + e - \beta\,(1-\pi(h,e))\,\mathbf{v}^f(h) \\
&\geq (1-\beta\pi(h,e))\left(\mathbf{v}^f(h) + \frac{e}{\beta\pi(h,e)}\right) - u(0) + e - \beta\,(1-\pi(h,e))\,\mathbf{v}^f(h) \\
&= (1-\beta)\,\mathbf{v}^f(h) - u(0) + \frac{e}{\beta\pi(h,e)} \\
&\geq \frac{e}{\beta\pi(h_0,e)}.
\end{aligned}$$

The first line uses the definition of $\mathbf{v}^f$ with deterministic depreciation; the next two lines use the definition of v^U when $a = e$; the fourth line uses the optimality condition for high-effort (13); and the last line uses the fact that $\mathbf{v}^f(h) \geq \frac{u(0)}{1-\beta}$, with equality holding when $h = 0$. The intuition for this result is that both w and π decrease as h depreciates, while the search effort cost e is constant. **Q.E.D.**

9 Appendix B: Sequential Formulation

History: Let $x^t = \{z_0, h_0, y_0, ..., z_t, h_t, y_t\}$ be a history up to time t, where $z_t \in \{z^e, z^u\}$ is the employment status, $h_t \in H$ is the level of human capital, and $y_t \in \{s, f\}$ is the outcome of the worker's activity. The initial condition $x_0 = (z_0, h_0, y_0)$ is exogenously given.

Contract: Let $\mathcal{W}(x_0) = \{\mathbf{c}, \mathbf{a}, \mathbf{d}, \mathbf{m}\} = \{c_t(x^t), a_t(x^t), d_t(x^t), m_t(x^t)\}_{t=0}^{\infty}$ to denote the contract, where:

[22] The proof for the general case, more cumbersome but equally instructive, is available upon request.

– $c_t(x^t)$ is the transfer function, with $c_t(x^t) \geq 0$ for any x^t. Denote by $\mathbf{c}(x^\tau)$ the continuation plan of transfers after history x^τ, i.e. $\mathbf{c}_t(x^\tau) = \{c_{t+n}(x^{t+n})\}_{n=0}^{\infty} \ / \ x^t$

– $a_t(x^t)$ is the action (effort choice), where

$$a_t\left(x^t\right) \in \begin{cases} \{0, e\} & \text{if } z_t = z^u, \\ e & \text{if } z_t = z^e, \end{cases}$$

i.e., employment is defined as a state where the worker is productive and production requires the high effort level e. Denote by $\mathbf{a}_t(x^\tau)$ the continuation plan of effort choices after node x^τ and by $\mathbf{A}_t(x^\tau)$ the set of all admissible continuation plans, after history x^τ.

– $d_t(x^t)$ is the activity. If $z_t = z_u$, then $d_t(x^t) \in \{search, train\}$. When $z_t = z_e$, $d_t(x^t)$ equals to a singleton that we might call "work". Once again, $\mathbf{d}_t(x^\tau)$ will denote the continuation plan of activities after node x^τ.

– $m_t\left(x^t\right) \in \{0, 1\}$ is a dummy variable for the use of the search-effort monitoring technology, with $\mathbf{m}_t(x^\tau)$ denoting the continuation plan contingent on history x^τ.

Define the expected continuation utility promised in equilibrium by the contract $\mathcal{W}$ after history x^t as

$$U_t\left(\mathcal{W}; x^t\right) = \mathbf{E}\left[\sum_{n=0}^{\infty} \beta^n u\left(c_{t+n}\left(x^{t+n}\right)\right) - v_{z_{t+n}}\left(a_{t+n}\left(x^{t+n}\right)\right) \mid \mathbf{a}_t\left(x^t\right), \mathbf{d}_t\left(x^t\right), \mathbf{m}_t\left(x^t\right), x^t\right].$$

we assume that $U_t(\mathcal{W}; x^t)$ is well defined for all $(\mathcal{W}; x^t)$.

Incentive compatibility: In our framework, the triple (z_t, h_t, y_t) is fully observable. We also assume that the activity d_t (search, train, work) is observable and enforceable by the planner, hence "contractible". Because of the existence of the monitoring technology, at every node with $m_t(x^t) = 1$ the effort chosen by the agent should be included in the set of contractible variables.

Define by $\mathbf{a}_t^{\mathbf{m}}(x^t) \subset \mathbf{a}_t(x^t)$ the sub-plan of actions which are not contractible under the monitoring plan $\mathbf{m}$. We then have that $a_t^m(x^t) = a_t(x^t)$ if and only if $m_t(x^t) = 0$. In order to generate the sub-plan $\mathbf{a}^{\mathbf{m}}$ we simply delete the element $a_t(x^t)$ from $\mathbf{a}$ whenever

$m_t(x^t) = 1$. We are now ready to define the set of incentive compatibility constraints. For all x^t we require

$$U_t\left(\mathbf{c}, \mathbf{a}, \mathbf{d}, \mathbf{m};\, x^t\right) \geq U_t\left(\mathbf{c}, \hat{\mathbf{a}}, \mathbf{d}, \mathbf{m};\, x^t\right), \qquad (IC(x^t))$$

where, $\hat{\mathbf{a}}_t(x^t)$ can differ from $\mathbf{a}_t(x^t)$ only on the non contractible components $\mathbf{a}_t^{\mathbf{m}}(x^t)$. Notice that in order to lighten notation, we have omitted the argument (x^t) form the continuation plans.

Planner problem: In the sequential representation of the contractual relationship, the planner solves

$$\begin{aligned} V^*(U_0, x_0) &= \sup_{\mathcal{W}(x_0)} \mathbf{E}\left[\sum_{t=0}^{\infty} \beta^t \left(r\left(h_t, z_t, m_t\left(x^t\right), d_t\left(x^t\right)\right) - c_t\left(x^t\right)\right) \mid \mathbf{a}, \mathbf{d}, \mathbf{m}, x_0,\right] \\ s.t. &: \\ U_0(\mathcal{W}; x_0) &\geq U_0 \text{ and } IC(x^t) \text{ for all } x^t \mid x_0 \end{aligned}$$

where the return function during employment is $r(h_t, z_e, m_t(x^t), d_t(x^t)) = w(h_t)$, and during unemployment is $r(h_t, z_u, m_t(x^t), d_t(x^t)) = -\kappa(m_t(x^t), d_t(x^t))$, with the costs given by $\kappa(0, \mathit{search}) = 0$, $\kappa(1, \mathit{search}) = \kappa^{JM} > 0$, $\kappa(0, \mathit{train}) = \kappa^{TR} > 0$, $\kappa(1, \mathit{train}) = +\infty$.

Options of the contract during unemployment: The Table below represents all the admissible combinations of effort, activity and monitoring the planner can implement at every node. The entry $\times$ in a cell means that this option is never chosen by a welfare maximizing planner, whereas the entry $*$ denotes an option that can be optimal at some point during the contract.

	$d_t = \mathit{search}$		$d_t = \mathit{train}$	
	$m_t = 0$	$m_t = 1$	$m_t = 0$	$m_t = 1$
$a_t = e$	$*$ (UI)	$*$ (JM)	$*$ (TR)	$\times$
$a_t = 0$	$*$ (SA)	$\times$	$\times$	$\times$

The last entry in the first line is due to the assumption that monitoring effort perfectly during training is prohibitively costly. The entries in the second line ($a_t = 0$) can be explained as follows. Choosing zero search effort and at the same time monitoring workers' effort is not optimal since in this case the moral hazard problem disappears: because

$\pi(0, h_t) = 0$, $y_{t+1} = s$ is never an equilibrium outcome, so the planner can implement $a_t = 0$ by threatening an infinite punishment, for example no benefits, off the equilibrium (i.e. whenever $y_{t+1} = s$). Choosing zero effort during training is never optimal since, whenever $a = 0$, the training technology is ineffective, hence the planner will always prefer to implement search without monitoring which is cheaper and leads to the same outcome ($y = f$).

The planner can therefore restrict attention to the four remaining options labeled respectively Unemployment Insurance (UI), Job-search Monitoring (JM), Training (TR), and Social Assistance (SA), described in more detail in the main text.

Recursive formulation: The state space can be described as a correspondence $\Gamma(h, z)$ from all the pairs of human capital and employment status $(h, z) \in H \times \{z^e, z^u\}$ to the set of attainable worker's lifetime utility given by

$$\Gamma(h, z) = \left\{U : \exists\, \mathcal{W}(x_0) \text{ satisfying } IC(x^t)\ \forall x^t \mid x_0;\ U_0(\mathcal{W}; x_0) = U,\ \ (h_0, z_0) = (h, z)\right\},$$

where we have omitted y_0 from the initial conditions since it is payoff irrelevant for both the agent and the planner.

It is easy to see that when u is unbounded, $\Gamma(h, z) = R$ for all (h, z). We will argue below (see the proof of Proposition 1 in Appendix C) that $\Gamma(h, z)$ is bounded above, hence the state space has a simple rectangular structure. It is therefore easy to show that a straightforward extension of the standard recursive-contracts methodology (e.g., Spear and Srivastava, 1987) delivers the recursive formulation of the principal-agent problem in terms of the triple (U, h, z) we propose in the text. Below we will show that the functions solving the Bellman equation are bounded, available upon request we also have a proof that the policy correspondence admits a measurable selection. The usual verification theorem hence implies that the recursive formulation of the problem fully characterizes the optimal program.

10 Appendix C: Proofs

PROOF OF PROPOSITION 1.

(i) Boundedness. Since the wage function $\omega(\cdot)$ is bounded, and $c \geq 0$, $\mathbf{V}^y$ and $\mathbf{W}$ are bounded above by $\frac{\omega_{\max}}{1-\beta}$. Clearly, if the inverse function of u, $g \equiv u^{-1}$, is bounded above then $\mathbf{W}$ is bounded below as well. This conclusion holds also for $\mathbf{V}^y$ because $\mathbf{V}^y(U,h)$ is the expected discounted sum of nonnegative returns minus the consumption payments. The returns are bounded since $\omega(\cdot)$ is bounded and if g is bounded above, the expected discounted value of the consumption payments is also bounded. Consider now the case where g is *unbounded above.* Let $U_{\max} = \lim_{c\to\infty} u(c)$. Notice that, since g is continuous, in order for g (and the value functions) to be unbounded it must be the case that $U_{\max} = \infty$. The idea of the proof is to show that we can, without loss of generality, restrict the state space for U to be bounded above, hence $U_{\max} < \infty$. First, we will show below that since $\lim_{c\to\infty} u'(c) = 0$, there will be a sufficiently large utility level U^* above which the optimal program always implements SA. Since SA induces constant utility forever, for all $U \geq U^*$ the policy function never delivers utilities above U. Second, the upper bound satisfies $\bar{U} = U^* + L$ where $L < \infty$ represents a sufficiently large number that allows to satisfy the incentive compatibility constraint (IC1) starting from any $U \leq U^*$. Since $\theta > 0$ and $\pi(0) > 0$ any $L \geq \max\left\{\frac{e}{\theta\beta}, \frac{e}{\pi(0)}\right\}$ will do the job.

Lemma A1: *There exists a value $U^* < \infty$ such that if $U \geq U^*$ then $V^y(U,h) \leq \frac{g((1-\beta)U)}{1-\beta}$ for all h, y.*

Proof: The idea here is that U^* is the level of promised utility above which SA dominates any other policy since at this value the cost of compensating the agent for his effort is too large in consumption terms. Clearly, any policy involving a positive effort choice (UI, JM, TR) is dominated by a policy that: i) can implement the effort $a = e$ this period without IC problem; and ii) by implementing the effort e, it obtains for sure a permanent job with wage $\omega_{\max}$. We want to show that for a sufficiently large U^* the planner will always prefer SA over this "dominating policy". The difference between these two options, expressed in terms of costs for the planner, is

$$\frac{c^*}{1-\beta} - \frac{\omega_{\max}}{1-\beta} - \frac{c^*_{SA}}{1-\beta}, \tag{14}$$

where the utility promised by the two policies to the agent must satisfy $U^* = \frac{u(c^*_{SA})}{1-\beta} = \frac{u(c^*)}{1-\beta} - e$. If we multiply by $1-\beta$ and use $g = u^{-1}$ to denote the inverse function of u, we can state the cost difference in (14) equivalently as

$$g\left(\left(1-\beta\right)\left(U^*+e\right)\right) - \omega_{\max} - g\left(\left(1-\beta\right)U^*\right). \tag{15}$$

By the mean value theorem, we have

$$g\left(\left(1-\beta\right)\left(U^*+e\right)\right) - g\left(\left(1-\beta\right)U^*\right) = g'\left(\left(1-\beta\right)\xi_U\right)\left(1-\beta\right)e$$

for some $\xi_U \in \left[U^*, U^*+e\right]$. Since $\lim_{U\to\infty} g'\left(\left(1-\beta\right)U^*\right) = \lim_{c\to\infty} \frac{1}{u'(c)} = \infty$, it must be that for U^* large enough $g'\left(\left(1-\beta\right)\xi_U\right) > \frac{\omega_{\max}}{(1-\beta)e}$ and hence the expression (15) becomes a positive number, i.e. SA is less costly than the dominating policy. **Q.E.D.**

Continuity. First, notice that the integral and Max operator in (5) deliver a continuous function as long as each V^i is continuous and Q^y has the Feller property. Second, given $\mathbf{W}$ and a generic $\mathbf{V}$, the value function associated to policy i takes the form

$$\begin{aligned} V^i_{\mathbf{V}}\left(U,h\right) &= \max_{(z,U^y)\in\Gamma^i(U,h)} -g\left(z\right) + \beta\left[p\left(h\right)\mathbf{W}\left(U^s,h\right) + \left(1-p\left(h\right)\right)\mathbf{V}\left(U^f,h\right)\right] \\ & \quad s.t.\ IC^i\left(U,h\right),\ PK^i\left(U,h\right) \end{aligned}$$

with $p\left(h\right) = \pi\left(h\right) \in \left(0,1\right)$ if $i = UI,\ JM$; $p\left(h\right) = \theta$ if $i = TR$; and $p\left(h\right) = 0$ if $i = SA$. Analogously, we have different incentive constraints for different policies i. Since the domain constraints $\Gamma^i\left(U,h\right)$ can always be chosen to be a continuous correspondence at every $\left(U,h\right)$, the feasibility correspondence $\Gamma^i\left(U,h\right) \cap IC^i\left(U,h\right) \cap PK^i\left(U,h\right)$ is continuous.[23] Since $g = u^{-1}$ is a continuous function, we can apply the Maximum Theorem to show that as long as $\mathbf{W}$ and $\mathbf{V}$ are continuous $V^i_{\mathbf{V}}\left(U,h\right)$ is also continuous. Moreover, since Q^y has the Feller property, and both π and ω are continuous in h, a direct application of Theorem 9.6 in SLP implies that $\mathbf{V}^y$ is a bounded and continuous function (jointly in U and h).

