TICKET PRICING*

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Abstract

Price discrimination among ticket service classes is analyzed when aggregate demand is known and individual preferences are private information. Serving customers in cheap second-class seats limits the seller's ability to extract surplus from expensive first-class seats because some switch to the lower class. Discrimination is greatest in the class with the largest variance in demand prices. The seller's incentives to limit substitution by altering the between-class quality spread and the pricing of complementary (concession) goods are also analyzed. These issues depend on comparing ''marginal'' with ''average'' customers, parallel to the provision of public goods. Finally, when capacity limitations require sequential servicing of buyers in ''batches'' (for example, theatrical productions), intertemporal price discrimination requires prices to decline over time, so customers with the greatest demand prices buy higher-priced tickets to earlier performances rather than wait for later performances. The rational policy can generate queues for early performances.

I. Introduction

Price discrimination represents some of the most interesting and challenging problems in microeconomics. The practice is widespread and hardly confined to traditional monopolists. It occurs in such highly competitive businesses as restaurants, airlines, hotels, bars, and private colleges, where many alternative sellers are available to customers and barriers to entry are nil. Price discrimination tends to be observed in activities where inventory/capacity constraints make the marginal costs of providing service to any one user smaller than the average cost. For example, so long as capacity is slack, the marginal user cost of hotel rooms or airplane seats to customers is trivial once the hotel has been built and the airplane has been configured. Charging different prices to different customers apparently has more general uses in the management of capacity than is suggested by economists' preoccupation with it for public utilities and natural monopolies.

Nowhere are these issues more sharply drawn than in ticket pricing, a

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problem of historical importance in public finance, industrial organization, and the analysis of price discrimination itself.[1] Ticket pricing practices are also of substantial interest as an important practical example of applied price theory and as a mechanism that sustains high incomes of performers.[2] We use the theory of second-degree price discrimination to analyze some commonly observed practices of ticket sellers under conditions of deterministic demand.

The next section analyzes the pricing of discrete classes of service: first- and second-class seats on airplanes and trains; boxes, grandstands, and bleachers at athletic events; and different quality seats at theaters, concerts, and the opera. Price discrimination serves to sort customers with different tastes to various service classes. The analysis of tickets neatly illustrates how catering to any subset of customer tastes in one class constrains the revenues that can be extracted from other groups in other classes. While discriminatory price differentials separate buyers into more homogeneous groups, substitution opportunities in other classes invariably limit the extent to which surplus can be extracted from them. A point new to our analysis is that the relative variance of customers' preferences across service classes affects outcomes. For instance, discrimination is greater in first-class than in second-class service only if the relative (marginal) variance of demand prices is greater in first class.

We also show that virtually the same formal analysis carries over to many kinds of intertemporal price discrimination. Theater-size constraints require repeated production, a kind of ''batch processing'' of audiences of given size, until all demand is served. Class of service roughly corresponds to rank in the intertemporal queue: attending earlier performances is akin to first class, and attending later performances is like second class. Prices have to decline over time to sort customers by tastes. More ardent customers buy early if the intertemporal pattern of prices is declining slowly enough to keep them from waiting but fast enough to keep less ardent buyers from purchasing too early. There is also a sense in which it is possible to observe queuing for tickets to early performances even though there are no scalping opportunities.

Section III considers how the number of seats in each class is determined and shows a sense in which customers with the most intense preferences

[1] See Jules Dupuit, *On the Measurement of the Utility of Public Works* (1844), translated into English by R. H. Barbak in *International Economic Papers* (No 2, 1952); Harold Hotelling, *Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions*, 40 J Pol Econ 577 (1932); Harold Hotelling, *The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates*, 6 Econometrica 242 (1938).

[2] Sherwin Rosen, *The Economics of Superstars*, 71 Am Econ Rev 845 (1981).

for the service are the most ''exploited,'' as measured by price-cost margins. Section IV considers the determinants of class quality and the classic problem of Dupuit. Does the seller alter quality differences between classes of service to deter substitution and increase the interclass price premium? Depending again on the relative variances of customer valuations between service classes, the seller has incentives either to reduce the quality of second-class seats or increase the quality of first-class seats relative to socially optimal levels.

Section V considers the pricing of complementary goods sold on the premises (for example, food and drinks), given that tickets are offered to all potential buyers on the same terms. Ticket prices are lower and prices of complements are set above marginal cost when the average customer buys more complementary goods than the marginal customer. Complements are subsidized (sold below costs) and ticket prices raised when marginal customers of tickets consume more complements than the average customer. These marginal-average comparisons also apply to the choice of product quality.

II. Class of Service Pricing

A. The Problem

We consider variations of the following problem. A facility has two kinds of seats, high quality or first class, H, and low quality or second class, L. The seller chooses the number of seats, the quality of each class, and a pricing policy for complementary goods (food, souvenirs, programs, and so on) sold on the premises to ticket holders, as well as the price of tickets themselves. Customers either attend or do not attend a scheduled event (a specific concert or prizefight, an airplane trip between two cities on a given day, and so forth). All prefer first- to second-class service, but the willingness to pay for either type of ticket varies among customers. Personal preferences are completely described by a pair of reserve prices, r_h and r_l, for seats of quality H or L, conditional on the seat qualities and prices of complements chosen by the seller. The seller knows that the conditional demand prices are distributed with frequency $f(r_h, r_l)$ over the population but cannot identify the specific tastes of individual buyers. Ticket prices are set in advance and posted on equal terms to all buyers. The seller also knows how the reservation price distribution changes as seat quality and complementary goods prices vary.

The problem is solved in two steps. First, given the quantities and qualities of the two classes of seats and the price of complements, the seller chooses ticket prices to maximize revenue, knowing that buyers make

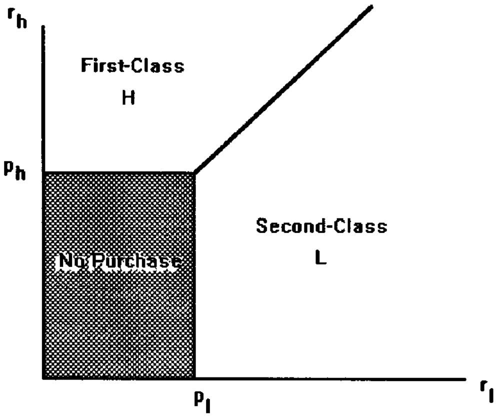

FIGURE 1

choices that maximize their utility. We assume the seller is selfish and does not allow others to gain financially from resale of tickets in secondary markets. Second, given the optimum pricing policy, the seller decides on the quantity and quality of seats and on the price of complements. This section considers the first step: the pricing of seats, given quantities and qualities. Later sections consider the other parts of the problem.

B. Ticket Pricing

If p_h and p_l are the prices charged for each kind of seat, a buyer chooses to purchase a ticket in service class H or L or not attend at all according to

$$\max\{r_h - p_h, r_l - p_l, 0\}. \quad (1)$$

Any price policy (p_h, p_l) partitions the (r_h, r_l) plane into the three regions shown in Figure 1.[3] All people whose reserve prices are less than either p_h or p_l do not purchase anything. Those whose reserve prices fall in the region marked H purchase a first-class ticket, and those in the region marked L purchase a second-class ticket.

[3] Note the family resemblance of this image to bundling problems; see W. Adams and Janet Yellen, *Commodity Bundling and the Burden of Monopoly,* 90 Q J Econ 475 (1976); Richard Schmalensee, *Gaussian Demand and Commodity Bundling,* 57 J Bus S211 (Supp 1984); R. Preston McAfee, John McMillan, and Michael D. Whinston, *Multiproduct Monopoly, Commodity Bundling, and Correlation of Values,* 103 Q J Econ 371 (1989). Here the customer is presented with a variety of choices and buys at most one of them, not a bundle.

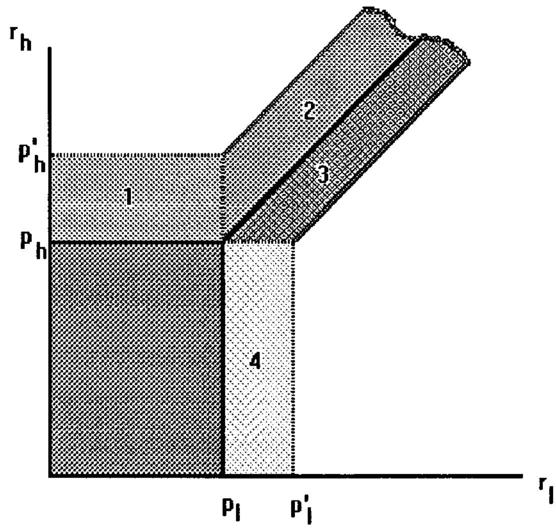

FIGURE 2

Figure 2 shows how changing the prices of each ticket type affects the partition of customers in the three regions. If p_h is increased to p'_h, some people who previously purchased a high-quality seat decide not to attend (those whose reserve prices fall in the area marked 1 of Figure 2). Those whose reserve prices for low-quality service are high enough switch to second-class service instead (area marked 2). If p_l is increased to p'_l, some people who previously bought a low-quality ticket choose not to attend (area marked 4). Other low-quality ticket buyers switch to a higher-quality seat (area marked 3).

The seller's problem is to partition the reservation price distribution to maximize revenue subject to seat capacity constraints. The basic solution is transparent when there are only a few types, and even more so when all buyers have the same tastes. In that case the frequency distribution degenerates to a point mass in the (r_h, r_l) plane, say (r_{1h}, r_{1l}), and the seller sets $p_h = r_{1h}$ and $p_l = r_{1l}$ and extracts surplus of all customers in each class. Both kinds of seats are rationed by availability, and prices are set so high that buyers are indifferent to attending or not.

Suppose now that there are also some type 2 buyers, located at point (r_{2h}, r_{2l}) in Figure 3, with the second type less willing to pay for either class of service. There are two possible solutions. First, the seller can exclude all type 2 customers by continuing the high price policy that extracts all sur-

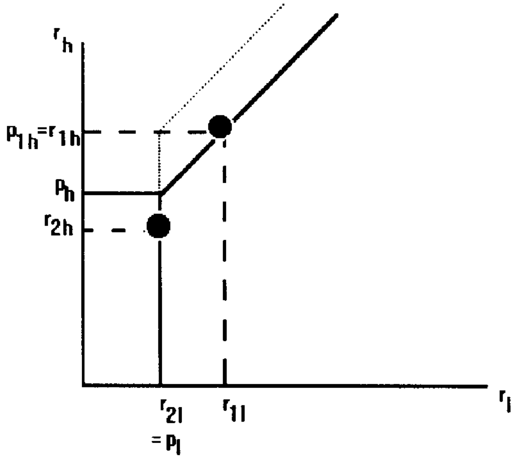

FIGURE 3

plus from type 1 customers. This is optimal if type 1 customers are sufficiently numerous relative to total seat capacity. Second, if there are not enough type 1 buyers to fill up the seats, the seller may want to serve type 2 customers. In this case the price of second-class service must be reduced to r_{2l} or less. But maintaining p_h at its previous price of p_{1h} flips all type 1 buyers into area 3 in Figure 2, and no first-class tickets are sold. This is the basic constraint of second degree price discrimination: catering to type 2 buyers affects the terms on which tickets can be sold to type 1 buyers.[4] To insure that type 1 buyers purchase the better seats while simultaneously inducing type 2 customers to enter the market, the seller must reduce the price of *both* high- and low-quality tickets.

Figure 3 depicts the revenue-maximizing policy that caters to both types. All rent is extracted from type 2 buyers, who strictly prefer low-quality seats. Type 1 customers get an equal amount of surplus from either kind of seat. It is impossible to extract more surplus than this from type 1 buyers so long as type 2 buyers are served. The seller chooses to sell to both types if the number of enthusiastic buyers is small relative to capacity and there are lots of cheap seats and many less enthusiastic buyers.

[4] Michael Mussa and Sherwin Rosen, *Monopoly and Product Quality*, 18 J Econ Theory 301 (1978), is the basic reference. With a qualification for corner solutions, that and subsequent work analyzes cases where customer tastes are homogeneous within classes. This problem differs because there is generally lots of heterogeneity within, as well as between, classes. Jean-Charles Rochet, *Ironing Sweeping and Multidimensional Screening* (working paper, Université des Science Sociales, Institute d'Economie Industrielle [IDEI], Toulouse, 1995), is a more formal version of a related problem.

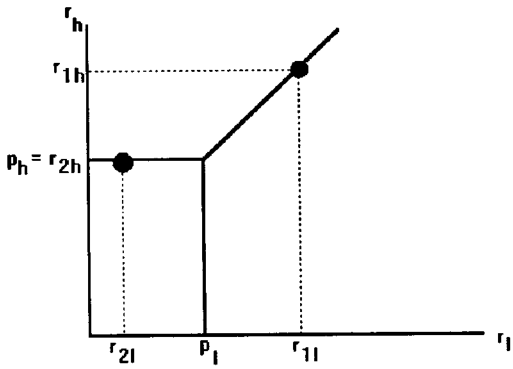

FIGURE 4

The seller's optimal assignment of customer types to service classes depends in important ways on how tastes are distributed. Consider Figure 4. Type 1 customers remain willing to pay more for either kind of seat than type 2 buyers, but differences in the willingness to pay for H are relatively small, and differences in the willingness to pay for L are large. Serving type 2 buyers now requires that they occupy and strictly prefer the *best* seats. Type 1 customers must occupy L seats (though some might purchase H as well). More generally, the relative variances in value among service qualities affect the optimal pricing policy because it has important consequences for the sorting of customers to classes, a point well known in the economics of selection. Nevertheless, all surplus is always extracted from customers with the lowest valuations. High valuation customers always pay less than their reserve prices, wherever they sit. We show this next.