Concavity (in U) is obtained from the "convexification" over human capital. Showing concavity for $h = 0$ is easy since $\omega\left(0\right) = 0$, $Q^s\left(\cdot,0\right) = Q^f\left(\cdot,0\right)$ and the fact that $Q^f\left(\cdot,0\right)$

[23] In particular, notice that from Theorem 3.4 in Stokey, Lucas and Prescott (SLP), the fact that we allow u to be unbounded does not create additional problems since for any finite U it is never optimal to promise $U^y = -\infty$. As a consequence we can w.lo.g. impose a lower bound on U^y for any $\left(U,h\right)$ and get a compact valued correspondence $\Gamma^i\left(U,h\right)$ for all i.

has a mass of probability one at 0 jointly imply that at $h = 0$ the optimal program implements SA whose value is a concave function.

For $h > 0$ the proof extends Aumann (1965), Proposition 6.2 to problem (5).

Lemma A2: *Let V be bounded, continuous in U and measurable in h and let $D \subset \Re$ a compact set. If $Q\left(\cdot, \bar{h}\right)$ is atomless for every $\bar{h}$, the function V defined as*

$$\begin{aligned}\mathbf{V}\left(U;\ \bar{h}\right) &= \sup_{(U(h))_{h\in H}} \int_H V(U(h), h)\, dQ\left(h; \bar{h}\right) \\ s.t. &: \quad U(h) \in D; \int_H U(h) dQ\left(h; \bar{h}\right) = U\end{aligned}$$

is concave in U for all $\bar{h} \in H$.

Proof: (Sketch) We have to show that the (ipo)graph of $\mathbf{V}$ is a convex set. Given V, define the correspondence

$$F^V(h) = GrV(\cdot, h) = \left\{x \in \Re^2 :\ x_1 \in D,\ -B \leq x_2 \leq V(x_1, h)\right\}.$$

We claim that the set

$$A^F\left(\bar{h}\right) = \left\{ \begin{array}{c} (u, v) \in \Re^2 :\ \exists\left(\bar{\mathbf{u}}, \bar{\mathbf{v}}\right) : \left(\bar{\mathbf{u}}, \bar{\mathbf{v}}\right)(h) \in F^V(h) \\ \int_H \bar{u}(h)\, dQ\left(h; \bar{h}\right) = u \\ \int_H \bar{v}(h)\, dQ\left(h; \bar{h}\right) = v \end{array} \right\}$$

is the graph of $\mathbf{V}$ given $\bar{h}$. Now consider the set

$$CW^V\left(u, v;\ \bar{h}\right) = \left\{ \begin{array}{c} (\mathbf{u}, \mathbf{v}) : H \rightarrow \Re^2\text{: for all } h\ \ (u(h), v(h)) \in coF^V(h) \\ \int_H v(h)\, dQ\left(h; \bar{h}\right) = v \\ \int_H u(h) dQ\left(h; \bar{h}\right) = u. \end{array} \right\}$$

When the set

$$\left\{(\mathbf{u}, \mathbf{v}) : H \rightarrow \Re^2\text{: for all } h\ \ (u(h), v(h)) \in coF^V(h)\right\}$$

is endowed with the weak-* topology, $CW^V\left(u, v;\ \bar{h}\right)$ is convex and compact. Hence, by the Krein-Mirman Theorem, it has an extreme point, the vector valued function $\left(\bar{\mathbf{u}}, \bar{\mathbf{v}}\right)$. Since the equality constraints are finitely many, we can use an extension of Propositions 6.1 and 6.2 in Aumann (1965), and show that $\left(\bar{\mathbf{u}}, \bar{\mathbf{v}}\right)$ takes values only at the extreme points of $coF(h)$. But then $\left(\bar{\mathbf{u}}, \bar{\mathbf{v}}\right)(h) \in extF(h)$ *a.e.* for all h. Since $F(h)$ is compact, then $extF(h) \subset F(h)$. This results implies that $A^{coF}\left(\bar{h}\right) = A^{F}\left(\bar{h}\right)$, which concludes the proof since $A^{coF}\left(\bar{h}\right)$ is clearly convex. **Q.E.D.**

(ii) Monotonicity in U. To show that $\mathbf{V}^y$ is decreasing in U when u is unbounded below notice that for any finite U in order to reduce marginally the continuation utility, the planner can replicate exactly the same payment scheme as the one under U from next period on and reduce this period payment to $c' > 0$ so that $\delta U = u(c) - u(c')$ for an arbitrarily small δ. This possibility is always guaranteed also in UI and TR by the fact that the IC constraint (IC1) can be written independently of c and that u is unbounded below. As a consequence, the associated Bellman operator maps decreasing functions into decreasing functions, and we can directly apply the line of proof of Theorem 9.7 of SLP to show that $\mathbf{V}^y$ is monotone in U.

Monotonicity in h. We first need a preliminary Lemma, which states formally the intuitive fact that the value of employment for the planner dominates the value of unemployment $\mathbf{V}^f$ in every state.

Lemma A3: *If $e_w = 0$, and u is unbounded below, then $\mathbf{W}(U,h) > \mathbf{V}^y(U,h)$ for any pair (U,h) and for any $y = s, f$.*

Proof: Let $\bar{\mathbf{V}}^y(U,h)$ be the function that dominates $\mathbf{V}^y(U,h)$ defined as

$$\begin{aligned}
\bar{\mathbf{V}}^y(U,h) &= \int \max_{U(h')\in D} \bar{V}(U(h'),h')Q^y(dh';h), \\
s.t. &: \\
U &= \int U(h')\,Q^y(dh';h),
\end{aligned}$$

where for any (U,h)

$$\bar{V}(U,h) = \max_{i\in\{JM,SA,TR,UI\}} \bar{V}^i(U,h)$$

with the dominating value for UI being defined as

$$\begin{aligned}
\bar{V}^{UI}(U,h) &= \max_{c,U^f,U^s} -c + \beta\left[\pi(h)\mathbf{W}(U^s,h) + (1-\pi(h))\mathbf{V}^f(U^f,h)\right] \\
s.t. &: \\
U &= u(c) + \beta\left[\pi(h)U^s + (1-\pi(h))\,U^f\right],
\end{aligned}$$

the dominating value for TR being defined as

$$\begin{aligned}
\bar{V}^{TR}(U,h) &= \max_{c,U^f,U^s} -c+\beta\left[\theta\mathbf{V}^s(U^s,h)+(1-\theta)\mathbf{V}^f(U^f,h)\right] \\
s.t. &: \\
U &= u\left(c\right)+\beta\left[\theta U^s+\left(1-\theta\right)U^f\right],
\end{aligned}$$

and the dominating values for JM and SA being defined as

$$\bar{V}^{JM}(U,h) = \bar{V}^{UI}(U,h),$$

$$\bar{V}^{SA}(U,h) = V^{SA}(U,h).$$

Note that $\bar{V}^y \geq V^y$ because in the "bar" policies the planner can save resources (i) by avoiding paying the direct costs κ^i, (ii) by avoiding satisfying the IC constraint in UI and TR, and (iii) when u is unbounded below, by reducing the transfer c, since there is no need to compensate the agent for her search/training effort, while at the same time promising the same $\left(U^s,U^f\right)$ as under the benchmark policies.

Now, if we replace $\mathbf{V}^y$ with the corresponding $\bar{\mathbf{V}}^y$ in each specific policy, we obtain new values for each programs $\overline{\overline{V}}^i$ that dominate the individual policy $\bar{V}^i$, hence $\overline{\overline{\mathbf{V}}}^y \geq \bar{\mathbf{V}}^y$ where

$$\begin{aligned}
\overline{\overline{\mathbf{V}}}^y(U,h) &= \int \max_{i(h'),U(h')} \overline{\overline{V}}^{i(h')}(U\left(h'\right),h')Q^y\left(dh';h\right), \\
s.t. &: \\
i(h') &\in \left\{JM,SA,TR,UI\right\}, \\
U(h') &\in D, \quad \text{and} \\
U &= \int U\left(h'\right)Q^y\left(dh';h\right).
\end{aligned}$$

Now, notice that since there is no incentive compatibility constraint and no effort to be compensated, consumption is always constant and we can simply decompose $\overline{\overline{\mathbf{V}}}^y(U,h)$ in two separate pieces, i.e.

$$\overline{\overline{\mathbf{V}}}^y(U,h) = K^y(h) - \frac{g\left(\left(1-\beta\right)U\right)}{1-\beta},$$

where $K^y(h)$ is the expected discounted wage return attainable starting from unemployment and human capital h. It is now easy to see that $K^y(h) < \frac{\mathbf{E}[\omega(h');h,s]}{1-\beta}$, where we used the law of iterated expectations to obtain the right-hand side. This inequality must hold since β, π, and θ are all less or equal than 1, and $Q^s \succeq_{FO} Q^f$. Recalling the expression for the value of employment in (10), we have shown that $\mathbf{W}(U,h) > \overline{\overline{\mathbf{V}}}^y(U,h) \geq \mathbf{V}^y(U,h)$. **Q.E.D.**

Remark: A comment on the assumption $e_w = 0$ is in order. If $e_w > 0$ it might be the case that SA becomes more attractive than employment to the planner for certain states. In particular, for large utility levels, the planner might indeed be willing to give up the wage returns in exchange for the possibility of not compensating the worker for her effort on the job. We want to rule out this possibility.

We are now ready to show our result on the monotonicity of $\mathbf{V}^y(U,h)$ with respect to h. Our aim is to show that the Bellman operator maps weakly increasing functions into themselves. The additional complication with respect to the standard case analyzed in SLP stems from the fact that the feasibility constraint is not necessarily monotone in h. Assume first UI is implemented at (U,h) (if TR is implemented the argument is even easier), and consider the case where h' is slightly above h. We show that the planner can gain at this higher human capital level by keeping c and U^f constant and by giving the agent $\hat{U}^s$ so that the promise-keeping constraint is satisfied, i.e.

$$U = u(c) - e + \beta\left[\pi(h')\hat{U}^s + (1-\pi(h'))\,U^f\right].$$

In order to recover the value $\hat{U}^s$ consider the following experiment: assume that under h', in case of success the planner gives U^s to the agent with probability γ so that $\gamma\pi(h') = \pi(h)$, and U^f otherwise (notice that $U^s \geq U^f$ since it might be the case that at some point during unemployment the agent supplies positive effort). Now, if we compute the return of the planner from next period on, we have

$$\pi(h')\left[\gamma W(U^s,h') + (1-\gamma)W(U^f,h')\right] + (1-\pi(h'))\mathbf{V}^f(U^f,h'),$$

and rearranging terms one gets

$$\pi(h)W(U^s,h') + (1-\pi(h))\mathbf{V}^f(U^f,h') + \left[\pi(h') - \pi(h)\right]\left[W(U^f,h') - \mathbf{V}^f(U^f,h')\right],$$

which is greater than the next period's planner return under h because – since wages are increasing in h– $W(U,h') \geq W(U,h)$ for any U, $\mathbf{V}^f$ is increasing in h by assumption, and $W(U,h) > \mathbf{V}^f(U,h)$ for any pair (U,h). Since both W and $\mathbf{V}^f$ are concave we can always find a contract that does not involve the use of such $\gamma-$lotteries which (weakly) dominates that described above. **Q.E.D.**

We can now apply directly Theorem 9.11 in SLP. Notice that what we show together with monotonicity is that if $\mathbf{V}_0^y \leq \mathbf{W}$ then $T\mathbf{V}_0^y \leq W$ where T is the Bellman operator.

(iv) The proof of differentiability with respect to in h is omitted for brevity, but it is available upon request.

(v) Straightforward from the expression in (10). **Q.E.D.**

Proof of Corollary 1.

(i) Boundedness, continuity, and monotonicity have been shown above. Strict concavity is obtained from the fact that $-g \equiv -u^{-1}$ is strictly concave and for any program both the incentive and promise keeping constraints are linear (hence convex). Hence, as long as $\mathbf{V}^y$ and $\mathbf{W}$ are concave V^i will be strictly concave (e.g. see the line of proof of Theorem 4.8 in SLP). Differentiability is obtained by a simple application of the Benveniste and Scheinkman Lemma (1979) to the problem defining V^i using the fact that $-g$ is concave and continuously differentiable (see Theorems 4.10 and 4.11 in SLP).

(ii) Straightforward from (6).

(iii) Again, for brevity, we omit the proof of differentiability with respect to h.

(iv) (Sketch) Clearly, $\mathbf{V}^y$ is differentiable at all interior points where it is linear and where it coincides with one specific V^i. It remain to show that it is differentiable also at all points where it 'just' touches the single policies. Denote U_0 one of such points. In this case we can apply the Benveniste and Scheinkman Lemma. To see that the conditions for its application are met notice that $\mathbf{V}^y(U,h) \geq V^i(U,h)$ for all U and that $\mathbf{V}^y(U_0,h) = V^i(U_0,h)$ with $\mathbf{V}^y$ concave and V^i concave an differentiable. **Q.E.D.**

Proof of Proposition 2.

The reason why the proof is somewhat involved is that one reason why the planner might want to implement, say, JM after SA is that this strategy allows the planner to avoid requiring the agent to supply the whole high effort level, as it permits eliciting from the agent only a fraction β of the high effort level e. The proof makes heavily use of randomizations in period t to show that the effort "level" βe can always be implemented through lotteries across SA and other policies within the same period, without the use of delays.

More formally, we want to show that SA is absorbing up to a measure zero event.

Proposition 2': *In an optimal WTW program, SA cannot be followed with positive probability by any other policy. That is, in each period t there cannot be a positive measure* μ_t *of human capital shocks* h_t *such that SA is implemented for all such* h_t *and at the same time there a positive measure* μ_{t+n} *of* h_{t+n} *at a future time* $t+n$*, for which another policy is implemented.*

Proof: We will show it by contradiction. Let us set w.l.o.g. $\mu_t = \mu_{t+1} = 1$ and consider the plan α where SA is *immediately* followed by UI. The case where SA is followed by JM is easier to show using the same line of proof: we can simply disregard incentive compatibility. At the end of the proof, we will show that SA cannot be followed by TR either.

We want to show that the stated sequence cannot be part of an optimal program since the planner can gain by implementing an alternative plan α' where in the initial period t it implements UI with probability β and SA with probability $(1-\beta)$. In the following periods, after UI the alternative plan α' implements exactly the same program which followed UI under α, whereas for the $(1-\beta)$ shocks SA is implemented forever.