C. General Solution When Tastes Are Ordered

The pricing problem can be characterized very neatly when preferences are well-ordered among customers. Think of the good as a bundle of characteristics, including the basic service itself and the ''comfort'' with which it is consumed. Each buyer has intensity of demand T for the service, and T is distributed as $g(T)$ in the population. A buyer's reserve prices for each type of seat are increasing functions of T, such as in[5]

[5] This specification is like a statistical factor analysis with one factor and no independent noise. Obviously the argument works if any independent noises have small enough variances to preserve the ordering of preferences and a recursive structure—see Figure 6 below. Ordering in this problem is identical to the usual ''single-crossing'' condition in information theory.

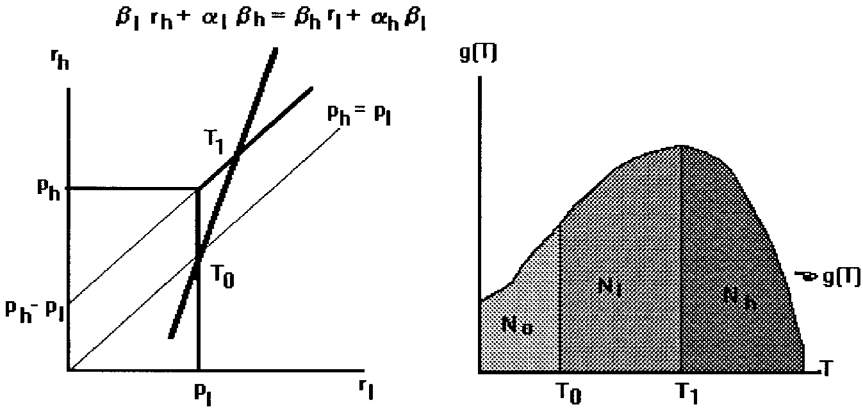

FIGURE 5

$$r_j = \alpha_j + \beta_j T \quad \text{for } j = h, \text{l}, \tag{2}$$

where the αs and βs are parameters that depend on the specific service, seat quality, and prices of complements. Parameters β_h and β_l are strictly positive. Solving (2) out for T, the distribution $g(T)$ implies that reserve prices are distributed over a positively sloped line in the (r_h, r_l) plane with equation $\beta_l r_h + \alpha_l \beta_h = \beta_h r_l + \alpha_h \beta_l$. Since T increases in passing from left to right over the line, the partition in Figure 1 implies that any buyers priced out of the market must have the smallest values of T. Customers with the most intense preferences buy the expensive seats if $\beta_h > \beta_l$. Customers with the most intense preferences purchase the less expensive seats if $\beta_h < \beta_l$.

The first panel in Figure 5 shows the market equilibrium when $\beta_h > \beta_l$. The second panel shows how customer tastes $g(T)$ are partitioned across seats. All customers above T_1 (the area marked N_h) purchase high-class service, those between T_0 and T_1 (the area marked N_l) purchase low-class service, and the rest (in the area marked N_0) do not purchase at all. A consumer with taste intensity T_1 gets equal surplus between H and L. This ‘‘marginal consumer’’ is defined by $r_h - p_h = r_l - p_l$. The value T_0 is the taste value of another marginal consumer, those who receive zero surplus, defined by $r_l = p_l$. Substituting from (2),

$$T_1 = \frac{(p_h - p_l) - (\alpha_h - \alpha_l)}{(\beta_h - \beta_l)} \tag{3}$$

and

$$T_0 = \frac{p_1 - \alpha_l}{\beta_1}. \tag{4}$$

Write $G(T) = \int_0^T g(t)dt$ as the cumulative distribution, with $G(\infty) = N$, where N is the number of people with positive tastes. The demand functions for each class of service are $N_h = N - G(T_1)$ and $N_l = G(T_1) - G(T_0)$. Equations (3) and (4) reveal that the demand for tickets in each class depend on the first-class price premium, $\Delta p = (p_h - p_l)$, and the price level of a second-class ticket, p_l. In the most interesting case when both classes of tickets are sold, the cut-point T_1 and the number of high-quality service users N_h are solely determined by Δp—see the first panel of Figure 5—and the problem has a recursive structure. The seller sets Δp to get the desired number of high-service buyers and then chooses the level of prices (both p_l and p_h) to get the desired margin at the low end.

Choosing Δp and p_l to maximize total revenue $p_h N_h + p_l N_l$ yields marginal conditions

$$\frac{\partial(p_h N_h + p_l N_l)}{\partial \Delta p} = N_h + \frac{\partial N_h}{\partial \Delta p} \geq 0 \tag{5}$$

and

$$\frac{\partial(p_h N_h + p_l N_l)}{\partial p_1} = (N_h + N_l) + p_l \frac{\partial N_0}{\partial p_l} \geq 0, \tag{6}$$

which hold with equality if the capacity constraints are not binding. Assuming second-order conditions, the first equation alone determines Δp. Then p_l is chosen to ration L buyers.[6] Note that if $\beta_h < \beta_l$ the recursion goes in the other direction. The value Δp uniquely determines the second-class margin and p_h rations H. These differences help us see that there are no general characterization results available for arbitrary reserve price distributions, specifically where tastes are not ordered.

[6] Many different corner cases are possible. An interesting variant by Steven Cheung, *Why Are Better Seats ''Underpriced''?* 15 Econ Inquiry 3 (1974), constrains first class to be at full capacity, to eliminate possibilities for customers to buy lower-class tickets and move to unoccupied first-class seats. If transactions costs of enforcing class property rights are high enough, this may require charging only one price for all seats and allowing random assignment, as apparently happens in movie theaters.

D. Application to Intertemporal Price Discrimination

Before analyzing the choice of quantity and quality, we pause to discuss an interesting application of the results so far[7] to help understand some commonly observed aspects of how ticket prices vary over time. Capacity constraints often require services to be rendered sequentially. For example, repeat performances of theatrical performances are necessary to overcome theater size limitations. The seller's pricing problem turns out to be conceptually identical to class of service pricing. Intuitively, the most ardent customers should be served first, and the less ardent customers after that. A declining pattern of prices over time sorts customers in the correct order. Prices cannot decline too fast, or else the high demanders wait for a better deal, and they cannot decline too slowly, or else the less ardent try to attend too early. They must decline somewhere in-between to separate buyers into homogeneous categories over time. The seller's willingness to serve less ardent buyers later limits the prices that can be charged to early performances, just as serving type 2 buyers in Figure 2 restricts the prices that can be charged to type 1 buyers.

1. *Queuing with Homogeneous Preferences.* Suppose a large number of identical customers have spot reservation price r for the service today and an impatience factor D. The theater can serve only K customers at a time, and K is small relative to total demand. A person is willing to pay rD^s today for a ticket to a performance s periods from now. Consumers prefer early to later performances because some entertainment services have durable elements—customers retain memories of the event and talk to each other about them. Assume that subjective discounting is so large that financial investments during the waiting period never produce enough interest to fully compensate waiting for tickets. This is appropriate for tickets where the sums are small relative to the transactions costs of using securities markets.

Were time-dated tickets sold up-front before the first performance, market clearing prices would be r for a ticket to the first performance, rD for a ticket to the second performance, and rD^t on performance t. Revenue is $r + rD + rD^2 + \ldots$, and customers are indifferent to when the service is obtained. Those attending later performances are paid to wait, getting their tickets at lower prices. The seller prefers this method if the rate of interest earned on the front money exceeds the rate of customer-time preference.

The answer is much different in the more relevant case (for entertainment services) where buyers are very impatient and D is smaller than the seller's

[7] This section is independent of the results on quantity and quality, so impatient readers can skip to the next section at this point and return, if desired, later on.

interest discount factor. If d is the seller's discount factor, the seller does better by holding back tickets till the day of each performance and selling them at price r. This strategy yields present value $r + rd + rd^2 + \ldots > r + rD + rD^2 + \ldots$ when $d > D$. The implicit market for waiting is shut down, and tickets might have to be rationed by queues or other nonprice means.

To prove this point, consider what happens at the last performance. Since everyone's reserve price is r that day, the seller charges r. Now consider the performance before the last. If customers expect the seller to reduce price tomorrow by $r(1 - D)$, an ''orderly market'' for that day's performance would seem to occur. For if buyers expect this intertemporal price pattern, they are indifferent to going to the current performance at price r or to tomorrow's performance at price rD. However, the price cannot be rD when the day of the last performance arrives. The seller charges r at the next performance because the buyers who were not served yesterday value the service at r today (and rD the day after that). Bygones are bygones, and all surplus is extracted by charging r once again. Buyers cannot credibly expect price to fall to rD on the next performance.

Working backward, the only feasible equilibrium requires a ''disorderly'' market for tickets to the next-to-last performance. Knowing the price will be r at the last performance, all buyers scramble to purchase the next-to-last day's ticket. However, the price on that day cannot be greater than r because no one will buy at a spot price higher than that. Therefore a ticket to the day before the last performance must also sell for r. The same argument applies to all performances. There is always excess demand for tickets to earlier performances when spot reserve prices are identical across customers. Excess demand declines over time as more and more customers are served, but nonprice rationing (scalping is not implied here, even though there is excess demand—the price is as high as it will go) is the seller's optimal policy. There is no other solution under the circumstances. By not paying the compensation necessary to get buyers to willingly wait, the producer extracts all surplus and charges r for each performance.[8]

This solution is forced on the seller. For the equilibrium price in later periods must be r whether the theater is selling the ticket at that point or someone else has tickets in their possession and offers them for sale. If some other party owns one, the equilibrium price is r when each day comes

[8] This formulation produces the appearance of high prices and excess demand with independent preferences. The same phenomena can occur when preferences are sufficiently interdependent that demand is upward-sloping in part of its range. See Gary S. Becker, *A Note on Restaurant Pricing and Other Examples of Social Influences on Prices,* 99 J Pol Econ 1109 (1991). Demand is never upward-sloping here because ''social'' factors do not affect demand.

along. Why should the seller give a gift to someone else by selling a lower priced ticket in advance? Why not just hold it and sell for r when the time comes? The seller credibly commits to selling a ticket to any future performance at price r.

Notice that organizing a market for waiting is impossible if customers differ in their rates of time preference but have the same value of r. Though it is socially efficient for the impatient buyers to attend earlier performances and the more patient ones to attend later performances, that outcome is not usually achieved. The reason is the same as before. The price of later performances at the time the service is rendered must be r whether impatient or patient persons buy them. But then everyone bids r for early performances irrespective of their impatience. More patient people have no incentive to wait because waiting cannot be compensated and the market cannot be separated.

2. *Heterogeneous Buyers.* Market separation is possible when buyers differ in their spot reservation prices because the end-period unraveling problem can be controlled. A policy of declining prices over time allows the waiting market to clear in all but the final period. Different preference groups are served sequentially.[9] People who desire the service the most are willing to buy early if they expect that the price will not fall too quickly. Sorting customers by preferences in this way allows the seller to extract greater surplus from all customers as a whole, so the commitment is credible.

Assume two groups of buyers, one with reserve price r_1 and the other with reserve price $r_2 < r_1$. Everyone has the same rate of time preference. Again, the seller knows the distribution of reservation prices but cannot identify them individually. We might as well assume that the marginal cost of each performance (production batch) is zero. Customers choose when to purchase—now, later, or not at all—and have rational expectations about the path of future prices. The problem for the seller is to maximize total revenue, given buyers' timing decisions.

Ticket prices cannot be rising over time because all buyers desire early performances and there are gains from resale at higher prices by people

[9] There is a close connection between this problem and the ''Coase Conjecture.'' See R. H. Coase, *Durability and Monopoly* 15 J Law & Econ 143 (1972); Nancy Stokey, *Intertemporal Price Discrimination*, 93 Q J Econ 355 (1981). Capacity constraints imply that the seller can credibly commit to a policy of intertemporal price discrimination. Jeremy Bulow, *An Economic Theory of Planned Obsolescence,* 100 Q J Econ 729 (1986), and Charles Kahn, *The Durable Goods Monopolist and Consistency with Increasing Costs,* 54 Q J Econ 275 (1986), treat a related intertemporal problem with increasing cost of production technologies. The capacity constraint makes our problem somewhat different and much more transparent than theirs.

who managed to obtain them. The seller leaves money on the table by adopting such a policy. Type 2 customers do not purchase unless price $P \leq r_2$. Any constant price policy that charges more than r_2 is not credible, for when all type 1 customers have been served, the seller has incentives to reduce the price to pick off the business of type 2 buyers. But if type 1 buyers anticipate this, they delay purchase, and no one is served at all. Hence any constant price policy that caters to type 2 buyers can only charge r_2, leaving substantial rents available to type 1 buyers that the seller wants to capture. Constant prices pool purchase timing of all types: everyone prefers early performances, and there are gains from trade among types who manage to obtain such tickets. Again, the seller gives money away by adopting this policy. Note the similarity of this logic to that underlying Figure 3.

The only interesting policy has price declining over time. A solution where type 1 customers purchase early at a price larger than r_2 and type 2 customers purchase later at price r_2 is feasible. It extracts more surplus from type 1 customers, but their option to delay purchase limits the extent to which the seller can exploit them. To see this, assume the existence of a pricing policy that serves consumer types sequentially. We know from above that once all type 1 customers have attended, the seller faces a homogenous group of type 2 buyers and rationally prices all tickets at r_2 from that date forward. Production ceases when all such buyers have been served.