Two remarks on the general case: (1) When α contemplates that, after SA, UI is implemented only for a subset of shocks h_{t+1} with measure $\mu_{t+1} < 1$, then the randomization in the alternative plan α' should be amended as follows: in period t, UI is implemented with probability $\mu_{t+1}\beta$ and from then on the program follows exactly what α suggested. With probability $\left(1-\mu_{t+1}\beta\right)$ the new plan α' implements SA in the first period. In the second period, after SA the program implements SA only with probability $(1-\beta)$, whereas with probability $\left(1-\mu_{t+1}\beta\right)-(1-\beta)=\beta\left(1-\mu_{t+1}\right)$ the program follows what was implemented under α. In general, if for each $h_t\, UI$ is implemented with probability $\mu_{t+1}(h_t)$ - since $h_{t+1} \leq h_t$ for any such h_{t+1} - there must be a measure $\mu = \int \mu_{t+1}(h_t)\, dQ^f(\cdot, h_0)$ (a fortiori a measure $\beta\mu$) of h_t shocks such that implementing UI in period t is cheaper than some UI implemented in period $t+1$ under α. Finally,

(2) in the case where α implements UI at some future period $t+n$ with $n>1$, the initial randomization under α' must be adjusted so that only for a measure β^n of shocks it implements UI and it implements SA for the remaining (measure $1-\beta^n$ of) shocks.

The optimal payments under the plan α are $c_t(h) = c_{t+1}(h,h') = c$ since SA does not involve incentives and the agent is insured against human capital shocks. These payments, together with the continuation utilities U_{t+2}^y contingent on the period $t+1$ search-activity outcome $y = s, f$, and the human capital shocks must satisfy

$$\begin{aligned} U_t^\alpha &= u(c) + \beta(u(c) - e) + \\ &+ \beta^2 \int_H \int_H \beta\left[\pi(h') U_{t+2}^s(h,h') + (1-\pi(h') U_{t+2}^f(h,h')\right] dQ^f(\cdot;h)\, dQ^f(\cdot;h_0). \end{aligned}$$

where, for notational simplicity, we set $h' = h_{t+1}$. Incentive compatibility implies that for all h' we must have

$$U_{t+2}^s(h,h') - U_{t+2}^f(h,h') \geq \frac{e}{\pi(h')\beta}, \tag{16}$$

Consider now the program α'.[24] Under this alternative, the agent receives the initial payment $\hat{c}_t$ independent on the randomization, hence her expected discounted utility is

$$\begin{aligned} U_t^{\alpha'} &= u(\hat{c}_t) - \beta e + \int_{H_\beta} \beta\left[\pi(h)\hat{U}_{t+1}^s(h) + (1-\pi(h))\hat{U}_{t+1}^f(h)\right] dQ^f(\cdot;h_0) \\ &+ \int_{H_{1-\beta}} \beta\left[\hat{U}_{t+1}(h)\right] dQ(\cdot;h_0), \end{aligned}$$

where H_β has measure β and $H_{1-\beta}$ has measure $1-\beta$. Now set $\hat{c}_t = c$, $\hat{U}_{t+1}(h) = -\frac{c}{1-\beta}$, and for $y = s, f$ set $\hat{U}_{t+1}^y(h) = \int U_{t+2}^y(h,h')\, dQ^f(\cdot,h) \equiv \mathbf{E}\left[U_{t+2}^y(h,h')\right]$.

Substitute these terms into the general formulation of the agent's utility under α'. We get:

$$\begin{aligned} U_t^{\alpha'} &= u(c) - \beta e + (1-\beta)\beta u(c) + \beta u(c) \\ &+ \int_{H_\beta} \beta\left[\pi(h)\mathbf{E}U_{t+2}^s(h,h') + (1-\pi(h))\mathbf{E}U_{t+2}^f(h,h')\right] dQ^f(\cdot;h_0). \end{aligned}$$

[24] Recall that the new program α' implements

$$\begin{aligned} \text{With probability } \beta &: \quad UI \to \pi(h)\, Empl + (1-\pi(h))\, Unempl \\ \text{With probability } 1-\beta &: \quad SA \to SA \to SA \ldots \end{aligned}$$

It is easy to see that these payments deliver the same utility to the agent. We now have to check whether incentive compatibility is satisfied. Notice that we have

$$\mathbf{E}U^{s}_{t+2}\left(h,h'\right)-\mathbf{E}U^{f}_{t+2}\left(h,h'\right)=\mathbf{E}\left[U^{s}_{t+2}\left(h,h'\right)-U^{f}_{t+2}\left(h,h'\right)\right]\geq\mathbf{E}\left[\frac{e}{\pi\left(h'\right)\beta}\right]\geq\frac{e}{\pi\left(h\right)\beta},$$

where the second inequality uses (16) and the last one uses the fact thatπ is increasing in h.

What is now left to show is that the planner can gain by following α' instead of α. Under α the planner net returns are

$$-c-\beta c+\beta^{2}\int_{H}\mathbf{E}\pi\left(h'\right)\mathbf{W}\left(U^{s}_{t+2}\left(h,h'\right),h'\right)+\left(1-\pi\left(h'\right)\right)\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,h'\right),h'\right)dQ^{f}dQ^{f},$$

whereas under the new contract α' they are

$$-c-\beta c+\beta^{2}\int_{H}\pi\left(h\right)\mathbf{W}\left(\mathbf{E}U^{s}_{t+2}\left(h,h'\right),h\right)+\left(1-\pi\left(h\right)\right)\mathbf{V}^{f}\left(\mathbf{E}U^{f}_{t+2}\left(h,h'\right),h\right)dQ^{f}.$$

It suffices to show that for every h we have

$$\begin{aligned}&\mathbf{E}\pi\left(h'\right)\mathbf{W}\left(U^{s}_{t+2}\left(h,h'\right),h'\right)+\left(1-\pi\left(h'\right)\right)\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,h'\right),h'\right)\\ \leq\;&\pi\left(h\right)\mathbf{W}\left(\mathbf{E}U^{s}_{t+2}\left(h,h'\right),h\right)+\left(1-\pi\left(h\right)\right)\mathbf{V}^{f}\left(\mathbf{E}U^{f}_{t+2}\left(h,h'\right),h\right).\end{aligned}$$

To see that this is the case, for any given h, we perform a fictitious randomization (whose realization we call x') extracted from the distribution $Q^{f}\left(\cdot,h\right)$ so that for all x' we have $U^{y}_{t+2}\left(h,x'\right)=U^{y}_{t+2}\left(h,h'\right)$ for $y=s,f$. Clearly, $\mathbf{E}U^{y}_{t+2}\left(h,x'\right)=\mathbf{E}U^{y}_{t+2}\left(h,h'\right)$.

We now have that, for all $h\geq h'$ and related x',

$$\begin{aligned}&\pi\left(h'\right)\mathbf{W}\left(U^{s}_{t+2}\left(h,h'\right),h'\right)+\left(1-\pi\left(h'\right)\right)\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,h'\right),h'\right)\\ \leq\;&\pi\left(h\right)\mathbf{W}\left(U^{s}_{t+2}\left(h,h'\right),h'\right)+\left(1-\pi\left(h\right)\right)\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,h'\right),h'\right)\\ \leq\;&\pi\left(h\right)\mathbf{W}\left(U^{s}_{t+2}\left(h,x'\right),h\right)+\left(1-\pi\left(h\right)\right)\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,x'\right),h\right),\end{aligned}$$

where the first line is due to the fact that π increases with h and, since the incentive constraint (16) is binding, it must be that for all pairs $\left(h,h'\right)$

$$\mathbf{W}\left(U^{s}_{t+2}\left(h,h'\right),h'\right)\geq\mathbf{V}^{f}\left(U^{f}_{t+2}\left(h,h'\right),h'\right).$$

Otherwise, for each $\left(h,h'\right)$ the planner could have generated utility $U\left(h,h'\right)=u\left(c\right)+\beta U^{f}_{t+2}\left(h,h'\right)$ - which is the utility the agent gets in period $t+1$ under UI by supplying the effort e and receiving the reward $U^s_{t+2}(h,h') - U^f_{t+2}(h,h') = \frac{e}{\beta\pi(h')}$ - simply by implementing SA. This would generate a net return of $\mathbf{V}^f\left(U^f_{t+2}(h,h'),h'\right)$ contradicting the optimality of α. The second line is due to the fact that both $\mathbf{W}$ and $\mathbf{V}^f$ are increasing in h and $U^s_{t+2}(h,x') = U^y_{t+2}(h,h')$.

Hence, for all h we have

$$
\begin{aligned}
& \pi(h)\mathbf{W}\left(\mathbf{E}U^s_{t+2}(h,h'),h\right) + (1-\pi(h))\mathbf{V}^f\left(\mathbf{E}U^f_{t+2}(h,h'),h\right) \\
\geq\ & \mathbf{E}^x\pi(h)\mathbf{W}\left(U^s_{t+2}(h,x'),h\right) + (1-\pi(h))\mathbf{V}^f\left(U^f_{t+2}(h,x'),h\right) \\
\geq\ & \int_H \pi(h')\mathbf{W}\left(U^s_{t+2}(h,h'),h'\right) + (1-\pi(h'))\mathbf{V}^f\left(U^f_{t+2}(h,h'),h'\right) dQ^f
\end{aligned}
$$

and we are done. The first inequality is due to the fact that both $\mathbf{W}$ and $\mathbf{V}^f$ are concave, the second inequality descends from the above discussion and from the fact that x' is extracted from $Q^f(\cdot,h)$, the same distribution from which h' is extracted.

It is easy to see that we can follow exactly the same line of proof to show that SA cannot be followed by training (TR): by using the same randomization and the same relationship between the payments in α and α' the agent will obviously get the same lifetime utility, and incentive compatibility will be satisfied. The planner will gain by the fact that both $\mathbf{V}^f$ and $\mathbf{V}^s$ are concave, increasing in h and whenever the incentive constraint $U^s_{t+2}(h,h') = U^f_{t+2}(h,h') + \frac{e}{\beta\theta}$ is binding, it must be that for all(h,h')

$$
\mathbf{V}^s\left(U^s_{t+2}(h,h'),h'\right) \geq \mathbf{V}^f\left(U^f_{t+2}(h,h'),h'\right),
$$

otherwise α cannot be optimal as SA would dominate TR in these contingencies. Strict dominance is clearly guaranteed as long as either of $\mathbf{W}$ and $\mathbf{V}^y$ are strictly increasing in h and/or by the fact that at some equilibrium point they are strictly concave. **Q.E.D.**

Proof of Proposition 3.

In order to show the absorbing property of JM note that the first order conditions are $\mathbf{V}^f_U(U) = \mathbf{V}^f_U\left(U^f\right)$. This implies that $U^f = U$ is an optimal policy. As a consequence implementing the same policy, i.e. JM, every period is part of an optimal program. Clearly, whenever $\mathbf{V}^f$ is strictly concave the absorbing policy is the unique optimal one. That SA is an absorbing policy has already been shown in Proposition 2 for the general case.

We now show that UI is absorbing. The first order conditions and the envelope condition under UI are:

$$\begin{aligned}
\mathbf{V}_U^f(U) &= V_U^{UI}(U) = -\frac{1}{u'(c)}, \\
-\mathbf{V}_U^f\left(U^f\right) &= \frac{1}{u'(c)} - \mu\frac{\pi}{1-\pi}, \\
-\mathbf{W}_U(U^s) &= \frac{1}{u'(c)} + \mu,
\end{aligned} \tag{17}$$

where $\mu \geq 0$ is the multiplier on the IC constraint.

Lemma A4: *At any U where UI is optimal we either have $\mathbf{V}_U^f(U) = V_U^{UI}(U) = \mathbf{V}_U^f\left(U^f\right)$ for $U^f < U$ or $\mu > 0$. In particular, if $\mathbf{V}^f$ is strictly concave to the left of U, then the incentive compatibility constraint is binding with $\mu > 0$.*

Proof. If the incentive compatibility is not binding, by the first order conditions and the strict concavity of $\mathbf{V}^f$ to the left we have $U^f \geq U$. Moreover, the special form of $\mathbf{W}$ implies $u(c) = z = (1-\beta)U^s + e_w$. If we now use the promise keeping constraint (with $U \leq U^f$) we obtain $U^s \leq \frac{e-e_w}{(1-\beta)+\beta\pi} + U^f$. Since both $e_w \geq 0$ and $(1-\beta) > 0$ the incentive compatibility cannot be satisfied and this leads to a contradiction. The incentive compatibility constraint must hence be binding. **Q.E.D.**

Now notice that since each function V^i is continuous, if for different levels of utility different policies are preferred, the value functions must cross each other.

Lemma A5: *For all U we have $V_U^{SA}(U) \geq V_U^{UI}(U)$. Hence, if at U_0 UI is optimal then never implementing SA is optimal. Moreover, V^{SA} and V^{UI} can cross each other at most once.*

Proof. The first order and envelope conditions for the program SA are:

$$\begin{aligned}
V_U^{SA}(U) &= -\frac{1}{u'(c^{SA})} = \mathbf{V}_U^f\left(U_{SA}^f\right) \\
\text{and } U &= u\left(c^{SA}\right) + \beta U_{SA}^f.
\end{aligned}$$

From (17) we have that the optimality conditions for UI imply

$$\begin{aligned}
V_U^{UI}(U) &= -\frac{1}{u'(c^{UI})} \leq \mathbf{V}_U^f\left(U_{UI}^f\right) \\
\text{and } U &\geq u\left(c^{UI}\right) + \beta U_{UI}^f.
\end{aligned}$$

If $c^{SA} \leq c^{UI}$ by envelope we are done. Now, assume instead that $c^{SA} > c^{UI}$. In order to satisfy the promise keeping and incentive constraints under UI we must have $U^f_{UI} > U^f_{SA}$. Hence, the concavity of $\mathbf{V}^f$ and the envelope condition imply

$$-\frac{1}{u'(c^{UI})} \leq \mathbf{V}^f_U\left(U^f_{UI}\right) \leq \mathbf{V}^f_U\left(U^f_{SA}\right) = -\frac{1}{u'(c^{SA})}$$

or $u'\left(c^{UI}\right) \leq u'\left(c^{SA}\right)$ which is a contradiction.

From Lemma A4 whenever at U_0 UI is implemented we have two possibilities: *(a)* We might have $\mu > 0$. In this case, since the optimality conditions imply $\mathbf{V}^f_U(U_0) < \mathbf{V}^f_U\left(U^f_0\right)$ it cannot be optimal to implement SA ever. This is so since any lottery implementing U^f_0 solves $\mathbf{V}^f_U\left(U^f_0\right) = \mathbf{V}^f_U(U(x))$ for all $U(x)$ in such lottery. The concavity of $\mathbf{V}^f$ implies that $U(x) < U_0$ and the above result implies that SA can never be implemented at any of such $U(x)$. *(b)* The other possibility is that $\mathbf{V}^f$ is linear to the left of U_0 and $\mathbf{V}^f_U(U_0) = V^{UI}_U(U_0) = \mathbf{V}^f_U\left(U^f_0\right)$ for a $U^f_0 < U_0$. Since SA can be implemented only for utility levels larger than U_0 $\mathbf{V}^f$ must be linear to its right as well. In this case for any contract that implements SA with some probability, we can find another contracts that (weakly) dominates it and never implements SA as it is formed by all $U(x) \leq U_0$. **Q.E.D.**

Lemma A6: *Let U_0 such that $V^{JM}(U_0) = V^{UI}(U_0)$. Then we have $V^{JM}_U(U_0) \geq V^{UI}_U(U_0)$. Hence, if at U_0 UI is optimal then never implementing JM is optimal. Moreover, V^{JM} and V^{UI} can cross each other at most once.*

Proof.