Again, the solution is found by working backward. Consider the rational pricing policy at the last performance where type 1 customers are present. Any price above r_2 on that day excludes type 2 customers. However, the remaining type 1 customers know that price will fall to r_2 tomorrow. If the seller tried to charge r_1, all of these people would defer purchase because they would anticipate positive rent of amount $(r_1 - r_2)D$ the next day and no rent today. Define P_0 as the maximum price the seller can charge today to induce purchase by type 1 customers. P_0 yields equal surplus of type 1 buyers on either day: $r_1 - p_0 = (r_1 - r_2)D$, or

$$P_0 = r_1 - (r_1 - r_2)D < r_1. \tag{7}$$

This condition is virtually identical to the one depicted in Figure 3 for class of service pricing, with today's ticket on the vertical axis and tomorrow's ticket on the horizontal axis.

When all buyers were alike, the backward logic led us to conclude that the price of prior performances could not rise because it was already a limit price. However, P_0 is less than the limit price of type 1 customers. Consider the optimal price P_1 on the day prior to the one where all type 1 customers are served. There are two performances available to serve the remaining $2K$

type 1 buyers. Since earlier performances are always preferable to later ones, the seller can charge more for the first of these two production dates. The equal surplus condition between these two performances for type 1 buyers is $(r_1 - P_1) = (\mathrm{r}_1 - P_0)D$, or

$$P_1 = r_1 - (r_1 - r_2)D^2. \tag{8}$$

Continuing in this way, if there are $t + 1$ consecutive performances remaining at which only type 1 buyers are served, the price on that day must be

$$P_t = r_1 - (r_1 - r_2)D^{t+1}. \tag{9}$$

If there were a third group with an even larger reservation price, the optimal price for that group would be anchored by the price in equation (9) at the first performance they are served. Linking up all such equations, we see that there is a chain-letter effect of extending the run to include lower-reservation-value-customers. Knowing that the seller has incentives to serve such customers affects the amounts higher-value customers are willing to pay for earlier performances. But then the length of the run (this determines total market capacity here) becomes another decision variable for the seller. The price is r_1 if the seller can commit to closing the show after all type 1 customers have been served. This yields more profit than extending performances to type 2 customers if r_2 is small enough or there are not many of these kinds of customers.

One way sellers such as musicians and traveling theater companies commit to limiting performances in a market is by organizing a national tour and committing to a fixed schedule, selling tickets to precisely identified venues and performance dates in advance (‘‘one-performance only’’ per city). Since substitution across city/venues on the tour usually is trivial, customers in each location cannot wait for prices to drop because the promoter avoids serving those with small demand prices. The seller can raise ticket prices closer to the reservation levels of more ardent buyers. In the theaters on Broadway, typically the actors in the earlier runs are more talented than those who replace them later on or in later runs. This also increases the willingness to pay for earlier performances.

Another way price declines is through discrete changes in marketing format, for example, release to videotape, foreign venues, and outlying theaters for movies; and out-of-town productions for Broadway and the like. The cause is often related to capacity constraints among first-run venues. For if a current production is not very successful, there is much option value in yanking it from the first-run theater and trying an unknown new production that might be much more successful. The price is not reduced in that theater, but the less successful product is priced down when shifted to another market. For such things as book production, capacity constraints are irrelevant. Hardbacks are the ''first-class'' product, and paperbacks are the ''second-class'' product. Book pricing combines both class-of-service and intertemporal price discrimination: this analysis carries through almost exactly.

III. Choice of Quantities

Return now to the class of service problem, and assume that the venue can be resized and/or reconfigured to change the size of each class at cost $C(N_h, N_l)$, where both marginal costs C_h and C_l are positive. What is the optimal number of seats in each class? The seller's problem is to maximize net revenue $p_h N_h + p_l N_l - C(N_h, N_l)$, where the demand functions N_h and N_l for each class are found by superimposing the contour map of the frequency function $f(r_h, r_l)$ on the price partition of Figure 1 and calculating the integrals

$$N_h = \int_0^{p_l} \int_{p_h}^{\infty} f(r_h, r_l)\, dr_h dr_l + \int_{p_l}^{\infty} \int_{p_h - p_l - r_l}^{\infty} f(r_h, r_l)\, dr_h dr_l \tag{10}$$

and

$$N_l = \int_{p_l}^{\infty} \int_0^{p_h - p_l - r_l} f(r_h, r_l)\, dr_h dr_l. \tag{11}$$

Figure 2 implies that raising the price has two effects on own demand. Some customers *leave* the market entirely (areas 1 or 4), and others *switch* from one class to the other (areas 2 or 3). In addition, Figure 2 reveals that both cross-demand effects consist only of switchers and are symmetrical: $\partial N_h/\partial p_l = \partial N_l/\partial p_h$ in (7) and (8). Writing U_h and U_l for the leavers component of the own price effects (the integrals over regions 1 or 4), we have that $U_h = \partial N_h/\partial p_h + \partial N_l/\partial p_h$ and $U_l = \partial N_l/\partial p_l + \partial N_h/\partial p_l$.

The first-order conditions for this problem are

$$(p_h - C_h)\partial N_h/\partial p_j + (p_l - C_l)\partial N_l/\partial p_j \geq N_j \quad \text{for } j = \text{h, l}. \tag{12}$$

Second-order conditions require $\nabla = (\partial N_h/\partial p_h)\,(\partial N_l/\partial p_l) - (\partial N_h/\partial p_l)\,(\partial N_l/\partial p_h) > 0$. Solving the pair of equations in (12) as equalities for $(p_h - C_h)$ and $(p_l - C_l)$, and using the restrictions above,

$$(p_h - c_h) - (p_l - c_l) = \frac{[U_l/N_l - U_h/N_h]N_h N_l}{\nabla}. \tag{13}$$

The terms U_l/N_l and U_h/N_h are the fractions of class L and H consumers who leave the market rather than switch to the other class when own prices rise. Using the price-cost wedge as an index, we find that greater monopoly

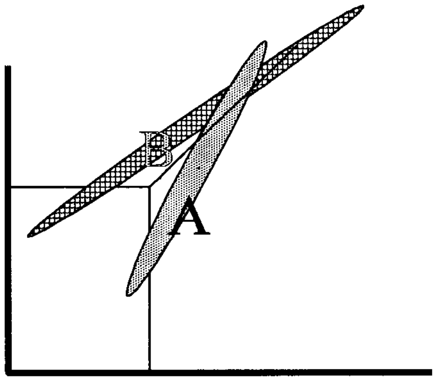

FIGURE 6

power is exercised in the class where relatively fewer customers drop out of the market. These customers are trapped by the seller because they prefer to switch to another class of service rather than not participate at all.