Clearly, if at U_0 V^{JM} and V^{UI} have the same slope we are done.

So assume they have different slope. It is easy to see that in this case at U_0 none of the two programs can be optimally implemented with probability one.

Now, notice the following: First, it must be that at U_0 $\mu > 0$. Otherwise $V^{JM}(U_0) < V^{UI}(U_0)$. As a consequence we have $V^{JM}_U(U) = \mathbf{V}^f_U\left(U^f_{JM}\right)$ and $V^{UI}_U(U) < \mathbf{V}^f_U\left(U^f_{UI}\right)$. Which implies that if $U^f_{JM} \leq U^f_{UI}$ then $V^{JM}_U(U_0) > V^{UI}_U(U)$. Which from the envelope and the strict concavity of u implies $c^{JM} < c^{UI}$. Second, let $u\left(c^{JM}_0\right) = z^{JM}_0$ the payment implemented in the optimal program at U_0 under JM. We then have

$$U_0 = z^{JM}_0 - e + \beta\left[\pi\frac{z^{JM}_0 - e_w}{1-\beta} + \pi U^f_{JM}\right].$$

For $V^{JM}(U_0) = V^{UI}(U_0)$ to be true it must be that $\frac{z_0^{JM}-e_w}{1-\beta} - U^f_{JM} < \frac{e}{\beta\pi}$ otherwise again UI would have been strictly preferred. This implies that $U_0 < z_0^{JM} + \beta U^f_{JM}$ while from incentive compatibility we have $U_0 \geq z_0^{UI} + \beta U^f_{UI}$ hence whenever $U^f_{JM} \leq U^f_{UI}$ then $z_0^{JM} > z_0^{UI}$. But this is a contradiction. So, the only possibility is $U^f_{JM} > U^f_{UI}$.

Now, notice that the optimal program cannot deliver U^f_{JM} by implementing JM with positive probability. This is so since from the first order conditions we have that for any $U(x)$ included in the randomization we have

$$V_U^{JM}(U_0) = \mathbf{V}_U^f\left(U^f_{JM}\right) = V_U^{JM}(U(x))$$

the strict concavity of V^{JM} implies that $U_0 = U(x)$. This leads to a contradiction to the fact that at U_0 was not optimal.

(a) One possibility is that is that U^f_{JM} is generated by implementing UI with positive probability. Let $\lambda > 0$ such probability, we have $U^f_{JM} = \lambda U^{UI}_{JM} + (1-\lambda) U^{SA}_{JM}$. From Lemma A5 it must be that $U^{SA}_{JM} \geq U^f_{JM} \geq U^{UI}_{JM}$ with strict inequality whenever $\lambda < 1$. In particular, we have $V_U^{JM}(U_0) = \mathbf{V}_U^f\left(U^f_{JM}\right) = V_U^{UI}\left(U^{UI}_{JM}\right)$ with $U^{UI}_{JM} \leq U^f_{JM}$. In this case, if $U^f_{JM} \leq U_0$, we are done. This is so since $U^f_{JM} \leq U_0$ implies $U_0 \geq U^{UI}_{JM}$. And this means that V^{JM} has the same slope of V^{UI} at $U_0 \geq U^{UI}_{JM}$. Hence the result is obtained by strict concavity of the V^i. Now, assume that $U^f_{JM} > U_0$. Recall that $U^{UI}_{JM} \leq U^f_{JM}$. The only interesting case is again when $U^{UI}_{JM} > U_0$. We saw that at U_0, neither UI nor JM can be implemented. Moreover, we know that

$$\mathbf{V}_U^f(U_0) \geq \mathbf{V}_U^f\left(U^{UI}_{JM}\right) = V_U^{JM}(U_0) = V_U^{UI}\left(U^{UI}_{JM}\right).$$

Notice that by strict concavity at the left of U_0 UI cannot be implemented. And Lemma A5 rules out the possibility that SA is implemented at U_0 or at its left. Since JM cannot be either, we have a contradiction to the fact that $U^{UI}_{JM} > U_0$ hence again $U^{UI}_{JM} \leq U_0$ and we are done.

(b) The remaining case is that after JM the program implements SA almost surely. We hence have $V_U^{JM}(U_0) = \mathbf{V}_U^f\left(U^f_{JM}\right) = V_U^{SA}\left(U^f_{JM}\right)$. Let z_0^{JM} be the constant number that solves

$$U_0 = \frac{z_0^{JM}}{1-\beta} - e - \beta\pi\frac{e_w}{1-\beta}. \tag{18}$$

For this last case we need to investigate what happens under UI.

(b1) Suppose first that U_{UI}^f is implemented by SA with probability one. If we denote by z_0^{UI} the payment under UI and by z^f and z^s the (constant because of full insurance) payments from next period onward, we have

$$U_0 = z_0^{UI} + \beta\left[\pi\frac{z^s}{1-\beta} + (1-\pi)\frac{z^f}{1-\beta}\right] - e - \beta\pi\frac{e_w}{1-\beta}. \quad (19)$$

The reason why we have the same value $-e-\beta\pi\frac{e_w}{1-\beta}$ is that in both cases the expected future effort cost are the same. For the sake of contradiction, assume now that $z_0^{UI} < z_0^{JM}$. Since g' is convex, the first order condition and envelope for UI

$$g'(z_0^{UI}) = \pi g'(z^s) + (1-\pi)g'(z^f)$$

imply $z_0^{UI} \geq \left[\pi z^s + (1-\pi)z^f\right]$. But then the right hand side of (18) must be strictly lower than that of (19). This contradicts that U_0 must be the same in both cases. Hence, we must have that $z_0^{UI} \geq z_0^{JM}$ which, by the envelope condition, implies the desired result.

Now consider the general case where UI is implemented for any $n \leq \infty$ periods, and then the program switches to JM from period $n+1$ onward. It is easy to see that since both JM and SA are absorbing the proof goes through with only minor changes when we consider the possibility that at any period there is a probability μ_t^{JM} of switching to JM and μ_t^{SA} of switching to SA. If fact, we will simplify notation and assume $e = e_w$. We have

$$\begin{aligned}
U_0 &= \beta^0(1-\pi)^0(z_0-e) + \beta\pi\frac{z_1^s-e}{1-\beta} + \beta(1-\pi)U_1^f \\
&= z_0 - e + \beta\pi\frac{z_1^s-e}{1-\beta} + \beta(1-\pi)\left\{z_1 - e + \beta\left[\pi\frac{z_2^s-e}{1-\beta} + (1-\pi)U_2^f\right]\right\} \\
&= z_0 - e + \beta\pi\frac{z_1^s-e}{1-\beta} + \beta(1-\pi)(z_1-e) + \beta^2(1-\pi)\pi\frac{z_2^s-e}{1-\beta} + \beta^2(1-\pi)^2U_2^f \\
&= \ldots \\
&= \sum_{t=0}^{\infty}(1-\pi)^t\beta^t\left[(z_t-e) + \beta\pi U_{t+1}^s\right],
\end{aligned}$$

with $U_t^s = \frac{z_t^s-e}{1-\beta}$. Recall that when JM becomes optimal, it is absorbing, and $z_t^{JM} - e = (1-\beta)U_t$. Hence, if after n periods the contract switches to JM, we have

$$U_0 = \sum_{t=0}^{n}(1-\pi)^t\beta^t\left[(z_t-e) + \beta\pi U_{t+1}^s\right] + (1-\pi)^{n+1}\beta^{n+1}\left[\frac{\left(z_{n+1}^{JM}-e\right) + \beta\pi U_{n+2}^s}{1-\beta(1-\pi)}\right].$$

Since by the first order conditions $(1-\beta)\, U^s_{n+2} = z^{JM}_{n+1} - e$, the above expression can be simplified to

$$U_0 = \sum_{t=0}^{n} (1-\pi)^t \beta^t \left[(z_t - e) + \beta\pi U^s_{t+1}\right] + (1-\pi)^{n+1}\beta^{n+1}\left[\frac{z^{JM}_{n+1}}{1-\beta} - \frac{e}{1-\beta}\right]. \qquad (20)$$

Recalling that $U^s_t = \frac{z^s_t - e}{1-\beta}$, the above expression derives U_0 in UI as a convex combination of future payments $z_t' s$ with weights $(1-\beta)\beta^t(1-\pi)^t$, and $\beta^t(1-\pi)^t\beta\pi$, for $t = 0, 1, ..$ (we'll call them $k^{i\prime}_t s$) minus $\frac{e}{1-\beta}$.

Recall now that under JM in period zero we have

$$U_0 > \frac{z^{JM}_0}{1-\beta} - \frac{e}{1-\beta}.$$

Thus, we must be able to write z^{JM}_0 as a convex combination of such $z_t' s$ with the same weights as in (20). At the same time, under any circumstance (no matter whether the incentive compatibility constraint is binding or not) we have

$$g'(z^f_t) = \pi g'(z^s_{t+1}) + (1-\pi) g'(z^f_{t+1}), \text{ for any } t = 0, 1, ...$$

We now can repeatedly use the fact that

$$g'(z^f_t) = (1-\beta)\left[g'(z^f_t) + \frac{\beta}{1-\beta}\left[\pi g'(z^s_{t+1}) + (1-\pi) g'(z^f_{t+1})\right]\right]$$

which delivers

$$g'(z^{UI}_0) = \sum_{t=0}^{\infty} \sum_{i=s,f} k^i_t g'(z_t)$$

with exactly the same $z_t' s$ and weights used in the promise keeping to write U_0 in the UI case:

$$(1-\beta) U_0 + e = \sum_{t,i} k^i_t z^i_t.$$

But then since $(1-\beta)U_0 + e > z^{JM}_0$ we must have, from the convexity of g', that $g'(z^{UI}_0) > g'(z^{JM}_0)$ and we are done. **Q.E.D.**

Proof of Corollary 2.

See Lemma A5 and A6. **Q.E.D.**

Proof of Proposition 4.

(i) From the first order conditions in (17), payments are decreasing as u is concave. Concavity of $\mathbf{V}$ also implies that $U^f \leq U$. From Proposition 3 we know that UI will be chosen next period as well, hence from the incentive compatibility we have $U^s_{t+1} = U^f + \frac{e}{\beta\pi} \leq U^s_t = U + \frac{e}{\beta\pi}$. Since the net wage c^e_t satisfies $c^e_t = (1-\beta) U^s_t + e_w$ we are done.

(ii) The first order conditions and the envelope condition under JM are as follows

$$\begin{aligned} \mathbf{V}'(U) &= \frac{dV^{JM}(U)}{dU} = -\frac{1}{u'(c)}, \\ -\mathbf{V}'\left(U^f\right) &= \frac{1}{u'(c)} = -\mathbf{W}'(U^s), \end{aligned}$$

hence unemployment payments and net wage are constant.

(iii) It is straightforward, hence we omit it. **Q.E.D.**

Proof of Proposition 5.

Recall that during JM the problem of the planner is

$$\begin{aligned} V^{JM}(U_0,h) &= \max_{U^f,U^s,z} -g(z) - \beta\left[\pi(h)\mathbf{W}(U^s,h) + (1-\pi(h))\mathbf{V}^f\left(U^f,h\right)\right] \\ & \quad s.t. \\ U_0 &= z - e + \beta\left[\pi(h)U^s + (1-\pi(h))U^f\right]. \end{aligned}$$

By the envelope theorem we have

$$\begin{aligned} V^{JM}_h(U_0,h) &= \pi'(h)\beta\left[\mathbf{W}(U^s_{JM},h) - \mathbf{V}\left(U^f_{JM},h\right)\right] + \\ & \quad +\pi(h)\mathbf{W}_h(U^s_{JM},h) + (1-\pi(h))\mathbf{V}^f_h\left(U^f_{JM},h\right), \end{aligned} \tag{21}$$

where the subscript JM indicates that they are the optimal choices under JM.

Consider now the UI program. Using incentive compatibility and the promise keeping constraint into the objective function of the planner, we have[25]

$$V^{UI}(U_0,h) = \max_{U^f_{UI}} -g(U_0-\beta U^f)+\beta\left[\pi(h)\mathbf{W}\left(U^f + \frac{e}{\beta\pi(h)}, h\right) + (1-\pi(h))\mathbf{V}^f(U^f,h)\right].$$

[25] Clearly, if the incentive compatibility is not binding then $V^{UI}_h(U_0,h) = V^{JM}_h(U_0,h)$ as they solve essentially the same problem and κ^{JM} does not depend on h.

Differentiating the value function with respect to h, we obtain

$$\begin{aligned}
V_h^{UI}(U_0,h) &= \pi'(h)\,\beta\left[\mathbf{W}\left(U_{UI}^f+\frac{e}{\beta\pi(h)},h\right)-\mathbf{V}\left(U_{UI}^f,h\right)\right]-\beta\pi(h)\frac{e}{\beta\pi(h)^2}\mathbf{W}_U'\left(U_{UI}^f+\frac{e}{\beta\pi},h\right)\\
&\quad+\pi(h)\,\mathbf{W}_h\left(U_{UI}^s,h\right)+(1-\pi(h))\,\mathbf{V}_h^f\left(U_{UI}^f,h\right)\\
&= \beta\left[\mathbf{W}\left(U_{UI}^f+\frac{e}{\beta\pi},h\right)-\mathbf{V}\left(U_{UI}^f,h\right)\right]-\frac{e}{\pi}W'\left(U_{UI}^f+\frac{e}{\beta\pi},h\right)\\
&\quad+\pi(h)\,\mathbf{W}_h\left(U_{UI}^s,h\right)+(1-\pi(h))\,\mathbf{V}_h^f\left(U_{UI}^f,h\right)\\
&= \beta\left[\mathbf{W}\left(U_{UI}^s,h\right)-\mathbf{V}\left(U_{UI}^f,h\right)\right]-\beta W'\left(U_{UI}^s,h\right)\left(U_{UI}^s-U_{UI}^f\right)\\
&\quad+\pi(h)\,\mathbf{W}_h\left(U_{UI}^s,h\right)+(1-\pi(h))\,\mathbf{V}_h^f\left(U_{UI}^f,h\right),
\end{aligned}$$

where we used the subscript UI notation for the optimal choices and the last line uses the IC constraint. From the separable form of $\mathbf{W}$ displayed in (10) we can make the following two simplifying observations. First, $\mathbf{W}_h(U,h)$ does not depend on U^s hence it must be the same in the two policies UI and JM and can be omitted when comparing the two slopes. Second, the h component of $\mathbf{W}$ can be omitted when comparing the two policies and only the part $\mathbf{W}(U,0)=-\frac{u^{-1}((1-\beta)U)}{1-\beta}$ can be retained.

Therefore, in light of (21), to prove that $V_h^{UI}(U_0,h)\geq V_h^{JM}(U_0,h)$ we need to show that

$$\begin{aligned}
&\left[\mathbf{W}\left(U_{UI}^s,0\right)-\mathbf{V}(U_{UI}^f,h)\right]-\mathbf{W}_U'(U_{UI}^s,0)\left(U_{UI}^s-U_{UI}^f\right)\\
\geq\ &\left[\mathbf{W}\left(U_{JM}^s,0\right)-\mathbf{V}\left(U_{JM}^f,h\right)\right]+(1-\pi(h))\left[\mathbf{V}_h^f\left(U_{JM}^f,h\right)-\mathbf{V}_h^f\left(U_{UI}^f,h\right)\right].
\end{aligned}\tag{22}$$

Since $\mathbf{V}^f$ is submodular we would be done if we showed that *(i)* $U_{JM}^f\geq U_{UI}^f$, so that the last term of the right hand side of (22) is non-positive, i.e.

$$(1-\pi(h))\left[\mathbf{V}_h^f\left(U_{JM}^f,h\right)-\mathbf{V}_h^f\left(U_{UI}^f,h\right)\right]\leq 0,$$

and that *(ii)*

$$\left[\mathbf{W}\left(U_{UI}^s,0\right)-\mathbf{V}(U_{UI}^f,h)\right]-\mathbf{W}_U'(U_{UI}^s,0)\left(U_{UI}^s-U_{UI}^f\right)\geq\mathbf{W}\left(U_{JM}^s,0\right)-\mathbf{V}\left(U_{JM}^f,h\right),$$

that is, the version of condition (22) without the last term in the right hand side.

(i) From the first order conditions and envelope, we have

$$\mathbf{W}'_U\left(U^s_{JM},h\right) = \frac{1}{u'\left(c^{JM}\right)} = \mathbf{V}^f_U(U^f_{JM},h),$$

$$\mathbf{W}'_U\left(U^s_{UI},h\right) < \frac{1}{u'\left(c^{UI}\right)} < \mathbf{V}^f_U(U^f_{UI},h).$$

Now assume, for the sake of contradiction that $U^f_{UI} \geq U^f_{JM}$. Since $\mathbf{V}^f$ is concave, then it must be that

$$\frac{1}{u'\left(c_{JM}\right)} = \mathbf{V}^f_U(U^f_{JM},h) = \mathbf{V}^f_U(U^f_{UI},h) > \frac{1}{u'\left(c_{UI}\right)}$$

which implies $c_{JM} < c_{UI}$. Moreover, the first order conditions and the concavity of $\mathbf{W}$ imply $U^s_{UI} \geq U^s_{JM}$. Notice that we found a contradiction since all payments in JM are lower than those under UI and the agent gets the same utility U in the two cases.

(ii) As a preliminary result we want so show the following:

Lemma A6: *If at U_0 the incentive compatibility constraint is binding, we must have $U^f_0 \leq U_0$.*

Proof: Notice that in any future date we can only have two cases. Either the incentive compatibility constraint is binding (and we implement UI) or it is slack. The latter possibility can occur either because we implement JM or because we have a slack incentive constraint under UI, or again because we implement SA. Now, for all t denote $U_{t+1} = U^f_t$, c_t the consumption payment during unemployment at any future date t, and consider the low of motion for U_t. Whenever the incentive compatibility is binding or SA is implemented we have

$$U_t = u\left(c_t\right) + \beta U_{t+1}.$$

In all cases where the incentive compatibility is not binding we have

$$\begin{aligned} U_t\left(h_t\right) &\leq u\left(c_t\left(h_t\right)\right) + \beta\left[\pi\left(h_t\right)U^s_t\left(h_t\right) + \left(1-\pi\left(h_t\right)\right)U_{t+1}\left(h_t\right)\right] \\ &= u\left(c_t\left(h_t\right)\right) + \beta\left[\pi\left(h_t\right)\frac{u\left(c_t\left(h_t\right)\right)}{1-\beta} + \left(1-\pi\left(h_t\right)\right)U_{t+1}\left(h_t\right)\right], \end{aligned}$$

where the last inequality comes form the first order conditions and the peculiar form of $\mathbf{W}$, and the equality from the fact that as long as the program implemented is not SA

we would have to deduct the effort cost e. Since by assumption the period zero constraint is binding, we have

$$U_0 = u(c_0) + \beta U_0^f. \tag{23}$$

We want to show that $U_1 = U_0^f \leq U_0$. From the above discussion we have

$$U_1 \leq \mathbf{E}_1 \sum_{t=0}^{n} \frac{\beta^t \gamma_{t+1}(h_{t+1})}{1-\beta} u(c_{t+1}(h_t)) + \mathbf{E}_1 \beta^{n+1} \chi_{t+n} U_{t+2+n}(h_{t+1+n}),$$

where for each history $(h_1, \ ..., \ h_{t+1})$

$$\gamma_{t+1}(h_{t+1}) = \prod_{n=1}^{t} [(1-\pi(h_n)) d_n(h_n) + (1-d_n(h_n))] \left(1 + \left[\frac{\beta\pi(h_{t+1})}{1-\beta}\right] d_{t+1}(h_{t+1})\right)$$

with $d_s(h_s) = 1$ iff in period s the incentive compatibility constraint is slack and $d_s(h_s) = 0$ otherwise. Moreover we used the fact that by the envelope theorem c_t does not depend on the h_t shock (as it is h_{t-1} measurable). Since $U_{t+2+n}(h_{t+1+n})$ is bounded above by assumption and $\chi_{t+n} \leq 1$, we must have that $\limsup_{n\to\infty} \mathbf{E}_1 \beta^{n+1} \chi_{t+n} U_{t+2+n}(h_{t+1+n}) \leq 0$. This yields

$$\begin{aligned} & \limsup_{n\to\infty} \mathbf{E}_1 \sum_{t=0}^{n} \frac{\beta^t \gamma_{t+1}(h_{t+1})}{1-\beta} u(c_{t+1}(h_t)) + \mathbf{E}_1 \beta^{n+1} \chi_{t+n} U_{t+2+n}(h_{t+1+n}) \\ \leq \ & \limsup_{n\to\infty} \mathbf{E}_1 \sum_{t=0}^{n} \frac{\beta^t \gamma_{t+1}(h_{t+1})}{1-\beta} u(c_{t+1}(h_t)) \end{aligned}$$

with $\lim_{n\to\infty} \mathbf{E}_1 \sum_{t=0}^{n} \beta^t \gamma_{t+1}(h_{t+1}) = 1$. In addition, it should be noted that $u(c_{t+1}(h_t))$ is not correlated with $\gamma_{t+1}(h_{t+1})$. Finally, notice that from the envelope condition we have that $c_{t+1} \leq c_t$ for all t, hence $\limsup_{n\to\infty} \mathbf{E}_1 \sum_{t=0}^{n} \frac{\beta^t \gamma_{t+1}(h_{t+1})}{1-\beta} u(c_{t+1}(h_t)) \leq \frac{u(c_0)}{1-\beta}$. But then (23) delivers $U_1 \leq U_0$ as desired. **Q.E.D.**

We are now ready to show the last part of the proof. We will show first that since $U_{UI}^s > U_{UI}^f$, we can prove that, for $U \geq U_{UI}^f$, we must have:

$$\left[\mathbf{W}(U_{UI}^s, 0) - \mathbf{V}^f(U_{UI}^f, h)\right] - \mathbf{W}'(U_{UI}^s, 0)\left(U_{UI}^s - U_{UI}^f\right) \geq \mathbf{W}(U, 0) - \mathbf{V}^f(U, h). \tag{24}$$

The reason is the following. If we add and subtract $\mathbf{W}(U_{UI}^f, 0)$ from the left-hand side and rewrite the above inequality as

$$\left[\mathbf{W}(U_{UI}^s, 0) - \mathbf{W}(U_{UI}^f, 0)\right] - \mathbf{W}'(U_{UI}^s)\left(U_{UI}^s - U_{UI}^f\right) + \mathbf{W}(U_{UI}^f, 0) - \mathbf{V}^f(U_{UI}^f, h) \geq \mathbf{W}(U, 0) - \mathbf{V}^f(U, h).$$

The concavity of $\mathbf{W}$ and the fact that $U_{UI}^s > U_{UI}^f$ imply

$$\left[\mathbf{W}\left(U_{UI}^s, 0\right) - \mathbf{W}(U_{UI}^f, 0)\right] - \mathbf{W}'(U_{UI}^s)\left(U_{UI}^s - U_{UI}^f\right) \geq 0.$$

So we are left to show that $\mathbf{W}(U_{UI}^f, 0) - \mathbf{V}^f(U_{UI}^f, h) \geq \mathbf{W}(U, 0) - \mathbf{V}^f(U, h)$, or

$$\mathbf{V}^f(U, h) - \mathbf{V}(U_{UI}^f, h) \geq \mathbf{W}(U, 0) - \mathbf{W}(U_{UI}^f, 0).$$

Since $U_{UI}^f \leq U$ and since $\mathbf{V}$ is steeper than $\mathbf{W}$ for all h,[26] the above inequality is true.

The very last step of the proof requires showing that

$$\mathbf{W}(U, 0) - \mathbf{V}^f(U, h) \geq \mathbf{W}(U_{JM}^s, 0) - \mathbf{V}^f(U_{JM}^f, h).$$

Since $\mathbf{W}$ is flatter than $\mathbf{V}$ and since the first-order conditions during JM guarantee that $\mathbf{W}'(U_{JM}^s, h) = \mathbf{W}'(U_{JM}^s, 0) = \mathbf{V}'(U_{JM}^f, h)$, the concavity of both functions implies that $U_{JM}^s \geq U_{JM}^f$. And for any $U \geq U_{JM}^f$ the above inequality is satisfied. **Q.E.D.**

11 Appendix D: The U.S. Welfare System

In what follows, we list the pivotal ingredient of the U.S. welfare system which are then summarized into the "actual" U.S. WTW program of section 6.1.2.

Unemployment Insurance– The unemployment insurance replacement ratio in the U.S. varies across states. The state-determined weekly benefits generally replace between 50% and 70% of the individual last weekly pre-tax earnings. The regular state programs usually provide benefits up to 26 weeks. The permanent Federal-State Extended Benefits program, present in every State, extends coverage up to 13 additional weeks, for a combined maximum of 39 weeks. Weekly benefits under the extended program are identical to the regular program.[27]

TANF– The Temporary Assistance for Needy Families (TANF) program is the main cash assistance program for poor families with children under age 18 and at least one unemployed parent. It was implemented in 1996 as part of *The Personal Responsibility and*

[26] Recall that $\mathbf{W}$ has the same slope of V^{SA}, which is the flattest among the functions describing the different policies.

[27] Extended programs can be activated when unemployment is "relatively high" (i.e., the insured unemployment rate must be above $5-6\%$).

Work Opportunity Reconciliation Act (PRWORA) which, at the same time, eliminated all existing Federal assistance programs (the AFDC, in particular). The main innovations of the TANF program were three. First, the emphasis on encouraging self-sufficiency through work. TANF legislation specifies that, with few exceptions, recipients must participate to "work activities", such as un-subsidized or subsidized employment, on-the-job training, community service, job search, vocational training, or education directly related to work.

Second, the time-limit to benefits: families with an adult who has received TANF assistance for a total of five years are not eligible for further cash aid over their lifetime. A number of states, however, have also imposed a shorter limit over fixed calendar intervals (e.g. 24 months over any given 5-year period). See Moffitt (2001) for a detailed description of the TANF program.

Third, financial incentives were created for states to run mandatory active labor market programs for workers on the TANF rolls. Generally speaking, U.S. states followed one of two alternative strategies. Some programs emphasized short-term job search monitoring (the Labor Force Attachment approach, LFA thereafter). Others emphasized longer-term skill-building activities and training (the Human Capital Development approach, HCD thereafter). The programs based on the LFA approach started each worker on job-search assistance activities (e.g., classroom instructions on resume preparation, preparation for specific job interviews, supervision of individual workers' search activity), and only later moved workers still on welfare into either basic education (e.g., brush-up courses in math and reading skills, preparation for GED or high-school completion courses), or college-level courses, or vocational training (e.g., occupational training courses in automotive repair, nursing, clerical work, computer programming, cosmetology), usually for fairly brief periods. The programs based on the HCD approach reverse the order of the policies, starting workers on education/training and moving them later (but only for a short period) onto job-search monitoring. See NEWWS (2001, Box 1.2) for a more detailed description.

Food Stamp Program– The Food Stamp program provides monthly coupons to eligible low-income families which can be used to purchase food. Over 80% of TANF recipients also receive Food Stamps (DHHS, 2004). Once TANF benefits expire, households remain virtually without any other form of benefits and have the right to the maximum allotment of food stamps.

Unemployment Tax– The Federal Government imposes a net payroll tax on employers (FUTA) of 0.8% on the first \$7,000 of earnings paid annually to each employee.[28] States finance their welfare programs with an additional State Unemployment Tax. In 1996, the estimated national average tax rate as a fraction of total wages was 0.8% (House Ways and Means Committee, 1996).

EITC– The Federal Earned Income Tax Credit (EITC) is the major wage subsidy program in the United States. It is a refundable tax credit that supplements the earnings of low-income workers. It has a "trapezoid" structure as a function of annual earnings. In 1996, for a single-parent household with two children (the typical household on the welfare rolls) the subsidy rate was 40% up to \$741 per month. In the range \$741 – \$967, the subsidy is fixed at \$296. For monthly earnings over \$967, workers start paying a tax rate of 21% over and above the \$296 subsidy, until the break-even income such that the net subsidy is exactly zero, i.e. \$2,374. See Hotz and Scholtz (2001, Table 1) for details.

12 Appendix E: Numerical Algorithm

1. Grid for human capital $\mathcal{H}$
   (a) Set the grid over human capital $\mathcal{H} = \{h_{\min}, h_1, h_2, ..., h_{\max}\}$ with size $N_h = 30$.
   (b) Set the Markov transition matrices for human capital $Q^z(h', h)$, $z = s, f$, the job finding probability function $\pi(h)$, and the wage function $w(h)$ as described in the calibration.
   (c) Compute the gross value of employment recursively as

$$\Omega^n(h) = \sum_{h' \in \mathcal{H}} \left[w(h') + \beta \Omega^{n-1}(h') \right] Q^s(h', h),$$

   and define $\Omega(h) = \lim_{n \to \infty} \Omega^n(h)$.

2. Grid for promised utility $\mathcal{U}$

[28] The current gross FUTA tax is 6.2% but employers in states meeting certain requirements are eligible for a 5.4% credit.