This result is also related to the relative variances in reserve prices. Figure 6 shows two distributions of preferences with positive covariance. The one marked *A* has greater variance in preferences for first class than for second class. Marginal buyers of first-class seats switch to second class rather than not purchase anything when p_h rises. The difference between price and marginal cost is greater in H than in L in such a case. For distribution *B*, the marginal second-class buyers are less likely to cease purchasing. Second-class buyers are trapped, and the price-marginal cost spread is larger for them. When preferences are reasonably well ordered, as in Figures 5 and 6, buyers with the highest intensity of preferences are ''exploited'' the most, in the price-cost margin sense. This is accomplished by restricting seat availability in their preferred class by more (relative to costs) than in the other class. The degree of the restriction depends on the usual elasticity conditions and the responsiveness of interclass substitution to price. The seller balances revenue gains from lower seat availability and higher margins in first class against more business and a smaller margin in the less preferred class.

IV. Choice of Service Quality

A. Dupuit's Problem

A classic problem posed of Jules Dupuit[10] is whether sellers have incentives to change the quality of each type of service by investing in fancy

[10] See note 1 above.

accommodations and accoutrements. Dupuit thought that railroad companies of that day widened quality differences between classes of service to deter rich buyers from buying cheaper tickets. The uncomfortable, crowded, and dirty conditions of third-class travel and the plush, uncrowded, and comfortable conditions of first-class accommodations could be manipulated so that rich customers would be scared off from buying cheaper tickets: ''[H]aving deprived the poor of what is necessary, first class customers are given what is superfluous.''[11] Louis Phlips, whose analysis is in some ways related to ours, quotes L. Walras to the same effect.[12] Extravagant differences between steerage and first-class accommodations on ocean liners and airplanes, between boxes and standing room tickets at the opera, and between sky boxes and bleachers at ball games seem consistent with this idea. Increasing the differences in quality between classes limits substitution and allows higher prices to be charged to customers with high reservation prices.

The standard results in models without capacity constraints is that the seller produces the socially efficient quality for customers with the greatest willingness to pay and reduces lower-end qualities offered to customers who will not pay as much.[13] The reason is to deter substitution among those who are willing to pay the most. But those models are structured so that the seller always tailors unique, socially optimal goods to customers with the highest reservation prices in order to charge them the highest possible prices. Capacity constraints and indivisibilities complicate things because buyers of different tastes typically purchase the same class of service and therefore disagree about the appropriate quality of that class. This complication produces a weaker result. The quality of accommodations can be inefficient in either class of service, depending on the way preferences are distributed.

B. One Class of Service

Let us begin by briefly reviewing quality choice of a monopolist who offers only one class of service though buyers have different quality preferences. Spence[14] showed that this problem resembles choice of a public good: the seller must choose one overall quality even though each buyer prefers a different one. The outcome hinges on comparing the quality preferences of the average

[11] Dupuit is quoted in R. B. Ekelund, *Price Discrimination and Product Differentiation in Economic Theory: An Early Analysis*, 84 Q J Econ 268 (1970).

[12] Louis Phlips, *The Economics of Price Discrimination* 216 (Cambridge Univ Press 1983); L. Walras, *The State and the Railways* (1875), translated into English by P. Holmes in 13 J Pub Econ 81 (1980).

[13] See Mussa and Rosen (cited in note 4); Eric Maskin and John Riley, *Monopoly with Incomplete Information*, 15 Rand J Econ 171 (1984); Phlips (cited in note 12); Jean Tirole, *The Theory of Industrial Organization* (MIT Press 1988).

[14] Michael A. Spence, *Monopoly, Quality and Regulation*, 6 Bell J Econ 417 (1975).

and marginal consumer within each class. The monopolist caters to the marginal customer's preferences, whereas the efficient quality level caters to the average customer's preferences. Quality is inefficient to the extent that the average and marginal customers have different tastes.

Let r be the maximum price a person is willing to pay for a seat of quality q in a one-class venue. Given q, r is distributed as $f(r, q)$ with cumulative distribution $F(r, q)$. The seller incurs cost $c(q)$ per occupied seat to change quality, with $c'(q) > 0$. Given ticket price P and seat quality q, demand is $N = \int_p f(r, q)dr$. Profit is $\int_p (P - c(q))f(r, q)dr$, where P is also the reserve price of the marginal buyer. Maximizing profit subject to the capacity constraint requires

$$c'(q) = \frac{\int_p f_q(r, q)\,dr}{f(P, q)} = -N\frac{\partial N/\partial q}{\partial N/\partial P} = N\frac{dP}{dq}. \tag{14}$$

The marginal benefit of quality to the seller is the equilibrium size of the audience times the amount the *marginal* customer, the one whose reserve price is P, is willing to pay for an increment of quality.

The socially efficient quality maximizes consumer surplus of all customers. This is $\int_{r^*}(r - c(q))f(r, q)dr$, where r^* is the socially efficient marginal buyer, the person whose presence just fills venue capacity. The social analogue of the price term in monopoly revenue is the conditional expectation of r given that $r \geq r^*$. The socially optimal choice of q satisfies

$$c'(q) = \frac{\int_{r^*}(r - r^*)\,f_q(r, q)\,dr}{\int_{r^*} f(r, q)\,dr}. \tag{15}$$

The right-hand side is the average amount customers who attend are willing to pay for an increment of quality. Private and social choice of quality are identical if all consumers have the same value of r. Otherwise, observed quality is greater or less than the socially efficient level as average willingness to pay exceeds or falls short of marginal willingness to pay. On which side it falls depends on the specific details of each problem.

C. Two Classes of Service

The marginal and average comparison carries over to the multiple service class problem. Again, service quality in each class is socially optimal if all customers within the class have identical preferences. Otherwise, general characterization results are not available for arbitrary distributions of preferences. However, we can illustrate Dupuit's intuition for why sellers might inefficiently widen service quality between classes for preference distributions of the form depicted in Figure 6, where class preferences are well-enough ordered between customers that the pricing problem is (approximately) recursive.

In such circumstances customers with the highest general tastes for the basic service never leave the market when prices of their most preferred class of seat increases. The seller can charge them more and deter substitution to the other class by increasing the interclass quality difference. For preferences where the variance in r_h exceeds the variance in r_l (distribution A in Figure 6), the quality of second-class service is reduced, just like the standard second-degree price discrimination result. However, for preferences where the variance ordering is reversed (distribution B in Figure 6), the quality of first-class service is reduced. This is why it is not generally possible to say how quality is affected for arbitrary taste distributions.

For this problem the joint frequency of preferences must be written as $f(r_h, r_l; q_h, q_l)$, where q_j is the quality of service class j. An increase in q_h pushes the cloud of points in Figure 6 generally upward. An increase in q_l pushes it generally rightward. The Figure 1 partition and Figure 5 imply that the demand functions are recursive, so the number of tickets demanded by customers with the most intense preferences depends only on the price difference $\Delta p = p_h - p_l$. From results above, preference distributions like A imply demand functions of the form $N_h = N_h(\Delta p, q_h, q_l)$ and $N_l = N_l(\Delta p, p_l, q_h, q_l)$, while distributions like B imply the form $N_h = N_h(\Delta p, p_h, q_h, q_l)$ and $N_l = N_l(\Delta p, q_h, q_l)$.