(a) Set the size of the grid $N_U = 400$ and the maximum order of the Chebyshev polynomials $N_{cheb} = 20$

(b) Set the upper and lower bounds for the grid over promised lifetime utilities as

$$\begin{aligned} U_{\min} &= \frac{u\left(c_{\min}\right) - e}{1-\beta}, \\ U_{\max} &= \frac{u\left(w\left(h_{\max}\right)\right) - e_w}{1-\beta}, \end{aligned}$$

(c) Define the grid points over promised utility $\mathcal{U} = \{U_{\min}, U_1, U_2, ..., U_{\max}\}$ as

$$U_k = U_{\min} + \cos\left(\frac{2k-1}{2N_U}\pi\right) * \left(U_{\max} - U_{\min}\right)$$

for $k = 1, ..., N_U$, where we set the lower and upper bounds $(U_{\min}, U_{\max})$ as

$$U_{\min} = \frac{\ln\left(\bar{c}^{SA}\right) - e}{1-\beta}, \qquad U_{\max} = \frac{\ln\left(w\left(h_{\max}\right)\right)}{1-\beta}.$$

(d) Use the recursion

$$\begin{aligned} T(1,k) &= 1, \\ T(2,k) &= \cos\left(\frac{2k-1}{2N_U}\pi\right), \\ T(n_{cheb},k) &= 2\cos\left(\frac{2k-1}{2N_U}\pi\right) T(n_{cheb}-1,k) - T(n_{cheb}-2,k), \end{aligned}$$

for $n_{cheb} = 3, ..., N_{cheb}$ to determine the Chebyshev polynomials on the grid points.

3. Absorbing States

(a) Define a function for the value of social assistance and for the associated consumption as

$$\begin{aligned} V^{SA}(U) &= -\frac{c^{SA}(U)}{1-\beta} \\ c^{SA}(U) &= u^{-1}\left((1-\beta)U\right) \end{aligned}$$

that can take values also outside the grid $\mathcal{U}$.

(b) Define a function for the net value of employment for the Planner, and the associated consumption as

$$\mathbf{W}(U,h) = \Omega(h) - \frac{c^{EMP}(U)}{1-\beta}$$
$$c^{EMP}(U) = u^{-1}((1-\beta)U + e_w)$$

that can take values also outside the grid $\mathcal{U}$.

4. Convergence check

(a) If the iteration number $iter = 1$, then guess two initial matrices $\mathbf{V}_M^z(U_k,h)$ with $z = s, f$ defined over the grid points only. If $iter > 1$, then the matrices are inherited from the algorithm (see step 9 below).

(b) Compute the parameter vectors $\Theta_{iter}^z(h)$ of dimension N_{cheb} for the Chebyshev approximations of $\mathbf{V}_M^z(U_k,h)$ off the grid points of $\mathcal{U}$ and call the Chebyshev functions $\mathbf{V}^s(U,h)$ and $\mathbf{V}^f(U,h)$.

(c) If $iter > 1$, verify if the convergence has been reached by comparing $\Theta_{iter}^z(h)$ with $\Theta_{iter-1}^z(h)$ for all h. We define the metric

$$dist = \max \left|\Theta_{iter}^z(h) - \Theta_{iter-1}^z(h)\right|, \text{ for } z = s, f \text{ and for } h \in \mathcal{H}$$

and we stop iterating when $dist < 0.000001$. If this convergence criterion has not been reached, we keep iterating.

5. Value of the programs $i = UI, JM, TR$ on the grid

(a) Use the bold functions $\mathbf{W}(U^s,h), \mathbf{V}^s(U^s,h)$, and $\mathbf{V}^f\left(U^f,h\right)$ to define the value for each program i on every grid point (U_k,h), only as a function of $\left(c^i, U^s, U^f\right)$.

(b) For every combination of point (U_k,h) on the grid, solve the maximization problem for UI as follows. From the (IC) constraint, obtain $U^s(U_k,h) = U^f(U_k,h) + e/\beta\pi(h)$, and from the (PK), set $c^{UI}(U_k,h) = u^{-1}\left(U - \beta U^f\right)$. Substituting these restrictions into the objective function $V^{UI}(U_k,h)$ defined in equation (7) in the main text, one obtains a simple unconstrained maximization problem. Use a Powell-type algorithm (without the need for computing derivatives) to obtain the maximizer $U^f(U_k, h)$. Call $V_M^{UI}(U_k, h)$ the value at the optimum for each grid point. It is useful to remark that both U^f and U^s in general are not on the grid $\mathcal{U}$, hence the need for the Chebyshev functions $\mathbf{V}^s$, and $\mathbf{V}^f$.

(c) For every combination of point (U_k, h) on the grid, solve the maximization problem for JM as follows. From the first-order condition of the problem in (8) in the main text, set $c^{JM}(U_k, h) = u^{-1}((1-\beta)U^s + e_w)$. Using this solution for the optimal payment into the (PK) constraint, we obtain

$$U^s = \frac{U + e - e_w - \beta(1-\pi(h))U^f}{1-\beta+\beta\pi(h)},$$

which allows one to write the objective function $V^{JM}(U_k, h)$ in (8) only as a function of one variable, U^f. Use a Powell-type algorithm (without the need for computing derivatives) to obtain the value $U^f(U_k, h)$ that solves the unconstrained maximization problem. Call $V_M^{JM}(U_k, h)$ the value at the optimum for each grid point. It is useful to remark that both U^f and U^s in general are not on the grid $\mathcal{U}$, hence the need for the Chebyshev functions $\mathbf{V}^s$, and $\mathbf{V}^f$.

(d) For every combination of point (U_k, h) on the grid, solve the maximization problem for TR as follows. From the (IC) constraint, obtain $U^s(U_k, h) = U^f(U_k, h) + e/\beta\pi(h)$, and from the (PK), set $c^{TR}(U_k, h) = u^{-1}\left(U - \beta U^f\right)$. Substituting these restrictions into the objective function $V^{TR}(U_k, h)$ defined in equation (9) in the main text, one obtains a simple unconstrained maximization problem. Use a Powell-type algorithm (without the need for computing derivatives) to obtain the maximizer $U^f(U_k, h)$. Call $V_M^{TR}(U_k, h)$ the value at the optimum for each grid point. It is useful to remark that both U^f and U^s in general are not on the grid $\mathcal{U}$, hence the need for the Chebyshev functions $\mathbf{V}^s$, and $\mathbf{V}^f$.

6. Upper envelope on the grid

   (a) For each grid point (U_k, h), compute the upper envelope matrix

$$UPV_M(U_k, h) = \max\left\{V^{SA}(U_k), V_M^{UI}(U_k, h)\, V_M^{JM}(U_k, h)\, V_M^{TR}(U_k, h)\right\}$$

and the associated optimal policy $i^*(U_k, h)$.

7. Convexification of the upper envelope on the grid

   (a) Use the revised simplex method to solve the following linear programming problem, for each pair of $\left(\bar{U}_k, \bar{h}\right)$ on the grid

$$
\begin{aligned}
& \max_{\{\lambda_k\}} \sum_{k=1}^{N_U} \lambda_k UPV_M\left(U_k, \bar{h}\right) \\
& s.t. \\
\sum_{k=1}^{N_U} \lambda_k &= 1 \\
\sum_{k=1}^{N_U} \lambda_k U_k &= \bar{U}_k \\
0 &\leq \lambda_k \leq 1 \text{ for all } k
\end{aligned}
$$

   (b) Denote by $\Lambda^*\left(U_k, h\right)$ the vector of probabilities, and by $coUPV_M\left(U_k, h\right)$ the convexified upper envelope matrix, for each point on the grid $\mathcal{U} \times \mathcal{H}$.

8. Randomization based on human capital shocks

   (a) Construct a function $coUPV\left(U, h\right)$ taking values both on and off the grid, using a piece-wise linear approximation of the matrix $coUPV_M\left(U_k, h\right)$, i.e.

$$
coUPV\left(U, h\right) = coUPV_M\left(U_{k-1}^*, h\right) + \frac{\left[coUPV_M\left(U_k^*, h\right) - coUPV_M\left(U_{k-1}^*, h\right)\right]}{U_k^* - U_{k-1}^*}\left(U - U_{k-1}^*\right)
$$

   where $\left(U_{k-1}^*, U_k^*\right)$ is the smallest pair of grid points that includes U.

   (b) For each pair $\left(U_k, h\right)$ on the grid, solve the constrained maximization problem

$$
\begin{aligned}
& \max_{U^z(h')} \sum_{h' \in \mathcal{H}} coUPV\left(U^z\left(h'\right), h'\right) Q^z\left(h', h\right) \\
& s.t. \\
U_k &= \sum_{h' \in \mathcal{H}} U^z\left(h'\right) Q^z\left(h', h\right)
\end{aligned}
$$

9. Updating of guess

   (a) Store the maximized objective functions in the previous step which represent the new guess for the matrices of values $\mathbf{V}_M^s\left(U_k, h\right)$ and $\mathbf{V}_M^f\left(U_k, h\right)$.

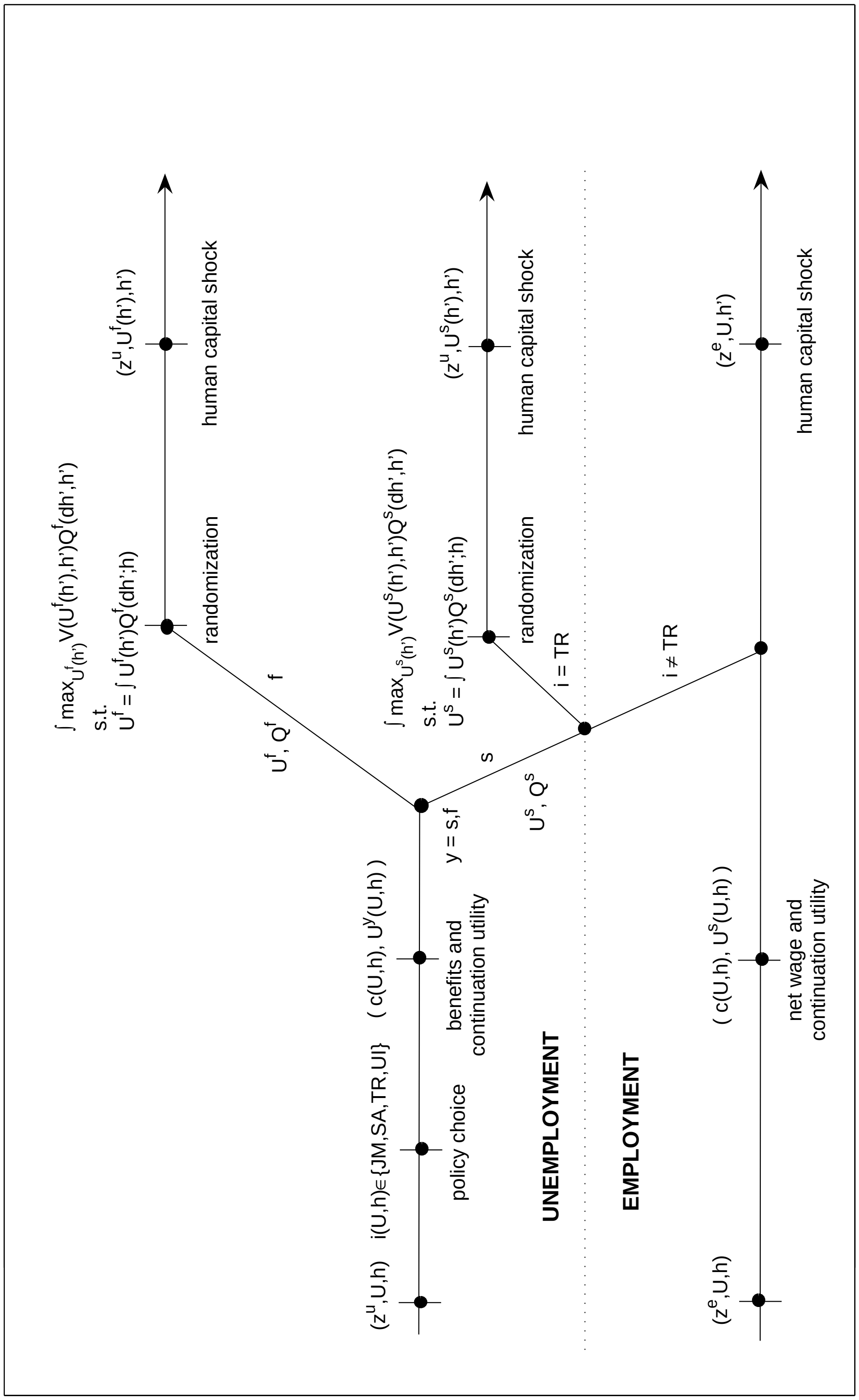


Figure 1: The timing of the dynamic principal-agent problem.

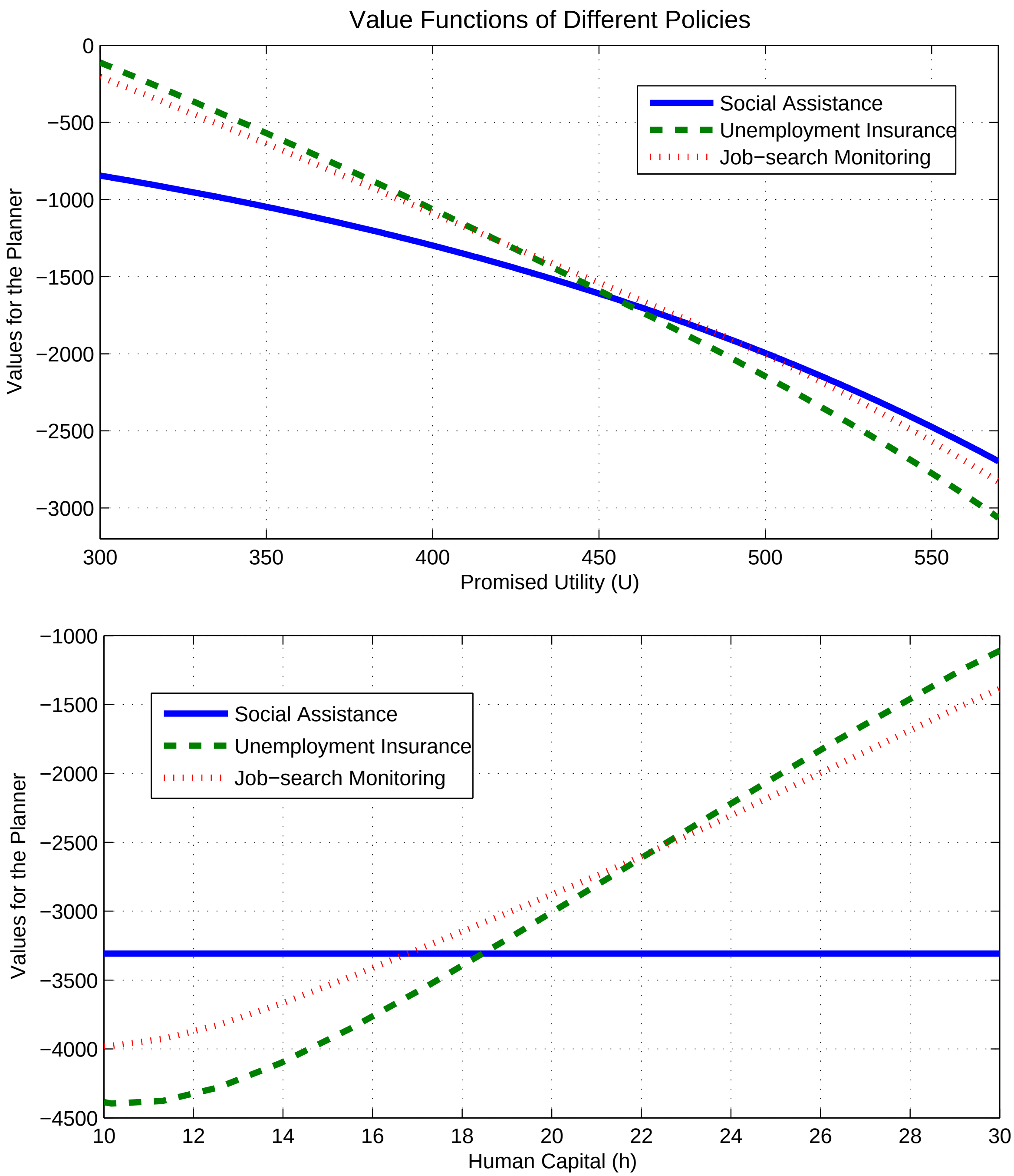


Figure 2: The value functions for UI, JM and SA plotted, separately, with respect to promised utility U and human capital h. Note the relative slopes of the value functions of the different policies, explained in Corollary 2 and Proposition 5.

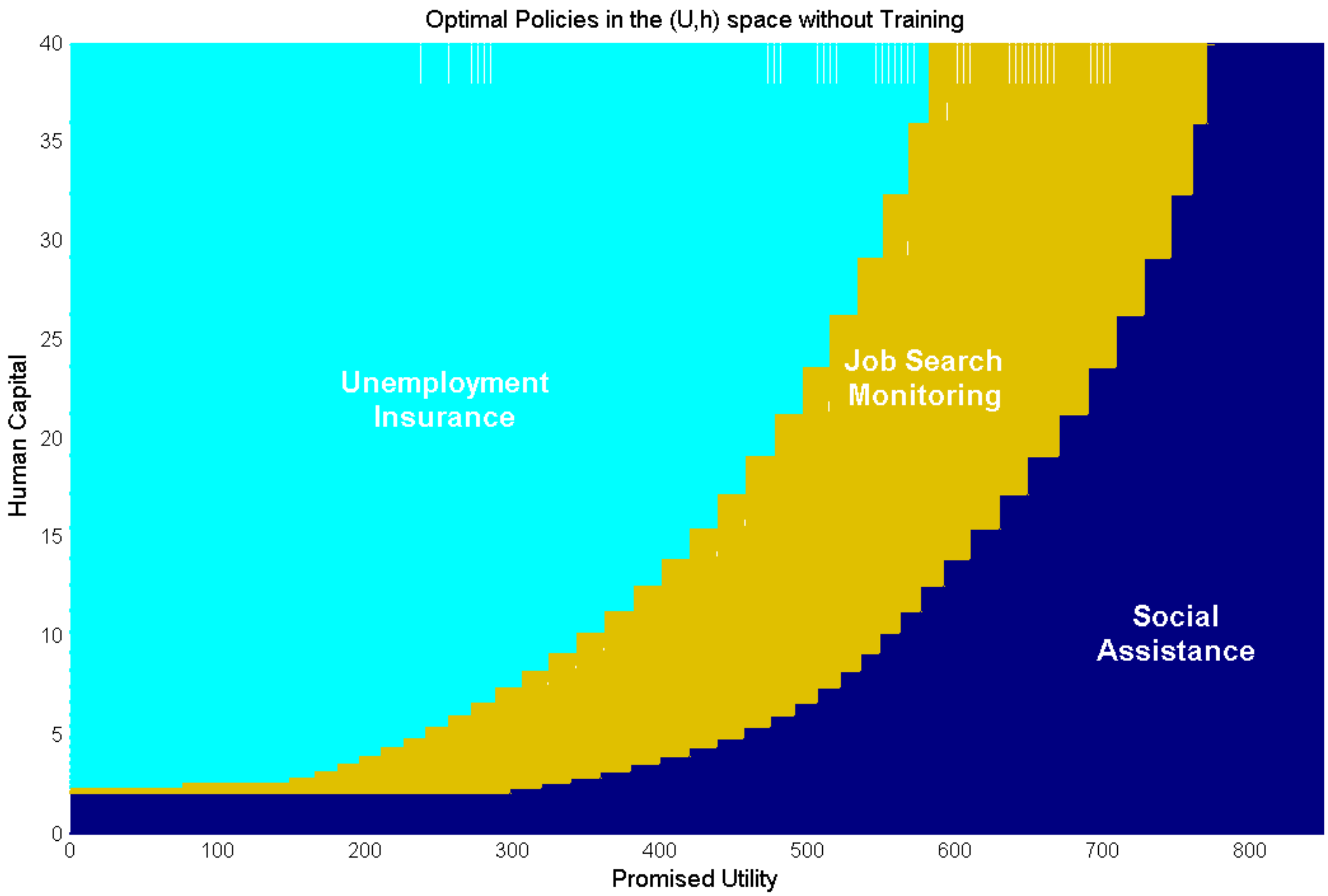


Figure 3: The policies of the optimal WTW program without training in the state space of human capital h and promised utility U.

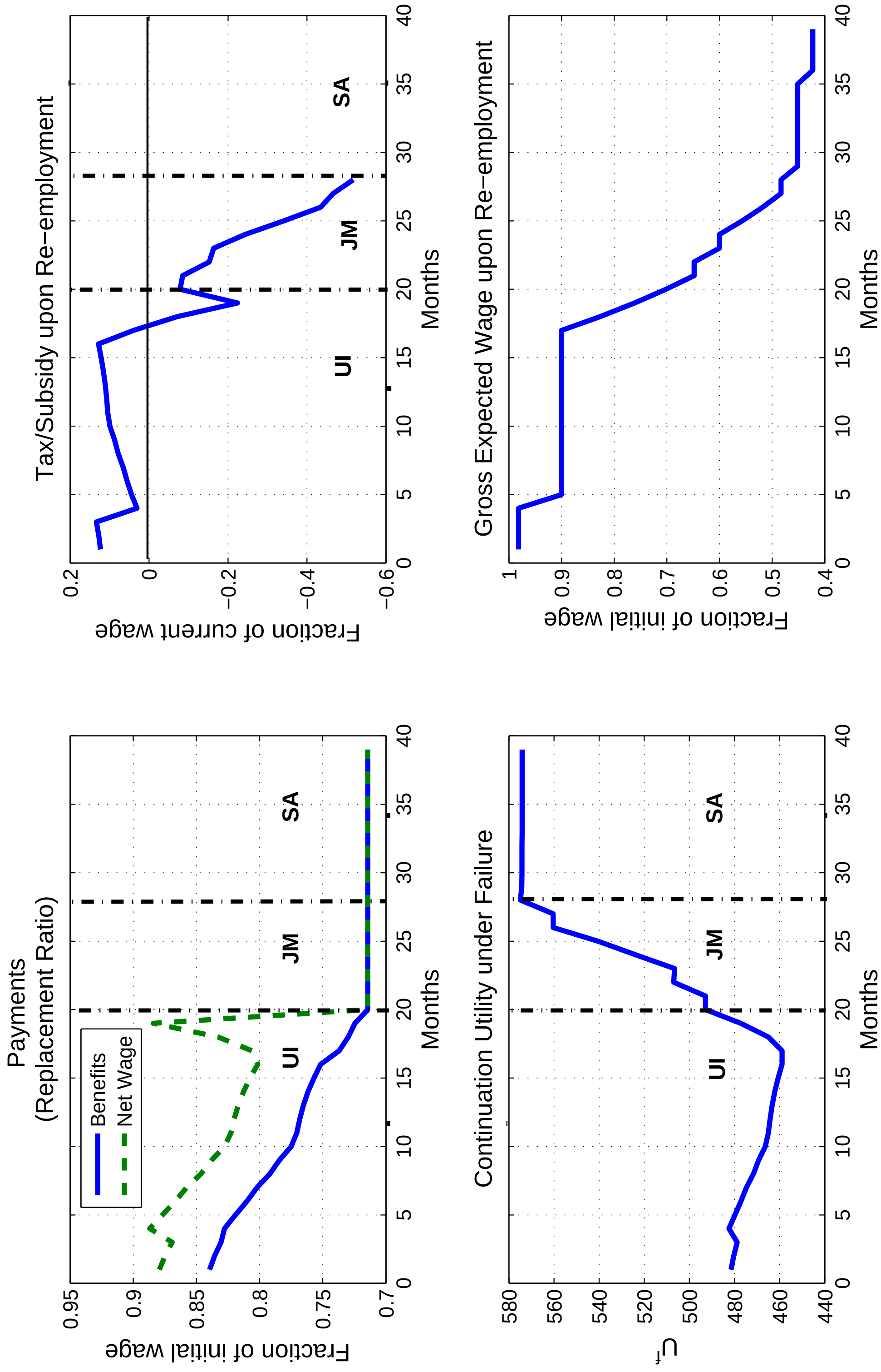


Figure 4: A representative history of the optimal WTW program without training policies.

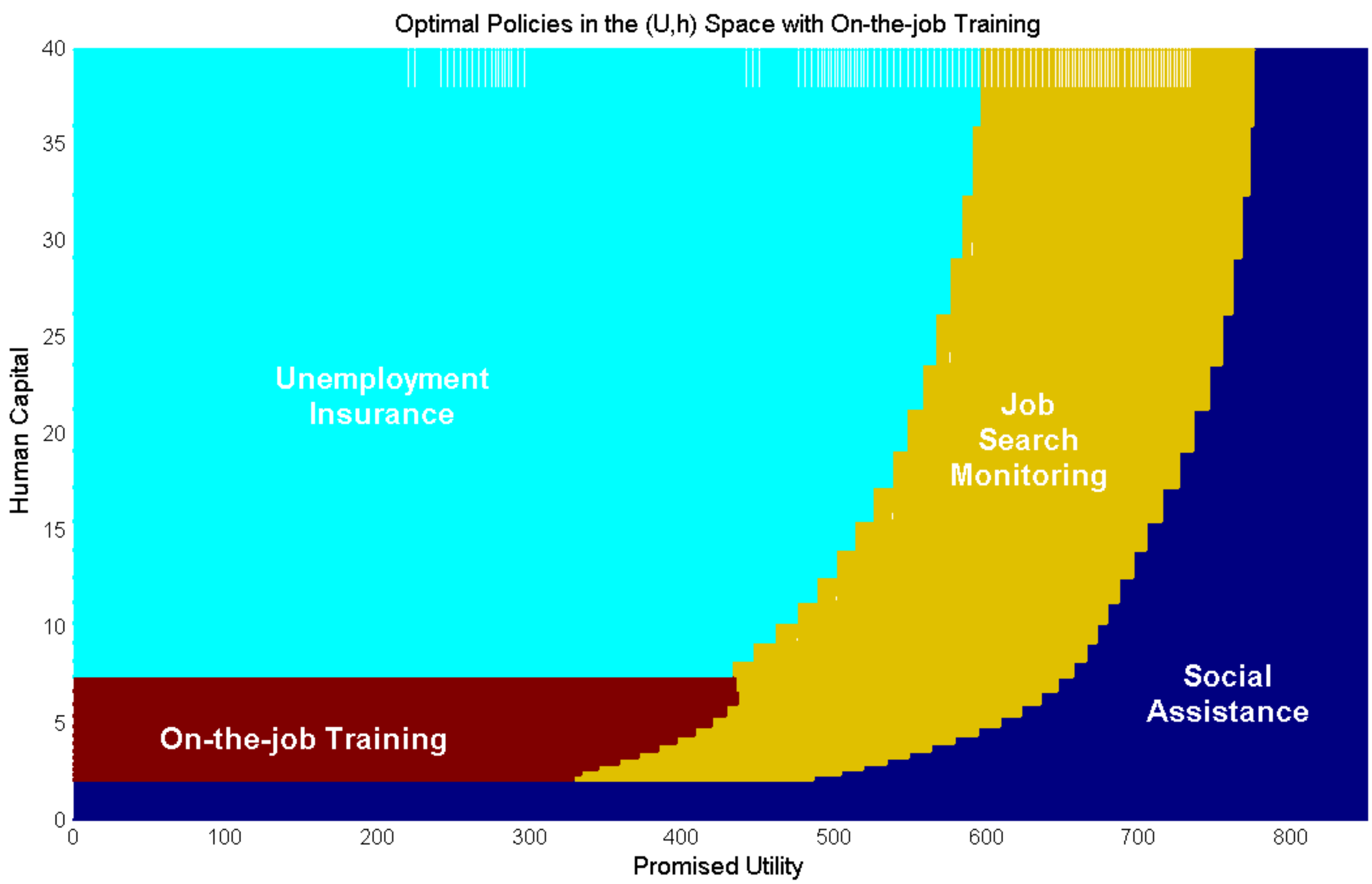


Figure 5: The policies of the optimal WTW program with on-the-job training in the state space of human capital h and promised utility U.