Consider A-preferences. Profit is $[\Delta p - c_h(q_h) + c_l(q_l)]N_h + [p_l - c_l(q_l)]N_l$, where $c_h(q_h)$ and $c_l(q_l)$ are the unit costs of improving seat quality. For a given quality configuration, the seller chooses Δp and p_l to maximize profits. Then qualities are chosen allowing for their effects on prices. Assuming that some low-valued customers are excluded at the optimal policy, the marginal conditions for the q's work out to be

$$
\begin{aligned}
c_h' N_h &= -N_h \frac{\partial N_h/\partial q_h}{\partial N_h/\partial \Delta p} - (N_h + N_l)\frac{\partial N_h/\partial q_h + \partial N_l/\partial q_h}{\partial N_l/\partial p_l} \\
&= N_h \frac{d\Delta p}{dq_h} + (N_h + N_l)\frac{dp_l}{dq_h}
\end{aligned}
\tag{16}
$$

and

$$c_l' N_l = -N_h \frac{\partial N_h/\partial q_l}{\partial N_h/\partial \Delta p} - (N_h + N_l) \frac{\partial N_h/\partial q_l + \partial N_l/\partial q_l}{\partial N_l/\partial p_l}$$
$$= N_h \frac{d\Delta p}{dq_l} + (N_h + N_l) \frac{dp_l}{dq_l}. \tag{17}$$

These expressions balance costs and benefits of increments of service quality. The first term on the marginal revenue side of (16) reflects how extra first-class service quality increases the between-class price spread. The second term might be called Dupuit's ''antisubstitution'' effect, that higher first-class service quality reduces interclass substitution and allows second-class prices to be increased. Both of these terms are positive. By contrast, the first term on the marginal revenue side of (17) for second-class quality is negative. Better second-class quality decreases Δp, so the antisubstitution effect tends to reduce second-class seat quality. The second (own) term is positive. Conceivably the negative effect could be so large that the seller has incentives to willfully reduce the quality of second class beyond its ''natural'' state, for example, keep it dirty and in a bad state of repair, as Dupuit claimed for railroads.

However, for *B*-preferences the marginal conditions *reverse the roles of H and L.* The seller has incentives to increase second-class quality because that is where the highest-taste customers sit. The antisubstitution effect tends to reduce first-class seat quality. While we are unaware of obvious examples where first-class ticket buyers are ''deprived'' in this sense, the point is that few general conditions are available for all distributions. Each problem must be considered case-by-case.

Analyzing if choice of quality levels are socially efficient requires comparing the expressions above with those corresponding to the consumer-surplus-maximizing solution (sketched in the Appendix). The cross effects of quality on price in the socially optimum marginal benefit calculations turn out to be symmetrically negative in both equations because an increase in one service quality reduces consumer surplus in the other quality. This is consistent with the fact that first-class quality may be excessive for *A*-preferences but not for *B*-preferences. But again, it is not possible to say much in general about these average-marginal customer comparisons within and between classes.

V. Pricing Complementary Goods

Dupuit's problem considers seat quality as a built-in and predetermined, take-it-or-leave-it offer to all customers. Other aspects of service quality are not so strictly tied in with the admission price but are chosen by ticket holders after arriving on the premises. Customers are given the rights to purchase complementary goods at prices fixed by the seller (food, drinks, recordings, and parking at concerts and ball games, popcorn at the movies). Since the amounts purchased are chosen optimally by each customer, they suit their preferences exactly. Nonetheless, an aspect of public goods choice remains. Ticket holders are a captive audience for complementary goods. To what extent is it in the seller's interests to gouge customers purchasing these additional services, give them away as promotional material, or just sell them at marginal cost?

The answer depends on the difference in demands for complementary goods by the average and marginal customer. If all customers have the same preferences for complementary goods, they are sold at marginal cost and all rents are extracted up-front in ticket prices. But when preferences are heterogeneous, extra profit results from pricing them above or below cost and adjusting ticket prices according to whether average demand for complements exceeds or falls short of marginal demand. This problem is related to several well-known monopoly pricing problems, specifically the ''loss leader'' problem and the multipart ''buffet''-pricing problem analyzed by Oi[15] and Carlton and Perloff.[16] We sketch a new development here for the standard (single-class-of-service) monopolist. It is Dupuit's problem with one more degree of freedom.

Let $u(x, z, \theta)$ be the utility function of a person who purchases a ticket in a given quality class, where x is all other goods consumed, z is the purchase of complementary goods at the venue, and θ is a taste parameter, distributed with a cumulative distribution function of $A(\theta)$. A consumer with income y who pays p for a ticket can purchase z on the premises at price w, so the budget constraint is $y - p = x + wz$ if a ticket is purchased. Once inside, the person chooses z to maximize u, and the marginal condition $u_z/u_x = w$ and the budget constraint imply a demand function $z = z(w, y - p, \theta)$ for customer θ. If the person does not attend, utility is exogenously determined at $v(y, \theta)$.

Let r be the maximum price a person of type θ will pay for the right to enter and buy the complementary good at price w. Then $r = r(w, y, \theta)$ is defined by $u[y - r - wz(w, y - r), z(w, y - r), \theta] = v(y, \theta)$. Furthermore, the envelope theorem implies $-dr/dw = z$: raising concession prices reduces the amount the customer is willing to pay for admission by the

[15] See Walter Oi, *A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly,* 85 Q J Econ 77 (1971).

[16] Dennis W. Carlton and Jeffrey M. Perloff, *Modern Industrial Organization* (Scott Foresman 1990).

amount of concessions purchased. Assume $r(w, y, \theta)$ is uniquely ordered in θ, with $r_\theta > 0$. Then θ is an index of demand intensity for the basic service.[17] If the seller charges price p for admission and sells concession goods at price w, customers of type θ^* are indifferent to attending, where $r(w, y, \theta^*) = p$, and the taste distribution $A(\theta)$ is partitioned into two parts. All those for whom $\theta \geq \theta^*$ choose to attend, and those for whom $\theta < \theta^*$ do not attend. Taste θ^* defines the marginal customer. Differentiating, we have $\partial\theta^*/\partial w = z/r_\theta > 0$ and $\partial\theta^*/\partial p = 1/r_\theta > 0$. Raising either price reduces attendance.