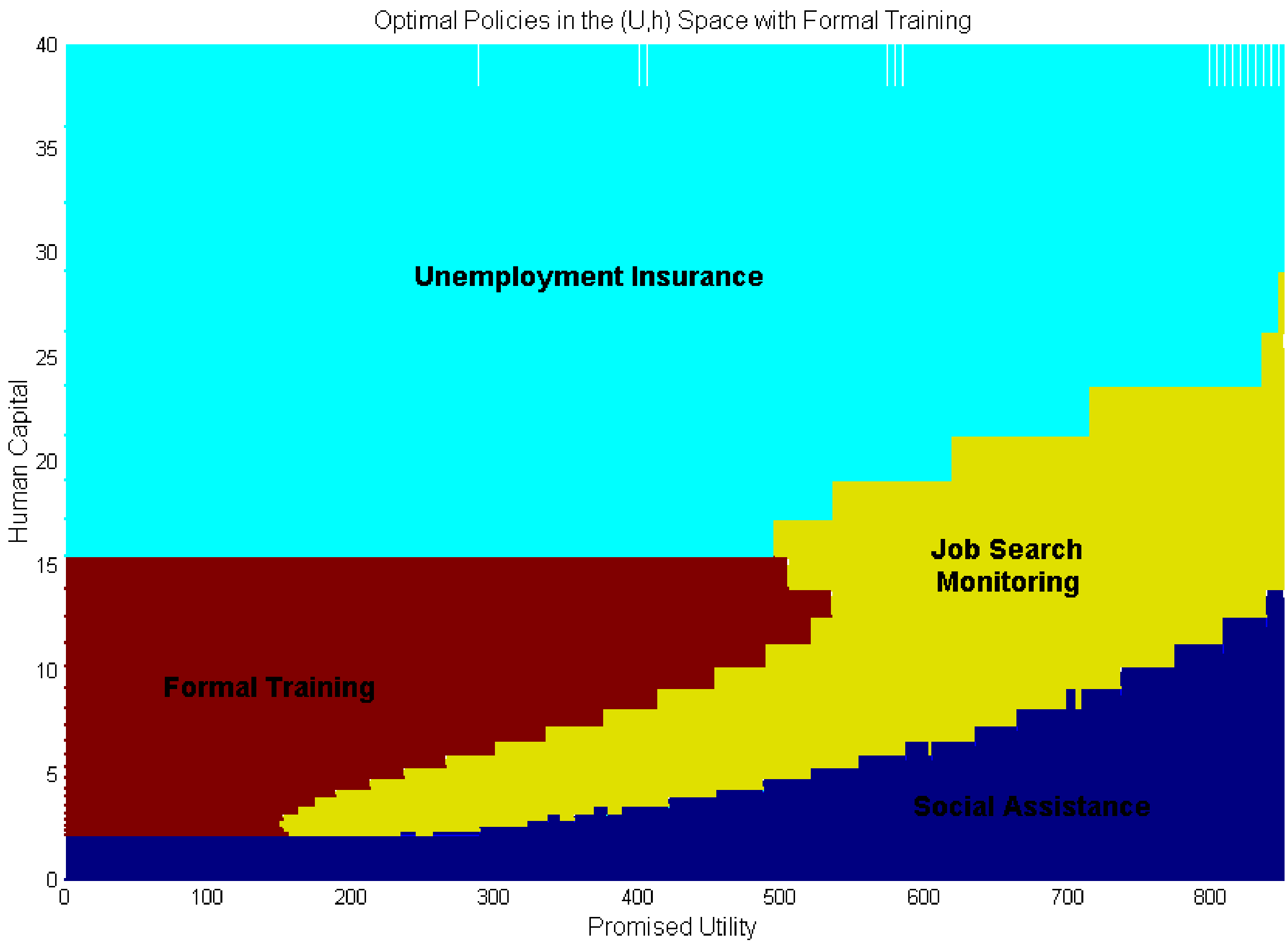


Figure 6: The policies of the optimal WTW program with formal training in the state space of human capital h and promised utility U.

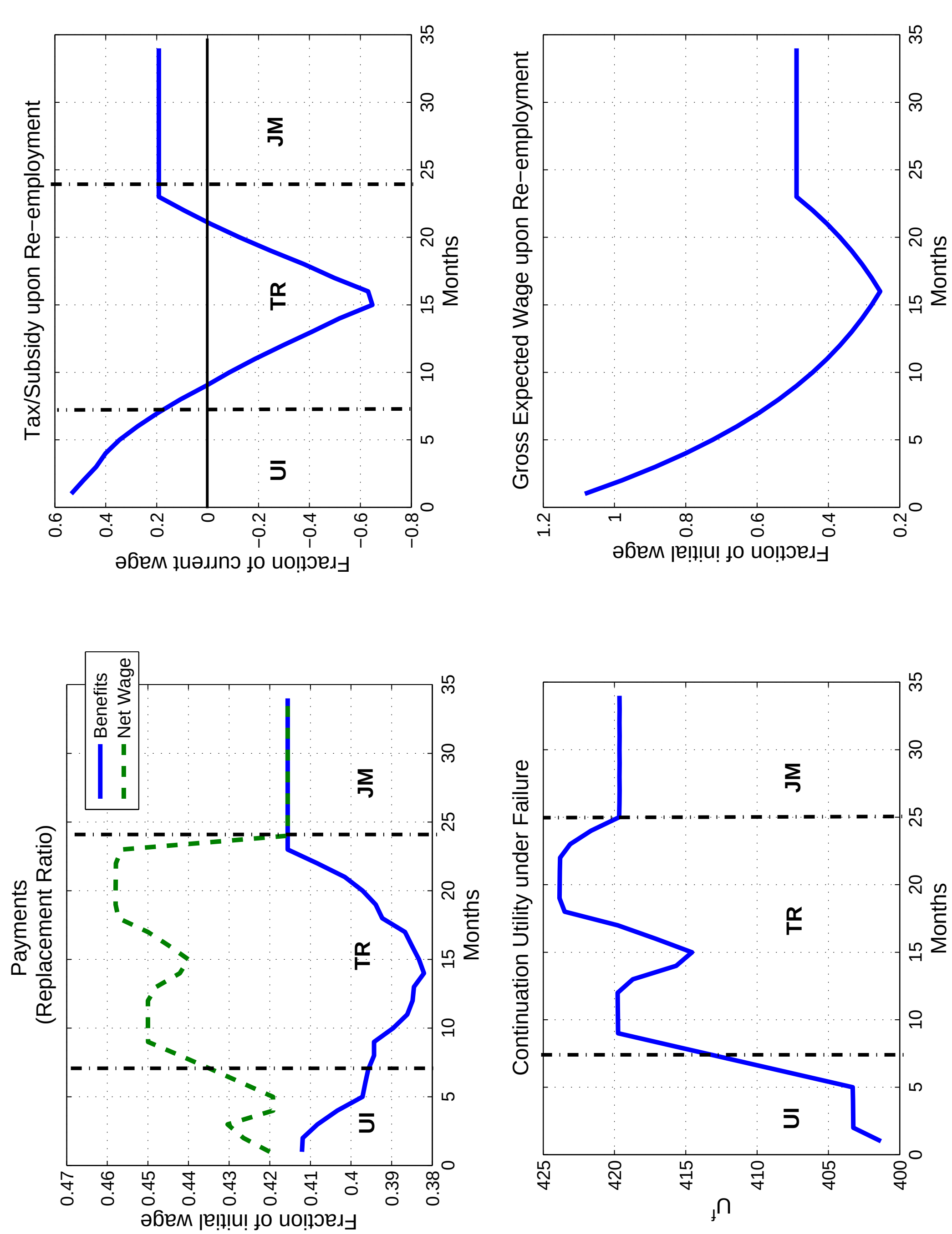


Figure 7: A representative history of the optimal WTW program with formal training.

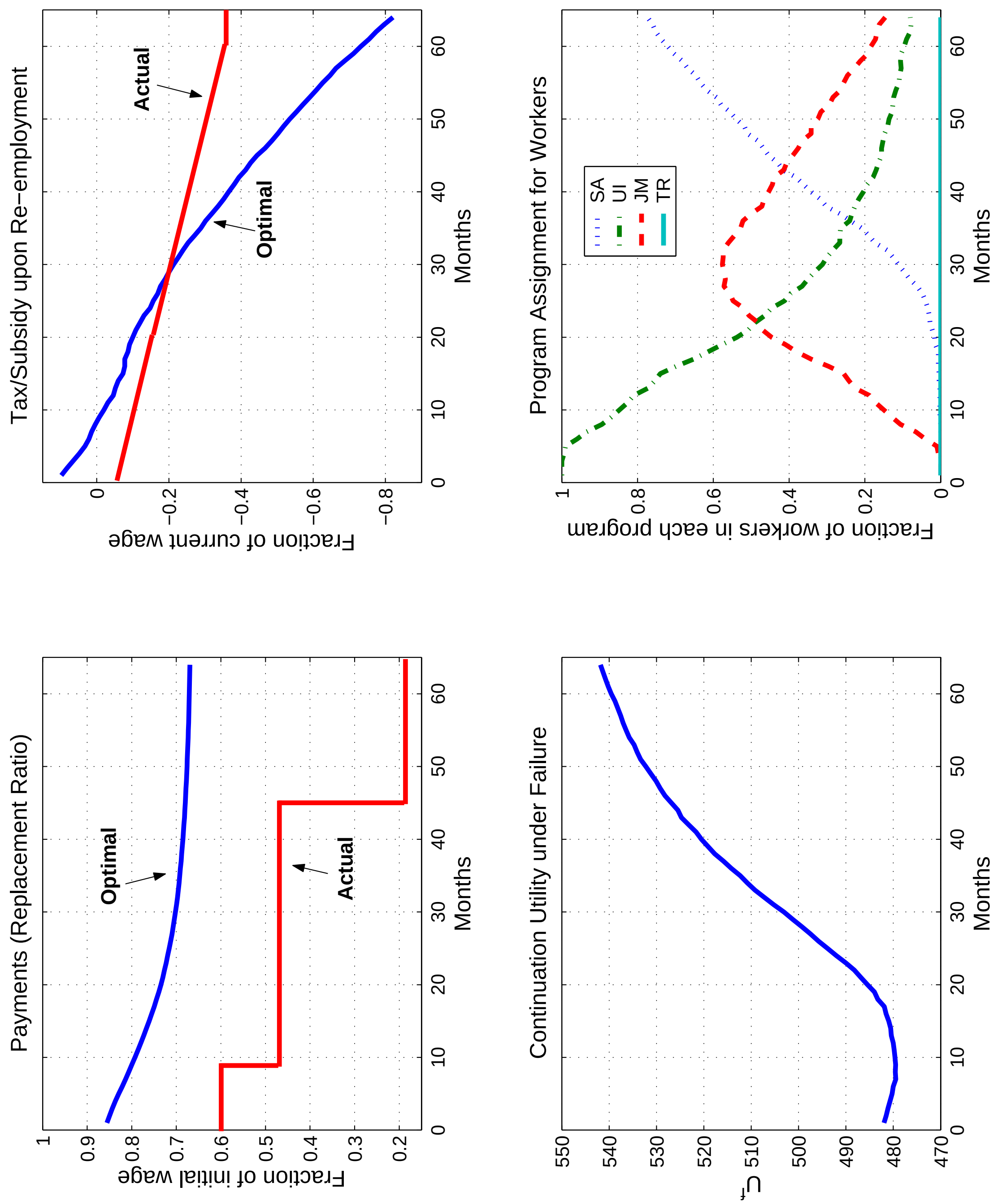
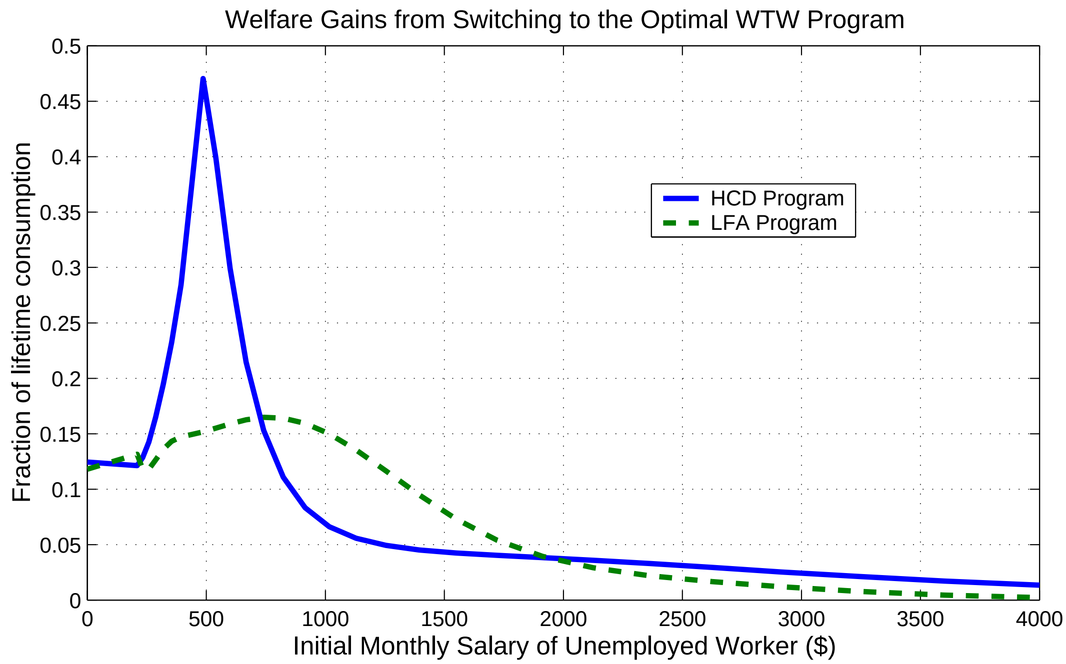


Figure 8: Features of the optimal WTW program compared to the actual U.S. welfare system (version with HCD programs).



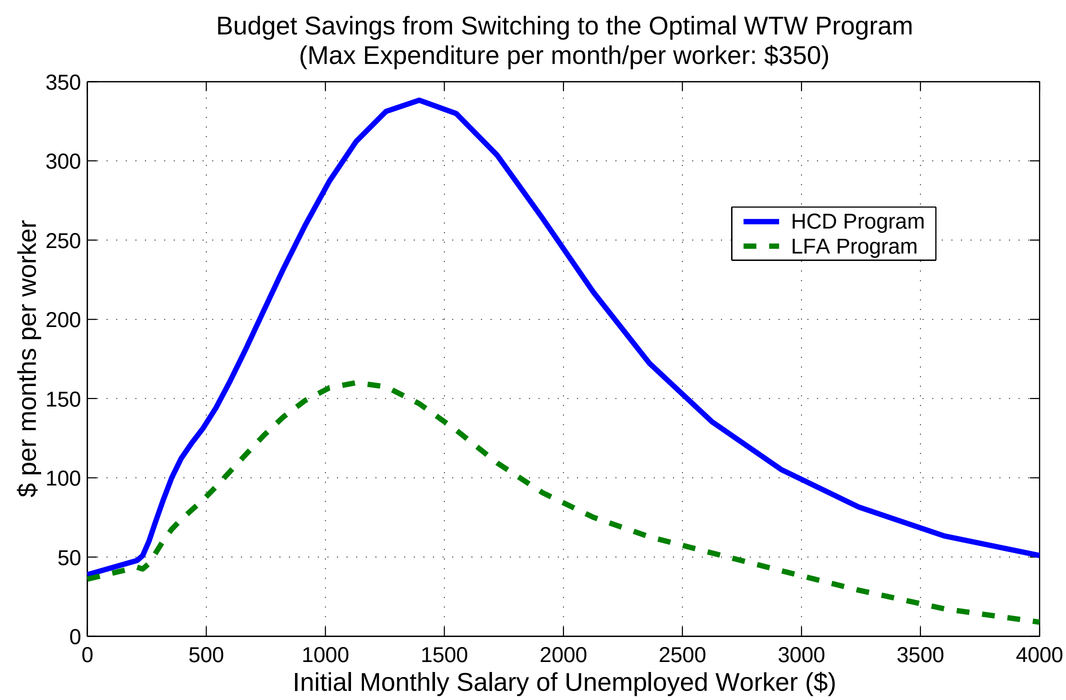


Figure 9: Welfare gains and budget savings of switching to the optimal WTW program from the current U.S. system.