Letting k be the (constant) marginal cost of supplying z, the seller's total profit is

$$\int_{\theta^*(p,w)} [p + (w - k)z(w, y - p, \theta)]\, dA(\theta). \tag{18}$$

Differentiating with respect to p and w and combining terms, it can be shown that

$$w - k = \frac{\int_{\theta^*} [z(\theta) - z(\theta^*)]\, dA(\theta)}{\int_{\theta^*} \left[-\frac{\partial z^s}{\partial w} + (z(w, y - p, \theta) - z(w, y - p, \theta^*)) \frac{\partial z}{\partial y} \right] dA(\theta)}. \tag{19}$$

The numerator in (19) can be expressed as $[E(z(\theta)|\theta \geq \theta^*) - z(\theta^*)]$ multiplied by audience size. It is the difference between the amount of concession goods purchased by the average consumer (those for whom $\theta \geq \theta^*$) and the marginal (θ^*) customer. The first term in the denominator is the slope of the Slutsky demand function for z, and the second term is the covariance between the income effect on concession demand from raising ticket prices and concession consumption. About this, little can be said. If either the income effect or the correlation is small, only the Slutsky effect remains, and price of concessions is larger or smaller than marginal cost according to whether the average person consumes more or less than the marginal person.

When the average customer buys more than the marginal customer, the seller extracts more rent (relative to marginal cost pricing of complements) by taxing the z-consumption of those who attend anyway. Higher prices are charged for complements, and the admission price is reduced to attract marginal customers, who do not buy many z-goods. It is another form of price discrimination when ticket prices must be offered on the same terms to all potential buyers. An example is the high price of popcorn at the movies and

[17] The joint distribution of y and θ should be considered, but this is cumbersome and is ignored.

wine in restaurants. Marginal customers have little taste for the stuff, but average customers do. Were it feasible to charge different prices for different quality seats at the movies, perhaps the price of popcorn would be closer to marginal cost.[18]

If marginal customers buy more concession goods than the average customer, *z*-goods are priced below cost, and ticket prices are increased. This does not unduly deter marginal customers from entering, while extracting greater rent from those with more intense preferences for the primary good who do not care so much about concessions. A practical example of this kind is bat day at ball games, a selective way of reducing price for customers with children. Most ‘‘promotions’’ of consumer goods are of this kind (free mugs at MacDonalds), and not only for ticket goods.

With more than one class of service, the analysis above carries over to each class separately if the seller can charge different concession prices to each class and prevent resale between classes. Typically the seller offers concession goods to all customers on the same terms, independent of class. The resulting marginal condition is similar to the one above but involves weighted averages over classes of the terms in both numerator and denominator. To the extent that these average and marginal comparisons conflict across classes, prices of concessions tend to be driven closer to marginal costs.

VI. Conclusion

Monopoly price determination always can viewed as the outcome of competition among customers to obtain the restricted quantities the seller makes available to them.[19] In our analysis customers are always on their demand curves. The competitive, arbitrage-like elements in the spatial and intertemporal sorting of customers to seats essentially duplicates the outcome of a competitive bidding (auction) market for available seats. In fact, prices and allocations are fully ‘‘competitive’’ in a natural sense when capacity is full. This simple model explains a number of commonly observed features of these markets, but many problems remain to be analyzed.

More analysis is needed for various aspects of bundling, such as sales of season tickets, and especially for stochastic aspects of preferences and customer arrivals. For example, some incentives for scalping are the result of prohibitions on resale to enforce bundling options offered by the seller to

[18] See, however, Luis Locay and Alvaro Rodriguez, *Price Discrimination in Competitive Markets,* 100 J Pol Econ 954 (1993), for another approach.

[19] This is one interpretation of John R. Lott and Russell D. Roberts, *A Guide to the Pitfalls of Identifying Price Discrimination,* 29 Econ Inquiry 14 (1991).

customers for either price discrimination or risk-sharing reasons. Other incentives arise because of changes in circumstances and incomplete information. The seller may not fully anticipate the demand for the service at the time tickets are sold and commitments are made. A promising approach conceptually is in terms of contingent claims market[20] with certain kinds of market incompleteness and errors in pricing due to imperfect information. In many ways the problem is related to the rationing role of prices in public utilities and to other peak-load and inventory problems with capacity constraints and uncertainty.[21]

APPENDIX

Socially Optimal Qualities

With Figure 6, *A*-preferences consumer surplus is defined as

$$\begin{aligned} Surplus = & \int_{r_l^*}^{\infty} \int_{\Delta r^* + r_l}^{\infty} (r_h - C_h(q_h)) f(r_h, r_l; q) \, dr_h dr_l \\ & + \int_{r_l^*}^{\infty} \int_0^{\Delta r^* + r_l} (r_l - C_l(q_l)) f(r_h, r_l; q) \, dr_h dr_l, \end{aligned}$$

where $\Delta r^* = r_h^* - r_l^*$ and the marginal customers satisfy $r_h^* = C_h(q_h)$ and $r_l^* = C_l(q_l)$. Taking care to properly differentiate under the double integrals,

$$\begin{aligned} \frac{\partial\ Surplus}{\partial q_h} = & -C_h^1(q_h) N_h + \int_{r_l^*}^{\infty} \int_{\Delta r^* + r_l}^{\infty} (r_h - C_h) \frac{\partial f}{\partial q_h} dr_h dr_l \\ & + \int_{r_l^*}^{\infty} \int_0^{\Delta r^* + r_l} (r_l - C_l) \frac{\partial f}{\partial q_l} dr_h dr_l = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial\ Surplus}{\partial q_l} = & -C_l^1(q_l) N_l + \int_{r_l^*}^{\infty} \int_{\Delta r^* + r_l}^{\infty} (r_h - C_h) \frac{\partial f}{\partial q_l} dr_h dr_l \\ & + \int_{r_l^*}^{\infty} \int_0^{\Delta r^* + r_l} (r_l - C_l) \frac{\partial f}{\partial q_l} dr_h dr_l = 0. \end{aligned}$$

If $dq_h > 0$ shifts $f(r_h, r_l)$ up and $dq_l > 0$ shifts it to the right, inspection of Figure

[20] See Milton Harris and Arthur Raviv, *Monopoly Pricing under Uncertainty*, 71 Am Econ Rev 347 (1981); Roger A. McCain, *Scalping: Optimal Contingent Pricing of Performances in the Arts and Sports*, 11 J Cultural Econ 1 (1987); Pascal Courty, *Equilibrium under Random Demand and Advance Production* (unpublished manuscript, Univ Chicago 1995); James D. Dana, *Advanced Purchase Discounts and Price Discrimination in Perfectly Competitive Markets* (unpublished manuscript, Northwestern Univ 1996).

[21] Robert B. Wilson, *Nonlinear Pricing* (Oxford Univ Press 1993); Michael A. Crew, Chitru S. Fernando, and Paul R. Kleindorfer, *The Theory of Peak Load Pricing: A Survey*, 8 J Regulatory Econ 215 (1995).

5 reveals that the cross derivatives in these expressions are negative. Analogous expressions apply to *B*-preferences.

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